Universität Augsburg University

Specular reflection and diffraction in the Casimir effect

Gert-Ludwig Ingold Universität Augsburg
Michael Hartmann
Benjamin Spreng

Paulo A. Maia Neto Universidade Federal do Rio de Janeiro
Vinicius Henning

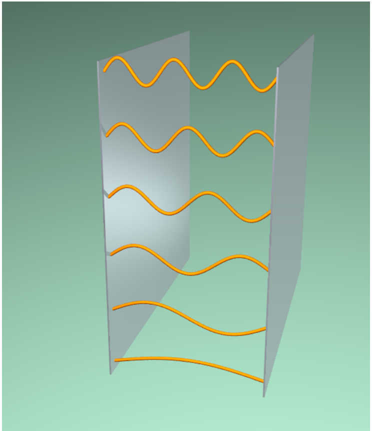
CAPES

DAAD

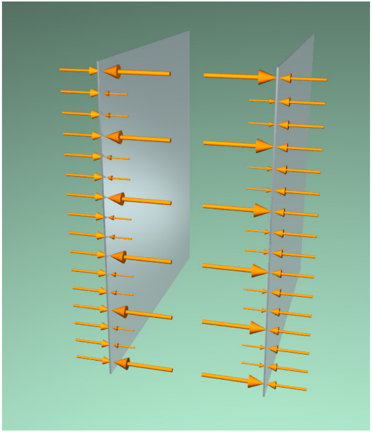
GEFÖRDERT VOM
Bundesministerium für Bildung und Forschung

Quantum fluctuations of the electromagnetic field

vacuum energy from modes



radiation pressure

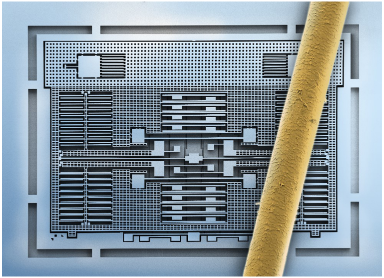


vacuum fluctuations under the influence of boundary conditions

Some applications

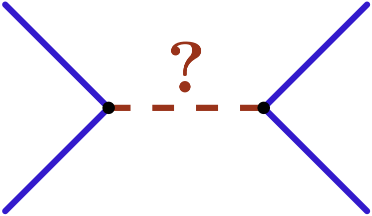
© Stefan Kühn (CC BY-SA 3.0)

colloids



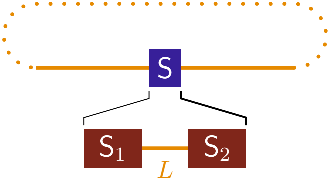
© Bosch Sensortec

MEMS and NEMS



search for fifth fundamental force

Scattering theoretical approach in one dimension



change of vacuum energy due to a scatterer

$$\Delta E_{\text{vac}} = \frac{i\hbar c}{4\pi}\int_0^\infty dk \ln\left(\det(S)\right)$$

$$\det(S) = \det(S_1)\det(S_2)\frac{1-\left[\bar{r}_1 r_2 e^{2ikL}\right]^*}{1-\left[\bar{r}_1 r_2 e^{2ikL}\right]}$$

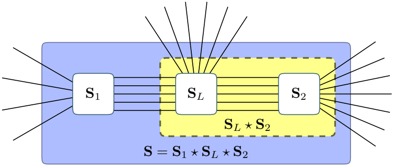
$$\Delta E_{\text{vac}} = \Delta E_{\text{vac}}^{(1)} + \Delta E_{\text{vac}}^{(2)} + \Delta E_{\text{vac}}(L)$$

Casimir energy

$$\Delta E_{\text{vac}}(L) = \Delta E_{\text{vac}} - \Delta E_{\text{vac}}^{(1)} - \Delta E_{\text{vac}}^{(2)} = \frac{\hbar c}{2\pi}\text{Im}\int_0^\infty dk\ \ln\left[1-\bar{r}_1 r_2 e^{2ikL}\right]$$

for a pedagogical presentation see GLI, A. Lambrecht, Am. J. Phys. **83**, 156 (2015)

Scattering theoretical approach with dissipation



$$\det \mathbf{S} = \det(\mathbf{S}_1)\det(\mathbf{S}_2)\det(\mathbf{S}_L)\frac{\det(\mathcal{D}_{21})}{\det(\mathcal{D}_{21})^*}$$

with $\mathcal{D}_{21} = \left(1 - \mathbf{S}_1^{ii}\mathbf{T}_{12}^{ii}\mathbf{S}_2^{ii}\mathbf{T}_{21}^{ii}\right)^{-1}$

Casimir energy at zero temperature

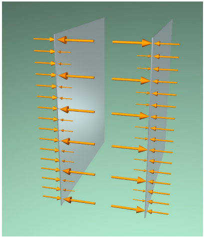
$$E_{\text{vac}}(L) = \hbar\int_0^\infty \frac{d\xi}{2\pi}\log\det\left[\mathbf{1} - \mathbf{S}_1^{ii}\mathbf{T}_{12}^{ii}\mathbf{S}_2^{ii}\mathbf{T}_{21}^{ii}(\xi)\right]$$

R. Guérout, GLI, A. Lambrecht, S. Reynaud, Symmetry **10**, 37 (2018)

Casimir free energy at finite temperature

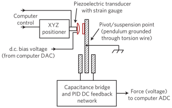
$$\mathcal{F}(L) = \frac{k_B T}{2}\sum_{n=-\infty}^{\infty}\log\det\left[\mathbf{1} - \mathbf{S}_1^{ii}\mathbf{T}_{12}^{ii}\mathbf{S}_2^{ii}\mathbf{T}_{21}^{ii}(|\xi_n|)\right] \qquad \xi_n = \frac{2\pi\hbar n}{k_B T}$$

Some experimental setups

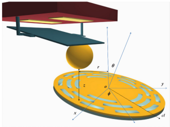


thy: Casimir (1948)
exp: Sparnaay (1958)

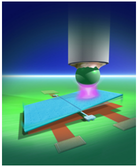
plane/plane



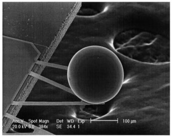
Lamoreaux group (2001)



Decca (2016)

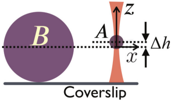


Capasso group (2001)

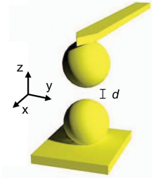


Mohideen group (1998)

plane/sphere



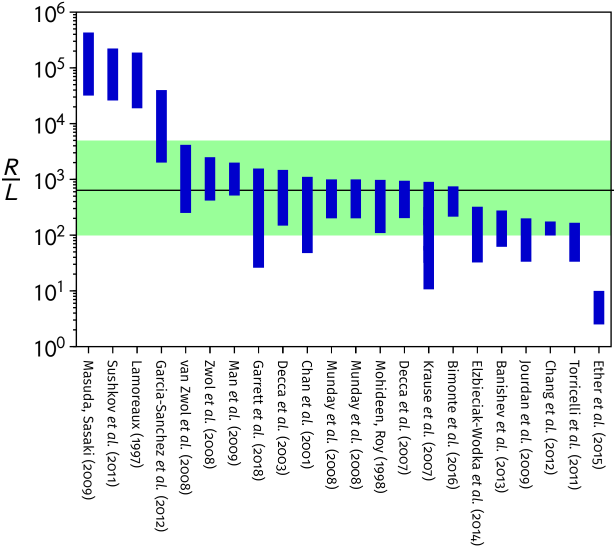
UFRJ group (2015)



Munday group (2018)

sphere/sphere

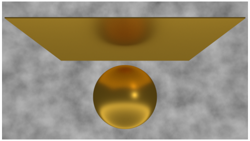
Experimental aspect ratios



aspect ratio

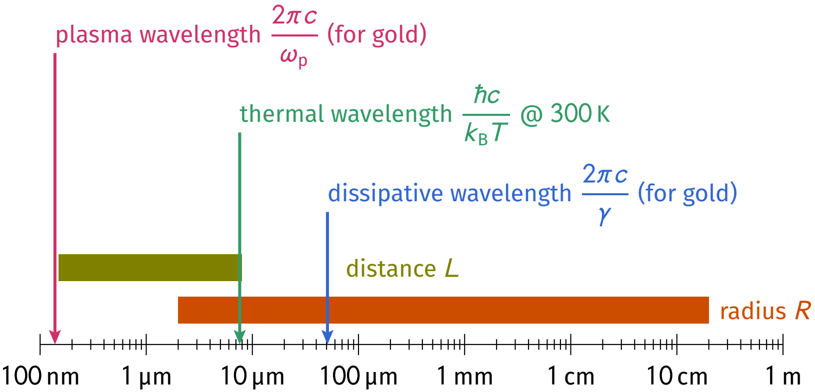
R ⟵ sphere radius

L ⟵ distance plane-sphere



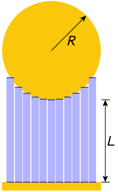
M. Hartmann, GLI, P. A. Maia Neto, Phys. Scr. **93**, 114003 (2018)

Length scales



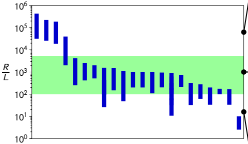
- even room temperature can be a very low temperature
- the sphere radius is often the (by far) largest length scale

Numerics for the sphere-plane geometry



proximity force approximation (PFA)

- Casimir force is non-additive
- PFA from semiclassics in k space and its leading correction



intermediate aspect ratios

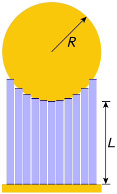
- accurate description of experiments
- theoretical understanding of corrections to PFA

M. Hartmann, GLI, P. A. Maia Neto, Phys. Rev. Lett. **119**, 043901 (2017)
Phys. Scr. **93**, 114003 (2018)

multipole expansion

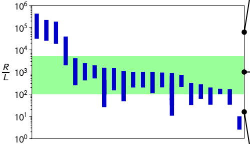
- ℓ_{max} increases linearly with aspect ratio
- numerics for larger aspect ratios becomes demanding

Numerics for the sphere-plane geometry



proximity force approximation (PFA)

- Casimir force is non-additive
- PFA from semiclassics in k space and its leading correction



intermediate aspect ratios

- accurate description of experiments
- theoretical understanding of corrections to PFA

M. Hartmann, GLI, P. A. Maia Neto, Phys. Rev. Lett. **119**, 043901 (2017)
Phys. Scr. **93**, 114003 (2018)

multipole expansion

- $\ell_{\max}$ increases linearly with aspect ratio
- numerics for larger aspect ratios becomes demanding

Proximity force approximation

free energy

$$\mathcal{F}_{\mathrm{PFA}} = -\frac{k_B T R}{4} \sum_{n=-\infty}^{+\infty} \sum_{p\in\{\mathrm{TE},\mathrm{TM}\}} \int_{|\xi_n|/c}^{\infty} \mathrm{d}\kappa\, \mathrm{Li}_2\left(r_p^{(1)} r_p^{(2)} e^{-2\kappa L}\right), \quad \xi_n = \frac{2\pi n k_B T}{\hbar}$$

force

$$F_{\mathrm{PFA}} = 2\pi R\, \mathcal{F}_{\mathrm{PP}}(L,T)$$

Lifshitz formula

with free energy in plane-plane geometry

$$\mathcal{F}_{\mathrm{PP}} = \frac{k_B T}{2} \sum_{n=-\infty}^{+\infty} \sum_{p\in\{\mathrm{TE},\mathrm{TM}\}} \int_{|\xi_n|/c}^{\infty} \frac{\mathrm{d}\kappa}{2\pi} \kappa \log\left(1 - r_p^{(1)} r_p^{(2)} e^{-2\kappa L}\right)$$

- finite temperature
- arbitrary materials through Fresnel coefficients r_p

Zero temperature and perfect reflectors

Universität Augsburg University

Casimir energy at zero temperature for perfect reflectors

$$\mathcal{E} = \mathcal{E}_{\text{PFA}} \left(1 + \beta_1 \frac{L}{R} + o(R^{-1})\right)$$

proximity force approximation

$$\mathcal{E}_{\text{PFA}} = \frac{\hbar c \pi^3 R}{720 L^2}$$

NTLO correction

$$\beta_1 = \frac{1}{3} - \frac{20}{\pi^2} \approx -1.693\ldots$$

- polarization mixing ?
- diffraction effects ?

Previous work

Universität Augsburg University

- semiclassics in real space $\longrightarrow \mathcal{E}_{\mathrm{PFA}}$

  M. Schaden, L. Spruch, Phys. Rev. A **58**, 935 (1998)

  R. L. Jaffe, A. Scardicchio, Phys. Rev. Lett. **92**, 070402 (2004)

  A. Scardicchio, R. L. Jaffe, Nucl. Phys. B **704**, 552 (2005)

  A. Bulgac, P. Magierski, A. Wirzba, Phys. Rev. D **73**, 025007 (2006)

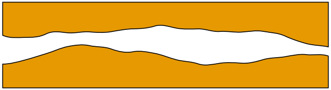
- asymptotic expansion of scattering formula in multipole basis $\longrightarrow \beta_1$

  M. Bordag, V. Nikolaev, J. Phys. A **41**, 164002 (2008)

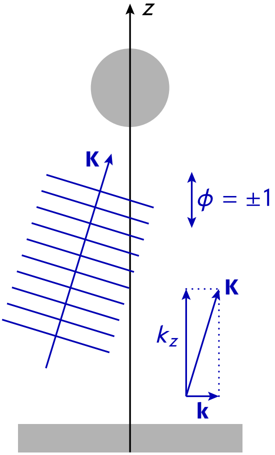
  L. P. Teo, M. Bordag, V. Nikolaev, Phys. Rev. D **84**, 125037 (2011)

- derivative expansion $\longrightarrow \beta_1$

  G. Bimonte, T. Emig, R. L. Jaffe, M. Kardar, EPL **92**, 50001 (2012)



Angular spectral representation



polarizations

$$\hat{\epsilon}_{\mathrm{TE}} = \frac{\hat{\mathbf{z}} \times \hat{\mathbf{K}}}{|\hat{\mathbf{z}} \times \hat{\mathbf{K}}|}\,, \qquad \hat{\epsilon}_{\mathrm{TM}} = \hat{\epsilon}_{\mathrm{TE}} \times \hat{\mathbf{K}}$$

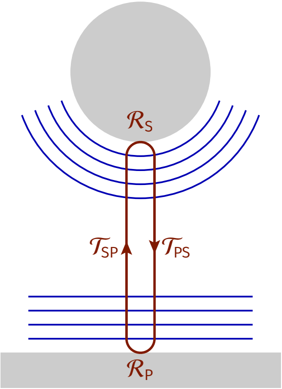
wave vector

$$K_z = \phi k_z\,, \qquad k_z = \sqrt{\frac{\omega^2}{c^2} - \mathbf{k}^2}$$

basis function

$$\langle x, y, z | \omega, \mathbf{k}, p, \phi \rangle = \hat{\epsilon}_p \sqrt{\frac{1}{2\pi} \left| \frac{\omega}{c k_z} \right|}\, e^{i(\mathbf{k}\cdot\mathbf{r} + \phi k_z z)}$$

Round-trip expansion



Wick rotation

$$\xi = i\omega\,, \qquad \kappa = \sqrt{\frac{\xi^2}{c^2} + \mathbf{k}^2}$$

free energy

$$\mathcal{F} = \frac{k_B T}{2} \sum_{n=-\infty}^{+\infty} \operatorname{tr} \log\left[1 - \mathcal{M}(|\xi_n|)\right]$$

round-trip operator

$$\mathcal{M} = \mathcal{T}_{SP}\mathcal{R}_P\mathcal{T}_{PS}\mathcal{R}_S$$

round-trip expansion

$$\mathcal{F} = -\frac{k_B T}{2} \sum_{n=-\infty}^{+\infty} \sum_{r=1}^{\infty} \frac{1}{r} \operatorname{tr} \mathcal{M}^r(|\xi_n|)$$

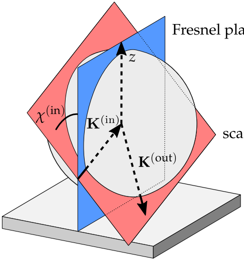
r round trips

$$\mathrm{tr}\mathcal{M}^r = \sum_{p_1,\dots,p_{2r}} \int \frac{\mathrm{d}\mathbf{k}_1 \dots \mathrm{d}\mathbf{k}_{2r}}{(2\pi)^{4r}} \mathrm{e}^{-(\kappa_1+\dots+\kappa_{2r})(L+R)}$$

$$\times \langle \mathbf{k}_1, p_1, + | \mathcal{R}_\mathrm{P} | \mathbf{k}_{2r}, p_{2r}, - \rangle \dots \langle \mathbf{k}_2, p_2, - | \mathcal{R}_\mathrm{S} | \mathbf{k}_1, p_1, + \rangle$$

- reflection matrix elements
  - plane: Fresnel coefficient, **k** conserved
  - sphere: Debye expansion
- saddle-point evaluation of the momentum space integrals

Scattering plane and Fresnel planes

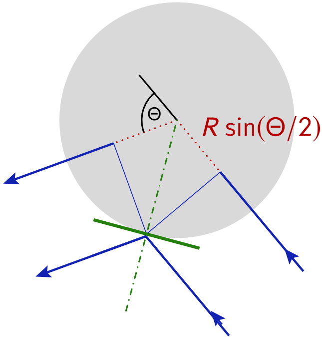


polarization bases

- $\hat{\epsilon}_{\mathrm{TE}} = \dfrac{\hat{\mathbf{z}} \times \hat{\mathbf{K}}}{|\hat{\mathbf{z}} \times \hat{\mathbf{K}}|} \quad \hat{\epsilon}_{\mathrm{TM}} = \hat{\epsilon}_{\mathrm{TE}} \times \hat{\mathbf{K}}$
- $\hat{\epsilon}_{\perp}^{(\mathrm{in})} = \dfrac{\hat{\mathbf{K}}^{(\mathrm{out})} \times \hat{\mathbf{K}}^{(\mathrm{in})}}{|\hat{\mathbf{K}}^{(\mathrm{out})} \times \hat{\mathbf{K}}^{(\mathrm{in})}|} \quad \hat{\epsilon}_{\parallel}^{(\mathrm{in})} = \hat{\epsilon}_{\perp} \times \hat{\mathbf{K}}^{(\mathrm{in})}$
- $\hat{\epsilon}_{\perp}^{(\mathrm{out})} = \dfrac{\hat{\mathbf{K}}^{(\mathrm{out})} \times \hat{\mathbf{K}}^{(\mathrm{in})}}{|\hat{\mathbf{K}}^{(\mathrm{out})} \times \hat{\mathbf{K}}^{(\mathrm{in})}|} \quad \hat{\epsilon}_{\parallel}^{(\mathrm{out})} = \hat{\epsilon}_{\perp} \times \hat{\mathbf{K}}^{(\mathrm{out})}$

$$\cos\left(\chi^{(in)}\right) = \hat{\epsilon}_{\mathrm{TE}}\left(\mathbf{K}^{(\mathrm{in})}\right) \cdot \hat{\epsilon}_{\perp} \qquad \cos\left(\chi^{(out)}\right) = \hat{\epsilon}_{\mathrm{TE}}\left(\mathbf{K}^{(\mathrm{out})}\right) \cdot \hat{\epsilon}_{\perp}$$

at saddle point: Fresnel plane and scattering plane coincide ($\chi^{(\mathrm{in/out})} = 0$)
for NTLO correction: tilt needs to be accounted for ($\chi^{(\mathrm{in/out})} \neq 0$)



scattering amplitude ⟶ geometrical optics

$$S_p^{\mathrm{WKB}} = (-1)^p \frac{\xi R}{2c} \exp\left[\frac{2\xi R}{c} \sin\left(\frac{\Theta}{2}\right)\right]$$

with leading corrections

$$S_p = S_p^{\mathrm{WKB}} \left(1 + \frac{1}{R} s_p + \mathcal{O}\left(R^{-2}\right)\right)$$

$$s_\perp = \frac{c}{2\xi} \frac{\cos(\Theta)}{\sin^3(\Theta/2)}$$

$$s_\parallel = -\frac{c}{2\xi} \frac{1}{\sin^3(\Theta/2)}$$

Matrix elements for reflection at large spheres

$$\langle \mathbf{k}^{(\text{out})}, p^{(\text{out})}, -|\mathcal{R}_S|\mathbf{k}^{(\text{in})}, p^{(\text{in})}, +\rangle \simeq \frac{\pi R}{\kappa^{(\text{out})}} \exp\left[\frac{2\xi R}{c}\sin\left(\frac{\Theta}{2}\right)\right]\rho_{p^{(\text{out})},p^{(\text{in})}}$$

perfect reflectors ($r_{\text{TM}} = 1, r_{\text{TE}} = -1$)

$$\rho_{\text{TM,TM}} = (A - B) + \frac{1}{R}\left(As_{\parallel} - Bs_{\perp}\right) \qquad \rho_{\text{TE,TE}} = -(A - B) - \frac{1}{R}\left(As_{\perp} - Bs_{\parallel}\right)$$

$$\rho_{\text{TE,TM}} = (C - D) + \frac{1}{R}\left(Cs_{\perp} - Ds_{\parallel}\right) \qquad \rho_{\text{TM,TE}} = (C - D) + \frac{1}{R}\left(Cs_{\parallel} - Ds_{\perp}\right)$$

$$A = \cos(\chi^{(\text{out})})\cos(\chi^{(\text{in})}) \qquad B = \sin(\chi^{(\text{out})})\sin(\chi^{(\text{in})})$$

$$C = \sin(\chi^{(\text{out})})\cos(\chi^{(\text{in})}) \qquad D = -\cos(\chi^{(\text{out})})\sin(\chi^{(\text{in})})$$

Saddle-point manifold

$$\mathrm{tr}\mathcal{M}^r = \left(\frac{R}{4\pi}\right)^r \int \mathrm{d}\mathbf{k}_0 \dots \mathrm{d}\mathbf{k}_{r-1}\, g(\mathbf{k}_0, \dots, \mathbf{k}_{r-1}) e^{-Rf(\mathbf{k}_0,\dots,\mathbf{k}_{r-1})}$$

saddle-point manifold

$$\mathbf{k}_0 = \dots = \mathbf{k}_{r-1} \equiv \mathbf{k}_{\mathrm{sp}}$$

consequences for leading order (PFA)

- scattering plane and Fresnel planes coincide ($\chi^{(\mathrm{in})} = \chi^{(\mathrm{out})} = 0$)
- no polarization mixing ($A = 1$, $B = C = D = 0$)

Saddle-point approximation and leading-order correction

Universität Augsburg University

$$\mathrm{tr}\mathcal{M}^r = \left(\frac{R}{4\pi}\right)^r \int \mathrm{d}\mathbf{k}_0 \dots \mathrm{d}\mathbf{k}_{r-1}\, g(\mathbf{k}_0, \dots, \mathbf{k}_{r-1}) e^{-Rf(\mathbf{k}_0,\dots,\mathbf{k}_{r-1})}$$
$$= \frac{R}{2r} \int_{\xi/c}^{\infty} \mathrm{d}\kappa_{\mathrm{sp}}\, \kappa_{\mathrm{sp}}^r \left[F_0 + \frac{1}{R} F_1 + o\left(R^{-1}\right)\right]$$

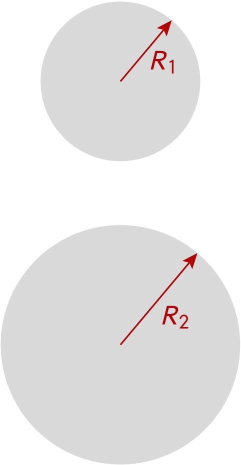
with

$$F_0 = g|_{\mathrm{sp}}$$

$$F_1 = g|_{\mathrm{sp}} \left(\sum_{ijk} \frac{2 f_{ijk} f_{\bar{i}\bar{j}\bar{k}} + 3 f_{ij\bar{j}} f_{\bar{i}k\bar{k}}}{24 \lambda_i \lambda_j \lambda_k} - \sum_{ij} \frac{f_{i\bar{i}j\bar{j}}}{8 \lambda_i \lambda_j} \right) + \sum_{ij} \frac{g_i f_{\bar{i}j\bar{j}}}{2 \lambda_i \lambda_j} + \sum_i \frac{g_{i\bar{i}}}{2 \lambda_i}$$

λ_i: eigenvalues of Hessian $\quad \bar{i} = r - i$

Saddle-point approximation



saddle-point approximation (F_0)
⟶ Lifshitz formula for the Casimir force

$$F \simeq 2\pi R_{\text{eff}} \mathcal{F}_{\text{PP}}(L, T)$$

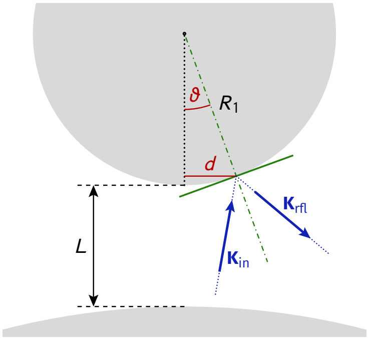
free energy per area in plane-plane geometry

$$\mathcal{F}_{\text{PP}}(L, T) = \frac{k_B T}{2} \sum_{n=-\infty}^{+\infty} \sum_{p \in \{\text{TE},\text{TM}\}} \int_{|\xi_n|/c}^{\infty} \frac{d\kappa}{2\pi} \kappa \times \log\left(1 - r_p^{(1)} r_p^{(2)} e^{-2\kappa L}\right)$$

effective radius

$$R_{\text{eff}} = \frac{R_1 R_2}{R_1 + R_2}$$

Effective area



- change of $k = |\mathbf{k}|$ allowed by Gaussian width

$$\delta k \sim \frac{1}{\sqrt{LR_1}}$$

- relevant angles for $\vartheta \ll 1$

$$\vartheta \lesssim \frac{\delta k}{2k_z}$$

- cutoff on k_z

$$k_z \sim 1/L$$

effective area $A \sim R_1 L \ll R_1 < R_2$

contribution from thermal fluctuations $A^{(T)} \sim \frac{\hbar c}{k_B T} R_1$

Leading-order correction

$$\mathrm{tr}\mathcal{M}^r = \left(\frac{R}{4\pi}\right)^r \int \mathrm{d}\mathbf{k}_0 \ldots \mathrm{d}\mathbf{k}_{r-1}\, g(\mathbf{k}_0, \ldots, \mathbf{k}_{r-1}) e^{-Rf(\mathbf{k}_0,\ldots,\mathbf{k}_{r-1})}$$
$$= \frac{R}{2r} \int_{\xi/c}^{\infty} \mathrm{d}\kappa_{\mathrm{sp}}\, \kappa_{\mathrm{sp}}^r \left[F_0 + \frac{1}{R}F_1 + o\left(R^{-1}\right)\right]$$

- diffractive corrections in $F_0 = g|_{\mathrm{sp}}$
- higher-order saddle-point approximation exploiting symmetry of functions f and g $\longrightarrow$

$$F_1 = g|_{\mathrm{sp}} \left(\sum_{ijk} \frac{f_{ijk} f_{\bar{i}\bar{j}\bar{k}}}{12\lambda_i\lambda_j\lambda_k} - \sum_{ij} \frac{f_{i\bar{i}j\bar{j}}}{8\lambda_i\lambda_j} \right) + \sum_i \frac{g_{i\bar{i}}}{2\lambda_i}$$

Diffractive correction

contribution arising from $F_0 = g|_{\mathrm{sp}}$

- leading term: proximity force approximation
- leading correction: diffraction

perfect reflectors, zero temperature

$$\mathcal{E}_p = \mathcal{E}_{\mathrm{PFA}} \left(\frac{1}{2} + \beta_{\mathrm{d},p} \frac{L}{R} \right)$$

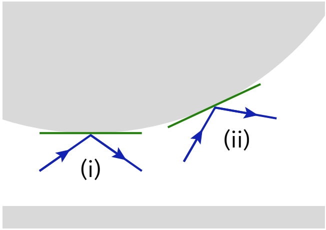
diffractive contribution from the two polarizations

$$\beta_{\mathrm{d,\,TE}} = -\frac{25}{2\pi^2} \qquad \beta_{\mathrm{d,\,TM}} = -\frac{5}{2\pi^2} = \frac{1}{5}\beta_{\mathrm{d,\,TE}}$$

total diffractive correction

$$\beta_{\mathrm{d}} = -\frac{15}{\pi^2}$$

Geometrical optics correction



$$F_1 = g\big|_{\rm sp}\left(\sum_{ijk}\frac{f_{ijk}f_{\bar{i}\bar{j}\bar{k}}}{12\lambda_i\lambda_j\lambda_k} - \sum_{ij}\frac{f_{i\bar{i}j\bar{j}}}{8\lambda_i\lambda_j}\right) + \sum_i \frac{g_{i\bar{i}}}{2\lambda_i}$$

- only the last term can account for polarization mixing
- one finds that the polarization mixing terms cancel each other
- F_1 can be interpreted as arising from specular reflection, but at a tilted tangent plane [process (ii)]

both polarizations contribute equally with a sum of

$$\beta_{\rm go} = \frac{1}{3} - \frac{5}{\pi^2}$$

Leading correction

diffraction, TE	$-\frac{25}{2\pi^2}$
diffraction, TM	$-\frac{5}{2\pi^2}$
geometrical optics, TE	$\frac{1}{6}-\frac{5}{2\pi^2}$
geometrical optics, TM	$\frac{1}{6}-\frac{5}{2\pi^2}$
total correction	$\frac{1}{3}-\frac{20}{\pi^2}$

Relevance of diffraction

Casimir energy at zero temperature for perfect reflectors

$$\mathcal{E} = \frac{\hbar c \pi^3 R}{720 L^2}\left[1 + \left(\frac{1}{3} - \frac{20}{\pi^2}\right)\frac{L}{R} + o(R^{-1})\right]$$

leading correction

	TE	TM
diffraction	74.8%	15.0%
geometrical optics	5.1%	5.1%

- dominant contribution due to diffraction in the TE mode
- diffraction contribution depends on polarization
- geometrical optics contribution independent of polarization

Electromagnetic vs. scalar field

Universität Augsburg University

leading correction to PFA based on electromagnetic field

$$\beta_{\mathrm{TE}} = \frac{1}{6} - \frac{15}{\pi^2} \qquad \beta_{\mathrm{TM}} = \frac{1}{6} - \frac{5}{\pi^2}$$

leading correction to PFA based on scalar fields

G. Bimonte, T. Emig, R. L. Jaffe, M. Kardar, EPL **97**, 50001 (2012)

L. P. Teo, M. Bordag, V. Nikolaev, Phys. Rev. D **84**, 125037 (2011)

Dirichlet boundary conditions $\quad \beta_{\mathrm{DD}} = \frac{1}{6}$

Neumann boundary conditions $\quad \beta_{\mathrm{NN}} = \frac{1}{6} - \frac{20}{\pi^2}$

total leading correction to PFA

$$\beta_1 = \beta_{\mathrm{TE}} + \beta_{\mathrm{TM}} = \beta_{\mathrm{DD}} + \beta_{\mathrm{NN}} = \frac{1}{3} - \frac{20}{\pi^2}$$

Conclusions

Universität Augsburg University

- The Casimir free energy in the sphere-plane geometry has been calculated semiclassically in momentum space for large spheres accounting fully for the electromagnetic field.
- The results allow for an interpretation in terms of geometrical optics and diffraction.
- The proximity force approximation has been derived for arbitrary temperatures and materials.
- The leading correction has been derived for perfect reflectors at zero temperature highlighting the role of diffraction.

references:

- B. Spreng, M. Hartmann, V. Henning, P. A. Maia Neto, GLI
  Phys. Rev. A **94**, 062504 (2018)
- V. Henning, B. Spreng, M. Hartmann, GLI, P. A. Maia Neto
  to appear in J. Opt. Soc. Am. B (2019); arXiv:1811.12856