Quantum dynamics after connecting two integrable spin chains

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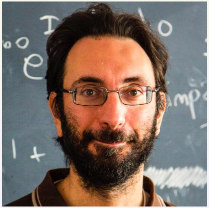
Credits: a BIG thanks to my collaborators





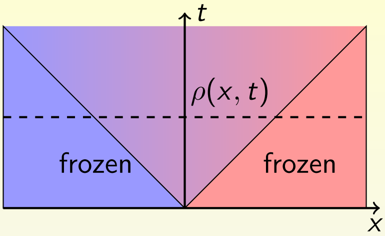






Inhomogeneous quenches

- Initial state is **not** translation invariant
- *Locally* an eigenstate of the Hamiltonian (far from GS!)
- *Examples* : inhomogeneous density $\rho(x,0)$ or energy profile $h(x,0)$
- Observables show a **quantum light-cone**



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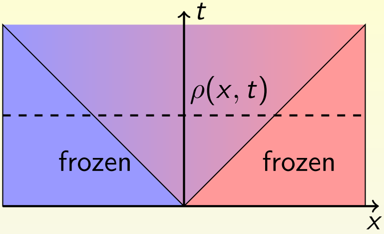
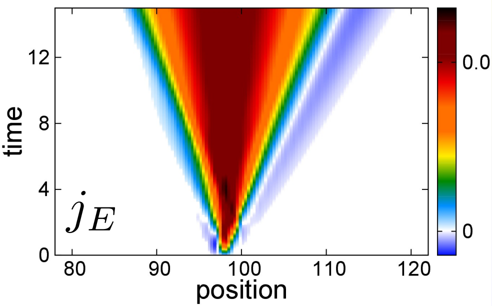
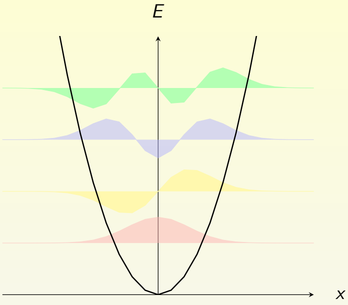


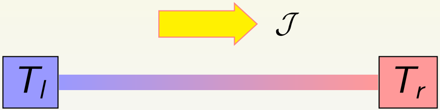


- XXZ with halves at different temperatures
- $\mathcal{J}(x,t)$ **(energy current)** is zero outside the light-cone [Vasseur, Karrash and Moore (2015)]

Motivations I

- Most of the time, real systems are **inhomogeneous**, like fermions in an harmonic external potential





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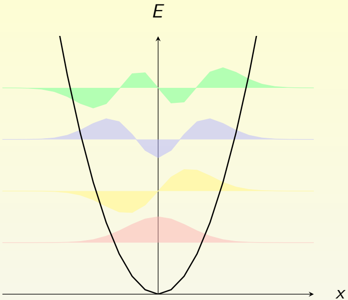
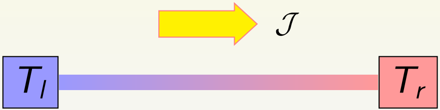


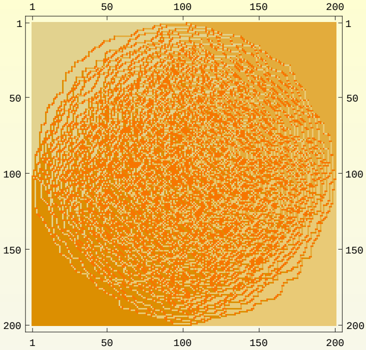


- **Transport** problem in one-dimensional systems (related to cold atom experiments)

- Are there **universal features** in the large time and space dynamics (like a CFT description à la Calabrese and Cardy)

Motivations II

- Density profiles $\rho(x,t)$ are related to limiting shapes of statistical mechanics model (see. Allegra, Dubail, Stephan and V., 2016)
- Quantum quench in imaginary time is related to the phenomena of **arctic curves** (Korepin, Izergin, Colomo, Pronko, Reshetikin)



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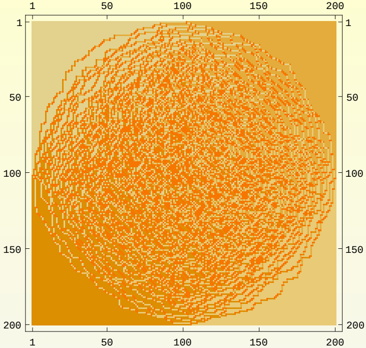

- Emergent hydrodynamical equations for the time evolution of the density profile (Sasha Abanov, see also Victor and Maurizio talk)

$$\partial_t n(x,k;t) - v(k)\partial_x n(x,k;t) = 0$$ Free evolution of the modes

$$\rho(x,t) = \int_{k_-(x/t)}^{k_+(x/t)} dk\ n(x,k,t)$$ Modes occupied: $v(k_\pm) = x/t$

Universality of the energy current at low energy

- Let us consider two **spin chains** with different temperatures β_l and β_r (say XXZ)
- Current profile is a function $\mathcal{J}(x/t)$ (Vasseur & Moore 2015; V., Stephan, Dubail and Haque 2016]

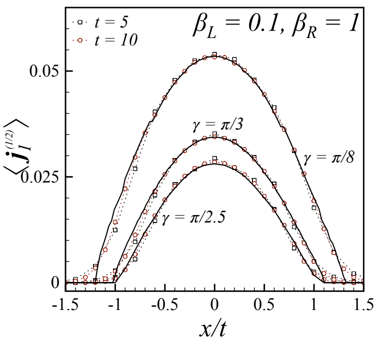

Figure: From Bertini et. al 2016; $\Delta = \cos\gamma$.

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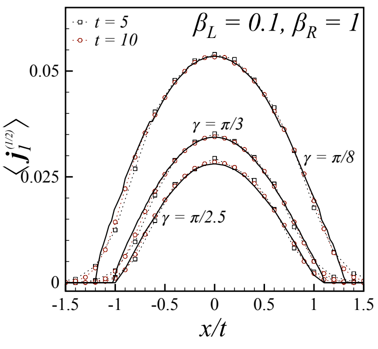

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- Strictly speaking the stationary state is reached only in the limit $x/t \to 0$, when all the particles have reached the point x. For $\beta_{l,r} \to \infty$, the curve should have the same maximum (Bernard, Doyon 2012)

Universality of the current at low energy

- For a **gapless** quantum system at low enough temperature [Bernard, Doyon 2012]

$$\mathcal{J}(0) = f(T_l) - f(T_r) = \frac{c\pi}{12}(T_l^2 - T_r^2)$$

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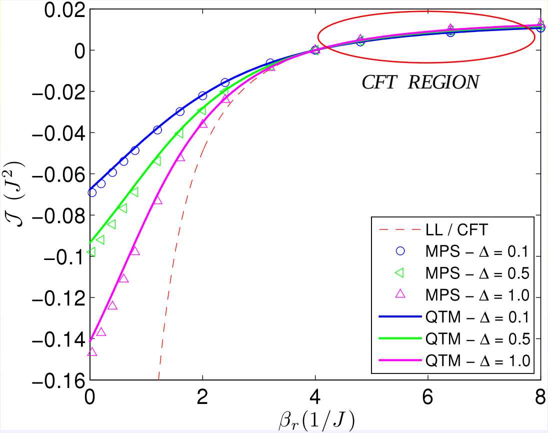

- **Example:** XXZ spin chain. Figure from De Luca, V., Mazza, Rossini (2014). Similar considerations in Karrash, Ilan and Moore (2013).

Remarks on thermalization

- **Remark 1.** A ballistic component of the energy current is persistent.
- **Remark 2.** Energy current might be protected by symmetries (like in XXZ spin chain)

$$\boxed{i[H_{XXZ}, h_i] = j_{i+1} - j_i} \tag{1}$$

$$[H_{XXZ}, j_i] = \kappa_{i+1} - \kappa_i \Rightarrow i[H_{XXZ}, \overbrace{\sum_{i=A}^{B} j_i}^{J_{AB}}] = \kappa_B - \kappa_A \tag{2}$$

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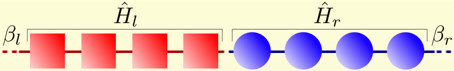
- Effect of integrability on thermal transport have been studied a lot (Mazur, Affleck, Pereira, Prosen, Sirker, Zotos,...)
- Such conservation law forbids thermalization at any β_l and β_r when coupling two XXZ spin chains, actually is implying (Vasseur, Karrash, Moore 2015)

$$\boxed{\langle J_{AB} \rangle = t[\langle \kappa \rangle_{\beta_l} - \langle \kappa \rangle_{\beta_r}]}$$

Energy current breaking locally integrability (with. A. Biella, A. De Luca, L. Mazza, D. Rossini and R. Fazio (PRB 2016))

- A model that breaks integrability locally



- We consider two XXZ spin chains with different anisotropy parameters ($\Delta_{l/r}$) and temperatures ($\beta_{l/r}$)

$$H = \sum_i \overbrace{J(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y - \Delta_i \sigma_i^z \sigma_{i+1}^z)}^{\Delta_i = \theta(-i)\Delta_l + \theta(i+1)\Delta_r} = H_l + H_r + V$$

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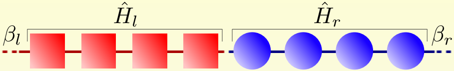

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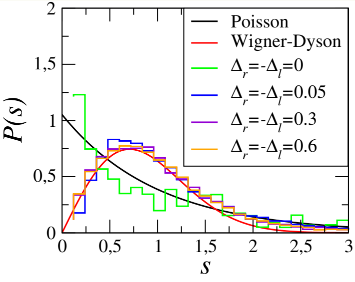
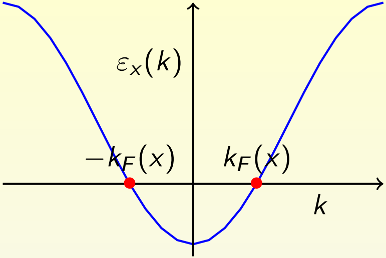


- Level statistics is Wigner Dyson for $\Delta_l \neq \Delta_r$ (model is non-integrable)

Low-energy analysis I

- The ground state of the non-homogeneous chain is at half filling
- We can **bosonize** the chain in a standard way with $k_F(x) = \pi/2$



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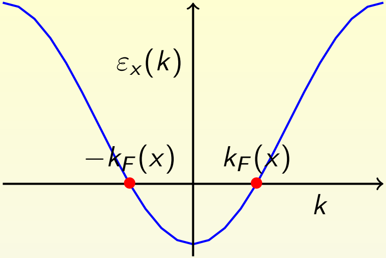

- Left and right moving fermions will have different (renormalized) Fermi velocities $u(x)$ (back-scattering)

$$H_{LE} = \overbrace{\frac{1}{2}\int_{-\infty}^{\infty} dx u(x) \left[\frac{(\partial_x\phi)^2}{K(x)} + K(x)(\partial_x\theta)^2\right]}^{\text{Inhom. Luttinger Liquid}} + \overbrace{\lambda \left(e^{i\sqrt{4\pi}\phi(x=0)} + h.c.\right)}^{\text{localized back scattering}}$$

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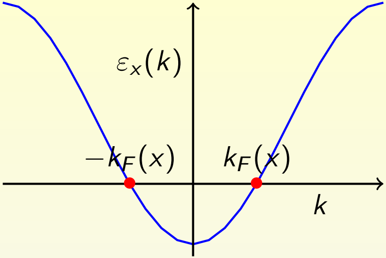

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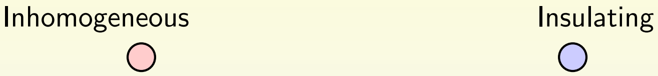
- With $K(x) = \frac{1}{2}\left[1 - \frac{1}{\pi}\arccos\Delta(x)\right]^{-1}$, the Luttinger parameter and $u(x)$ the renormalized Fermi velocity; $\Delta(x) = \theta(-x)\Delta_l + \theta(x)\Delta_r$.

Low energy analysis II: CFT

- The RG flow produced by the back scattering operator is very similar to the **Fisher-Kane** problem (K is the harmonic mean between K_l and K_r)



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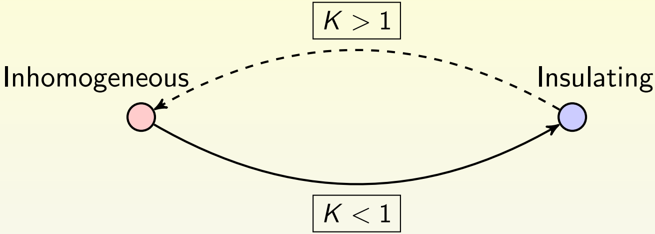

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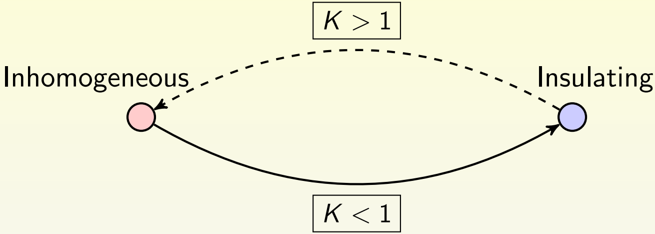

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- **Can be tuned to zero**, sitting at the Inh. fixed point! Energy current at this point can be characterized by CFT

Analytical and numerical results at low-energy

- At the Inh. fixed point, two free bosons with different compactification radius (Bacas et. al; Bernard, Doyon and V. 2014)

$$\boxed{J_E = \frac{\pi \mathcal{T}}{12}(T_l^2 - T_r^2)}\,, \qquad \mathcal{T} = \frac{4K_l K_r}{(K_l + K_r)^2}$$

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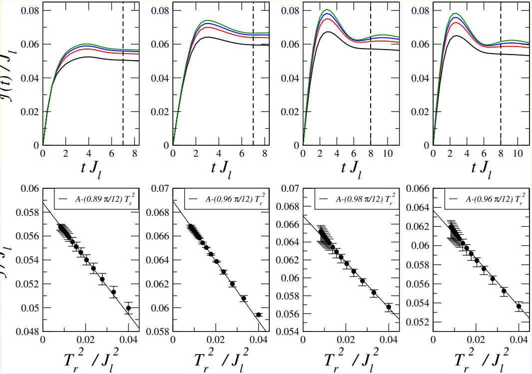


Relaxing the Fermi velocities matching

- Relaxing the Fermi velocity matching condition we can explore (numerically) the full RG flow

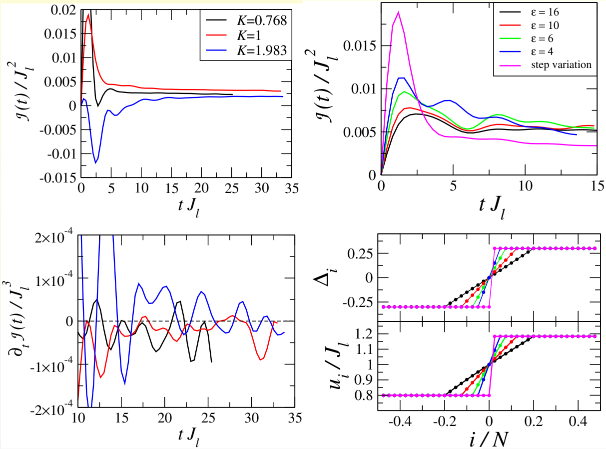


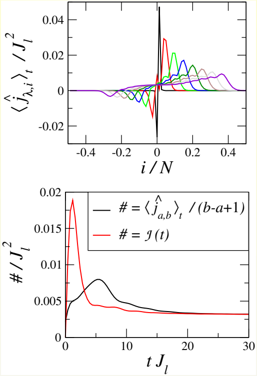
Figure: Current is decreasing very slowly from its CFT value in the times explored by numerics: this seems a prethermalization plateau

Higher energies prethermalization and thermalization: conjecture

- Equation of motion for the integrated current

$$\langle \partial_t J_{AB} \rangle = \langle \kappa \rangle_A - \langle \kappa \rangle_B + \langle \Theta \rangle_t$$

- In the time explored by numerics we observe mild deviations from linear growing of the integrated current **(prethermalization)**



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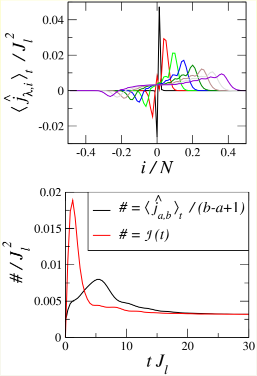

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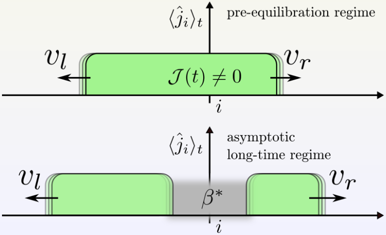


- At larger times, system has to *thermalize* (is not integrable) $\mathcal{J} \to 0$

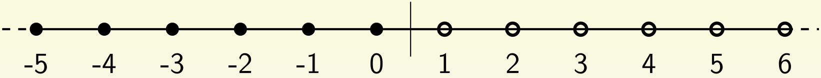
Conclusions/Summary I

1. Mild breaking of integrability leads to a **stable energy current** in a non homogeneous XXZ spin chain (different Δ's) for the times explored by the numerics
2. Energy current can be computed at the inh. fixed point **using CFT**
3. For times larger than the ones explored by the numerics, the systems is expected to thermalize

Preliminary: Domain wall quench and entanglement

(with J. Dubail, JM Stephan and P. Calabrese)

- Consider free fermions in a **DW** initial state (see N. Andrei and V. Eisler talks)



- Density profile can be obtained from *semiclassics* (or stationary phase argument)

$$\varepsilon'(k_s) = \frac{x}{t}, \quad \varepsilon(k) = -\cos k$$

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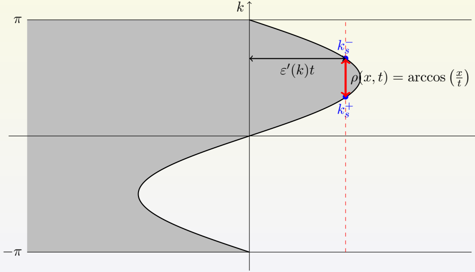

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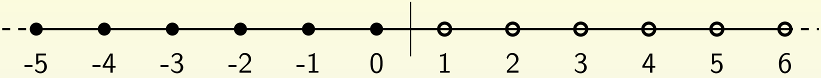

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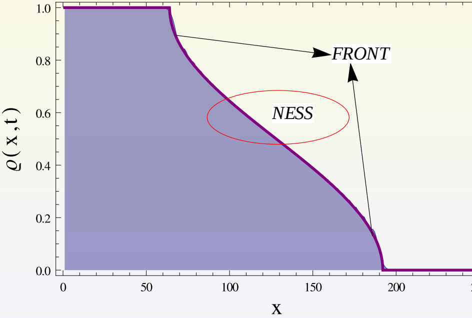
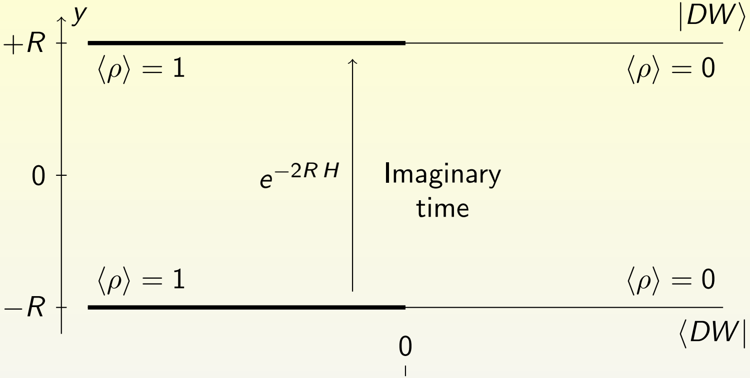


Imaginary time version of the quench

- DW initial state acts as a **boundary condition**



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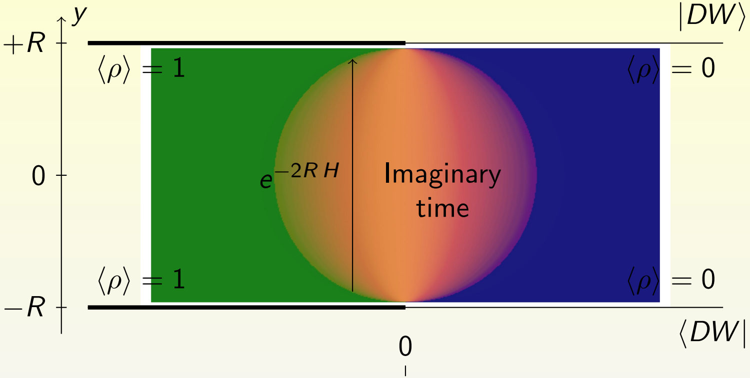

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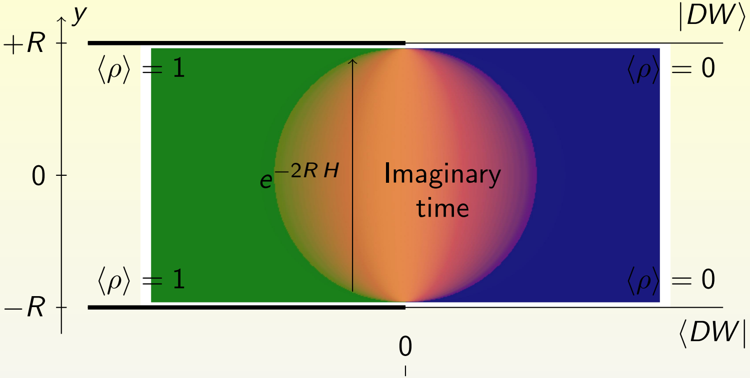


- **Fermionic propagator** obtained as

$$\langle c^\dagger(x,y)c(x',y,)\rangle = \frac{\langle DW|e^{-H(R-y)}c^\dagger(x)c(x')e^{-H(R+y)}|DW\rangle}{\langle DW|e^{-2HR}|DW\rangle}$$

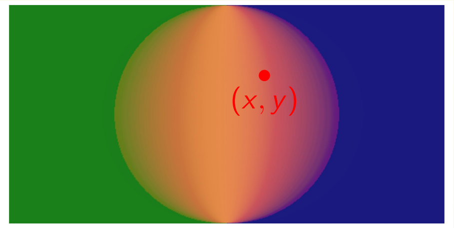
- **Real time** results from the analytic continuation $R \to 0$ and $y = it$

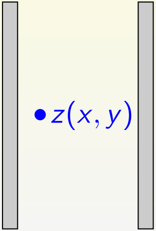
Stationary phase and CFT

- Correlators are obtained by **stationary phase** approximation around (and $-z^*(x,y)$)

$$z(x,y) = \arccos \frac{x}{\sqrt{R^2 - y^2}} + i \,\mathrm{arcth} \frac{y}{R}$$

- The stationary phase equation maps the inhomogeneous region in a strip where the fermionic correlation functions are **conformally invariant!**





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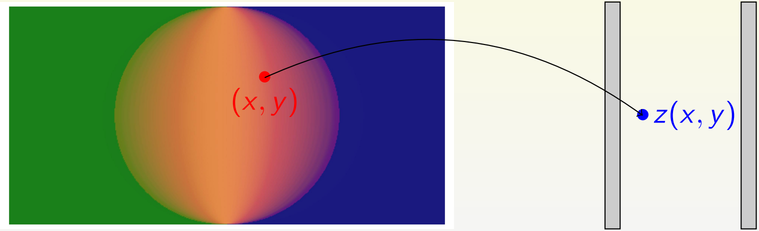

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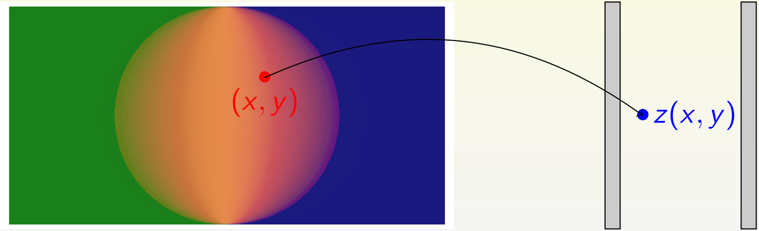

- The conformal invariant action on the strip has a non-trivial metric

$$S = \frac{1}{2\pi} \int_{strip} d^2 z\ e^{\sigma} [\psi_L^\dagger \overleftrightarrow{\partial}_z \psi_L + \psi_R^\dagger \overleftrightarrow{\partial}_{\bar{z}} \psi_R]$$

Application: entanglement entropy

- We can conformally map the strip into the upper half plane with

$$\boxed{g(z) = e^{iz}}$$

- and compute the **Reny entanglement entropies** using the Calabrese-Cardy formula

$$\boxed{S_n = \frac{n+1}{12n} \log \left[\overbrace{\sin[\rho(x,y)/\pi]}^{\text{cut.off}} \ \overbrace{d(x,y)}^{\text{conf. distance}} \right]}, \quad d(x,y) = e^{\sigma} \left| \frac{dg}{dz} \right|^{-1} \mathrm{Im} g(z)$$

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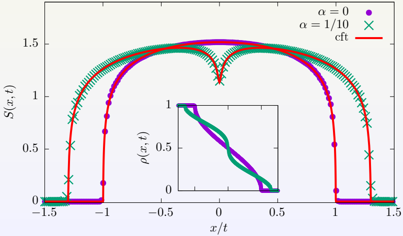
- Analytically continuing back in real time

$$S_n(x,t) \sim$$



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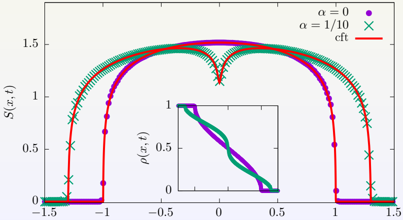
$$S_n(x,t) \sim \frac{n+1}{12\,n} \ln\left[t(1-(x/t)^2)^{3/2}\right]$$



Conclusion/Summary II

1. A new **field theoretical** (CFT) approach to inhomogeneous quenches
2. *Curvature* of the dispersion gives non-trivial **metric** of the CFT
3. **Example:** Entanglement entropy in the DW quench (hard to compute from lattice model)
4. Generalizable to many other protocols

Obrigado!