Cooperative shielding in many-body systems with long-range interaction

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Quantum Non-Equilibrium Phenomena, Natal 2016

Relevant publications

- (many body) Lea F. Santos, Fausto Borgonovi and Giuseppe Luca Celardo, PRL 2016
- (transport) G. L. Celardo , R. Kaiser, F. Borgonovi arXiv:1604.07868;
- (classical vs quantum) in preparation with R. Bachelard, L.F. Santos, G.L. Celardo

Motivations

- Out-of-equilibrium dynamics in many-body systems.
- Surge of interest in long range: cold atomic clouds, light harvesting complexes, exciton wires, ion traps, etc..
- Cooperativity and Long Range interaction: Emergent quantum properties, Macroscopic quantum tunnelling, Cooperative propagation of information.
- Long-range interacting systems: broken ergodicity, Long Relaxation Times, long-lasting out-of-equilibrium regimes, Abundance of Regular Orbits

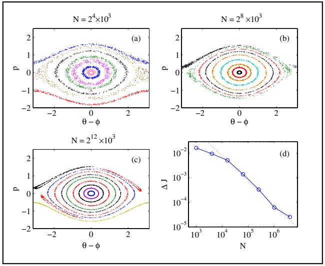
Many recent results

- Spreading of perturbations, correlations, entanglement, etc.. in many body systems.
- Lieb-Robinson bounds: for short range, spreading within a lightcone with exponential suppression outside.
- Fast propagation of perturbations, Experiments, P. Richerme et al., Nature 511 (2014) 198, P. Jurcevic et al, Nature, 511 (2014) 202
- Breakdown of Quasilocality in Long-Range Quantum Lattice Models Theory, J.Eisert, M van den Worm, S.R.Manmana M.Kastner, PRL 111, 260401 (2013); J. Schachenmayer et al, PRX 3, 031015 (2013); K. R. A. Hazzard, S. R. Manmana, M. Foss-Feig, and A. M. Rey, PRL 110, 075301 (2013), P. Hauke, L. Tagliacozzo PRL 111, 207202 (2013)
- but also "suppression" of long-range effects D.-M. Storch , M. van den Worm and M. Kastner, NJP 17 (2015) 063021

Long Range Interactions

- Statistical and Dynamical properties (A. Campa, T. Dauxois, D. Fanelli, S. Ruffo, "Physics of Long-Range Interacting Systems" , Oxford University Press, (2014)
- Non-Extensivity, Non-Additivity
- Ensemble inequivalence "Inequivalence of Ensembles in a System with Long-Range Interactions" Julien Barré, David Mukamel, and Stefano Ruffo Phys. Rev. Lett. 87, 030601 (2001)
- Broken Ergodicity (F.Borgonovi, G.L.Celardo, M.Maianti and E.Pedersoli "Broken Ergodicity in Classically Chaotic Spin Systems" Jour. of Stat. Phys. 116 , (2004) 235

- long range
  $V_{i,j} = \frac{J}{r_{ij}^{\alpha}} \quad \alpha < d$
- Abundance of Regular orbits and suppression of chaos R. Bachelard, C. Chandre, D. Fanelli, X. Leoncini, and S. Ruffo Phys. Rev. Lett. 101, 260603 (2008)



Out-of-equilibrium dynamics in MBS: Ion Traps

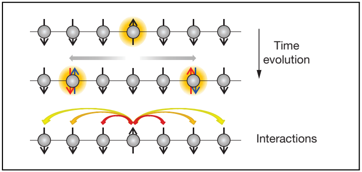
Many Body Hamiltonian:

$$H = B\sum_k \sigma_k^z + J\sum_{i<j} \frac{\sigma_i^x \sigma_j^x}{|i-j|^\alpha}$$

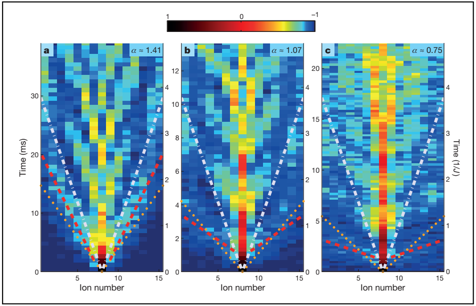
Experimental observation of spread of information and violation of Lieb-Robinson light cone.

Ion Traps

P. Jurcevic et al, Nature, 511 (2014) 202 .



Light Cones: Lieb-Robinson bounds



Red lines, fits to the observed magnon arrival times; white lines, light cone for averaged nearest-neighbour interactions; orange dots, after renormalization by the algebraic tail.

Non local propagation of correlations

Slow growth of bipartite entanglement

PHYSICAL REVIEW X **3**, 031015 (2013)

Entanglement Growth in Quench Dynamics with Variable Range Interactions

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(Received 2 June 2013; revised manuscript received 5 August 2013; published 13 September 2013)

Studying entanglement growth in quantum dynamics provides both insight into the underlying microscopic processes and information about the complexity of the quantum states, which is related to the efficiency of simulations on classical computers. Recently, experiments with trapped ions, polar molecules, and Rydberg excitations have provided new opportunities to observe dynamics with long-range interactions. We explore nonequilibrium coherent dynamics after a quantum quench in such systems, identifying qualitatively different behavior as the exponent of algebraically decaying spin-spin interactions in a transverse Ising chain is varied. Computing the buildup of bipartite entanglement as well as mutual information between distant spins, we identify linear growth of entanglement entropy corresponding to propagation of quasiparticles for shorter-range interactions, with the maximum rate of growth occurring when the Hamiltonian parameters match those for the quantum phase transition. Counterintuitively, the growth of bipartite entanglement for long-range interactions is only logarithmic for most regimes, i.e., substantially slower than for shorter-range interactions. Experiments with trapped ions allow for the realization of this system with a tunable interaction range, and we show that the different phenomena are robust for finite system sizes and in the presence of noise. These results can act as a direct guide for the generation of large-scale entanglement in such experiments, towards a regime where the entanglement growth can render existing classical simulations inefficient.

Cone-light features for long-range

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PAPER

Interplay of soundcone and supersonic propagation in lattice models with power law interactions

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Abstract

We study the spreading of correlations and other physical quantities in quantum lattice models with interactions or hopping decaying like $r^{-\alpha}$ with the distance r. Our focus is on exponents α between 0 and 6, where the interplay of long- and short-range features gives rise to a complex phenomenology and interesting physical effects, and which is also the relevant range for experimental realizations with cold atoms, ions, or molecules. We present analytical and numerical results, providing a comprehensive picture of spatio-temporal propagation. Lieb–Robinson-type bounds are extended to strongly long-range interactions where α is smaller than the lattice dimension, and we report particularly sharp bounds that are capable of reproducing regimes with soundcone as well as supersonic dynamics. Complementary lower bounds prove that faster-than-soundcone propagation occurs for $\alpha < 2$ in any spatial dimension, although cone-like features are shown to also occur in that regime. Our results provide guidance for optimizing experimental efforts to harness long-range interactions in a variety of quantum information and signaling tasks.

Cooperative Shielding

We will show that such apparently contradictory behavior is caused by a general property of long-range interacting systems (Cooperative Shielding). It refers to shielded subspaces inside of which the evolution is unaffected by long-range interactions for a long time. As a result, the dynamics strongly depends on the initial state: if it belongs to a shielded subspace, the spreading of perturbation satisfies the Lieb-Robinson bound and may even be suppressed, while for initial states with components in different subspaces, the propagation may be quasi-instantaneous.

The Shielding effect

- Let us consider a system:

$$H = H_0 + V, \qquad \text{with} \qquad [H_0, V] = 0$$

with V highly degenerate $V|v_k\rangle = v|v_k\rangle$, $k = 1, .., g$.

- $|\psi_0\rangle = \sum_{k=1}^{g} c_k |v_k\rangle$

V contribute to the dynamics only with a global phase

$$|\psi(t)\rangle = e^{-iHt/\hbar}|\psi_0\rangle = e^{-ivt/\hbar} e^{-iH_0 t/\hbar}|\psi_0\rangle$$

We have shielding from V!!

- H_0 describes completely the dynamics (within a constant phase) we may call it, emerging Hamiltonian

Many open questions for non trivial cases:

- What if $[H_0, V] \neq 0$?
- What if the spectrum of V is not degenerate?
- What's the connection with long-range interacting systems ?
- Is this a cooperative effects ? (how it scales with the number of particles?)
- What is and how to find, if any, the emergent Hamiltonian?

Cooperative Shielding in many-body systems

Experimentally accessible spin 1/2 1–d Hamiltonian:

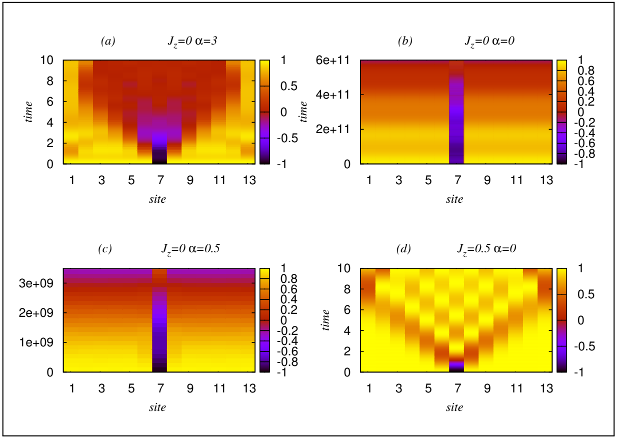
$$H = H_0 + V,$$
$$H_0 = \sum_{n=1}^{L} (\mathcal{B} + h_n)\sigma_n^z + \sum_{n=1}^{L-1} J_z \sigma_n^z \sigma_{n+1}^z,$$
$$V = \sum_{n<m} \frac{J}{|n-m|^\alpha} \sigma_n^x \sigma_m^x.$$

- transverse field: $h_n \in [-W/2, W/2]$.
- $\alpha < 1$: long range. $\alpha > 1$: short range. For $\alpha = 0$:

$$V = J \sum_{n<m} \sigma_n^x \sigma_m^x = \frac{JM_x^2}{2} - \frac{JL}{2} \quad \text{where} \quad M_x = \sum_n \sigma_n^x$$

has a degenerate spectrum, and its eigenvalues are given by,
$V_b = J(L/2 - b)^2/2 - JL/2, \quad \text{where} \quad b = 0, 1, \ldots L/2$

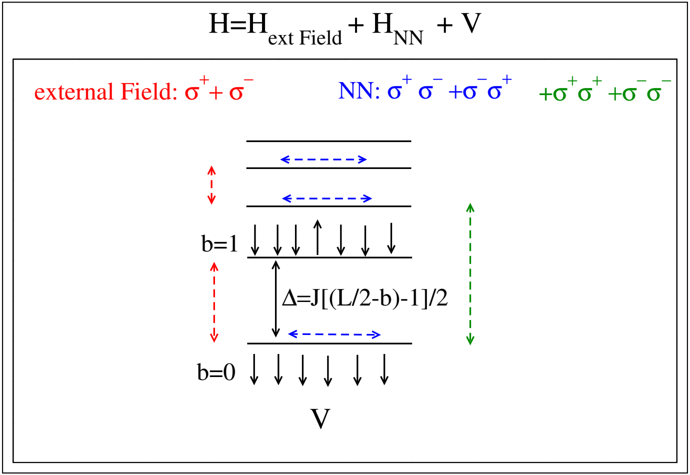
Light-cones , constant transversal magnetic field



Initially : All spins up along x, but the central spin (-x);
a) short range $\longrightarrow$ light-cone;
b) infinite range $\longrightarrow$ localization without disorder;
c) long range $\longrightarrow$ localization without disorder;

d) infinite range & nearest neighbor $\longrightarrow$, light cone (effective short range)

Effective couplings ($\alpha = 0$)



Leaking Probability 1

In each band there is a g-degeneracy $k = 1, .., g$.

$$V|V_{bk}\rangle = V_b|V_{bk}\rangle$$

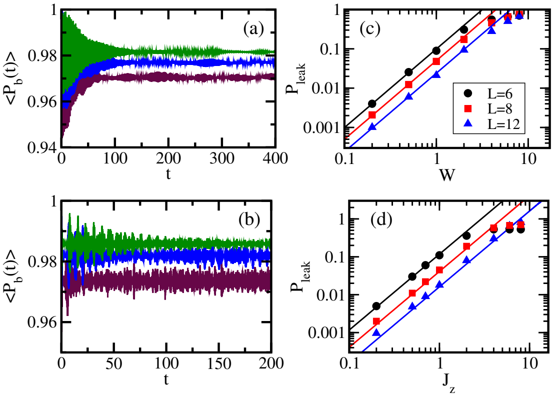
Taking a random superposition of degenerate eigenstates inside one band b one may ask for the leaking probability to go outside that band due to H_0,

$$|\psi(0)\rangle = \sum_k c_k|V_{bk}\rangle \rightarrow |\psi(t)\rangle$$

$$P_b(t) = \sum_k |\langle V_{bk}|\psi(t)\rangle|^2 \quad P_{leak} = \lim_{t\to\infty} 1 - P_b(t)$$

Leaking Probability experiments: $L = 10, 12, 14$

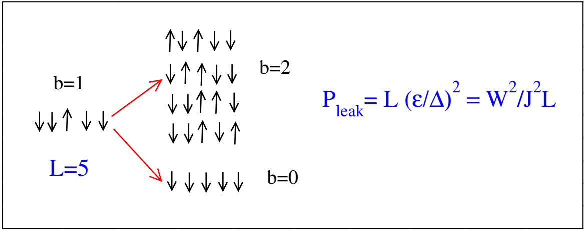
- No nearest neighbor interaction $J_z = 0$, and presence of a transversal field
- No transversal field but nearest neighbor interaction $J_z \neq 0$



Leaking Probability, pert. theory

Estimation for $P_{leak} = 1 - P_b$: Given a state coupled with amplitude ϵ to another state separated by an energy Δ, the probability to find the system in the second state is $\approx (\epsilon/\Delta)^2$ for $\epsilon/\Delta \ll 1$.

$$H_0^W = \sum_{n=1}^{L} h_n \sigma_n^z = \sum_{n=1}^{L} \frac{h_n}{2}(\sigma_n^{+,x} + \sigma_n^{-,x}), \quad \text{with} \quad \langle h_n^2 \rangle = W^2/12 \quad (1)$$



$P_{leak} \propto (W/J)^2/L$ for random field and no NN interaction.
$P_{leak} \propto (J_z/J)^2/L$ for NN interaction only (and no random field).

The emergent Hamiltonian

Consider the total Hamiltonian $H = H_0 + V_{LR}$ can be written in the basis of $V_{LR} = \sum_b V_b \sum_j |V_{bj}\rangle\langle V_{bj}|$,

$$H = \sum_{b,b'} \sum_{j,k} |V_{bj}\rangle\langle V_{bj}|H_0|V_{b'k}\rangle\langle V_{b'k}| + \sum_b V_b \sum_j |V_{bj}\rangle\langle V_{bj}|$$

taking the mean field approximation one gets

The Zeno Hamiltonian

$$H_z = \sum_{bj} (V_b + \langle V_{bj}|H_0|V_{bj}\rangle)|V_{bj}\rangle\langle V_{bj}|$$

Analogy with Quantum Zeno Effect

Consider the total Hamiltonian $H = H_s + KH_{meas}$, which one may interpret as a quantum system described by H_s that is contin- uously observed by an "apparatus" characterized by KH_{meas} . In the limit of strong coupling, $K \to \infty$, a superselection rule is induced that splits the Hilbert space into the eigensubspaces of KH_{meas}. Each one of these invariant quantum Zeno sub- spaces is specified by an eigenvalue and is formed by the corresponding set of degenerate eigenstates of KH_{meas} . The dynamics becomes confined to these P subspaces and dictated by the Zeno Hamiltonian

P. Facchi and S. Pascazio: Phys. Rev. Lett. 89 (2002)

$$H_z = \sum_b P_b H_0 P_b + V_b P_b = diag(H_0) + \sum_b V_b P_b$$

where P_b are the projectors on the eigensubspace of V corresponding to V_b.

Dynamics inside one specific band

- Specifically we should consider the diagonal matrix elements of H_0 : $\langle V_{bj}|H_0|V_{bj}\rangle$
- H_0 is a sum of magnetic field $\mathcal{B} \neq 0$ and a nearest neighbor (NN) coupling $J_z \neq 0$
- In absence of NN coupling, ($J_z = 0$) one has $\langle V_{bj}|H_0|V_{bj}\rangle = 0$ i.e. **freezing!**
- on the other hand for $\mathcal{B} = 0$ and $J_z \neq 0$ one has

$$H_z = V_b \sum_j |V_{bj}\rangle\langle V_{bj}| + \frac{J_z}{4} \sum_{n=1}^{L-1} \left(\sigma_n^+ \sigma_{n+1}^- + \sigma_n^- \sigma_{n+1}^+\right)$$

and one has an **effective NN interaction**

Zeno Fidelity

To substantiate that the dynamics becomes indeed controlled by the Zeno Hamiltonian as L increases, we consider an initial random state inside one V-band

$$|\Psi(0)\rangle = \sum_k c_k |V_{bk}\rangle$$

and the overlap of its evolution under both the total Hamiltonian H and H_z (fidelity)

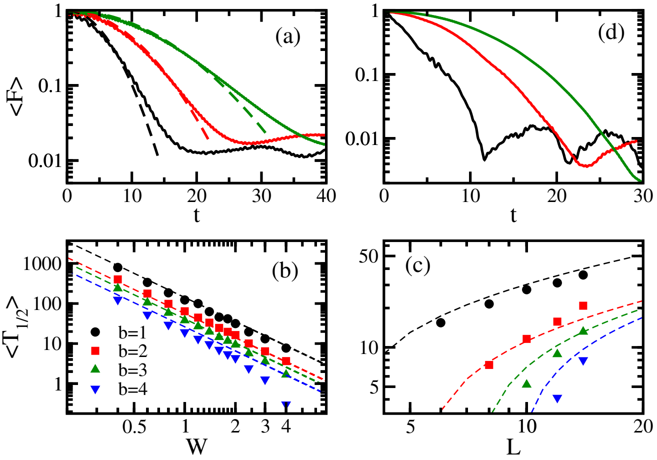
$$F(t) = |\langle\Psi(0)|e^{iH_z t/\hbar} e^{-iHt/\hbar}|\Psi(0)\rangle|^2.$$

It is clear that if in some limit $H \to H_z$ then $F(t) \to 1$. A perturbative argument suggests that

$$T_{1/2} \propto J\sqrt{L}/W^2$$

Numerical Experiments

Upper : fidelity
black L=10
red L=12
green L=14
left $J_z = 0$, $W = 2$
right $J_z = 1$, $W = 0$
....
Lower : shielding time
left $T_{1/2}$ vs W
right $T_{1/2}$ vs L

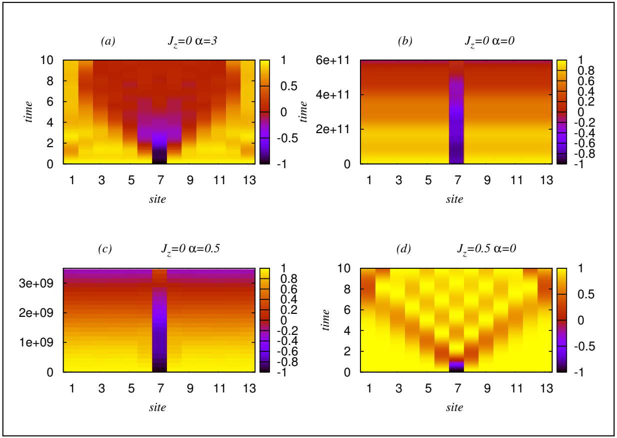


Spreading of perturbation

Starting with an initial state inside one specific band the dynamics is dictated by the Zeno Hamiltonian up to $T_{1/2}$. In other words we will have a spreading of perturbation freezed or effectively short-ranged despite the presence of long range interaction.

This is not a peculiarity of $\alpha = 0$.

Light-cones , constant transversal magnetic field



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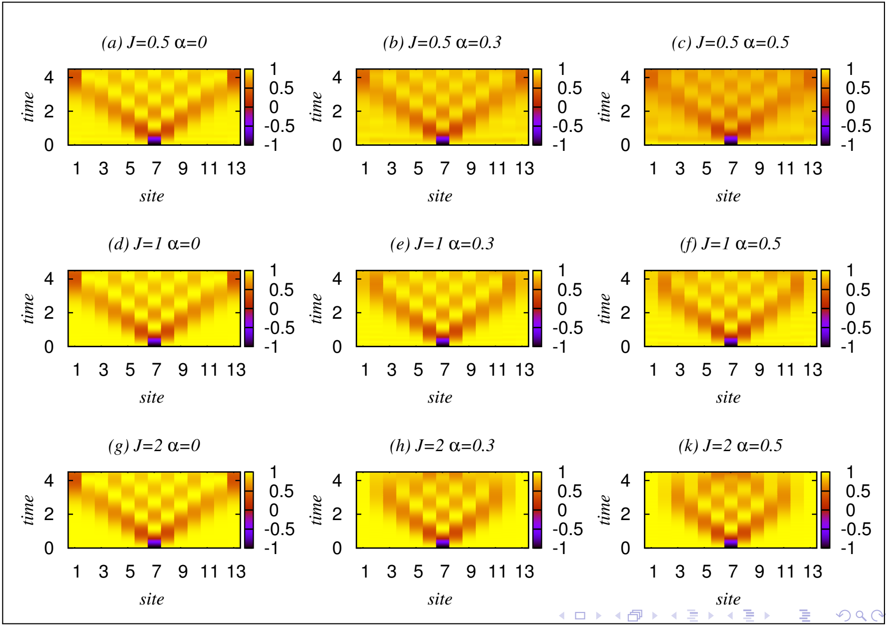
d) infinite range & nearest neighbor $\longrightarrow$, light cone (effective short range)

Shielding : independence of J coupling



Conclusions and Perspectives

1. Shielding is a novel cooperative effect : since it preserves invariant coherent subspaces, and its robustness increases with the system size, it might be essential in building efficient quantum devices able to work at room temperature.
2. Shielding allows to control quantum dynamics since the spreading of correlations strongly depends on the initial state.
3. Transport properties are strongly affected from shielding (arXiv:1604.07868)
4. Is it possible to have Classical Shielding? in preparation with L. Celardo, L. F. Santos & R. Bachelard

Old stuff

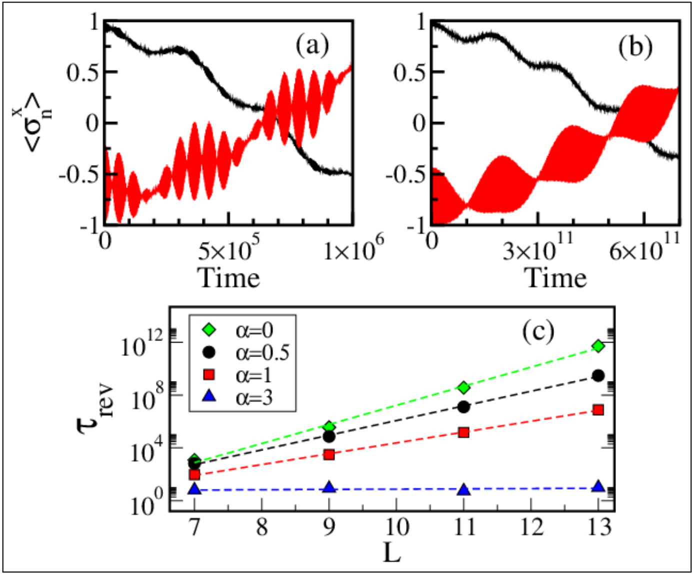
II. MODEL

We consider a system of N particles of spin l, described by the Hamiltonian

$$\hat{H} = \frac{\eta}{2}\sum_{i=1}^{N}\sum_{j\neq i}\hat{S}_i^x\hat{S}_j^x - \frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i}\hat{S}_i^y\hat{S}_j^y, \qquad (1)$$

where $-1 < \eta \leqslant 1$ is the anisotropy constant. We define

Magnetic reversal time



Shielding in tight binding models

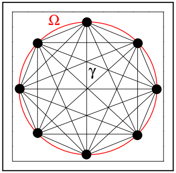
1d Anderson model with long range hopping

$$H = H_0 + V_{\mathrm{LR}}$$

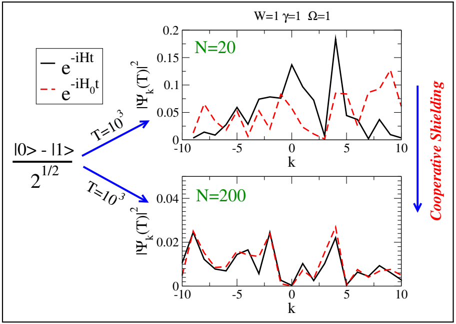
$$H_0 = Anderson \quad Model$$

$$H_0 = \sum_i \epsilon_i^0 |i\rangle\langle i| - \Omega \sum_i |i\rangle\langle i+1| + h.c.$$

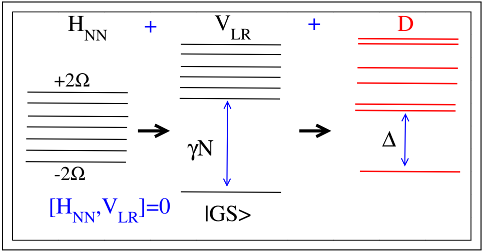
$$V_{\mathrm{LR}} = -\gamma \sum_{i \neq j} \frac{|i\rangle\langle j|}{r_{\mathrm{i,j}}^{\alpha}}$$



Shielding with Disorder?



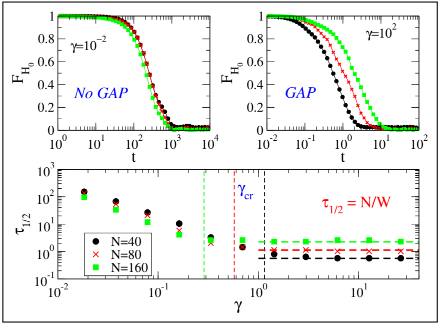
Induced gap and fidelity



Fidelity: (Loschmidt echo)

$$F(t) = |\langle\psi_0|e^{iH_0 t/\hbar} e^{-iHt/\hbar}|\psi_0\rangle|^2$$

$|\psi_0\rangle$: random superposition of excited states.



Classical Shielding : a full integrable model

Let us consider the Hamiltonian ($\alpha = 0$, $J > 0, J_x > 0$) with Kac's renormalization factor,

$$H_{LR} + H_{SR} = -\frac{J}{2N} \sum_{k=1}^{N} \sum_{j \neq k} S_k^x S_j^x - J_x \sum_{k=1}^{N-1} S_k^x S_{k+1}^x \qquad (2)$$

Classical Shielding