Quench(es) of symmetry broken ground states

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Introduction: Thermalization I

$$H(\{\lambda\}) \qquad \Rightarrow \qquad |G(\{\lambda\})\rangle$$

At $t = t_0$

$$H(\{\lambda\}) \qquad \Rightarrow \qquad H(\{\lambda'\})$$

$$H(\{\lambda'\})\,|G(\{\lambda\})\rangle \neq \varepsilon(\{\lambda\})\,|G(\{\lambda\})\rangle$$

$$|G(\{\lambda\}, t)\rangle = \exp\left[-iH(\{\lambda'\})t\right]|G(\{\lambda\})\rangle \neq |G(\{\lambda\})\rangle$$

Introduction: Thermalization II

Let A a subsystem of U

$$|G(\{\lambda\}, t)\rangle \Rightarrow \rho_A(\{\lambda\}, t)$$

For t large enough $\rho_A(\{\lambda\}, t) \rightarrow \rho_A^{(s)}$

Integrable Models	Non Integrable Models
$\rho_A^{(s)} = \rho_A(\{\lambda\}, \{\lambda'\})$	$\rho_A^{(s)} = \rho_A(T, \ldots)$

The Problem

Hypothesis: H is Integrable

Given $\{\lambda\}$ for which we have a degenerate ground state

$$H(\{\lambda\})\,|G(\{\lambda\})\rangle = \varepsilon\,|G(\{\lambda\})\rangle \qquad H(\{\lambda\})\,|\tilde{G}(\{\lambda\})\rangle = \tilde{\varepsilon}\,|\tilde{G}(\{\lambda\})\rangle$$

$$\varepsilon = \tilde{\varepsilon} \qquad |\,\langle\tilde{G}(\{\lambda\})|\,|G(\{\lambda\})\rangle\,|^2 \neq 1 \qquad \rho_A(\{\lambda\}) \neq \tilde{\rho}_A(\{\lambda\})$$

$$\rho_A^{(s)} = \tilde{\rho}_A^{(s)} \ ?$$

The model

Ferromagnetic Ising Model

$$H(\gamma, h) = -\sum_j \left(\frac{1+\gamma}{2} \sigma_j^x \sigma_{j+1}^x + \frac{1-\gamma}{2} \sigma_j^y \sigma_{j+1}^y \right) - h \sum_j \sigma_j^z$$

$$P_z = \bigotimes_i \sigma_i^z \qquad [H(\gamma, h), P_z] = 0$$

Each non degenerate eigenstate of $H(\gamma, h)$
must be also eigenstate of P_z

For each non degenerate eigenstate of $H(\gamma, h)$
we must have $M_x = \langle \sigma_i^x \rangle = 0$

The ferromagnetic phase

Thermodynamic Limit

$$\gamma > 0 \ \& \ h < 1 \quad \Rightarrow \text{Ferromagnetically ordered phase}$$

$$|e_{\gamma,h}\rangle \ \& \ |o_{\gamma,h}\rangle$$

$\|e_{\gamma,h}\rangle$	$\|o_{\gamma,h}\rangle$
$P_z \|e_{\gamma,h}\rangle = \|e_{\gamma,h}\rangle$	$P_z \|o_{\gamma,h}\rangle = - \|o_{\gamma,h}\rangle$
$H(\gamma, h) \|e_{\gamma,h}\rangle = \varepsilon(\gamma, h) \|e_{\gamma,h}\rangle$	$H(\gamma, h) \|o_{\gamma,h}\rangle = \varepsilon(\gamma, h) \|o_{\gamma,h}\rangle$

$$|g^{u,v}_{\gamma,h}\rangle = u \, |e_{\gamma,h}\rangle + v \, |o_{\gamma,h}\rangle \quad \Rightarrow \quad M_x(u,v) = \langle \sigma_i^x \rangle_{g_{u,v}} \neq 0$$

Local distinguishability between states I

Are the different ground state locally distinguishable?

I.e. given a subsystem A

$$\rho_{\gamma,h}^{u,v}(A) \neq \rho_{\gamma,h}^{u',v'}(A) \ \ ?$$

For the two symmetric states

$$|e_{\gamma,h}\rangle = |g_{\gamma,h}^{1,0}\rangle \text{ and } |o_{\gamma,h}\rangle = |g_{\gamma,h}^{0,1}\rangle$$

$$\forall A$$

$$\rho_{\gamma,h}^{1,0}(A) = \rho_{\gamma,h}^{0,1}(A) \equiv \zeta_{\gamma,h}^{(s)}(A)$$

Local distinguishability between states II

$$\rho^{u,v}_{\gamma,h}(A) = \frac{1}{2^l} \sum_{\{\mu_i\}} \langle g^{u,v}_{\gamma,h} | \hat{O}^{\{\mu_i\}}_A | g^{u,v}_{\gamma,h} \rangle \hat{O}^{\{\mu_i\}}_A$$

$$\hat{O}^{\{\mu_i\}}_A = \sigma^{\mu_1}_{i_1} \otimes \sigma^{\mu_2}_{i_2} \otimes \ldots \otimes \sigma^{\mu_l}_{i_l}$$

$$\rho^{u,v}_{\gamma,h}(A) = \zeta^{(s)}_{\gamma,h}(A) + (uv^* + vu^*)\chi^{(a)}_{\gamma,h}(A)$$

$\chi^{(a)}_{\gamma,h}(A)$ is an Hermitian and traceless matrix made by the sum of all the operators $\hat{O}^{\{\mu_i\}}$ that do not commute with P_z

Distance between symmetric and non symmetric state: General expression

$$\|\rho_{\gamma,h}^{0,1}(A) - \rho_{\gamma,h}^{u,v}(A)\| = (u^* v + v^* u) \sum |\lambda_\chi|$$

λ_χ are eigenvalues of $\chi_{\gamma,h}^{a}(A)$

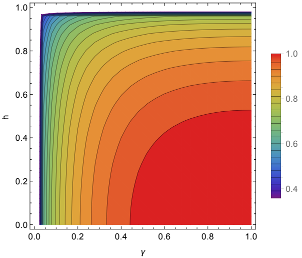
$$\mathrm{T}(1) = \sqrt{(M_{\gamma,h}^{x})^2 + (M_{\gamma,h}^{y})^2}$$

$$\mathrm{T}(2,d) = \sqrt{(M_{\gamma,h}^{x})^2 + (M_{\gamma,h}^{y})^2 + (M_{\gamma,h}^{xz,d})^2 + (M_{\gamma,h}^{yz,d})^2}$$

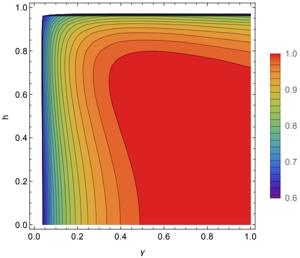
where $M_{\gamma,h}^{\mu} = \langle \sigma_i^{\mu} \rangle$ and $M_{\gamma,h}^{\mu\nu,d} = \langle \sigma_i^{\mu} \sigma_{i+d}^{\nu} \rangle$.

Results: Static case

$$M^y_{\gamma,h} = M^{yz,d}_{\gamma,h} = 0 \qquad M^x_{\gamma,h} = \frac{\sqrt{2}(\gamma^2(1-h^2))^{1/8}}{(1+\gamma)^{1/2}} \qquad M^{xz,d}_{\gamma,h} = \frac{h}{1+\gamma} M^x_{\gamma,h}$$



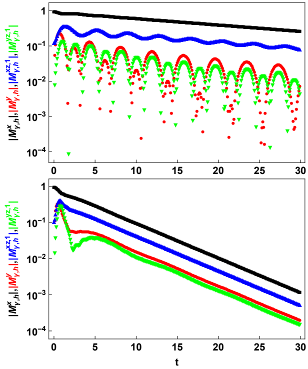
Single Spin



Two next neighbor spins

Dynamic case

$$M^{x}_{\gamma,h_0,h_1}(t),\ M^{y}_{\gamma,h_0,h_1}(t),\ M^{xz,d}_{\gamma,h_0,h_1}(t),\ M^{yz,d}_{\gamma,h_0,h_1}(t) \neq 0$$



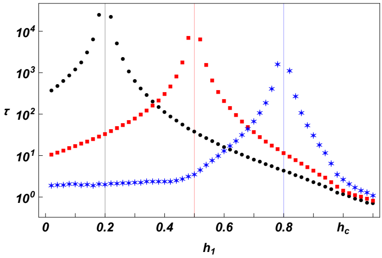
$\gamma = 0.5$
$h_0 = 0.2$
$h_1 = 0.5$

$\gamma = 0.8$
$h_0 = 0.2$
$h_1 = 0.8$

The steady state has no memory on the initial superposition

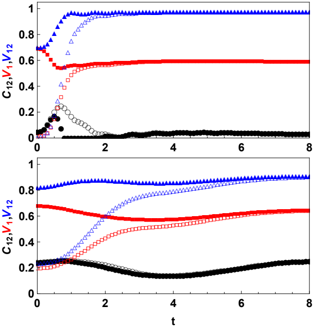
Analysis of the exponential decay

$$M^{\alpha}_{\gamma,h_0,h_1}(t) \propto e^{-t/\tau}$$



No signature of the quantum phase transition in the behavior of τ

Time dependent Entanglement



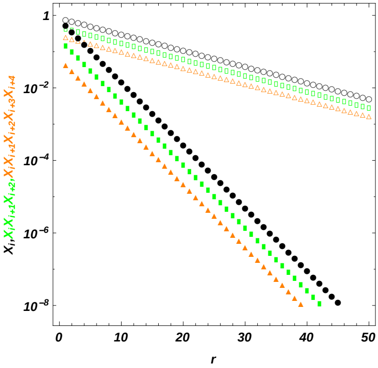
$\gamma = 0.8$
$h_0 = 0.2$
$h_1 = 1.2$

$\gamma = 0.25$
$h_0 = 0.3$
$h_1 = 0.7$

The concurrence is enanched in the time evolution of the symmetry broken ground states

More Proofs

Larger symmetry broken correlation functions in the steady state



All symmetry broken correlation functions evaluated in the steady state are zero

Mutual Information I

A. Hamma, S. M. G., F. Illuminati Phys. Rev. A 93, 012303 (2016)

$$\mathcal{I}_\alpha(A|B) = S_\alpha(A) + S_\alpha(B) - S_\alpha(AB)$$

$$S_\alpha(A) = \frac{1}{1-\alpha} \log_2(Tr(\rho^\alpha))$$

A non vanishing Mutual Information between two very far away subsystems A & B signals the presence of a non vanishing global entanglement

Mutual Information II

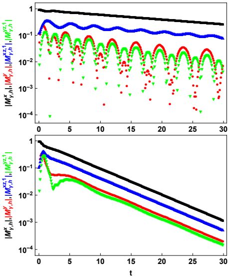
1) Without any quench (Stationary)

$$\mathcal{I}_2(\infty) = \log_2\left[1 + \frac{(M^x_{\gamma,h})^4(1-(uv^*+v*u)^4)}{(1+(M^z_{\gamma,h})^2+(M^x_{\gamma,h})^2(uv^*+v*u)^2)^2}\right]$$

In the maximally symmetry broken state $\mathcal{I}_2(\infty) = 0$

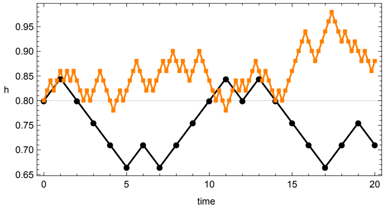
In the symmetric state $\mathcal{I}_2(\infty) \neq 0$

2) Quenched system



In the limit of $t \to \infty$, $\mathcal{I}_2(\infty)$ vanishes for all the ground states

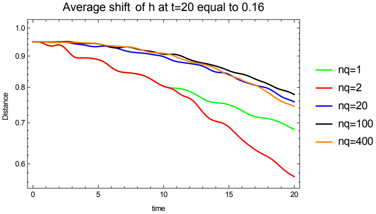
Repeated quenches I

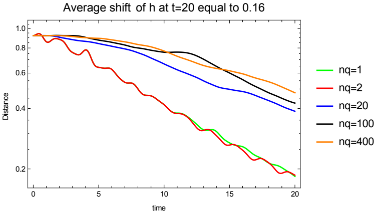


External Field described by a random walk with a fixed δ_h

δ_h are chosen to have an equal average displacement among the samples with a different numbers of quenches at the end of the simulations

Repeated quenches II





1) The distance between the state that breaks the symmetry and the symmetric one is better preserved moving from one / two quenches to several quenches ~ 20

2) By further increasing the number of quench, further improvements are not seen

3) Quantum Zeno effect?

Conclusion

In the presence of a quench

1) The informations about the superposition parameters are completely erased by the time evolution. All the ground state of a precise set of initial parameters arrive at the same local steady state (partial thermalization).

2) Evidence of the fragility of the states that show a nonzero global entanglement even in the presence of a unitary evolution.

3) Evidence of a temporary increment of the spin-spin entanglement (concurrence) that generalize and increase the effect known for the static situation below the factorization point $h_f = \sqrt{1-\gamma^2}$

In the presence of repeated quench

4) The reduction of the distance between the different states is slowed