Generating Superextreme Waves in Linear Optics

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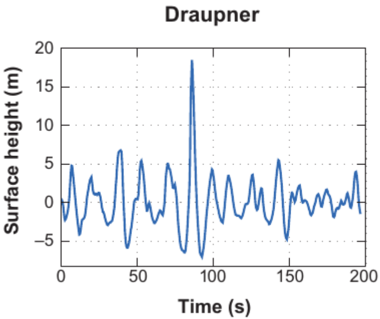
Overview

Rogue Waves



New Year wave recorded at the Draupner platform in the North Sea on January 1, 1995. The crest height is $\approx$ 18.5 m and exceeds the Significant Wave Height of 11.8 m by a factor of 1.54.

SWH = 4 X the standard deviation of the surface elevation.

Characteristics

- High-amplitude waves that appear more often than predicted by the Gaussian statistics.
- Draper (1964, 1971) gave an early account of the phenomenon and Mallory (1974) provided the first discussion of the giant waves in the Agulhas current.
- They have unusually steep, solitary or tightly grouped profiles implying a relatively broadband frequency content compared with normal waves and also suggesting possible connections with solitons - solitary waves that propagate without spreading in water because of a balance between dispersion and nonlinearity.

A First Approach

Random linear superposition of many plane waves with differing direction and wavelength [Longuet-Higgins, 1957] offers a statistical explanation for the occurrence of freak waves.

By the central limit theorem, the sea surface height H must be a Gaussian random variable with some standard deviation σ.

In the limit of a narrow frequency spectrum the crest height then follows a Rayleigh distribution.

$$P_{\text{Rayleigh}} = \exp\left(-H^2/2\sigma^2\right)$$

Rare but Not Infrequent

The likelihood of observing even a freak wave in hundred of years should be essentially non-existent!

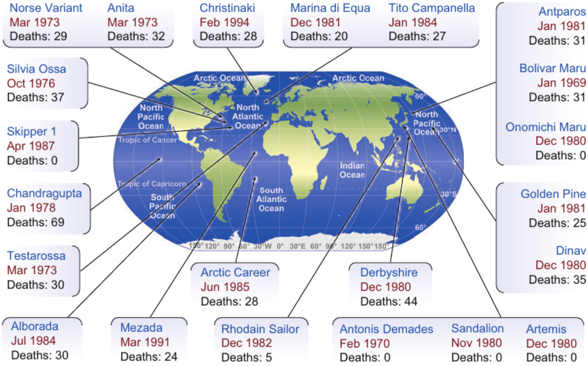


Figure 1

Locations of 22 supercarriers assumed to be lost after collisions with rogue waves between 1969 and 1994. Figure copyright C. Kharif and E. Pelinovsky, used with permission.

Two Main Generation Mechanisms

Linear Theory

- simple linear superposition of sinusoidal waves
- linear focusing due to the presence of currents in ocean or change in the fluid depth

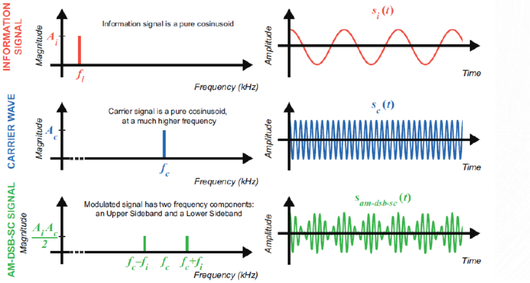
Not able to predict many events (it seems!)

Nonlinear Corrections

- nonlinear effect on crossing seas
- one-dimensional modulation instability (Benjamin-Feir index)
- modulation instability by varying currents
- modulation instability by wind blowing

It seems to predict higher events better!

Modulational Instability



1d wave - carrier that is modulated

Narrow band assumption at f_c

Instability in this modulation may grow in time or space

Nonlinear Schrödinger Equation - nonlinear diffraction

Linear mechanisms

Heller, Kaplan & Dahlen. *J. Geophys. Res. (2008)* Refraction of a Gaussian seaway.

Hohmann, Kuhl, Stockmann, Kaplan & Heller. *Phys. Rev. Lett. (2010)* Freak waves in the linear regime: a microwave study.

Arecchi, Bortolozzo, Montina & Residori. *Phys. Rev. Lett.(2011)* S. Granularity and inhomogeneity are the joint generators of *optical rogue waves*.

Metzger, Fleischmann & Geisel. *Phys. Rev. Lett. (2014)* Statistics of extreme waves in random media.

Liu, van der Wel, Rotenberg, Kuipers, Krauss, Di Falco & Fratalocchi. *Nat. Phys. (2015)* Triggering extreme events at the nanoscale in photonic seas.

Mathis, Froehly, Toenger, Dias, Genty & Dudley. *Sci. Rep. (2015).* Caustics and rogue waves in an *optical sea*.

SuperExtreme Waves

This is a new class of waves exhibiting an enhanced wave amplification, as shown experimentally and theoretically in a water wave tank.

Chabchoub, Hoffmann, Onorato, & Akhmediev. *Phys. Rev. X (2012)*

Super rogue waves: observation of a higher-order breather in water waves. .

They correspond to higher-order solutions of the focusing nonlinear Schrödinger equation.

Chabchoub, Hoffmann, Onorato, Slunyaev, Sergeeva, Pelinovsky & Akhmediev. *Phys. Rev. E (2012).*

Observation of a hierarchy of up to fifth-order rogue waves in a water tank.

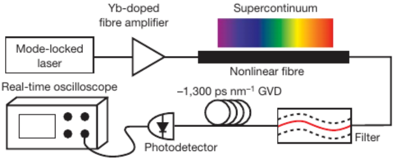
In optics, superextreme light pulses exceeding by far usual rogue waves criteria have been recently predicted to occur in a CO_2 laser under a harmonic modulation.

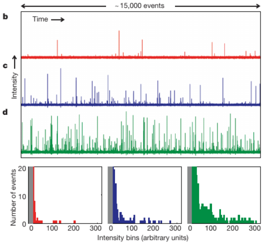
Bonatto & Endler. *Phys. Rev. E (2017).*

Extreme and superextreme events in a loss-modulated CO2 laser: nonlinear resonance route and precursors.

Optical Rogue Waves

There are striking phenomenological and physical similarities between the extreme events of this optical system and oceanic rogue waves Solli, Ropers, Kooneth & Jalali. Nature 2007.





The supercontinuum radiation used is generated by sending picosecond seed pulses at 1,064 nm through a highly nonlinear microstructured optical fibre with matched zero-dispersion wavelength. The output is red-pass filtered at 1,450 nm and stretched so that many thousands of events can be captured with high resolution in a single-shot measurement.

Modulation Instability as an Important Ingredient

The nonlinear processes responsible for supercontinuum generation amplify the noise present in the initial laser pulse.

For long pulses and continuous wave input radiation, modulation instability - an incoherent nonlinear wave mixing process - broadens the spectrum from seed noise in the initial stages of propagation so that the output spectrum is highly sensitive to the initial conditions.

As the NLSE also describes optical pulse propagation in nonlinear media, they investigate this numerically using the generalized NLSE (neglecting absorption).

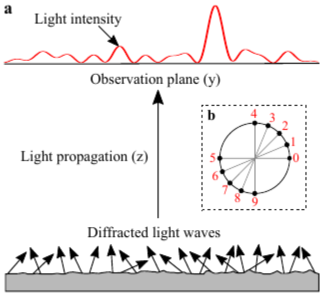
This equation has been successfully used to model supercontinuum generation in the presence of noise and is capable of qualitatively explaining their experimental results.

SuperExtreme Optical Waves

- The term superextreme has been introduced with a precise meaning related to higher-order solitons.
- Here it is used in the sense of very large deviations from normal distributions in the simplest case of linear one-dimensional light diffraction.
- By studying phases spatial memory-dependence on diffracted light waves, we generate superextreme waves in the linear regime.

Phases Distribution

We consider the propagation of N random waves with an uniform initial phase distribution and distinct degrees of correlation. A diffraction pattern is observed in the far field (Fraunhofer plane).



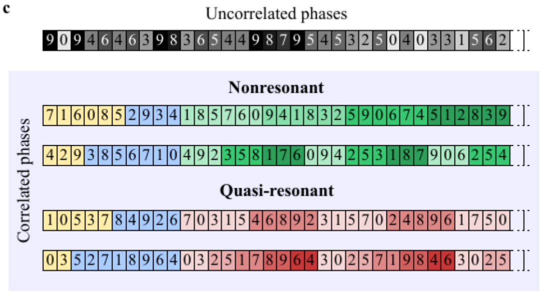
$$\frac{E(\beta)}{E_0} = \text{sinc}\left(\frac{\beta}{2}\right)\sum_{j=1}^{N} e^{[i\beta j + i\phi_j]}$$

$$\beta = ka\sin\theta \text{ and } \phi_j = \frac{2\pi}{L}n_j$$

n_j is an integer and a the pixel width.

Sequence of Phases

Take a random permutation of the L distinct phases. Keep fixed the last M phases and perform a random permutation of the first $(L - M)$ phases filling up the sequence from left to right successively.



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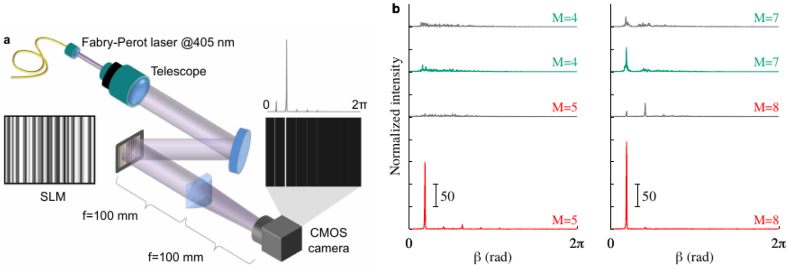
Light with Tunable Non-Markovian Phase Imprint

Robert Fischer,[1,*] Itamar Vidal,[2,4] Doron Gilboa,[4] Ricardo R. B. Correia,[3] Ana C. Ribeiro-Teixeira,[3] Sandra D. Prado,[3] Jandir Hickman,[3] and Yaron Silberberg[4]

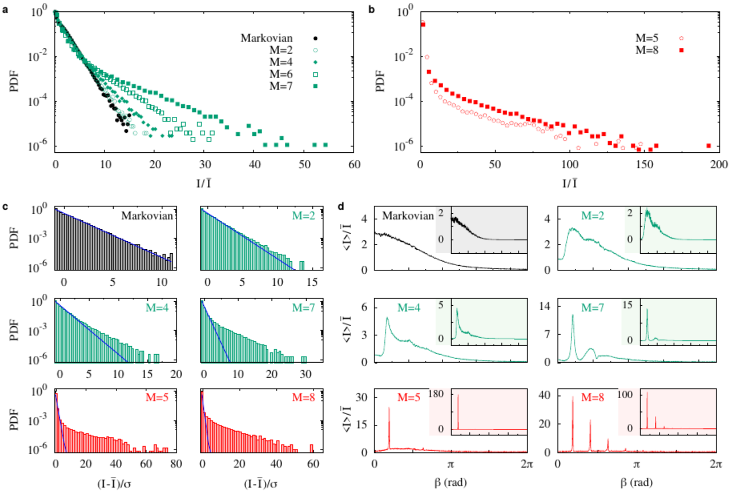
Experimental Details

- We used a 5.0 mW fiber-coupled cw FabryPerot laser operating at 405 nm (Thorlabs- MCLS1), whose output is sequentially collimated with a 75$\times$ beam expander.
- The resulting wavefront illuminates the active area of the SLM 1280$\times$720 pixels of 9.5 mm pitch (Cambridge- SDE1280) containing the designed spatial phase patterns.
- The SLM provides, according to an 8-bit gray level, the phase structure randomly generated in a computer.
- This diffracted wavefront illuminates an f = 100 mm cylindrical lens, which in turn images the intensity pattern of the diffracted beam onto a CCD camera (Point Grey - CMLN-13S2M-CS) with 1280$\times$960 pixels of 3.5 mm pitch.
- Images excluding the central maxima were registered for a set of 1000 realizations up to the first diffraction order.

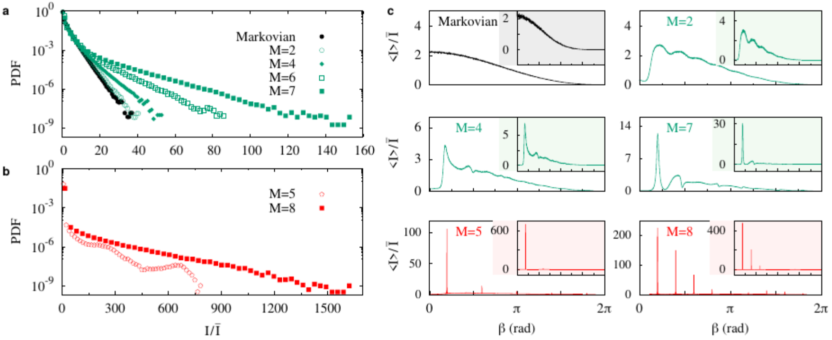
Experimental Setup



Results from the Experiment



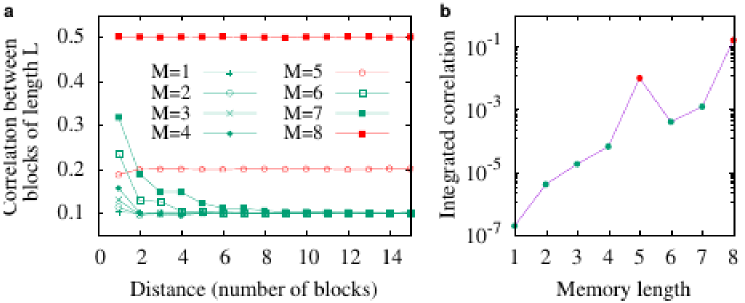
Results from the Simulation



Correlation Length

$$C_d = \frac{1}{L}\left\langle \phi_i \phi_{i+dL} \right\rangle_{\text{realizations}} \qquad \text{d=1,2,3}\cdots$$

$$\overline{C} = \frac{1}{n}\sum_{d=1}^{n}\left(C_d - \frac{1}{L}\right)^2$$



Final Considerations

Novelty

Memory effects of spatially distributed random phases can increase the probability of constructive interference of random linear waves. The spatial phase structure, albeit being disordered, can exhibit long-range correlations leading to extreme waves.

Superextreme Optical Waves in a linear diffraction

Up to date, superextreme events have been observed in both hydrodynamical and optical systems, only when strong nonlinearities are present. Here, we have reported the observation of superextreme optical waves in a linear and one-dimensional light diffraction experiment.

Linear X Nonlinear Mechanisms

We show that for suitable memory lengths, the linear interference model can generate waves with amplitudes as high as those observed in systems with strong nonlinearities. The superextreme optical waves exceed by far the usual criterion of rogue waves classification.

Thank you for your attention.