Secularly growing loop corrections in strong background field

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IIP, NATAL (July 2015)

Motivation

- de Sitter (dS) space solves

$$R_{\alpha\beta} - \frac{1}{2} R\, g_{\alpha\beta} = \Lambda\, g_{\alpha\beta} + \langle T_{\alpha\beta} \rangle, \tag{1}$$

- Is $\langle T_{\alpha\beta} \rangle$ is relevant or not? Common wisdom is that it is not! I will try to convince you that this is a wrong intuition even for massive fields.
- There is UV divergence in $\langle T_{\alpha\beta} \rangle$. The same as in flat space $\langle T_{\alpha\beta} \rangle \propto g_{\alpha\beta}$. Leads to the renormalization of Λ.
- On top of that there also can be non–trivial fluxes in $\langle T_{\alpha\beta} \rangle$, because dS metric is time dependent — the situation is non–stationary.

Motivation

- dS has $SO(D, 1)$ isometry. Similar to the Poincare invariance in Minkowski space.
- If dS isometry is respected, then all contributions to $\langle T_{\alpha\beta} \rangle \propto g_{\alpha\beta}$ — no fluxes. No particle production?!

- Is the dS isometry always respected (on tree–level or in the loops)? For all initial states? If there is such a ground state that always respects dS isometry, is this state stable under non–symmetric perturbations?
- We restrict our attention to massive fields — the most comfortable situation for those who believe in eternal dS. We will see that even in this case the situation is not so obvious.

Motivation

- In any interacting, non–conformal QFT (even massive) on dS there are secular IR effects.
- Secular growth of loop corrections is practically inevitable in non–stationary situations (Landau and Lifshitz, X-th volume).
- This growth is the IR effect. No modifications of UV physics.

- IR effects are non–local. Hence, there is a dependence on coordinate systems, if no correct match between initial and boundary conditions is made.
- The IR effects that we are discussing below do not cover all interesting large scale effects in dS space. For massless non–conformal fields there are additional IR effects.

Motivation

- We consider IR effects in the following situations:

- dS space interacting massive scalar QFT (review arXiv:1309.2557). See also A.Polyakov et al. and E.Mottola et al.
- QED on strong electric field background beyond the background field approximation (arXiv:1405.5225). See also E.Mottola et al.
- Loop correction to Hawking radiation (arXiv:1509. ...). See the talk of F.Popov.

Adiabatic catastrophe

- Suppose one would like to find:

$$\langle \mathcal{O} \rangle_{t_0}(t) = \left\langle \Psi \left| \overline{T}\, e^{i\int_{t_0}^{t} dt' H(t')}\, \mathcal{O}\, T\, e^{-i\int_{t_0}^{t} dt' H(t')} \right| \Psi \right\rangle, \quad (2)$$

e.g. $\langle T_{\mu\nu} \rangle$ or $\langle J_\mu \rangle$.

- Here $H(t) = H_0(t) + V(t)$.
- T — time–ordering, $\overline{T}$ — anti–time–ordering.
- t_0 — initial moment of time, $|\Psi\rangle$ — initial state, $\langle \Psi | \mathcal{O} | \Psi \rangle (t_0)$ is supposed to be given.

Adiabatic catastrophe

- Transferring to the interaction picture:

$$\langle \mathcal{O} \rangle_{t_0}(t) = \left\langle \Psi \left| S^{+}(+\infty, t_0)\, T\left[\mathcal{O}_0(t)\, S(+\infty, t_0)\right] \right| \Psi \right\rangle . \quad (3)$$

Here $S(t_2, t_1) = T\, e^{-i \int_{t_1}^{t_2} dt' V_0(t')}$; $\mathcal{O}_0(t)$ and $V_0(t)$ are the above defined operators in the interaction picture.

- Slightly changing the problem:

$$\langle \mathcal{O} \rangle_{t_0}(t) = \left\langle \Psi \left| S^{+}_{t_0}(+\infty, -\infty)\, T\left[\mathcal{O}_0(t)\, S_{t_0}(+\infty, -\infty)\right] \right| \Psi \right\rangle . \quad (4)$$

Here t_0 is the time moment after which the interactions, $V(t)$, are adiabatically turned on.

Adiabatic catastrophe

- When does the dependence on t_0 disappear? Otherwise we have adiabatic catastrophe and breaking of various symmetries: E.g. correlation functions stop to depend only on $|t_1 - t_2|$.

- The dependence on t_0 disappears when the situation is or becomes stationary.

Adiabatic catastrophe

- The seminal example of the stationary situation is when the free Hamiltonian H_0 is time independent and has a spectrum bounded from below: $H_0 \left| vac \right\rangle = 0$ and $\left| \psi \right\rangle = \left| vac \right\rangle$.

- In fact, in the latter case by adiabatic turning on and then switching off $V(t)$ we do not disturb the ground state:

$$\left\langle vac \left| S^+(+\infty, -\infty) \right| excited\ \ state \right\rangle = 0,$$

while

$$\left| \left\langle vac \left| S^+(+\infty, -\infty) \right| vac \right\rangle \right| = 1.$$

It does not matter when one adiabatically turns on interactions. The dependence on t_0 disappeared!

- Furthermore, in the latter case we obtain:

$$\langle \mathcal{O} \rangle (t) =$$

$$\sum_{sta} \langle vac \left| S^{+}(+\infty, -\infty) \right| sta \rangle \; \langle sta \left| T \left[\mathcal{O}_0(t) \, S(+\infty, -\infty) \right] \right| vac \rangle =$$

$$= \langle vac \left| S^{+}(+\infty, -\infty) \right| vac \rangle \; \langle vac \left| T \left[\mathcal{O}_0(t) \, S(+\infty, -\infty) \right] \right| vac \rangle =$$

$$= \frac{\langle vac \left| T \left[\mathcal{O}_0(t) \, S(+\infty, -\infty) \right] \right| vac \rangle}{\langle vac \left| S(+\infty, -\infty) \right| vac \rangle}.$$

  This way we arrive at having only the T–ordered expressions and then can use Feynman technique.

- Other situation when the dependence on t_0 disappears if there is a stationary state (e.g. thermal density matrix in flat space–time).

Adiabatic catastrophe

- Is there a stationary state if a background field is never switched off? What is that state, if it is present? What if there is no stationary state?
- How does the dependence on t_0 reveals itself? t_0 does not appear in UV renormalization! In UV limit one always can use the Feynman technique, because high frequency modes are not sensitive to background fields.

- To answer the above questions one has to calculate directly:

$$\langle \mathcal{O} \rangle_{t_0}(t) = \left\langle \Psi \left| S^+_{t_0}(+\infty, -\infty)\, T\left[\mathcal{O}_0(t)\, S_{t_0}(+\infty, -\infty) \right] \right| \Psi \right\rangle \quad (5)$$

for various choices of $\mathcal{O}$.

- Schwinger notations: S — "$+$" vertexes, S^+ — "$-$" vertexes:

$$\begin{aligned} D^{++}(1,2) &= \left\langle \Psi \left| T\left(\phi(1)\ \ \phi(2)\right)\right| \Psi \right\rangle , \\ D^{--}(1,2) &= \left\langle \Psi \left| \overline{T}\left(\phi(1)\ \ \phi(2)\right)\right| \Psi \right\rangle , \\ D^{+-}(1,2) &= \left\langle \Psi \left|\phi(1)\, \phi(2)\right| \Psi \right\rangle , \\ D^{-+}(1,2) &= \left\langle \Psi \left|\phi(2)\, \phi(1)\right| \Psi \right\rangle . \end{aligned} \tag{6}$$

Every field is characterized by a matrix of propagators.

- After Keldysh's rotation of ϕ_+ and ϕ_-, we obtain:

$$D^{R,A}(1,2) = \theta\left(\pm\Delta t_{1,2}\right)\left(D^{+-}(1,2) \;-\; D^{-+}(1,2)\right) = \\ = \theta\left(\pm\Delta t_{1,2}\right)\left[\phi(1)\;,\;\phi(2)\right] \quad (7)$$

— state independent Retarded and Advanced propagators. They characterize only the spectrum of excitations.

- The Keldysh propagator:

$$D^{K}(1,2) = \frac{1}{2}\left(D^{+-}(1,2) \;+\; D^{-+}(1,2)\right) = \\ = \frac{1}{2}\left\langle \Psi \left| \left\{ \phi(1)\;,\;\phi(2) \right\} \right| \Psi \right\rangle . \quad (8)$$

Adiabatic catastrophe

- If we have spatially homogeneous non–stationary state: $\phi(t,\vec{x}) = \int d^{D-1}\vec{p}\,\left(a_{\vec{p}}\, e^{i\,\vec{p}\,\vec{x}}\, g_p(t) + h.c.\right)$, for the case of real scalar field, then

$$\int d^{D-1}\vec{p}\, e^{-i\,\vec{p}\,(\vec{x}_1-\vec{x}_2)}\, D^K\left(t_1, t_2, |\vec{x}_1 - \vec{x}_2|\right) \equiv D^K_p\left(t_1, t_2\right) =$$

$$= \left(\frac{1}{2} + \left\langle a^+_{\vec{p}}\, a_{\vec{p}}\right\rangle\right) g_p(t_1)\, g^*_p(t_2) + \left\langle a_{\vec{p}}\, a_{-\vec{p}}\right\rangle g_p(t_1)\, g_p(t_2) + c.c.$$

— carries information about background state!

- In QED, global de Sitter and black hole collapse case the formulas are a bit different, but the situation is conceptually the same.

Adiabatic catastrophe

- In a free theory $\left\langle a^+_{\vec{p}}\, a_{\vec{p}} \right\rangle = const, \quad \langle a_{\vec{p}}\, a_{-\vec{p}} \rangle = const.$ All time dependence is gone into harmonic functions — $g_p(t)$.
- If the initial state is the ground one: $|\Psi\rangle = |ground\rangle$ and $a_p\,|ground\rangle = 0$, we obviously have that $\left\langle a^+_{\vec{p}}\, a_{\vec{p}} \right\rangle = \langle a_{\vec{p}}\, a_{-\vec{p}} \rangle = 0.$
- However, if one turns on interactions, then $\left\langle a^+_{\vec{p}}\, a_{\vec{p}} \right\rangle$ and $\langle a_{\vec{p}}\, a_{-\vec{p}} \rangle$ start to depend on time.
- All quasi–classical results (non–interacting fields, background field approximation) follow from the tree–level propagator:

$$D^K_p\,(t_1\,, t_2) = \frac{1}{2}\,\Big(g_p(t_1)\,g^*_p(t_2) \;+\; g^*_p(t_1)\,g_p(t_2)\Big)\,. \qquad (9)$$

E.g. $\langle T_{\mu\nu}\rangle_0$ in de Sitter space and black hole collapse, and $\langle J_\mu\rangle_0$ in QED.

- Say for $\lambda\phi^3$ (or $\lambda\phi^4$) theory at loop level, as $t = \frac{t_1+t_2}{2} \to +\infty$, we obtain that

$$D^K_p(t_1, t_2) = \left(\frac{1}{2} + n_p(t)\right) g_p(t_1)\, g^*_p(t_2) + \kappa_p(t)\, g_p(t_1)\, g_p(t_2) + c.c.$$

- At one loop level $\lambda\phi^3$

$$n_p(t) \propto \lambda^2 \int d^{D-1}\vec{q}_1 \int d^{D-1}\vec{q}_2 \iint_{t_0}^{t} dt_3\, dt_4\, \delta\left(\vec{p} + \vec{q}_1 + \vec{q}_2\right) \times$$
$$\times g^*_p(t_3)\; g_p(t_4)\; g^*_{q_1}(t_3)\, g_{q_1}(t_4)\, g^*_{q_2}(t_3)\, g_{q_2}(t_4) + O\left(t_1 - t_2\right),$$
$$\kappa_p(t) \propto -\lambda^2 \int d^{D-1}\vec{q}_1 \int d^{D-1}\vec{q}_2 \iint_{t_0}^{t} dt_3\, dt_4\, \delta\left(\vec{p} + \vec{q}_1 + \vec{q}_2\right) \times$$
$$\times g^*_p(t_3)\; g^*_p(t_4)\; g^*_{q_1}(t_3)\, g_{q_1}(t_4)\, g^*_{q_2}(t_3)\, g_{q_2}(t_4) + O\left(t_1 - t_2\right).$$

Secular growth of loop corrections

- If there is no background field, then $g_p \propto \frac{e^{-i\,\epsilon(p)\,t}}{\sqrt{\epsilon(p)}}$ and

$$n_p(t) \propto \lambda^2 \,(t - t_0) \int d^{D-1}\vec{q}_1 \int d^{D-1}\vec{q}_2 \delta\,(\vec{p} + \vec{q}_1 + \vec{q}_2) \times \\ \times \delta\left(\epsilon(p) \;+\; \epsilon(q_1) \;+\; \epsilon(q_2)\right). \quad (10)$$

Hence, $n_p(t) = 0 = \kappa_p(t)$ due to energy conservation.

- There is no energy conservation in time–dependent background fields (or energy is not bounded from below), then we generically obtain:

$$n_p(t) \propto \lambda^2 \,(t - t_0) \times (\text{production rate}),$$

$$\kappa_p(t) \propto -\lambda^2 \,(t - t_0) \times (\text{backreaction on the ground state rate}).$$

The RHS is the collision integral.

Side remark on QED with constant electric field

- In QED with $\vec{E} = const$ formulas a bit different. Harmonics are $g_p(t) = g(p + eEt)$.
- All expressions are invariant under $p \to p + a$ and $t \to t - a/eE$.

- As the result, beyond the background field approximation, for photons we obtain that:

$$n_p(t) \propto e^2 \, (t - t_0) \times (\text{production rate}), \qquad \kappa_p(t) = 0. \tag{11}$$

Because of that t_0 cannot be taken to past infinity. Hence, we have adiabatic catastrophe for any initial state.

- For charged fields $n_p^{\pm}$ and $\kappa_p^{\pm}$ are time–dependent, but do not grow as $t - t_0 \to \infty$. However, the one–loop contribution to the current is growing with time.

Side remark on black hole collapse

- Harmonics are much more complicated, but at the final stage of the collapse they depend on $\omega\, e^{-t/2r_g}$.
- Invariance under $\omega \to \omega\, a$ and $t \to t + 2r_g \log a$.
- As the result, if the collapse had started at $t = 0$, then we obtain

$$\begin{aligned} n_p(t) &\propto \lambda^2\, t \times (\text{production rate}), \\ \kappa_p(t) &\propto -\lambda^2\, t \times (\text{backreaction rate}). \end{aligned} \tag{12}$$

- Change of the Hawking's thermal spectrum? Information paradox?
- See the talk of F.Popov.

de Sitter space, expanding patch

- In expanding Poincare patch: $g_p(t) = \eta^{\frac{D-1}{2}} h(p\eta)$, where $\eta = e^{-t}$ and $h(p\eta)$ is a Bessel function.
- There is invariance under $p \to p\, a$ and $\eta \to \eta/a$.

- For the case of massive scalars, $m > \frac{D-1}{2}$, in the limit $p\eta \to 0$, we obtain that

$$n_p(\eta) \propto \lambda^2 \log\left(\frac{m}{p\eta}\right) \times (\text{production rate}),$$
$$\kappa_p(\eta) \propto -\lambda^2 \log\left(\frac{m}{p\eta}\right) \times (\text{backreaction rate}). \qquad (13)$$

- No divergence, but there is secular growth.

- Contracting Poincare patch is just time–reversal of the expanding one.
- For the case of massive scalars, $m > \frac{D-1}{2}$, in the limit $p\eta_0 \to 0$ and $p\eta \to +\infty$, we obtain that

$$n_p(\eta) \propto \lambda^2 \log\left(\frac{m}{p\eta_0}\right) \times (\text{production rate}),$$
$$\kappa_p(\eta) \propto -\lambda^2 \log\left(\frac{m}{p\eta_0}\right) \times (\text{backreaction rate}). \tag{14}$$

Here $\eta_0 = e^{t_0}$ is the time after which interactions are adiabatically turned on.

- In this case — IR divergence and, hence, adiabatic catastrophe for any initial state.
- In global de Sitter there is also adiabatic catastrophe for any initial state.

- D–dimensional dS space is the hyperboloid,

$$-X_0^2 + X_1^2 + \dots + X_D^2 = 1, \quad H = 1,$$

in $(D+1)$–dimensional Minkowski space

$$ds^2 = -dX_0^2 + dX_1^2 + \dots + dX_D^2.$$

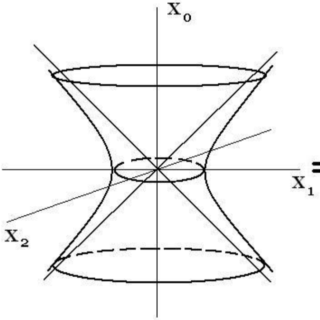
- dS isometry is the Lorentz rotation group of the ambient Minkowski space–time.
- Induced metric in the expanding Poincare patch (EPP):

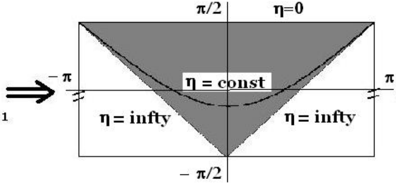
$$ds^2 = -dt^2 + e^{2t}\, d\vec{x}^2 = \frac{1}{\eta^2}\left[-d\eta^2 + d\vec{x}^2\right].$$

- Here $\eta = e^{-t}$. Then $\eta = +\infty$ — past infinity, while $\eta = 0$ — future infinity.

Penrose diagram for the 2D case

- Grey region is EPP.
- White region is contracting Poincare patch (CPP) — time reversal of EPP.





Discussion

- We consider model example:

$$S = \int d^D x \sqrt{|g|} \left[g^{\alpha\beta} \partial_\alpha \phi \, \partial_\beta \phi + m^2 \phi^2 + \frac{\lambda}{3} \phi^3 + \dots \right]. \tag{15}$$

- When $m > 0$ there is a dS invariant state. We restrict ourselves to this case.
- However, from the phenomenological point of view the most interesting case is $m = 0$ and the graviton. In these cases the presence of dS invariant ground state is still under discussion. We do not consider these issues here.

Free harmonics in de Sitter space

- Any harmonic function in dS:

$$g_p(\eta) = \eta^{\frac{D-1}{2}} h_{i\,\mu}(p\eta), \quad \mu = \sqrt{m^2 - \left(\frac{D-1}{2}\right)^2}. \qquad (16)$$

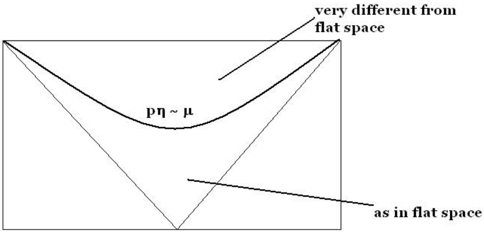
- $h_{i\,\mu}(t)$ is a solution of Bessel equation with $i\,\mu$ as the index:

$$h_{i\mu}(x) = \begin{cases} A_1 \frac{e^{i\,x}}{\sqrt{x}} + A_2 \frac{e^{-i\,x}}{\sqrt{x}}, & x \gg |\mu| \\ B_1\, x^{i\mu} + B_2\, x^{-i\mu}, & x \ll |\mu| \end{cases} \qquad (17)$$

- During this talk we consider $m > \frac{D-1}{2}$ (μ is real).
- If $A_2 = 0$ in (17) — Bunch–Davies (BD) or in–harmonics $h_{i\mu}(x) \sim H^{(1)}_{i\,\mu}(x)$ (Hankel function).
- If $B_2 = 0$ — out–harmonics $h_{i\mu}(x) \sim J_{i\mu}(x)$ (Bessel function).

Free harmonics in de Sitter space

- Due to the expansion of EPP every harmonic experiences a red shift.
- In UV limit harmonics do not feel the curvature of dS and behave as in flat space $\sim e^{\pm p\eta}$.
- In IR limit they behave very different from flat space case $\sim (p\eta)^{\pm i\,\mu}$.
- BD modes — proper UV behavior. Any other type of harmonics — wrong UV behavior.



Discussion

- Expand solution of $\left[\Box(g)+m^2\right]\ \phi=0$ as

$$\phi\left(\eta,\vec{x}\right)=\int d^{D-1}\vec{p}\ \left[\hat{a}_{\vec{p}}\, g_p\left(\eta\right)\ e^{-i\,\vec{p}\,\vec{x}}+h.c.\right].$$

- Ground state $\hat{a}_{\vec{p}}\ |0\rangle=0$.
- If dS isometry is respected, then any propagator is

$$D\left(\eta_1,\eta_2,|\vec{x}_1-\vec{x}_2|\right)=D\left(Z_{12}\right).$$

Here

$$Z_{12}=1+\frac{|\eta_1-\eta_2|^2-|\vec{x}_1-\vec{x}_2|^2}{2\eta_1\eta_2}$$

is the hyperbolic distance. $Z_{12}=\cos L_{12}$, where L_{12} is the geodesic distance.

Discussion

- If $h_{i\mu}(p\eta)$ are related to BD modes via a Bogolubov rotation, then the dS isometry is respected at tree–level. These are so called α–vacua for $m > 0$.
- For BD $Z_{12} = 1$ (or $L_{12} = 0$) is the only singularity of $D\left(Z_{12}\right)$. The same UV singularity as in flat space.
- For other α–vacua we have extra $Z_{12} = -1$ singularity. That is due to wrong UV behavior — linear combination of $e^{\pm i\, p\, \eta}$.

Discussion

- What should one do with these growing with time quantum corrections?
- Note that if background field is on for long enough, then $\lambda^2(t - t_0) \sim 1$ and quantum corrections are of the same order as classical contributions; $n_p \sim 1$ — classical effects.

- We need to sum up leading corrections from all loops: sum $\left(\lambda^2(t - t_0)\right)^n$ and drop off e.g. $\lambda^4(t - t_0) \ll \lambda^2(t - t_0)$.
- Does the dependence on t_0 disappear after the summation?
- We did this summation in de Sitter space (expanding and contracting Poincare patches) and in QED with constant field background.

Summation of leading loop corrections

- To do the summation one has to solve the system of the Dyson–Schwinger equations for propagators and vertexes in the IR limit.
- In all the above listed cases vertexes do not receive growing with time corrections. Also retarded and advanced propagators do not secularly growing correction. Hence, to sum up leading corrections we put them to be of tree–level form.
- Ansatz for the Keldysh propagator:

$$D_p^K(t_1, t_2) = \left(\frac{1}{2} + n_p(t)\right) g_p(t_1)\, g_p^*(t_2) + \kappa_p(t)\, g_p(t_1)\, g_p(t_2) + c.c.$$

- As the result we obtain a system of Boltzmann type of equations for n_p and κ_p.
- Solution of these equations, with specified initial conditions, solves the problem of the summation of such corrections.

Summation of leading loop corrections

- Dyson–Schwinger equations are covariant under simultaneous Bogolyubov rotations of harmonics and n_p and κ_p.
- Hence, to sum up leading IR corrections we have to find harmonics for which there is such a solution that $\kappa_p = 0$. Otherwise there is no hope for stationary state!
- Inspiration from the non–stationary theory for superconductors.