True SYK or (con)sequences

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IIP 07/31/19
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'Truth...or consequences!'

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Outline
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1. Holographic conjecture and condensed matter physics
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2. 'Bona fide' vs 'analogue' holography (graphene, metamaterials, etc.)
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3. SYK model: saddle-point analysis
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4. Beyond saddle-point: Schwarzian/Liouville
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5. Further generalizations
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6. Summary
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Standard model of condensed matter
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\[ H = T_e + T_i + U_{ee} + U_{ei} + U_{ii} \]

Long-ranged Coulomb
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Long-ranged Coulomb

Interaction effects:
- uninteresting (Fermi liquid)
- interesting, yet already known 2-particle (e-e, e-h) instabilities
- interesting and unknown: 'non-Fermi liquids',..
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Long-ranged Coulomb

Interaction effects:
- uninteresting (Fermi liquid)
- interesting, yet already known 2-particle (e-e, e-h) instabilities
- interesting and unknown: 'non-Fermi liquids',..

Purely electronic: \( T_i, U_{ei}, U_{ii} \rightarrow 0, \)

(Super)strongly interacting: \( T_e \rightarrow 0 \) ('Flat band')
Quest for elusive NFL
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\textbf{d=1}: no room for FL, Tomonaga-Luttinger liquid (and beyond)

- diagrammatic calculations ('parquet'), bosonization, exact solutions,...
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**d>1**: FL is robust at weak/short-ranged couplings, exact criteria for NFL are unknown
- diagrammatic and (functional) RG approaches, higher-dimensional bosonization, DMFT, ...
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\(d=1\): no room for FL, Tomonaga-Luttinger liquid (and beyond)
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\(d>1\): FL is robust at weak/short-ranged couplings, exact criteria for NFL are unknown
- diagrammatic and (functional) RG approaches, higher-dimensional bosonization, DMFT, ...

- new (still untested) tool: holography ('AdS/CMT')
Holography primer

- **Boundary** (quantum) theory → **Bulk** (semi) classical gravity (+ other fields)

\[
S = \frac{1}{2 \kappa_5^2} \int d^4x \sqrt{-g} \left[ R - 2\Lambda + \mathcal{L}_m + \mathcal{L}_{\text{cs}} \right],
\]

\[
\mathcal{L}_m = -\frac{Z_G}{4} G_{\mu\nu} G^{\mu\nu} - \frac{1}{2} D_\mu \Phi^e D^\mu \Phi^e - \frac{Z_A}{4} A_{\mu\nu} A^{\mu\nu} - \frac{Z_B}{4} B_{\mu\nu} B^{\mu\nu} - \frac{Z_{AB}}{2} A_{\mu\nu} B^{\mu\nu},
\]

\[
\mathcal{L}_{\text{cs}} = -\frac{d}{2} \alpha_1 \epsilon^{\mu_1 \mu_2 \mu_3} A_{\mu_1 \nu} A_{\mu_2 \nu} - \frac{d}{2} \alpha_2 \epsilon^{\mu_1 \mu_2 \mu_3} A_{\mu_1 \nu} B_{\mu_2 \nu}.
\]

Feynman diagrams

Classical Einstein-type eqs
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\]

\[
\mathcal{L}_{\text{cos}} = -\vartheta_1(\alpha)e^{\alpha \lambda} A_{\mu\nu} A_{\lambda\sigma} - \vartheta_2(\alpha)e^{\alpha \lambda} A_{\mu\nu} B_{\lambda\sigma}.
\]

Feynman diagrams

Classical Einstein-type eqs

- d=4 Q=4 SU(N) SYM <-> type-IIB superstrings (d=5 supergravity) (t’Hooft, Suskind, Maldacena, Witten, Gubser, Klebanov, Polyakov, …)
  - SUSY,
  - multi-component (focusing on N>>1),
  - Lorentz and scale-invariant,
  - boundary theory: very strongly interacting
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\[
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Feynman diagrams

- **Classical Einstein-type eqs**

- **d=4 Q=4 SU(N) SYM</- type-IIB superstrings (d=5 supergravity)**
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- multi-component (focusing on N>>1),
- Lorentz and scale-invariant,
- boundary theory: very strongly interacting

- How much of that can be relevant to condensed matter systems?
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\[ \mathcal{L}_\text{os} = -\theta_1(\alpha)e^{\mu_A \alpha \nu} A_{\mu \nu} A_{\lambda \sigma} - \theta_2(\alpha)e^{\mu A \alpha \nu} A_{\mu \nu} B_{\alpha \beta}. \]

- Why it would not work:
  - non-SUSY,
  - only a few components \((N\sim 1)\),
  - Lorentz, scale, translationally, and/or rotationally non-invariant,
  - boundary theory: only moderately interacting \((T\sim U)\),...
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- Why it would not work:
  - non-SUSY,
  - only a few components (N~1),
  - Lorentz, scale, translationally, and/or rotationally non-invariant,
  - boundary theory: only moderately interacting (T~U),...

- Why it might still work:
  - emergent effective (local) geometry,
  - perturbation theory/RG in d+1 dimensions --> classical EOMs in d+2,
  - tensor networks,...
Holographic correspondence: evidence (?)
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- Data fitting:
  Optical conductivity in cuprates
  (non-SUSY, N~1, T~U)

\[ \sigma(\omega) \sim \omega^{-2/3} \]

G, Horowitz and J. Santos, 1302.6586

\[ 2 < \omega \tau < 8 \]
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  \[ \frac{\eta}{s} \] ratio (>1/4π)
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  J. Rameau et al, 1409.5820
  Indirect (Im G)?
  Universal KSS bound?
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  2d Bose-Hubbard model

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  - E. Katz et al, 1409.3841
    $1/N$?

  - I. Kiritsis et al, 1510.00020
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- (Almost) exact methods (MC):
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- Not just qualitative:
  but quantitative (sic!) agreement:

\[ \rho_s(T = 0) = C\sigma_{DC}(T_c)T_c. \]
Status of AdS/CMT (a.k.a. non-AdS/non-CFT)
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- Some isolated critique: ..., DVK 1404.7000, 1502.03375, 1603.09741
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- New directions:
  ● strong coupling hydrodynamics,
  ● quantum chaos and information scrambling,
  ● SYK and beyond,...
Holography: physical origin?
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- Dynamical renormalization (energy/length/information) scale, \( \text{RG}=\text{GR} \)
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- Emergent extra dimension:
- Dynamical renormalization (energy/length/information) scale, ‘RG=GR’

- **Emergent geometry:**
  - Thermodynamics of phase transitions (Fisher/Ruppeiner),
  - Quantum information theory, tensor networks (Bures),
  - Bloch bands, dynamical time evolution (Berry),
  - Quantum Hall and other topological states (Fubini/Study),...
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- **Geometric nature** of certain physical observables:
- Hall conductance = 1st Chern class (Niu-Thouless,...),
- Entanglement entropy = Area of extremal surface (Ryu-Takayanagi),
- What else?
Holography light
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- **Fixed** classical metric,
- Non-SUSY and **N-irrelevant** (equiv. to 0\(^{th}\) order in 1/N),
- The bulk 'dual' is **not dynamical** ('boundary problem')

Can still explain certain **apparent holography-like** features without invoking new principles of nature
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Desktop realizations:

• Strained graphene and other 2d Dirac (semi)metals
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- \textbf{3d Topological insulators}/gapped Dirac materials (?)

Potentially problematic:
- Curved 3d space
- Fermi liquid on a 2d boundary is more robust than in 1d
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- **Hyperbolic metamaterials** (optical/IR)
Graphene: scotch tape-induced relativity
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- Linear dispersion: \( E = v_F \mathbf{p} \quad v_F = 10^6 \text{m/s} (= c/300) \)
- Spinor wavefunction (pseudospin \( \frac{1}{2} \)) \( \rightarrow \) Dirac equation
- ‘Fine structure’ constant: \( \frac{e^2}{hc} \sim 1 \)
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- **Desktop realizations** of fundamental phenomena:
  - Klein tunneling,
  - ‘zitterbewegung’,
  - Veselago lense,
  - atomic collapse,
  - chiral symmetry breaking (excitonic insulator),
    magnetic catalysis (Quantum Hall ferromagnetism),…
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  - (non-) abelian gauge fields and solitons,
  - Mimicking gravity and cosmology,
  - Analogue holographic correspondence
Elastic strain in graphene

- **Hopping** Hamiltonian
  \[ H = - \sum_{i,n} t(r_i, r_{i+n}) a_{r_i}^\dagger b_{r_{i+n}} + \text{H. c.} \]

- **Strain** tensor
  \[ \varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \frac{1}{2} \left( \frac{\partial u_k}{\partial x_i} \frac{\partial u_k}{\partial x_j} \right) \]

- **Elastic energy**
  \[ \mathcal{H}_{\text{elastic}} = \frac{\kappa}{2} \int d^2\vec{r} \left[ \nabla^2 h(\vec{r}) \right]^2 + \int d^2\vec{r} \left\{ \frac{\lambda}{2} \left[ \sum_i u_{ii}(\vec{r}) \right]^2 + \mu \sum_{ij} [u_{ij}(\vec{r})]^2 \right\} \]

- **Stress** engineering

- **Induced fermion mass**
  via hybridization with substrate

F. Guinea et al, ‘11

S. Tang et al, ‘13
N. Levy et al, ’10
Emergent pseudo-(gravi)magnetic field

- **Vector potential**
  \[
  A_x(R) - i A_y(R) = \frac{1}{q v_F} \sum_n \delta t(r, r + n) e^{iK \cdot n} \approx \frac{\hbar \beta}{2qa} (\epsilon_{xx} - \epsilon_{yy} + 2i \epsilon_{xy})
  \]

- **Higher order terms**
  \[
  A_x^{(c)} = -\frac{3a^2 V_{pp}^0}{8q v_F} \left[ (\frac{\partial^2 h}{\partial y^2})^2 - (\frac{\partial^2 h}{\partial x^2})^2 \right],
  \]
  \[
  A_y^{(c)} = -\frac{3a^2 V_{pp}^0}{4q v_F} \left[ \frac{\partial^2 h}{\partial x \partial y} \left( \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial x^2} \right) \right]
  \]

  \[\beta = -\frac{\partial \log t(r)}{\partial \log r} \bigg|_{r=a}\]

  - M.A.H. Vozmediano et al, ‘10;
  - A.L. Kitt et al, ‘12;
  - F. de Juan et al, ‘12

Position-dependent Fermi velocity (?)

- **Emergent gravity**: Weitzenbock geometry
  \[
  H_- = -\sigma^3 f_k^a \sigma^a [\partial_k + i A_k], \quad a = 1, 2; k = 1, 2
  \]
  \[
  H_+ = -\sigma^2 \left( \sigma^3 f_k^a \sigma^a [\partial_k - i A_k] \right) \sigma^2.
  \]

  \[\mathcal{H} = i \sigma^3 H_- = -ie f_k^a \sigma^a \circ [\partial_k + i A_k]\]

  \[^{g_{\mu \nu}} = e^\mu_a e^n_b \eta^{ab}\]

  \[^{g_{\mu \nu}^{\text{graphene}}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & g_{ij} \end{pmatrix}\]

  - G. Volovik and M. Zubkov, ‘13
  - A. Iorio and P. Pais, ‘15
Holographic boundary propagator

- Fermion action:

\[ S = \int drdt d^d x \sqrt{|\det g|} \bar{\psi} \gamma^a \alpha^d (i \partial_{\mu} + \frac{i}{8} \omega^{bc}_{\mu} \left[ \gamma_b, \gamma_c \right] + A_{\mu} - m) \psi \]

- Background metric:

\[ ds^2 = -f(z) dt^2 + g(z) dz^2 + h(z) d\bar{x}^2 \]

- Radial Schroedinger's eq.:

\[ \frac{\partial^2 \psi}{\partial r^2} = V(r) \psi \]

\[ \psi_\pm (r, \omega, k) \sim \frac{1}{V^{1/4} (r)} e^{\pm \int_0^r \sqrt{V}(r') \, dr'} \]

- WKB solutions:

\[ G(\tau, x) \sim \exp (-S_0(\tau, x)) \]

- Asymptotic behavior:

\[ S(\tau, x) = L \omega \int du \sqrt{g_{uu} + g_{\tau\tau} \left( \frac{d\tau}{du} \right)^2 + g_{xx} \left( \frac{dx}{du} \right)^2} \]

\[ S(\tau, x) = L \omega^2 \int_{u_0}^{u_t} du \sqrt{\frac{g_{uu}}{r(u)}} \quad mR \gg 1 \]

\[ r(u) = \omega^2 - k_x^2 / g_{xx}(u) - k_\tau^2 / g_{\tau\tau}(u) \]

\[ \tau = L k_\tau \int_{u_0}^{u_t} du \frac{g_{uu}}{g_{\tau\tau} r(u)} \quad x = L k_x \int_{u_0}^{u_t} du \frac{g_{uu}}{g_{xx} r(u)} \]

\[ u_t = \left( \omega / \sqrt{k_\tau^2 + k_x^2} \right)^{1/\alpha} \]
Bulk-edge correspondence

- Flat metric
  \[ dl_{flat}^2 = dr^2 + r^2 d\phi^2 \quad ds^2 = d\tau^2 + dl^2 \]
  \[ S_{flat}(\tau, x) = m\sqrt{\tau^2 + 4R^2 \sin^2(x/2R)} \quad G(\tau, x) \sim \exp(-S_0(\tau, x)) \]

- Surface of rotation
  \[ dl_{sor}^2 = dr^2[1 + (\frac{\partial h(r)}{\partial r})^2] + r^2 d\phi^2 \]
  \[ S_{sor}(\tau, x) = m\sqrt{\tau^2 + (Rx\eta)^2/(\eta+1)} \]

- Boundary propagator: 1d bosonization
  \[ G_{bos}^{\pm}(\tau, x) \sim \exp[-\int \frac{dk}{2\pi} \frac{2 + U_k}{\epsilon_k} (1 - e^{\pm ikx - \epsilon_k t})] \]
  \[ \epsilon_k = k\sqrt{1 + U_k} \quad U(x) \sim 1/x^\sigma \]

- Matching x-asymptotics:
  \[ \eta = (1 - \sigma)/(1 + \sigma) \]
  (time-of-flight, tunneling, noise power spectrum, etc).
Bulk-edge correspondence: more examples

- Generalized Beltrami trumpet:

$$dl_{\log}^2 = dr^2 + R^2 \exp(-2(r/R)^\lambda) d\phi^2$$

$$dl^2 = d\rho^2/\rho^2 + \rho^2 d\phi^2$$

$$S_{\log}(\tau, x) = m \sqrt{\tau^2 + R^2 (\ln x/a)^2/\lambda}$$

Cf., semi-local regime:

$$S_{s-1}(\tau, x) = \sqrt{(1 - \nu_0)^2 (\ln \tau/a)^2 + m^2 x^2}, \quad \text{AdS}_2 \times \mathbb{R}^d.$$  

- $\lambda = 1$  
  Luttinger:  
  $$G(0, x) \sim 1/x^{m_R}$$

- $\lambda = 2/3$  
  Coulomb interaction in 1d:  
  $$G(0, x) \sim \exp(-\text{const} \ln^{3/2} x)$$

**Underlying physics**: another manifestation of the equivalence principle?

"Curvature in the bulk = Phantom force at the boundary"
String holoography meets its optical namesake
String holography meets its optical namesake

- **Artificial metric** in electrically and/or magnetically active media

\[
\gamma_{ij} = g_{ij} / |g_{\tau\tau}| = \epsilon_{ij} / \det \hat{\epsilon} = \mu_{ij} / \det \hat{\mu}
\]

\[
\epsilon_{ij} = \mu_{ij} = \sqrt{-\hat{g}} g_{ij} / |g_{\tau\tau}|
\]

\[
\frac{\omega^2}{c^2} \vec{D}_\omega = \vec{\nabla} \times \vec{\nabla} \times \vec{E}_\omega \quad \text{and} \quad \vec{D}_\omega = \hat{\epsilon}_\omega \vec{E}_\omega
\]

\[
\frac{\omega^2}{c^2} = \frac{k_z^2}{\varepsilon_1} + \frac{k_x^2 + k_y^2}{\varepsilon_2}
\]

W. Lu et al.,'10, T. Mackay and A. Lakhtakia, '10
String holography meets its optical namesake

• Artificial metric in electrically and/or magnetically active media

\[
\gamma_{ij} = \frac{g_{ij}}{|g_{TT}|} = \frac{\epsilon_{ij}}{\det \epsilon} = \frac{\mu_{ij}}{\det \mu}
\]

\[
\epsilon_{ij} = \mu_{ij} = \sqrt{-\hat{g}g_{ij}}/|g_{TT}|
\]

\[
\frac{\omega^2}{c^2} \tilde{D}_\omega = \nabla \times \nabla \times \tilde{E}_\omega \quad \text{and} \quad \tilde{D}_\omega = \hat{\epsilon}_\omega \tilde{E}_\omega
\]

• Hyperbolic metamaterials

- Rindler and event horizons, black/white/worm-holes,
- inflation, Big Bang, Rip, and Crunch,
- metric signature transitions, end-of-time, multiverse,…

I.Smolyaninov et al, 1201.5348, 1510.07137

W.Lu et al, ’10,
T.Mackay and A.Lakhtakia, ’10
String holography meets its optical namesake

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  \[ \gamma_{ij} = \frac{g_{ij}}{|g_{\tau\tau}|} = \frac{\epsilon_{ij}}{\text{det} \epsilon} = \frac{\mu_{ij}}{\text{det} \mu} \]
  \[ \epsilon_{ij} = \mu_{ij} = \sqrt{-\hat{g}} g_{ij} / |g_{\tau\tau}| \]
  \[ \frac{\omega^2}{c^2} \vec{D}_\omega = \nabla \times \nabla \times \vec{E}_\omega \quad \text{and} \quad \vec{D}_\omega = \vec{\epsilon}_\omega \vec{E}_\omega \]
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- Hyperbolic metamaterials
  - Rindler and event horizons, black/white/worm-holes,
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- Analogue holography
  I.Smolyaninov et al, 1201.5348, 1510.07137

 DVK 1411.1693
Attainable geometries

- **Dispersion** of extraordinary waves

\[
\frac{\omega^2}{c^2} \hat{D}_\omega = \vec{\nabla} \times \vec{\nabla} \times \vec{E}_\omega \quad \text{and} \quad \hat{D}_\omega = \vec{\varepsilon}_\omega \vec{E}_\omega
\]

\[
\omega^2 = k_z^2/\varepsilon_{xy} + k_{xy}^2/\varepsilon_{zz}
\]

\[
ds^2 = -\varepsilon_{xy} dz^2 - \varepsilon_{zz} (dx^2 + dy^2)
\]

- **Attainable 2+1 geometries**

\[
ds^2 = \frac{d\tau^2}{u^{2\alpha}} + R^2\frac{du^2}{u^{2\beta}} + \frac{dx^2}{u^{2\gamma}}
\]

\[
ds^2 = u^{2\theta/d}\left(\frac{d\tau^2}{u^{2\zeta}} + \frac{L^2du^2 + dx^2}{u^2}\right)
\]

\[
\zeta = \frac{1 - \beta + \alpha}{1 - \beta + \gamma}, \quad \theta = \frac{1 - \beta}{1 - \beta + \gamma}
\]

Hyperscaling-violation metrics

I.Smolyaninov, E.Narimanov,'09...
Prospective boundary dual

• Fluctuating elastic membrane: (coupled in- and out-of-plane modes)

\[ F = \int d^d x \left[ \frac{\kappa}{2} (\nabla^2 h)^2 + \mu v_{\alpha\beta}^2 + \frac{\lambda}{2} v_{\alpha\alpha}^2 \right] \]

\[ v_{\alpha\beta} = \partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha + \partial_\alpha h \partial_\beta h \]

\[ \Delta F \sim \int d^d k k^{4-\eta} |h_k|^2 \]

• Effective out-of-plane action:

\[ S_{boundary} = \frac{1}{2\nu} \int d^2 k k^{2+\theta/\zeta} |\phi_k|^2 \]

\[ \psi(x) \sim \exp[i\phi(x)] \]

\[ G_\omega(x) \sim \exp[-\sqrt{cL\omega} |x/cL|^{\theta/\zeta}] \]

• Optical field correlations: (speckle interferometry)

\[ <E_\omega(x)E_{-\omega}^*(0)> \propto \exp(-\omega |x|) \]

• Noise power spectrum and other moments of the boundary field distribution function can be related to the bulk ‘metric’

• Practical realizations: Co nanoparticles in kerosene, PMMA on gold, InGaAs (m)/GaAs(d) ,…
The rise of SYK model
The rise of SYK model

- spin glasses (Georges/Parcollet/Sachdev ´89; Sachdev/Ye ´92),
- randomized Majoranas (Kitaev ´15),
- toy holography (Sachdev ´15, Maldacena, Stanford, Shenker, Gross, Polchinski, Rosenhaus ´16...)
The rise of SYK model

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Original, Dirac:

\[
H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell
\]

S. Sachdev,
1506.05111
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q-Generalized, Majorana:

$$H = i^{q/2} \sum_{\alpha_1...\alpha_q}^{N} J^{\alpha_1...\alpha_q} \chi^{\alpha_1} ... \chi^{\alpha_q}$$

$$S = \sum_{i}^{L} \sum_{\alpha}^{N} \chi_i^{\alpha} \partial_\tau \chi_i^{\alpha} - i^{q/2} \sum_{i_a,\alpha_a}^{i_1...i_q} J^{\alpha_1...\alpha_q} \chi_i^{\alpha_1} ... \chi_i^{\alpha_q}$$

S. Sachdev, 1506.05111
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Original, Dirac:

\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell} c_i^\dagger c_j^\dagger c_k c_\ell \]

q-Generalized, Majorana:

\[ H = i q/2 \sum_{\alpha_1...\alpha_q} J_{\alpha_1...\alpha_q} \chi_{i_1}^{\alpha_1} \ldots \chi_{i_q}^{\alpha_q} \]

\[ S = \sum_{i} \sum_{\alpha} X_i^\alpha \partial_\tau X_i^\alpha - i q/2 \sum_{i_a,\alpha_a} J_{i_1...i_q}^{\alpha_1...\alpha_q} \chi_{i_1}^{\alpha_1} \ldots \chi_{i_q}^{\alpha_q} \]

Disorder averaging:

\[ \langle J_{\alpha_1...\alpha_q} J_{\beta_1...\beta_q} \rangle = \frac{J^2(q-1)!}{N^{q-1}} \prod_{\alpha} \delta_{\alpha_i\beta_i} \]
Spreading SYK-ness: non-random models
Spreading SYK-ness: non-random models

- QM of tensors with \((D+1)n^D\) components, vector rep. of \(O(n)^D\):

\[
S = \int \mathcal{D} \left[ \sum_{c} \left( \sum_{a} \psi_{ac}^{\dagger} \frac{d}{dt} \psi_{ac} \right)^{D+1/2} \sum_{a_1, \ldots, a_D} \psi_{a_0}^{a_1} \cdots \psi_{a_D}^{a_2} \prod_{c_1 < c_2} \delta_{c_1 c_2} \right].
\]

Witten '16, Gurau '16...

- Tetrahedron model (\(D=3\)):

\[
H_1^t = \frac{g}{(N_a N_b N_c)^{1/2}} c_{a_1 b_1 c_1}^{\dagger} c_{a_2 b_2 c_1}^{\dagger} c_{a_1 b_2 c_2} c_{a_2 b_1 c_2}.
\]

\(a = 1, \ldots, N_a; b = 1, \ldots, N_b; c = 1, \ldots, N_c\), symmetry \(U(N_a) U(N_b) O(N_c)\)
Spreading SYK-ness: non-random models

- QM of tensors with \((D+1)n^D\) components, vector rep. of \(O(n)^D\) :

\[ S = \int dt \left[ \frac{1}{2} \sum_c \left( \sum_{a^c} \frac{d}{dt} \psi_{a^c} \right)^2 - q^{(D+1)/2} \frac{J}{n^{D(D-1)/4}} \sum_{a^0, a^D} \psi_{a^0} \ldots \psi_{a^D} \prod_{c_1 < c_2} \delta_{a^{c_1} a^{c_2}} \right]. \]

Witten '16, Gurau '16...

- Tetrahedron model (\(D=3\)):

\[ H_1^t = \frac{g}{(N_a N_b N_c)^{1/2}} c_{a_1 b_1 c_1}^\dagger c_{a_2 b_2 c_2}^\dagger c_{a_1 b_2 c_2} c_{a_2 b_1 c_2}. \]

\(a = 1, \ldots N_a; \; b = 1, \ldots N_b; \; c = 1, \ldots N_c\), symmetry \(U(N_a) \; U(N_b) \; O(N_c)\)

- \(N_a = N = 3, \; N_b = 2 = M = 2, \; N_c = L \gg 1\) designer unit cell

\[ H = \sum_j H_j, \quad H_j = U \hat{n}_j^2 + \sum_{\tilde{\epsilon} = \tilde{x}, \tilde{y}} J \left( \vec{S}_j \cdot \vec{S}_{j+\tilde{\epsilon}} - \frac{1}{4} \hat{n}_j \hat{n}_{j+\tilde{\epsilon}} \right) - K \left( \epsilon_{a\beta\epsilon\gamma\sigma} c_{j,x,y}^\dagger c_{j,x,y}^\dagger + \gamma c_{j,x,y} c_{j,x,y} + H.c. \right) \]

Wu et al, 1802.04293
SYK model: key properties
SYK model: key properties

- $N \gg 1$: simple diagrammatics (´melonic´ graphs)

\[ G = G_b + \ldots \]

- Replica-symmetric (not a spin-glass) mean-field states

- Reparametrization invariance $t \rightarrow f(t)$, Liouville quantum mechanics and Schwarzian action for fluctuations about mean-field

- Maximally chaotic behavior and fast scrambling (akin to black holes)
SYK model: key properties

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- Maximally chaotic behavior and fast scrambling (akin to black holes)

Prospective holographic dual:
- Pure AdS\(_2\) (naïve)

- Dilaton (Jackiw-Teitelboim) gravity in AdS\(_2\) (+ infinite number of massive scalars)?

- AdS\(_3\)? (Jevicki et al)
Saddle-point analysis of (generalized) SYK model
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- G-Σ functional:

\[
Z = \int DG\mathcal{D}\Sigma (\text{Det}[F[\partial_\tau] + \Sigma])^N \\
\exp(N \int_{\tau_1,\tau_2} G\Sigma - A[G]))
\]

\[
A = N \sum_{k} \int_{\tau_1,\ldots,\tau_k} J_k^2(\tau_1,\ldots,\tau_k) G^q(\tau_1,\tau_2) \ldots G^q(\tau_{k-1},\tau_k)
\]
Saddle-point analysis of (generalized) SYK model

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\]

- Equations of motion:

\[
\int_{\tau} (F(\partial_{\tau})\delta(\tau_1, \tau) + \Sigma(\tau_1, \tau)) G(\tau, \tau_2) = \delta(\tau_1 - \tau_2)
\]

\[
\Sigma(\tau_1, \tau_2) = \frac{1}{N} \frac{\delta A}{\delta G(\tau_1, \tau_2)}
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Saddle-point analysis of (generalized) SYK model

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\[ A = N \sum_{k}^{\infty} \int_{\tau_1, \ldots \tau_k} J_k^2(\tau_1, \ldots \tau_k) G^q(\tau_1, \tau_2) \ldots G^q(\tau_{k-1}, \tau_k) \]

- Equations of motion:

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\[ \Sigma(\tau_1, \tau_2) = \frac{1}{N} \frac{\delta A}{\delta G(\tau_1, \tau_2)} \]

- IR asymptotic (T << J):

\[ \int_\tau G(\tau_1, \tau) \frac{\delta A}{\delta G(\tau, \tau_2)} = \delta(\tau_1 - \tau_2) \]
Saddle-point analysis of (generalized) SYK model

- **G-Σ functional:**

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A = N \sum_{k} \int_{\tau_1, \ldots, \tau_k} J^2_k(\tau_1, \ldots, \tau_k) G^q(\tau_1, \tau_2) \ldots G^q(\tau_{k-1}, \tau_k)
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\]

- **IR asymptotic (∏ << J):**

\[
G(\tau_1, \tau_2) \rightarrow G_f = [f'(\tau_1)f'(\tau_2)]^\Delta G(f(\tau_1), f(\tau_2))
\]

\[
\Sigma(\tau_1, \tau_2) \rightarrow \Sigma_f = [f'(\tau_1)f'(\tau_2)]^{1-\Delta} \Sigma(f(\tau_1), f(\tau_2))
\]

- **Reparametrization symmetry:**
Saddle-point analysis of (generalized) SYK model

- G-Σ functional:

\[ Z = \int DGDΣ(Det[F[∂τ] + Σ])^N \exp(N\int_{τ_1, τ_2} GΣ - A[G])) \]

\[ A = N \sum_k \int_{τ_1, ..., τ_k} J_k^2(τ_1, ..., τ_k)G^q(τ_1, τ_2) ... G^q(τ_{k-1}, τ_k) \]

- Equations of motion:

\[ \int_τ (F(τ)δ(τ_1, τ) + Σ(τ_1, τ))G(τ, τ_2) = δ(τ_1 - τ_2) \]

\[ Σ(τ_1, τ_2) = \frac{1}{N} \frac{δA}{δG(τ_1, τ_2)} \]

\[ \int_τ G(τ_1, τ)\frac{δA}{δG(τ_1, τ_2)} = δ(τ_1 - τ_2) \]

\[ G(τ_1, τ_2) \rightarrow G_f = [f'(τ_1)f'(τ_2)]^ΔG(f(τ_1), f(τ_2)) \]

\[ Σ(τ_1, τ_2) \rightarrow Σ_f = [f'(τ_1)f'(τ_2)]^{1-Δ}Σ(f(τ_1), f(τ_2)) \]

- IR asymptotic (T << J):

- Reparametrization symmetry:

- Mean-field solution:
  (finite T)
Saddle-point analysis of (generalized) SYK model

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A = N \sum_{k} \int_{τ_1, ..., τ_k} J_k^2(τ_1, ..., τ_k)G^q(τ_1, τ_2) ... G^q(τ_{k-1}, τ_k)
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\[
Σ(τ_1, τ_2) = \frac{1}{N} \frac{δA}{δG(τ, τ_2)}
\]

\[
\int_τ G(τ_1, τ) \frac{δA}{δG(τ, τ_2)} = δ(τ_1 - τ_2)
\]

- IR asymptotic (T << J):

- Reparametrization symmetry:

- Mean-field solution: (finite T)

\[
G_0(τ_1, τ_2) = \left(\frac{\pi}{β \sin(\pi δτ_{12}/β)}\right)^{2Δ} \quad δτ_{12} = τ_1 - τ_2
\]

\[
G_{β=∞}(τ_1, τ_2) = -β^{Δ} |J(τ_1 - τ_2)|^{-2Δ} \text{sgn}(τ_1 - τ_2), \quad Δ = \frac{1}{q}
\]
Saddle-point analysis of (generalized) SYK model

- G-Σ functional:

\[ Z = \int DGD\Sigma(Det[F[\partial_\tau] + \Sigma])^N \exp(N \int_{\tau_1, \tau_2} G\Sigma - A[G]) \]

\[ A = N \sum_{k} \int_{\tau_1, \ldots, \tau_k} J_k^2(\tau_1, \ldots, \tau_k) G(\tau_1, \tau_2) \ldots G(\tau_{k-1}, \tau_k) \]

- Equations of motion:

\[ \int_{\tau} (F(\partial_\tau)\delta(\tau_1, \tau) + \Sigma(\tau_1, \tau))G(\tau, \tau_2) = \delta(\tau_1 - \tau_2) \]

\[ \Sigma(\tau_1, \tau_2) = \frac{1}{N} \frac{\delta A}{\delta G(\tau_1, \tau_2)} \]

\[ \int_{\tau} G(\tau_1, \tau) \frac{\delta A}{\delta G(\tau, \tau_2)} = \delta(\tau_1 - \tau_2) \]

- IR asymptotic (T << J):

\[ G(\tau_1, \tau_2) \to G_f = [f'(\tau_1)f'(\tau_2)]^\Delta G(f(\tau_1), f(\tau_2)) \]

\[ \Sigma(\tau_1, \tau_2) \to \Sigma_f = [f'(\tau_1)f'(\tau_2)]^{1-\Delta}\Sigma(f(\tau_1), f(\tau_2)) \]

- Reparametrization symmetry:

\[ G_0(\tau_1, \tau_2) = \left( \frac{\pi}{\beta \sin(\pi \delta\tau_{12}/\beta)} \right)^{2\Delta} \delta\tau_{12} = \tau_1 - \tau_2 \]

\[ G_{\beta=\infty}(\tau_1, \tau_2) = -b^\Delta |J(\tau_1 - \tau_2)|^{-2\Delta} \text{sgn}(\tau_1 - \tau_2), \quad \Delta = \frac{1}{q} \]

- Mean-field solution: (finite T)

- Residual invariance: (SL(2,R) for T=0)

\[ \tan \left( \frac{\pi f(\tau)}{\beta} \right) \to \frac{a \tan \left( \frac{\pi f(\tau)}{\beta} \right) + b}{c \tan \left( \frac{\pi f(\tau)}{\beta} \right) + d} \quad \text{ad}-bc=1 \]
Holographic matching
Holographic matching

- Equivalent geometry
  (charged BH in $AdS_2 \times R^d$):

$$ds^2 = \frac{R_2^2}{\zeta^2} \left[ - (1 - \zeta^2 / \zeta_0^2) dt^2 + \frac{d\zeta^2}{(1 - \zeta^2 / \zeta_0^2)} \right]$$

$$T = \frac{1}{2\pi \zeta_0}$$

$$A = \mathcal{E} \left( \frac{1}{\zeta} - \frac{1}{\zeta_0} \right) dt$$
Holographic matching

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  \[ T = \frac{1}{2\pi \zeta_0} \quad A = \mathcal{E} \left(\frac{1}{\zeta} - \frac{1}{\zeta_0}\right) dt \]

- Probe fermion bulk action:
  \[ S = i \int d^2x \sqrt{-g} \left( \bar{\psi} \Gamma^\alpha D_\alpha \psi - m \bar{\psi} \psi \right) \]

- Fermion dimension:
  \[ \Delta = \frac{1}{2} - \sqrt{m^2 R_2^2 - q^2 \mathcal{E}^2} \]

- Bulk fermion propagator:
  \[ G_{IR}(\omega, q) = \frac{\psi_-(z, \omega, q)}{\psi_+(z, \omega, q)} \bigg|_{z \to z_0} \sim e^{-S(\omega, q)} \]

  \[ S(\omega, q) = \frac{1}{2} \int_{z_0}^{z_0} dz \sqrt{g(z)} V(z) \]
  \[ V(z) = m^2 + \frac{q^2}{h(z)} + \frac{\omega^2}{f(z)} \]
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  (charged BH in $AdS_2 \times R^d$):

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$$T = \frac{1}{2\pi \zeta_0} \hspace{2cm} A = \mathcal{E} \left( \frac{1}{\zeta} - \frac{1}{\zeta_0} \right) \, dt$$

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$$S = i \int d^2x \sqrt{-g} \left( \bar{\psi} \Gamma^\alpha D_\alpha \psi - m \bar{\psi} \psi \right)$$

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$$G_{IR}(\omega, q) = \frac{\bar{\psi}_-(z, \omega, q)}{\psi_+(z, \omega, q)} \bigg|_{z \to z_0} \sim e^{-S(\omega, q)}$$

$$S(\omega, q) = 2 \int_{z_0}^{z_t} dz \sqrt{g(z)} V(z) \hspace{2cm} V(z) = m^2 + \frac{q^2}{h(z)} + \frac{\omega^2}{j(z)}$$

- Thermodynamics:

$$F(T), \quad S(N \to \infty, \ T \to 0) > 0$$

- Four-point functions (OTOC):

$$\langle [O(t, x), O(o, o)]^2 \rangle, \quad \text{etc.}$$
Beyond mean-field: Schwarzian dynamics
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- Correction to mean-field propagator:
  \[ \delta G = G_f(\tau_1, \tau_2) - G_0(\tau_1, \tau_2) \approx \]
  \[ \approx \frac{\Delta}{6} (\delta \tau_{12})^2 \text{Sch}\{f, \tau\} G_0(\tau_1, \tau_2) + \ldots \]

- Schwarzian derivative:
  \[ \text{Sch}\{F, \tau\} = \frac{F'''}{F'} - \frac{3}{2} \left( \frac{F''}{F'} \right)^2 \]
Beyond mean-field: Schwarzian dynamics

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- Non-reparametrization invariant action for soft mode:
  \[ A_0 = Tr \ln(1 - \partial_{\tau} G_f) = -M \int_{\tau} \text{Sch}\{\tan\frac{\pi f}{\beta}, \tau\} \]
  \[ \tau = (\tau_1 + \tau_2)/2 \]
Beyond mean-field: Schwarzian dynamics

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  \[ \tau = (\tau_1 + \tau_2)/2 \]

- Convenient change of variables:
  \[ f' = e^\phi \quad A_0 = \frac{M}{2} \int d\tau [\phi'(\tau)]^2. \]
Beyond mean-field: Schwarzian dynamics

- Correction to mean-field propagator:
  \[ \delta G = G_f(\tau_1, \tau_2) - G_0(\tau_1, \tau_2) \approx \frac{\Delta}{6} (\delta \tau_{12})^2 \text{Sch}\{f, \tau\} G_0(\tau_1, \tau_2) + \ldots \]

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  \[ \text{Sch}\{F, \tau\} = \frac{F'''}{F'} - \frac{3}{2} (\frac{F''}{F'})^2 \]

- Non-reparametrization invariant action for soft mode:
  \[ A_0 = \text{Tr} \ln(1 - \partial_\tau G_f) = -M \int_\tau \text{Sch}\{\tan \frac{\pi f}{\beta}, \tau\} \]
  \[ \tau = (\tau_1 + \tau_2)/2 \]

- Convenient change of variables:
  \[ f' = e^\phi \quad A_0 = \frac{M}{2} \int d\tau [\phi'(\tau)]^2. \]

- Regime of strong fluctuations:
  \[ T < 1/M = O(J/N) \ (<< J) \]
Beyond mean-field: Schwarzian dynamics

- Correction to mean-field propagator:
  \[ \delta G = G_f(\tau_1, \tau_2) - G_0(\tau_1, \tau_2) \approx \frac{\Delta}{6} (\delta \tau_{12})^2 \text{Sch}\{f, \tau\} G_0(\tau_1, \tau_2) + \ldots \]

- Schwarzian derivative:
  \[ \text{Sch}\{F, \tau\} = \frac{F'''}{F'} - \frac{3}{2} \left( \frac{F''}{F'} \right)^2 \]

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- Next order \(O(T/J)\) correction:
  \(\delta A \sim \frac{N}{J^2} \int_{\tau_1 \tau_2} \frac{(f'_1 f'_2)^2}{(\delta \tau_{12})^4} \ln \left( \frac{J^2 (\delta \tau_{12})^2}{f'_1 f'_2} \right)\) (non-local)
Generalized SYK models
Generalized SYK models

- **Generalized SYK**: other symmetry breaking terms are possible:

- **Time-dependent** SYK coupling $J_2$:

$$J_k^2(\delta\tau) = \delta_{k,2} \frac{J^{2-2\gamma}}{c^{2\gamma}}$$

- **Scale-invariant** IR solution with dimension:

$$\Delta = \frac{1-\gamma}{2q}$$
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- Reparametrization symmetry is broken \textit{spontaneously} AND \textit{explicitly}:
  \[ \delta A = \frac{\Gamma}{J} \int_{\tau_1 \tau_2} (\delta \tau_{12})^2 \ln(J \delta \tau_{12}) G^{2q}_{f}(\tau_1, \tau_2) \text{Sch}\{ \tan \frac{\pi f}{\beta}, \tau \} \approx \frac{\Gamma}{J} \int_{\tau_1 \tau_2} \frac{(f'_1 - 1)(f'_2 - 1)}{(\delta \tau_{12})^2} \]

- Cf. Caldeira-Leggett with Ohmic dissipation
SL(2,R)-symmetric Hamiltonians
SL(2,R)-symmetric Hamiltonians

- **Algebra** generators:

\[
\{L_0, L_{\pm 1}\} = \pm L_{\pm 1}, \quad \{L_{-1}, L_1\} = 2L_0
\]

- **Hamitonian** as Casimir:

\[
H = \frac{1}{2}L_0^2 - \frac{1}{4}(L_1L_{-1} + L_{-1}L_1)
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  \]

- Particular realization
  (one out of many):
  \[
  L_{-1} = \pi_f, \quad L_0 = f \pi_f + \pi_\phi, \\
  L_1 = f^2 \pi_f + 2f \pi_\phi + A(\phi) - B(\phi) \pi_f - \frac{C(\phi)}{\pi_f}
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- Particular realization (one out of many):

- Particle in \textbf{curved} geometry which is subjected to \textbf{magnetic} and \textbf{electric} fields (nominally 2D but \textbf{effectively} 1D):

\[ H = \frac{1}{2} g^{\phi \phi} \pi_\phi^2 + \frac{1}{2} g^{ff} (\pi_f - A_f)^2 + \Phi \]

\[ g^{ij} = \text{diag}[1, B(\phi)], \quad A_i = (0, \frac{A(\phi)}{B(\phi)}), \quad \Phi = C(\phi) - \frac{A(\phi)^2}{4B(\phi)} \]
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- Further reduction (Liouville-like):
  \[ A(\phi) = 2ae^\phi, \quad B(\phi) = be^{2\phi}, \quad \text{and} \quad C(\phi) = C \]
  \[ H = \frac{1}{2} \pi_\phi^2 + \frac{b}{2} \pi_f^2 e^{2\phi} - ae^\phi \pi_f + \frac{1}{2} c \]
  \[ ds^2 = \hat{d}\phi^2 + e^{-2\phi} df^2 \quad H^2 \]
Liouvillian quantum mechanics
Liouvillian quantum mechanics

- **Standard** Liouville theory: 
  \( a = \pi_f = \mu \sim J \), \( b=c=0 \)

  \[
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  \]

  D.Bagrets, A.Altland, A.Kamenev, 
  1607.00694, 1702.08902
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  \[ \psi_k \sim K_{2ik}(\sqrt{z}) \]
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Z(\beta) = \int_{\phi(-\beta/2) = \phi_0}^{\phi(\beta/2) = \phi_0} d\phi e^{-\int_\tau L(\phi) =}
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- SYK density of states (many-body)

\[ \rho(\epsilon) = \frac{1}{2\pi i} \int_{\beta} e^{\beta E} Z(\beta) \sim e^{S_0} \sinh(2\pi \sqrt{\epsilon}) \]
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  F = -\frac{1}{\beta} \ln Z(\beta) = E_0 - \frac{S_0}{\beta} - \frac{2\pi^2 M}{\beta^2} + \frac{\pi^2 \mu N}{6\beta^3 J^2} + \frac{3}{2\beta} \ln \beta J
  \]

- Higher order functions (e.g., OTOC)
Sick SYK cousin: Morse potential
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- Hamiltonian:

\[ H = \frac{1}{2} \pi_\phi^2 + \frac{b}{2} \pi_f^2 e^{2\phi} - a e^{\phi} \pi_f + \frac{1}{2} c \]
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  (incl. bound)

  \[ \psi_k \sim e^{-\phi/2} W_{\lambda,ik}(z) \quad z = 2\lambda e^\phi \]

  \[ \psi_n(z) \sim z^{\lambda-n-1/2 - z/2} L_n^{2\lambda-2n-1}(z) \]

\[ \epsilon_n = -(n - \lambda + 1/2)^2 \]

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\[ \Omega \sim \mu \beta / M \]
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- Gaussian approximation:
(including 'dissipative' term)
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\delta S = \frac{M}{2} \sum_n (\omega_n^2 + \Omega^2 + \Gamma|\omega_n||\phi_n|^2)
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Sick SYK cousin: Morse potential

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  \[ \rho(\epsilon) \approx \frac{\lambda - \frac{1}{2}}{2\pi} \sum_n \delta(E_n - \Omega(n + \frac{1}{2})) \sim M \]

DVK 1905.04381
Schwarzian correlations: Liouville vs Morse
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- Energy-stress:
  \[ T(\tau) = M(f'''' - (2\pi/\beta)^2 f') \]

- Correlation:
  \[ <T(\tau)T(0)> = M \sum_n \frac{e^{2\pi\tau/\beta}(\omega_n^2 - (2\pi/\beta)^2)\omega_n^2}{\omega_n^2 + \Omega^2 + \Gamma|\omega_n|} \]
  \[ \sim Mmax[1/\beta^3, \Omega^3] \sin \Omega \tau e^{-\Gamma \tau/2} \]

- Energy fluctuations:
  \[ <(\delta E)^2> = \frac{\partial^2}{\partial \beta^2} \ln Z(\beta) \sim Mmax[1/\beta^3, \Omega^3] \]
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\[ < G_f(\tau_1, \tau_2) > \approx \sum_n e^{-E_n \tau} N_1(E_n) \sim 1/\tau, \quad M < \tau < \frac{1}{\Omega} \]

\[ < G_f(\tau_1, \tau_2)G_f(\tau_3, \tau_4) > \sim 1/t^4 \quad \text{cf.} \quad 1/t^{3/2} \text{ and } 1/t^6 \]

- Long-time universal:

behavior ( q = 2)
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- Long-time universal behavior (q = 2)

- Lyapunov exponents:

\[ \frac{< G(\tau_1, \tau_3) G(\tau_2, \tau_4) >}{< G(\beta/2, 0) >^2} = 1 - O(\beta/M)e^{\lambda_{L} t} \]

Liouville

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Morse

\[ \lambda_L = \frac{2\pi}{\beta} (1 - O(\alpha)) \]
Bulk theory: JT gravity and beyond
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\[ I_{JT}[g, \Phi] = -\frac{1}{4\pi} \int_D \Phi (R + 2) \sqrt{g} \, d^2x - \frac{1}{2\pi} \int_{\partial D} \Phi K \, d\ell \]

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- 'Particle in magnetic field' problem (effectively 1D, too):

\[
I[X] = \int_0^\beta d\tau \left( \frac{1}{2} g_{\alpha\beta} \dot{X}^\alpha \dot{X}^\beta - \gamma \omega_\alpha X^\alpha \right)
\]

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G(x_1, x_0; \beta) = \int_{X(0) = x_0}^{X(\beta) = x_1} DX \, e^{-I[X]}
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A. Kitaev and J. Suh,
1711.08467, 1808.07032
Bulk theory: JT gravity and beyond

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A.Kitaev and J.Suh, 1711.08467, 1808.07032
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- Other equivalent AdS_3 sections?

A.Kitaev and J.Suh, 1711.08467, 1808.07032
Seeking to develop global SYK-ness
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**Generically:** $z < \infty$
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- doped (no particle-hole symmetry): charge fluctuations
- supersymmetric
- coupled
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Y.Gu, X.-L.Qi, and D.Stanford, 1609.07832

S.-K.Jian and H.Yao, 1703.02051

S.Banerjee and E.Altman, 1610.04619

X.Chen et al, 1705.03406
Seeking to develop global SYK-ness

- SYK: Ultra-local physics, $z = \infty$  
  Kondo systems? Cuprates??

  Generically: $z < \infty$

- doped (no particle-hole symmetry): charge fluctuations
- supersymmetric
- coupled

$$D \sim v^2_b t_L$$ (Sachdev, Hartnoll, Lucas,...)

- Diffusive energy transport, hydrodynamic universal bound

- Despite (postulated) single-particle ultra-locality:

$$G_{ij}(\tau) \sim \frac{sgn \tau}{|\tau|^{2/q}} \delta_{ij}$$
Thickening and sickening the SYK model
Thickening and sickening the SYK model

\[ S = \sum_{i} \sum_{\alpha} \chi_i^\alpha \partial_\tau \chi_i^\alpha - i^{q/2} \sum_{i_{\alpha}, i_{\beta}} J_{i_{\alpha} \ldots i_{\beta}} \chi_{i_1}^{\alpha_1} \ldots \chi_{i_q}^{\alpha_q} \]

Time-dependent and/or non-local disorder correlations:

- standard SYK on NL sites:

\[ F_{i_1 \ldots i_q, j_1 \ldots j_q}(\tau_{12}) = J^2 \prod_{\alpha} \delta_{i_\alpha, j_\alpha} \]

- L copies of N-site SYK:

\[ F_{i_1 \ldots i_n, i_1 \ldots i_n}(\tau_{12}) = J^2 \prod_{\alpha} \delta_{i_\alpha, i_\alpha} \prod_{n}^{q-1} \delta_{i_n, i_n} \]
Thickening and sickening the SYK model

\[ S = \sum_i \sum_{\alpha} \chi_i^\alpha \partial_{\tau} \chi_i^\alpha - i^{q/2} \sum_{i_\alpha, \alpha} J_{i_1 \ldots i_q}^{\alpha_1 \ldots \alpha_q} \chi_{i_1}^{\alpha_1} \ldots \chi_{i_q}^{\alpha_q} \]

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- L copies of N-site SYK:
  \[ F_{i_1 \ldots i_n i_1 \ldots i_n}(\tau)_{12} = J^2 \prod_{\alpha} \delta_{i_\alpha i_\alpha} \prod_{\alpha} \delta_{i_\alpha j_\alpha} \]

- **Algebraic** space and/or time correlations:
  \[ J_{ij}^2(\tau) \sim \tau^{-2\alpha} \]
  \[ J_{ij}^2(\tau) \sim |i - j|^{-2\beta} \]
  \[ J_{ii}^2(\tau) \sim (\tau^2 + a^2|i - j|^2)^{-\gamma} \]
Mean-field analysis
Mean-field analysis

- Partition function:

\[ Z = \int DG_{ij}(\tau) D\Sigma_{ij}(\tau) Pf(\partial_\tau - \Sigma) \]

\[ \exp(N \sum_{i,j} \int_{\tau_1, \tau_2} (G_{ij}(\tau_{12})\Sigma_{ij}(\tau_{12}) - \frac{1}{q} J_{ij}^2(\tau_{12})G_{ij}^{q}(\tau_{12}))) \]

- Saddle-point equation:

\[ \sum_j \int_{\tau_3} (\delta_{ij} \partial_\tau \delta(\tau_{13}) - \Sigma_{ij}(\tau_{13}))G_{jk}(\tau_{32}) = \delta_{ik}\delta(\tau_{12}) \]

\[ \Sigma_{ij}(\tau_{12}) = J_{ij}^2(\tau_{12})G_{ij}^{q-1}(\tau_{12}) \]

- Asymptotic IR regime:

\[ \int_{\tau_3, x_3} G_{x_{13}}(\tau_{13})J^2_{x_{32}}(\tau_{32})G_{x_{32}}^{q-1}(\tau_{32}) = \delta(\tau_{12})\delta(x_{12}) \]
### Mean-field analysis

- **Partition function:**

\[
Z = \int D G_{i j}(\tau) D \Sigma_{i j}(\tau) Pf (\partial_{\tau} - \Sigma) \\
\exp(N \sum_{i,j} \int_{\tau_1,\tau_2} (G_{i j}(\tau_{12})\Sigma_{i j}(\tau_{12}) - \frac{1}{q} J^2_{i j}(\tau_{12}) G^q_{i j}(\tau_{12})))
\]

- **Saddle-point equation:**

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\sum_{i,j} \int_{\tau_3} (\delta_{i j} \partial_{\tau_1} \delta(\tau_{13}) - \Sigma_{i j}(\tau_{13})) G_{j k}(\tau_{32}) = \delta_{i k} \delta(\tau_{12})
\]

\[
\Sigma_{i j}(\tau_{12}) = J^2_{i j}(\tau_{12}) G^q_{i j} - 1(\tau_{12})
\]

- **Asymptotic IR regime:**

\[
\int_{\tau_3, x_3} G_{x_{13}}(\tau_{13}) J^2_{x_{32}}(\tau_{32}) G^q_{x_{32}} - 1(\tau_{32}) = \delta(\tau_{12}) \delta(x_{12})
\]

- **Scaling-invariant** IR behavior for:

\[
\frac{d}{Z}(q - 2) + 2[J] - 2 < 0
\]

- **z** = \(\infty\) or **q**=2: holds for any **[J]<1**
- **z**=1 and **q**>2: **only** for **d**=1 and **[J]=0**
- **Generic** **z>1, q>2, d>0**: **[J]<0 (non-unitary?)**
Mean-field solutions
Mean-field solutions

- Ultra-local (original SYK): \[ G_{ij}(\tau) \sim \frac{\text{sgn} \tau}{|\tau|^{2/q}} \delta_{ij} \] (Hartree-type contractions)
Mean-field solutions

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\[ G_{ij}(\tau) \sim \frac{\text{sgn} \tau}{|\tau|^{2/q}} \delta_{ij} \]  

(Hartree-type contractions)

- Factorizable:

\[ J_{ij}^2(\tau) \sim \tau^{-2\alpha} |i-j|^{-2\beta} \]

\[ G(\tau, x) \sim \frac{\text{sgn} \tau}{\tau^{2\Delta}} \frac{1}{|x|^{2\Delta_x}} \Delta_{\tau} = (1 - \alpha)/q \quad \Delta_{x} = (d - \beta)/q \]

\[ G(\omega, k) \sim |\omega|^{2\Delta_{\tau} - 1} k^{2\Delta_x - d} \]
Mean-field solutions

- Ultra-local (original SYK):
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  \[ \Delta_x = (d - \beta)/q \]
  \[ G(\omega, k) \sim |\omega|^{2\Delta_{\tau} - 1} k^{2\Delta_x - d} \]

- Lorentz-invariant:
  \[ J_{ij}^2(\tau) \sim (\tau^2 + a^2 |i-j|^2)^{-\gamma} \]
  \[ G(\tau, x) \sim \frac{\text{sgn} \tau}{(\tau^2 + x^2)^{\Delta}} \]
  \[ \Delta = (D - \gamma)/q \]
  \[ D = d + 1 \]
  \[ G(\omega, k) \sim (\omega^2 + k^2)^{\Delta - D/2} \]

- Other?

(bosonic case: Patashinsky, Pokrovsky ˚64)
Fluctuations about mean-field
Fluctuations about mean-field

- Reparametrization invariance (for $\alpha=\beta=\gamma=0$ only):

$$G(x_1, x_2) \rightarrow |g(x_1)g(x_2)|^{D/2q} G(f(x_1), f(x_2))$$

$$g = |\det \frac{\partial f^\mu}{\partial x^\nu}|^2$$

$\mu, \nu = 1, \ldots, D=d+1$
Fluctuations about mean-field

- Reparametrization invariance (for $\alpha=\beta=\gamma=0$ only):

\[ G(x_1, x_2) \rightarrow |g(x_1)g(x_2)|^{D/2q} G(f(x_1), f(x_2)) \]

\[ g = |\text{det} \partial f^\mu/\partial x^\nu|^2 \]

\[ \mu, \nu = 1, \ldots, D=d+1 \]

- Original SYK ($d=0$): Schwarzian

\[ S(f) = \frac{N}{J} \int \{ f, x \} = \frac{N}{J} \int (\frac{f'''}{f'} - \frac{3}{2}(\frac{f''}{f'})^2) \]

- Generalized SYK ($d>0$): non-local action

\[ \delta S(f) = \frac{N}{2} \int_k (k_\mu f^\mu)^2 (C\omega + \omega^2 / J)|k|^d \]

\[ C = 0(\alpha) + 0(\beta) \]

- Stress-energy correlations: no diffusive pole

\[ \langle T_{\mu\nu}(\omega, k)T_{\mu\nu}(-\omega, -k) \rangle = \frac{i\omega |k|^d (C + i\omega / J)^2}{C + (i\omega + D\omega^2 / J) / J} \]
2-body problem

- Fluctuations:
  \[ G = G_0 + g |G_0|^{(2-q)/2} \]
  \[ \Sigma = \Sigma_0 + \sigma |G_0|^{(q-2)/2} \]

  \[ \delta S(g, \sigma) = N \int (g_{12} \sigma_{12} - \frac{q-1}{2} F_{12} g_{12}^2 - \frac{\sigma_{12} \hat{K}_{12,34} \sigma_{34}}{2(q-1)}) \]

  \[ \delta S(g) = \frac{N(q-1)}{2} \int [\hat{K}^{-1}_{12,34} \hat{1}_{13} \hat{1}_{24} F_{12}) g_{34} \]

- Quadratic kernel:
  \[ \hat{K}_{12,34} = (q-1) G_{13} G_{24} |G_{34}|^{q-2} \]

- Diagonalization:
  \[ \int_{x_3, x_4} \hat{K}_{12,34} F_{34} \Psi_{34}(h|\omega, k) = \lambda_h(\omega, k) \Psi_{12}(h|\omega, k) \]

- Eigenstates (spin-zero):
  \[ \Psi_{12}(h|\omega, k) \sim |x_{12}|^{h-2\Delta} e^{i k \mu (x_1^\mu + x_2^\mu)/2} \]

- Eigenvalue equation:
  \[ \frac{\lambda_h}{x_{12}^{2\Delta-h}} = \int_{x_3, x_4} \frac{1}{x_{13}^{2\Delta} x_{24}^{2\Delta} x_{34}^{2D-2\Delta-h}} \lambda_h = \lambda_h(0,0) \]

  \[ (1-q) \frac{\Gamma(D-\Delta) \Gamma(-D/2+\Delta-h/2) \Gamma(-D/2+\Delta+\frac{h}{2}) \Gamma(D-\Delta-h/2)}{\Gamma(-D/2+\Delta) \Gamma(D-\Delta-h/2) \Gamma(D-\Delta-h/2)} = 1 \]

  Bosonic SYK:
  \[ (1-q) \frac{\Gamma(D-\Delta) \Gamma(-D/2+\Delta-h/2) \Gamma(-D/2+\Delta+\frac{h}{2}) \Gamma(D-\Delta-h/2) \Gamma(D-\Delta-h/2)}{\Gamma(-D/2+\Delta) \Gamma(D-\Delta-h/2) \Gamma(D-\Delta-h/2)} = 1 \]

No solutions for \( h=2, D \) or \( D+1 \) (stress-energy operator)

Prospective dual is not dominated by gravity?
OTOC functions and chaos

- Generic 2-body amplitude:
  \[ F_{12,34} = \langle \chi_\alpha^\alpha(\tau_1)\chi_\beta^\beta(\tau_2)\chi_\gamma^\gamma(\tau_3)\chi_\delta^\delta(\tau_4) \rangle \]

- Expansion over eigenstates:
  \[ F_{12,34} = \frac{1}{1 - K} F_{12,34}^0 = \sum \Psi_{12} \frac{1}{1 - \lambda} \langle \Psi_{34} | F^{(0)} \rangle \]
  \[ F_{12,34}^{(0)} = G_{13}G_{24} - G_{14}G_{23} \]

- Finite temperature basis:
  \[ \Psi_{12}(h|k) \sim \frac{e^{ik(x_1+x_2)/2 - \pi Th(\tau_1+\tau_2)}}{\cosh(\pi T\tau_{12})^{2\Delta_r-h}|x_1-x_2|^{2\Delta_r-h}} \]

- OTOC functions:
  \[ F(\tau, x) = \langle u\chi_x^\alpha(\tau)u\chi_0^\beta(0)u\chi_x^\alpha(\tau)u\chi_0^\beta(0) \rangle \]
  \[ u = e^{-H/4T} \]
  \[ \text{(Larkin and Ovchinnikov '69),} \]

- Chaos spreading:
  \[ F(\tau, x) \sim 1 - \frac{1}{N} e^{\lambda_L(\tau - |x|/v_B)} \]

- Lyapunov index:
  \[ \lambda_L = -2\pi Th \]
  \[ \text{where } h \text{ solves } \frac{\Gamma(3 - 2\Delta_r)\Gamma(2\Delta_r - h)}{\Gamma(1 + 2\Delta_r)\Gamma(2 - 2\Delta_r - h)} = 1 \]

- Original SYK (d=0): \( h = -1 \) (maximal chaos)
- For \( d > 0 \) and/or \( \alpha, \beta, \gamma \neq 0 \): \( h > -1 \) (no chaotic bound saturation)
New horizons
New horizons

- **Resonant** SYK model in momentum space

\[
H_k = \int_k \sum_{\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha},
\]
\[
H_U = \frac{1}{(2N)^{3/2}} \sum_{\alpha_a} \int_{k_a} U_{\alpha_a} (k_a) c_{k_1\alpha_1}^\dagger c_{k_2\alpha_2} c_{k_3\alpha_3} c_{k_4\alpha_4}
\]
\[
\mathcal{C}(k_{\alpha}, k'_{\alpha'}) = \mathcal{C}_0 (k_{\alpha}, k'_{\alpha'}) \frac{1}{2} \left[ \mathcal{K}_1(k_{\alpha}) \delta (\epsilon_k + \epsilon_2 - \epsilon_3 - \epsilon_4) \right]
\]

A. Patel and S. Sachdev, 1906.03265
New horizons

- Resonant SYK model in momentum space

\[ H_k = \int k \sum_{\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}, \]
\[ H_U = \frac{1}{(2N)^{3/2}} \sum_{\alpha_1} \int_{k} U_{\alpha_1}(k) c_{k_1\alpha_1}^\dagger c_{k_2\alpha_2} c_{k_3\alpha_3} c_{k_4\alpha_4} \]
\[ \mathcal{K}(k_\alpha, k'_\alpha) = \mathcal{K}_0(k_\alpha, k'_\alpha) \frac{1}{2} \mathcal{K}_1(k_\alpha) \delta(\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_3} - \epsilon_{k_4}) \]

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- Universal linear resistivity:

\[ \rho = \frac{m^*}{n e^2} \frac{1}{\tau} \]
\[ \frac{1}{\tau} = \alpha \frac{k_B T}{\hbar} \]

SYK = 1
New horizons

- Resonant SYK model in momentum space

\[ H_k = \sum_{\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}, \]
\[ H_U = \frac{1}{(2N)^{3/2}} \sum_{\alpha} \int_{k.a} U_{\alpha a}(k_a) c_{k_1\alpha_1}^\dagger c_{k_2\alpha_2}^\dagger c_{k_3\alpha_3} c_{k_4\alpha_4} \]

\[ \mathcal{K}(k_\alpha, k'_{\alpha'}) = \mathcal{K}_0(k_\alpha, k'_{\alpha'}) \left\{ \frac{1}{2} \mathcal{K}_1(k_\alpha) \delta(\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_3} - \epsilon_{k_4}) \right\} \]

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<table>
<thead>
<tr>
<th>Material</th>
<th>( n ) (10^{27} \text{m}^{-3})</th>
<th>( m^* ) (m_0)</th>
<th>( A_1 / d ) (\Omega / K)</th>
<th>( h / (2e^2 T_F) ) (\Omega / K)</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bi2212</td>
<td>0.23</td>
<td>6.8</td>
<td>8.4 ± 1.6</td>
<td>8.0 ± 0.9</td>
<td>1.1 ± 0.3</td>
</tr>
<tr>
<td>Bi2201</td>
<td>0.4</td>
<td>3.5</td>
<td>7 ± 1.5</td>
<td>8 ± 2</td>
<td>1.0 ± 0.4</td>
</tr>
<tr>
<td>LSCO</td>
<td>0.26</td>
<td>7.8</td>
<td>9.8 ± 1.7</td>
<td>8.2 ± 1.0</td>
<td>0.9 ± 0.3</td>
</tr>
<tr>
<td>Nd-LSCO</td>
<td>0.24</td>
<td>7.9</td>
<td>12 ± 4</td>
<td>7.4 ± 0.8</td>
<td>0.7 ± 0.4</td>
</tr>
<tr>
<td>PCCO</td>
<td>x = 0.17</td>
<td>8.8</td>
<td>2.4 ± 0.1</td>
<td>1.7 ± 0.3</td>
<td>0.8 ± 0.2</td>
</tr>
<tr>
<td>LCCO</td>
<td>x = 0.15</td>
<td>9.0</td>
<td>3.0 ± 0.3</td>
<td>3.0 ± 0.45</td>
<td>1.2 ± 0.3</td>
</tr>
<tr>
<td>TMTSF</td>
<td>P = 11 kbar</td>
<td>1.4</td>
<td>1.15 ± 0.2</td>
<td>2.8 ± 0.3</td>
<td>1.0 ± 0.3</td>
</tr>
</tbody>
</table>

SYK = 1

S. Sachdev, Montreal, July '19
Summary
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- The status of the **holographic conjecture** (especially in its broad, 'non-AdS/non-CFT', form) still remains largely undetermined.
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- The popular 'bottom-up' approach is prone to substituting some forms of 'analogue holography' for the ‘bona fide’ one. Apparent examples of the former could indeed be observed in various tangible systems (flexible graphene, optical metamaterials, etc.) but regardless of the validity of the holographic conjecture itself.
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- While not providing water-proof examples of genuine holographic correspondence, the d=0 SYK-like models offer an important insight into the properties of a whole sequence of the SL(2,R)-symmetric QM systems and their JT-like (effectively 1D) 'bulk' duals.
Summary

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- While not providing water-proof examples of genuine holographic correspondence, the d=0 SYK-like models offer an important insight into the properties of a whole sequence of the SL(2,R)-symmetric QM systems and their JT-like (effectively 1D) 'bulk' duals.

- Higher-dimensional ‘thickening’ tends to ‘sicken’ the salient SYK behavior. Still, the d>0 - dimensional SYK-like models can be viewed as interesting examples of soluble (super)strongly-interacting many-body systems with markedly NFL properties.