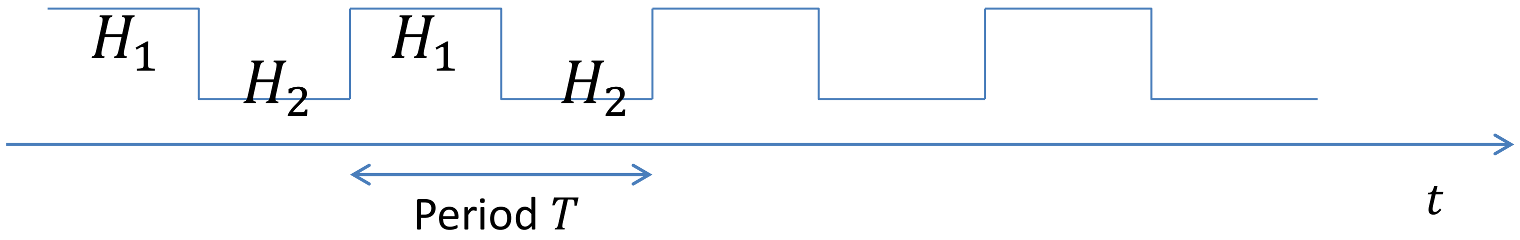
Periodic driving of many-body states—RMT theory for MBL

Wojciech De Roeck (K.U. Leuven-Belgium)

Based on collaboration with

- D. Abanin, F. Huveneers, W.W. Ho (periodic driving)
- F. Huveneers (RMT theory for MBL)

Periodic Driving of Many-Body Systems



Protocol: Switching between local many-body Hamiltonians H_1 and H_2 with period T.

Basic object: $U_T = e^{-\frac{i}{2}TH_2} e^{-\frac{i}{2}TH_1}$ one-cycle propagator (aka. Floquet operator)

*- Hypothesis: $U_T = e^{-iTH_*}$ for some (quasi)-local effective H_*

(quasi)-locality is key here: If not, always possible to take log of U_T!

If *- Hypothesis holds: 'Localization in Energy space': System does not heat up to featureless $T = \infty$ state (because H_* is a local conserved quantity)

But why believe *- Hypothesis: $U_T = e^{-iTH_*}$?

Because of famous BCH formula:

$$e^{-\frac{i}{2}TH_2} e^{-\frac{i}{2}TH_1} = e^{-\frac{i}{2}T(H_2+H_1)+\frac{1}{4}T^2[H_2,H_1]+ T^3\cdots + T^4\cdots}$$

seems to give H as a series in T with local terms (commutators)
general periodic pulse: similar Magnus-expansion for U_T
(I have this idea from D'Alesio-Polkovnikov)

However, series is known to converge if $T\|H_1\|, T\|H_2\| < 1$
hence for Hamiltoninans uniformly bounded in volume! This is of course not applicable to many-body context.

So, is *- Hypothesis true, at least for small T?

- I don't see a good reason to believe it, except for strongly disordered systems and of course for non-interacting systems
- However, we prove it 'asymptotically' as $T \to 0$, uniformly in volume

Setup:

- Periodicity: $H(t) = H(t+T)$
- $H(t) = \sum_{i=1}^{N} h_i(t)$, each $h_i(t)$ a finite range R around site i
- High frequency: $\frac{1}{T} \gg R^d \, max_{i,t} \; \|h_i(t)\|$
- N is nb. of sites and n is nb. of cycles (time)

Then there is $H_* = H_*(T)$ such that

A) Slow heating:

$$\|{U_T}^n H_* \, {U_T}^{-n} - H_*\| \leq \delta(T) \, N \, n$$

$\delta(T) = \; e^{-\frac{c}{T}}$ in $d = 1$ and $\delta(T) = \; e^{-\frac{c}{T \; \log(1/T)^3}}$ for $d > 1$

B) H_* governs evolution of local observables O:

$$\left\| {U_T}^n O {U_T}^{-n} \; - e^{inTH_*} O \; e^{-inTH_*} \right\| \leq \delta(T) \; c(O) n^{1+d}$$

Hence approx. valid up to time n≈ $\delta(T)^{1/(1+d)}$

Proof: KAM techniques + Cluster expansions + Lieb-Robinson bound

Idea: Do t-dependent transformation $e^{A(t)}$

$$e^{-A(t)}(i\partial_t - H(t))\, e^{A(t)} = \; i\partial_t - \widehat{H}(t)$$

such that the t-dependent part in $\widehat{H}$ is smaller than in H.

How?: Write $H(t) = D + V(t)$ and expand in $A(t)$:

$$e^{-A(t)}(i\partial_t - D - V(t))\, e^{A(t)} = i\partial_t - D - V(t) - i\partial_t \mathrm{A(t)} + \cdots$$

Set now $V(t) - i\partial_t \mathrm{A(t)} = 0$, then leading time-dependence is eliminated, $A(t) = i\int_0^T V \;\approx T$ and all omitted terms ($\cdots$) are higher order in T.

Further Idea: Keep iterating this until combinatorics blows up

Conclusion for periodic driving

- We prove rigorously that there is an effective Hamiltonian that governs the system for (quasi)-exponentially long times in $\frac{1}{T}$.
- This underpins theoretical work about Floquet engineering of states, often based on the case without interactions.
- Our work effectively provides error bounds for the truncated Magnus expansion.
- At the same time, very related work (also rigorous) appeared by Mori, Kuwahara, Saito.

A random matrix theory (RMT) for many-body localization (MBL)

Main question: How can we believe seriously in generic RMT and still end up with MBL, ie. manifest ergodicity breaking?

Answer: MBL is an instability of RMT in d=1

‘Ergodic’ many-body systems

Basic idea: Ergodic Hamiltonian ≈ GOE matrix

More precise: ETH for local operators V, e.g. $V = {\sigma^z}_i$ (site i)

$$\langle\psi|V|\psi'\rangle = \sqrt{\frac{v(\omega)}{\rho}}\,\eta_{\psi,\psi'}$$

Eigenstates $\psi \neq \psi'$
i.i.d. random numbers $\langle\eta\rangle = 0, \langle\eta\bar{\eta}\rangle = 1$
ρ: Many-body density of states
$\omega = E(\psi) - E(\psi')$

dynamic structure factor: $v(\omega) = \int dt\, \langle V(t)V(0)\rangle\, e^{-i\omega t}$

- The DOS ρ is e^{sN} with N nb of particles and s entropy density
- $v(\omega)$ is exp decaying at scale ‘energy per site’ (for single-site V). This is the only crucial difference with a GOE matrix.
- A crucial assumption: $v(\omega)$ smooth on scale of level spacing

MBL systems (NON-ergodic)

Basic idea: MBL Hamiltonian ≈ independent spins

More precise: l-spins ${\tau^z}_i$ as perturbations of physical spins ${\sigma^z}_i$

$$H = \sum_{i=1}^{N} h_i\, {\tau^z}_i + J_{i,j}\, {\tau^z}_i\, {\tau^z}_j + \cdots$$

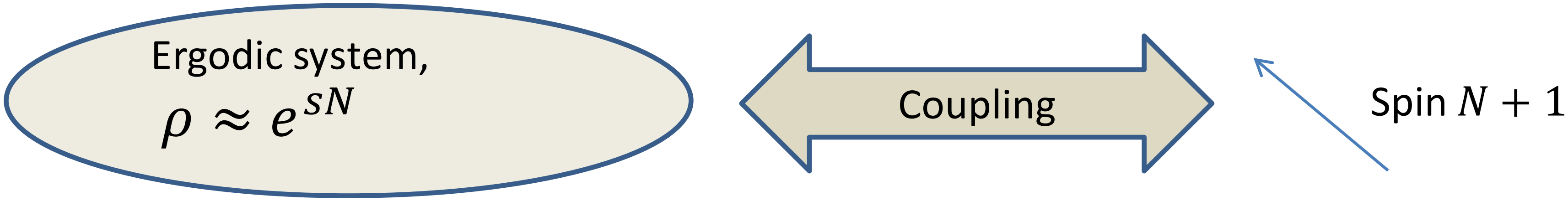
$h_i,\ J_{i,j},\ J_{i,j,k}$ parameters

Couplings $J_{i,j}$ decay in $|i-j|$

The ${\tau^z}_i$ are LIOMs (Local Integrals of motion): $[H,\ {\tau^z}_i] = 0.$

- Eigenstates labelled by ${\tau^z}_i = \pm 1$
- $\langle\psi|\ {\tau^z}_i|\psi'\rangle$ is hence 0 or 1: Manifest breaking of ETH
- MBL proven at strong disorder in d=1 (Imbrie) under modest spectral assumption.
- Many results suggesting it in much greater generality (Basko, Aleiner, Altschuler,…) but effect of rare regions?

Ergodic system + spin and the Ice-IX fallacy



Hamiltonian with ETH for V $\quad$ $gV\,\sigma^x{}_{N+1}$ $\quad$ $\mathrm{h}\,\sigma^z{}_{N+1}$

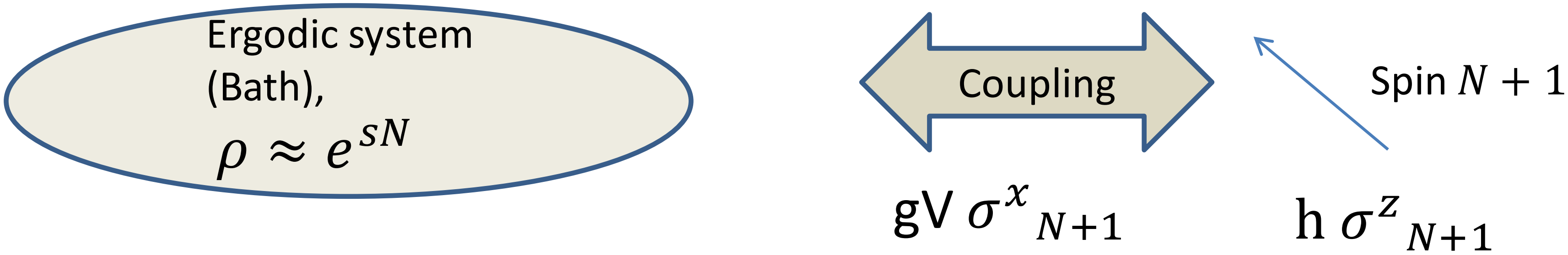
Spin states hybridize if $\dfrac{matrix\ element}{level\ spacing} \approx \dfrac{g\sqrt{\frac{v(\omega)}{\rho}}\,\eta}{\frac{1}{\rho}} \approx g\sqrt{\rho v(\pm 2h)} \gg 1$

Since $\rho \approx e^{sN}$, spin will thermalize for any reasonable g.

Likely the combined system will again be ergodic (because what else?)

Ice IX-fallacy (Kurt Vonnegut, via E. Altman): The extra spin increases ρ and one iterates the argument. Conclusion is that any number $N' \gg N$ of weakly coupled spins can be thermalized by the bath of length N. This contradicts the existence of even 1d MBL since there are always ergodic spots in the chain!!

A refined RMT theory for adding spins to an ergodic system



Step 1: Check $\frac{matrix\ element}{level\ spacing} \gg 1$

Step 2: If **Step 1** ok, then explicit form of new eigenstates ϕ (bath+spin)

$$\phi = \sum_{\psi,s} \frac{\sqrt{k(\omega)}}{\sqrt{\rho}} \eta_{\phi,\psi,s} \ (\psi \ \otimes s)$$

$(\psi \ \otimes s)$ basis of products of eigenstates ψ, s of bath, spin.
i.i.d. random numbers $\langle \eta \rangle = 0, \langle \eta \bar{\eta} \rangle = 1$
$\omega = E(\phi) - E(\psi) - E(s)$

Hybridization function $k(\omega) = \left(\frac{1}{f}\right)\left(1 + \left(\frac{\omega}{f}\right)^2\right)^{-1}$ with width f determined by $v(\omega)$ and h. In general, $f \searrow 0$ when $v(\pm 2h) \searrow 0$ (off-resonant coupling)

A refined RMT: continuation

Hence: form of new eigenfn: $\phi = \sum_{\psi,s} \frac{\sqrt{k(\omega)}}{\sqrt{\rho}} \eta_{\phi,\psi,s} \ (\psi \ \otimes s)$

All can be calculated from this form (assuming that all appearing η are independent). Hence we can iterate, assuming independence all the time. This procedure can only stop if

- 1) the hybridization condition (Step 1) fails
- 2) the structure function $v(\omega)$ is as narrow as the level spacing $1/\rho$: RMT inconsistent

The properties of the hybridization function $k(\omega)$ can be argued for convincingly (even some theorems).

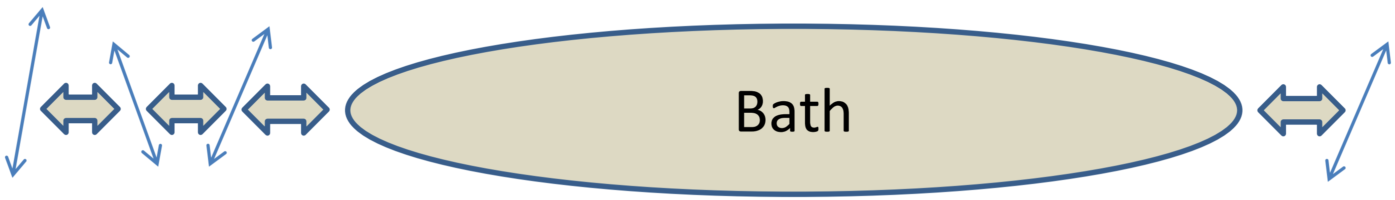
The real uncontrolled assumption is the independence of random η at each step.

Predictions from refined RMT approach

General picture: adding weakly coupled spins to bath: DOS ρ $\nearrow$ but width of $v(\omega)$ $\searrow$: Competition!

- In d=1, bath length N + L strongly disordered spins: for $L \sim N$, the system localizes (structure factors hit level spacing)
- This means that d=1 MBL in strongly disordered chain is stable against ergodic spots (of course we knew that already)
- MBL in d=1 is now understood as an *instability of baths*
- Explicit description of transition region, in agreement with rigorous results by Imbrie.
- For subexponentially decaying interactions, the system does not localize when coupling spins to a finite bath.
- d>1 systems do not localize when coupling spins to a finite bath.
- Problems with theory for fully ergodic systems: locality and dimensionality not reproduced very well.

Numerical Tests: Help from friends far away



localized spins (weak coupling) Extremely weak coupling

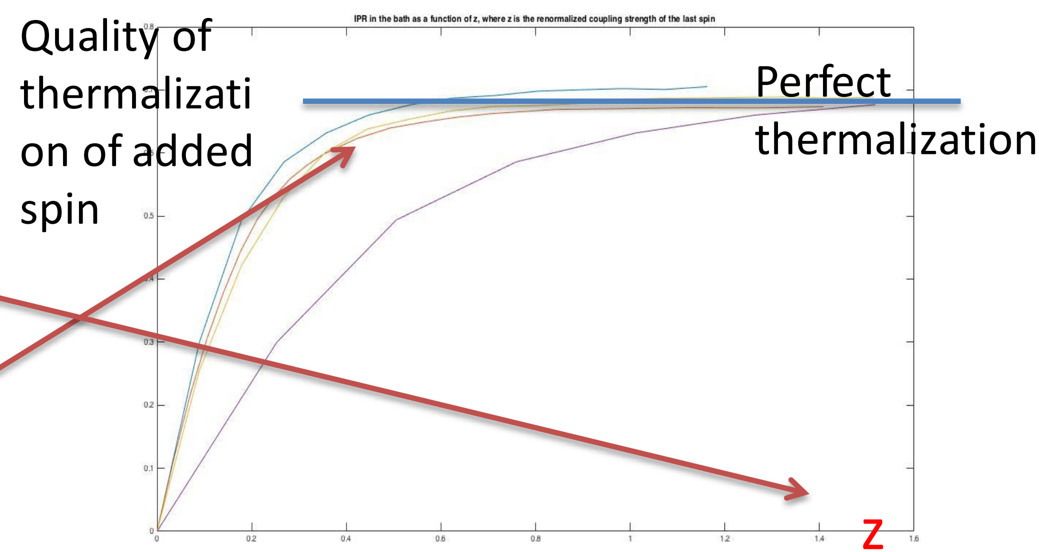
If our theory is correct, then the left spins (localized by themselves) should nevertheless help to thermalize the right spin *as long as they get thermalized themselves*.

Recall hybridization condition:

$$z \equiv g\sqrt{\rho v(\pm 2h)} \gg 1$$

3 different baths:

- Random Matrix
- Chain+ 1 weakly coupled spin
- Chain+ 3 weakly coupled spins



Conclusion on MBL from RMT

- We made a theory based on RMT: 'Assume as much chaos as possible'
- This theory still predicts the onset of MBL!
- Numerical tests are encouraging but need larger systems
- Theory predicts that MBL is rather fragile. It is only stable against zero-dimensional baths in d=1 with exponentially local interactions.
- Interesting to compare our theory with other (less microscopic) scaling theories by Huse et al, Vosk, Altman, Vasseur, Paramasweran, Potter, …

Thanks for your attention!