Spreading of correlations in long-range quantum lattice models

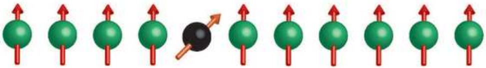
Michael Kastner

NITheP National Institute for Theoretical Physics

Universiteit Stellenbosch University

Quantum Non-Equilibrium Phenomena
Natal, 15 June 2016

Propagation 1



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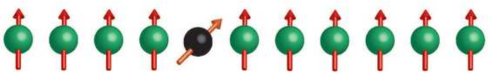

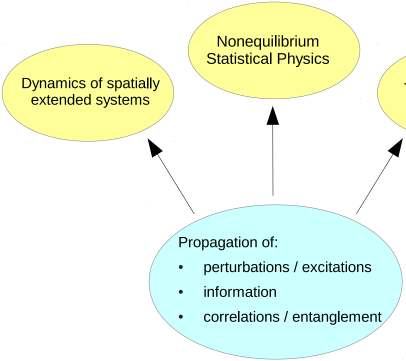

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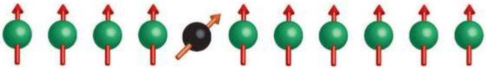

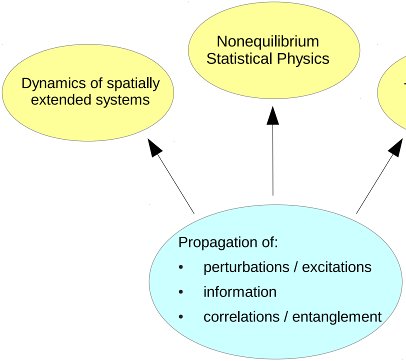

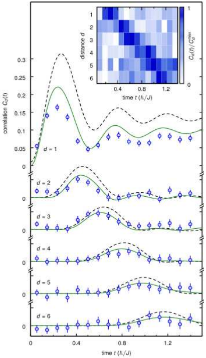

Cheneau *et al.*, Nature 2012

Plan / Approach

- General results based on Lieb-Robinson bounds
- Short- *vs.* long-range interactions
- Role of the initial state

- Surprising?
  (lightcone without relativity)
- Expected?
  (group velocity)

- Shape of horizon
- Decay outside the causal region
- Bound on velocity,…

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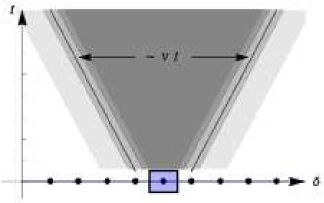

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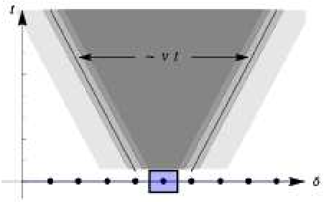

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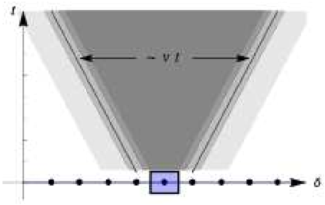

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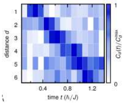

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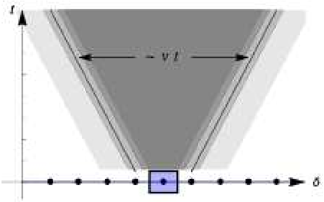

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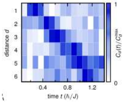

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Setting

- quantum lattice models; arbitrary lattice Λ in dimension D
- $\mathscr{H} = \bigotimes_{i\in\Lambda} \mathscr{H}_i$, $\dim \mathscr{H}_i < \infty$
- on-site + pair interactions, $H = \sum_i H_i + \sum_{i,j} V_{ij}$

short-range: finite range (e.g. nn) or exponentially decreasing ($\sim a\mathrm{e}^{-cr}$ with $c > 0$)

long-range: power law $\sim ar^{-\alpha}$ with $\alpha \geqslant 0$

strongly long-range: power law $\sim ar^{-\alpha}$ with $0 \leqslant \alpha < D$

Long-range interacting many-body systems:

- Astrophysical systems
  nonequivalent ensembles, negative specific heat
- Cavity QED
- Rydberg(-dressed) atoms
- Polar atoms or molecules
- Ion crystals in traps: $1/r^\alpha$

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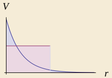

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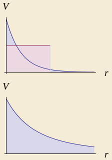

Fundamental forces: strongly long-range (Coulomb, gravity, ...)

Condensed matter physics: almost always short-range (screening!)

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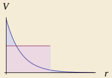

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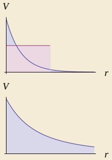

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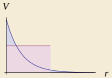

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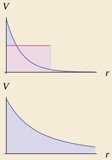

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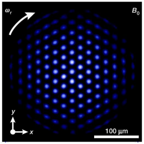

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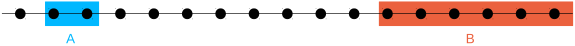
$$H = -J\sum_{i,j}\frac{\sigma_i^z\sigma_j^z}{|i-j|^\alpha} - \boldsymbol{h}\cdot\sum_i \boldsymbol{\sigma}_i$$

“quantum simulator”

Lieb-Robinson bounds



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E. H. Lieb & D. W. Robinson, CMP 1972

$$\|[O_A(t), O_B(0)]\| \leqslant C\,\|O_A\|\,\|O_B\|\,|A||B|\mathrm{e}^{(v|t|-d(A,B))/\xi}$$

(formally in the thermodynamic limit)

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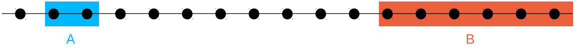

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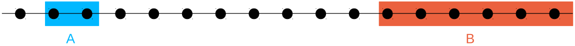

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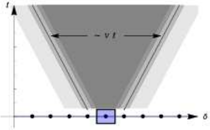

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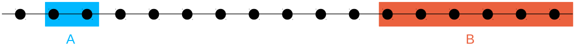

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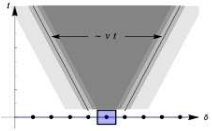

Why the commutator? Useful tool to

- quantify spreading of information, entanglement, **correlations**,...
- prove exponential clustering of ground state correlations
- prove Lieb-Schulz-Mattis in $D \geq 2$
- and more...

Caveat: independent of initial state and details of the Hamiltonian

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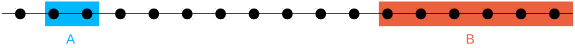

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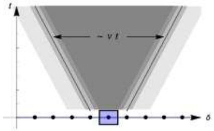

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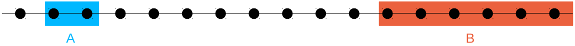

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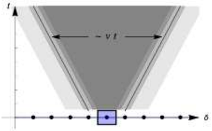

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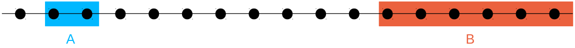

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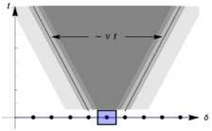

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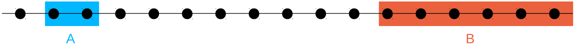

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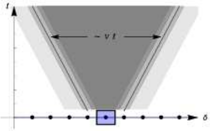

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M. Hastings & T. Koma, CMP 2006

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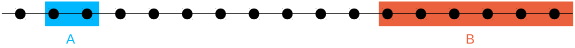

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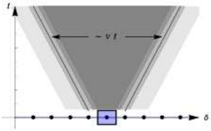

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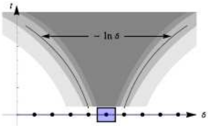

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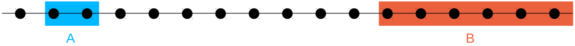

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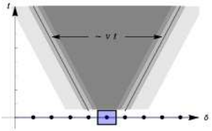

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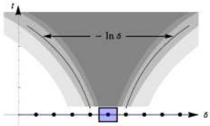

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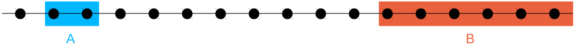

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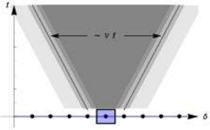

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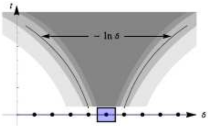

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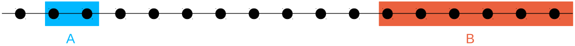

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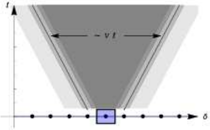

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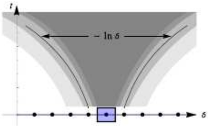

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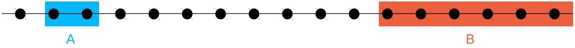

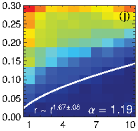

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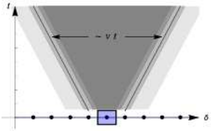

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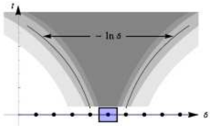

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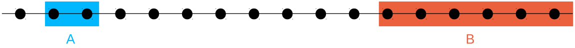

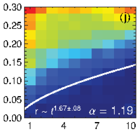

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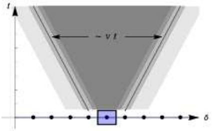

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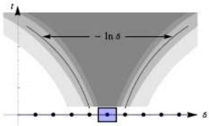
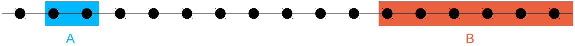


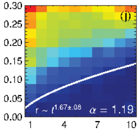
How sharp are the bounds?

Good *qualitative* agreement,

but prefactors and velocities are overestimated.

Lieb-Robinson bounds





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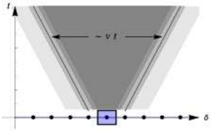

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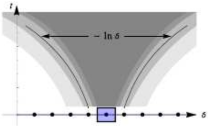

Bounds for $\alpha < D$?

Eisert, vd Worm, Manmana, Kastner, PRL 2013

Strongly long-range interactions: time scales

Example: long-range q-Ising model:

$$H = -J \sum_{i \neq j} \frac{\sigma_i^z \sigma_j^z}{|i-j|^\alpha} - h \sum_i \sigma_i^z$$

on *d*-dim. lattice

Analytic results for arbitrary *n*-point functions

M. K., PRL 2011; M. van den Worm, B. Sawyer, J. J. Bollinger, M. K., NJP 2013

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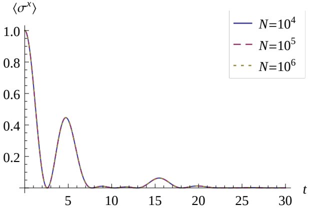

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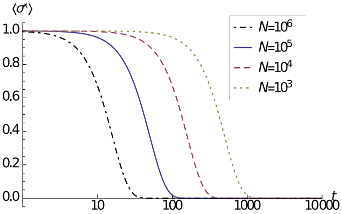

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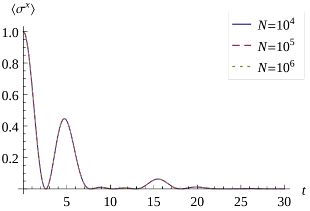

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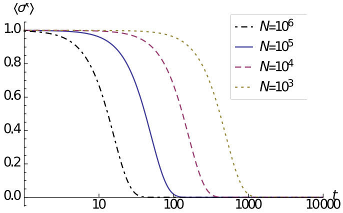

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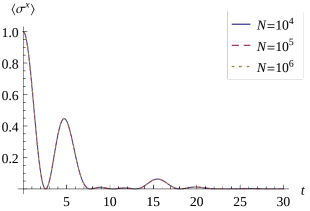

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Lieb-Robinson bound for strong long-range

Idea: bound in rescaled time $\tau = tN^{1-\alpha/D}$

D. Storch, M. van den Worm, M. K. NJP 2015

$$\left|\left|\left[O_A(\tau N^{\alpha/D-1}), O_B(0)\right]\right|\right| \leqslant C\,||O_A||\,||O_B||\,\frac{|A|\,|B|\,(e^{v|\tau|}-1)}{[d(A,B)+1]^{\alpha}}$$

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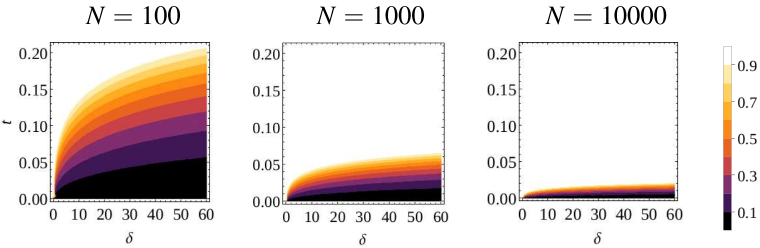

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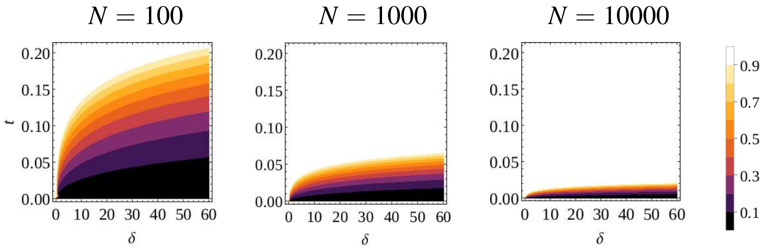
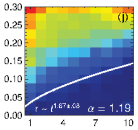


Do “real” systems/models behave like this?
Sometimes yes…



1D fermions with long-range hopping 1

$$H = -\frac{1}{2}\sum_{j=1}^{N}\sum_{l=1}^{N-1}\frac{c_j^\dagger c_{j+l}}{l^\alpha} + \mathrm{h.\,c.},$$

initial state $|10101\ldots\rangle$

Long-range systems can propagate correlations supersonically fast, but typically they don't want to!

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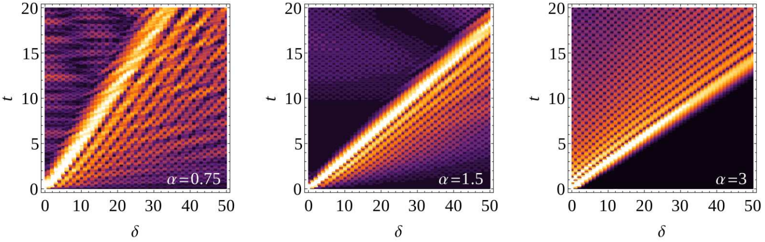

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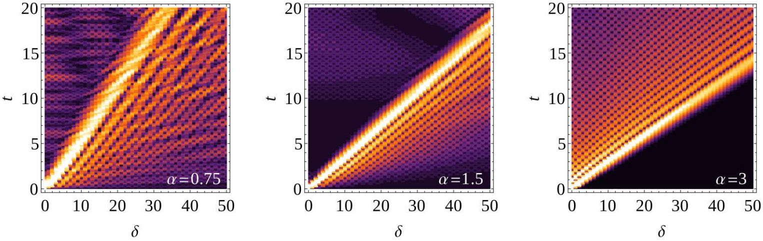

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$1D$ fermions with long-range hopping 2

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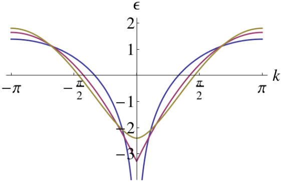
$$\epsilon(k) = -\sum_{l=1}^{N-1} \frac{\cos(kl)}{l^\alpha}$$



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$$\rho(v) = \frac{1}{2\pi} \int_0^{2\pi} \delta\left(v - \frac{\mathrm{d}\epsilon}{\mathrm{d}k}\right) \mathrm{d}k$$

Explains the observed propagation patterns

Related work: L. F. Santos, F. Borgonovi, G. L. Celardo, arXiv:1507.06649
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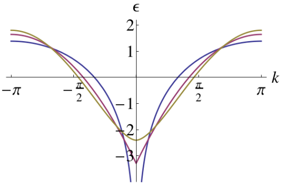

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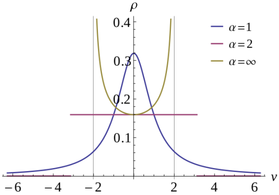

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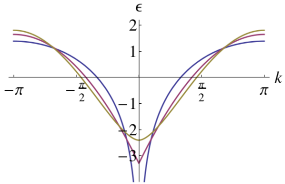

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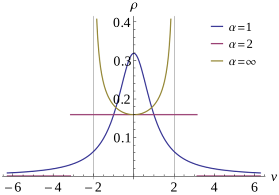

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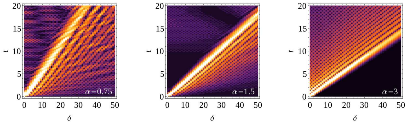

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Sharper bounds: exponentials of the coupling matrix

Improve bounds by considering details of the Hamiltonian:

$$||[O_i(t), O_j(0)]|| \leq 2\,||O_i||\,||O_j|| \left(\exp\left[2\kappa J\,|t|\right]_{i,j} - \delta_{i,j}\right),$$

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$$J_{k,l} = \|h_{kl}\|$$

$$\kappa = \sup_{n\in\Lambda}\sum_k J_{n,k}$$

- Functional form of propagation front not evident
- Analytically tractable for $1d$ translationally invariant systems
- Numerically efficient (polynomial in N) for other cases
- Sharper than other bounds; more detailed propagation fronts

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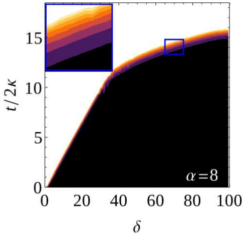

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Correlated initial states

Generalise bounds by considering details of the initial state:

Why the commutator? Useful tool to

- quantify spreading of information, entanglement, **correlations**,…

Tacit assumption: uncorrelated or exponentially clustering initial state

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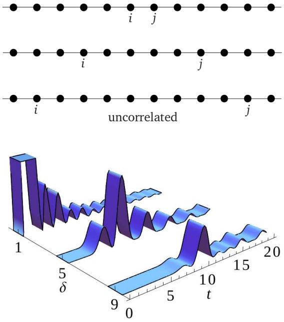

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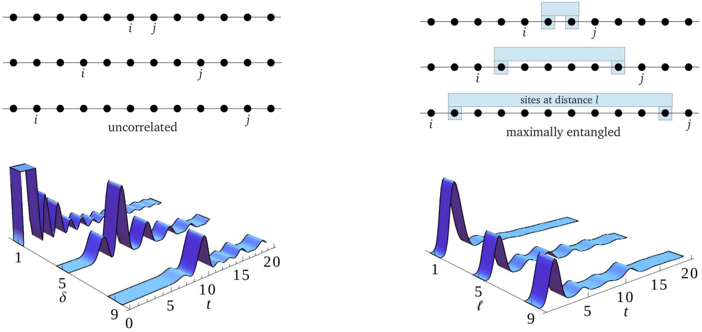

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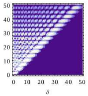
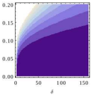
Applications:

- Quenching away from a quantum critical point
- Kondo at $T = 0$
- Quantum transport in dimerized open chains

Take-home message & outlook

short-range 

vs.

long-range 

- Different propagation patterns
- Long-range can be faster, but is often slow
- Lieb-Robinson bounds: estimating the spreading of correlations.
  - Bounds for $\alpha < D$
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Take-home message & outlook

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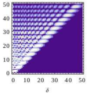

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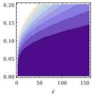

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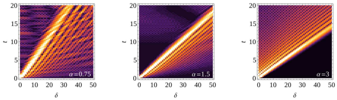

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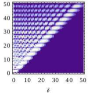
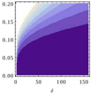
short-range *vs.* long-range





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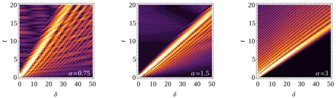

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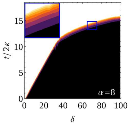

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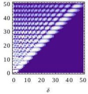
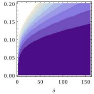
Take-home message & outlook

short-range  *vs.* long-range 

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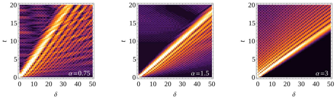

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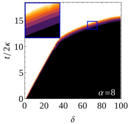

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