Gravitational Back-Reaction in Cosmology

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Early Cosmology

- **basic variables**

  ▷ cosmologically relevant spacetimes:

$$ds^2 = -dt^2 + a^2(t)\, d\mathbf{x} \cdot d\mathbf{x}$$

  $a(t)$ is the *scale factor* (" radius of the universe")

  ▷ time variation of the scale factor gives instantaneous values of the *Hubble parameter* $H(t)$ and the *deceleration parameter* $q(t)$ or the convenient *1st slow-roll parameter* $\epsilon(t)$:

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)} = \frac{d}{dt} \ln a(t)$$

$$q(t) \equiv -\frac{\dot{a}(t)\, \ddot{a}(t)}{\dot{a}^2(t)} = -1 - \frac{\dot{H}(t)}{H^2(t)} \equiv -1 + \epsilon(t)$$

  ▷ current values: $\epsilon_0 \sim 0.47 \pm 0.03$

  $H_0 \sim (67.3 \pm 1.2)\, km/sec\, Mpc \sim 2.2 \times 10^{-18} Hz$

- **primordial inflation**

  ▷ inflation $\equiv$ accelerated expansion $\equiv (H > 0\ ,\ \epsilon < 1)$

  ▷ a period of accelerated expansion can be a simple physical solution to some basic issues of cosmology.

- $\pi.\chi.$ **the horizon problem**

  (can detect cosmological history epochs in which the observable universe was in thermal equilibrium to 1 part in 10^{-5})

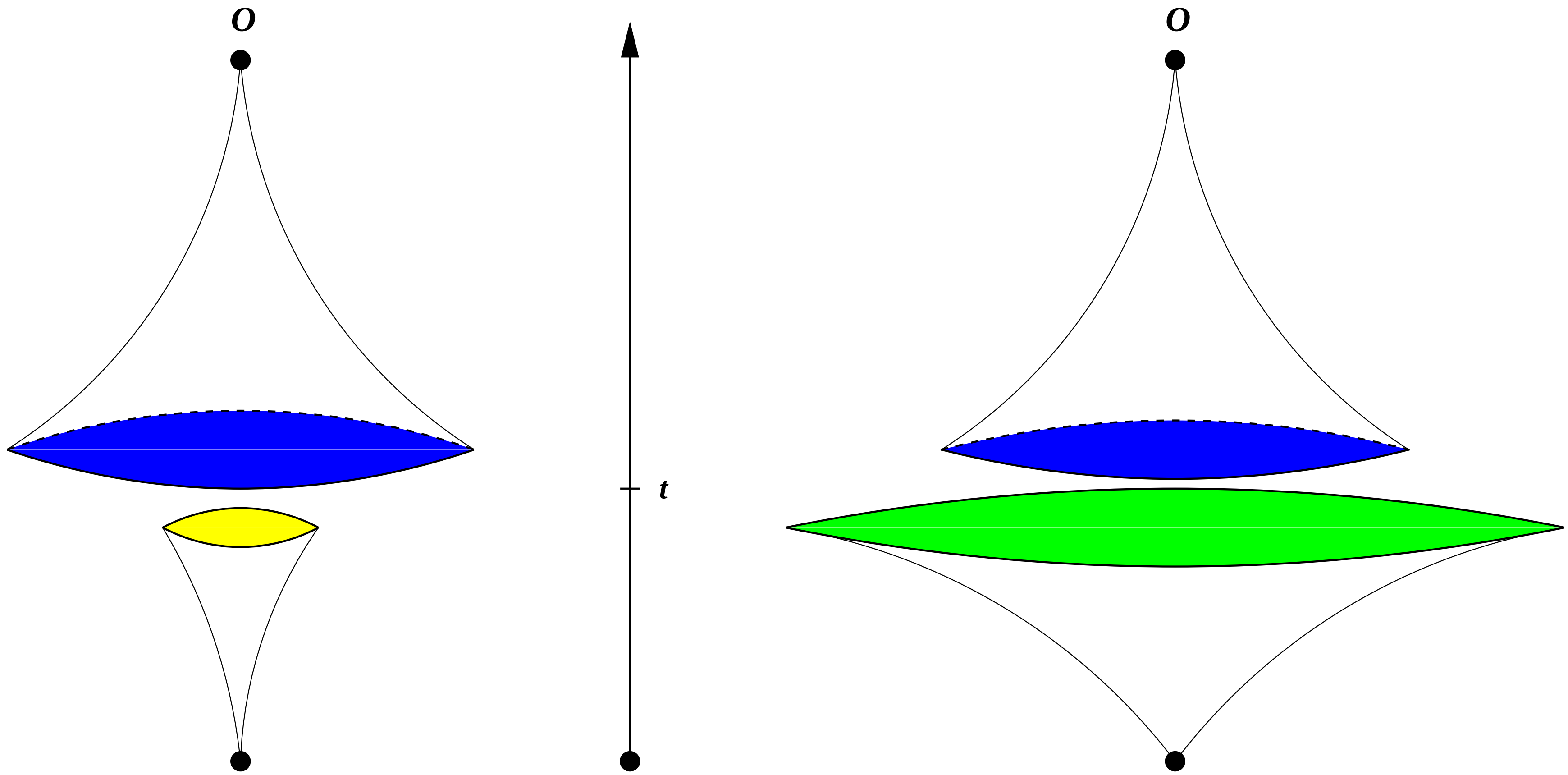
  ▷ horizon size is determined by light rays:

$$ds^2 = -dt^2 + a^2(t)\, dr^2 = 0 \quad \Rightarrow \quad dr = a^{-1}(t)\, dt$$

  ▷ for an event in some past time t:

$$R_{\text{past}} = \int_t^{t_0} dt\, a^{-1}(t) \quad , \quad R_{\text{future}} = \int_{t_i}^{t} dt\, a^{-1}(t)$$

- **the horizon problem**



Past & future horizons of an event at t **without** *(left)* and **with** *(right)* primordial inflation.

- **the horizon problem**

  ▷ when $q > 0$, ie when only radiation and matter domination, the *upper* limit dominates:

$$\Rightarrow \quad R_{\text{past}} \gg R_{\text{future}} \quad \Rightarrow \quad \textit{horizon problem}$$

  $\pi.\chi.$ at recombination or nucleosynthesis:

$$R^2_{\text{past}} \sim \left(2000 \text{ or } 10^9\right) R^2_{\text{future}}$$

  ▷ when $q < 0$, ie when inflation precedes radiation, the *lower* limit dominates:

$$\Rightarrow \quad a(t_i) \to 0 \text{ implies } R_{\text{future}} \to \infty \quad \Rightarrow \quad \textit{no horizon problem}$$

- **brief history via $\epsilon(t)$ and** N

  ▷ N $\equiv$ # of e-foldings wrt the end of inflation at t_I

  [$a(t) = a(t_I)\, e^{\mathrm{N}}$, $e^{60} \simeq 10^{26}$, $6000 Mpc$ **now** was $2m$ **then**]

  ▷ cosmological epochs:

  1. **late acceleration** : $\epsilon(t) \sim 0$, $59 \leq \mathrm{N} \leq 60$
  2. **matter domination** : $\epsilon(t) \sim 1.5$, $52 \leq \mathrm{N} \leq 59$
  3. **radiation domination** : $\epsilon(t) \sim 2$, $5 \leq \mathrm{N} \leq 52$
  4. **reheating** : $\epsilon(t) \sim ?$, $-5 \leq \mathrm{N} \leq 5$
  5. **primordial inflation** : $\epsilon(t) \sim 0$, $\mathrm{N} \leq -5$

  ▷ can observe early events, $\pi.\chi.$

  2. **recombination** at $t \approx 300,000\, years$
  3. **big bang nucleosynthesis** at $t \approx 1\, sec$

  ▷ current time is: $t_0 \approx 13,800,000,000\, years$

- **scalar-driven inflation**
  - ▷ what caused inflation?
  - ▷ the standard paradigm is the potential energy of a (minimally coupled) scalar field, the *inflaton*:

$$\mathcal{L} = \sqrt{-g}\left(-\frac{1}{2}\,g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi - V(\varphi) + \frac{R}{16\pi G}\right)$$

  - ▷ this can work if you are willing to ignore some issues

- **scalar-driven inflation issues**

$\triangleright$ The universe began with the scalar field approximately spatially homogeneous over more than a Hubble volume.

$\triangleright$ The scalar field potential is very flat.

$\triangleright$ The minimum of the scalar field potential has just the right value to leave the post-inflationary universe with almost zero vacuum energy.

$\triangleright$ The scalar field couples $strongly$ enough to ordinary matter to allow its kinetic energy to reheat the post-inflationary universe, but $not\ so\ strongly$ that loop corrections from ordinary matter to the effective potential endanger its flatness and nearly zero minimum.

- **gravity-driven inflation**

$\triangleright$ is there a more natural mechanism?

$\triangleright$ $fact$: gravitation plays the dominant role in shaping the cosmological evolution.

$\triangleright$ $question$: is there an "inflation causing mechanism" within gravitation?

$\triangleright$ $answer$: the presence of a bare and positive cosmological constant Λ provides such a mechanism; de Sitter spacetime is a solution of the field equations.

$\triangleright$ one should, therefore, first study its physical implications.

- **gravitational solution**

  ▷ de Sitter spacetime:

$$a(t) = e^{Ht} \quad , \quad H^2 = \frac{1}{3}\Lambda > 0$$

  ▷ its basic parameters are:

$$H(t) = H \quad , \quad q(t) = -1 \quad , \quad \epsilon(t) = 0$$

  ▷ physical lengths expand and momenta redshift:

$$x_{\text{phys}} = e^{Ht}\, x \quad , \quad k_{\text{phys}} = e^{-Ht}\, k$$

- **gravity-driven inflation: consequences**

$\triangleright$ because Λ is constant in $space$, no special initial condition is needed to start inflation.

$\triangleright$ however Λ is constant in $time$ as well.

$\triangleright$ classical physics cannot offer a natural mechanism for stopping inflation once it has begun.

$\triangleright$ $question$: can quantum physics provide such a mechanism?

$\triangleright$ $answer$: perhaps via real particle production.

- **particle production**

  ▷ *QFT* $\Rightarrow$ virtual pair $\Rightarrow$ $0 \to 2E$ (energy not conserved)

  ▷ *uncertainty principle* $\Rightarrow$ $\forall\, \Delta t \leq \frac{\hbar}{2E}$ violation is not detectable

  ▷ curved spacetimes $\Rightarrow$

  $$\text{energy :} \quad E(t, \mathbf{k}) = \sqrt{m^2\, c^4 + \hbar^2\, c^2\, |\mathbf{k}|^2\, a^{-2}(t)}$$

  $$\text{undetectability :} \quad \int_t^{t+\Delta t} dt'\, 2E(t', \mathbf{k}) \leq \hbar$$

  ▷ virtual particle lifetime Δt *increases* as:
  (i) the mass decreases,
  (ii) $a(t)$ grows

  ▷ when $\Delta t \to \infty$ we have real particle production

- **de Sitter particle production**

  ▷ massless lifetime bound:

$$2\,c\,|\mathbf{k}_{phys}| \times \Big[1 \,-\, e^{-H_0\,\Delta t} \Big] \;\leq\; H_0$$

  ▷ thus, massless virtual particles with:

$$c\,|\mathbf{k}_{phys}| \;=\; c\,|\frac{\mathbf{k}}{a(t)}| \;\leq\; H_0$$

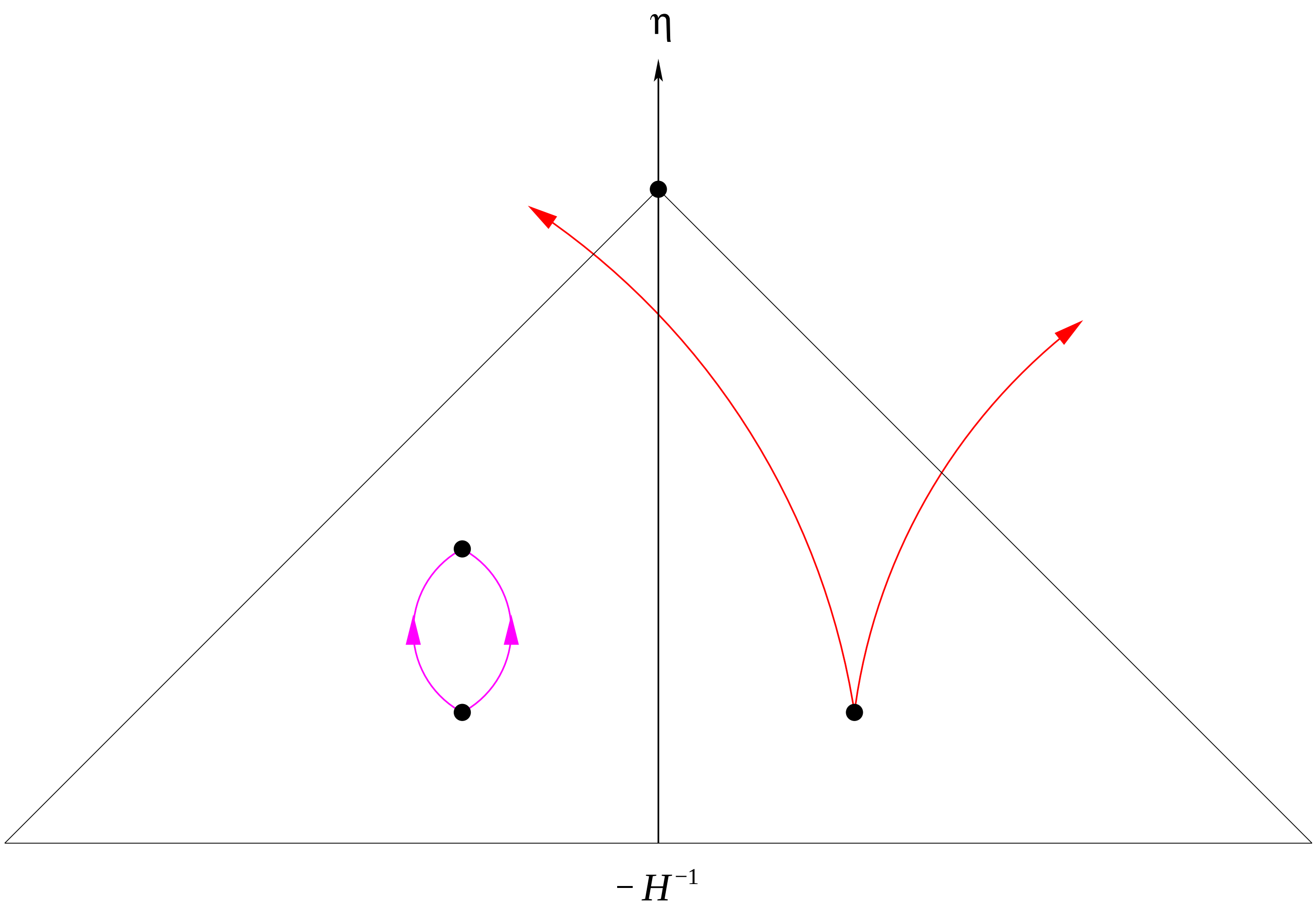
  may *never* recombine $\Rightarrow$ *real* particle production

  ▷ production rates for such massless particles are highly suppressed for growing $a(t)$ *unless* the particle has a *non*-conformally invariant classical lagrangian

(Conformal: $g_{\mu\nu} \to \Omega^2\, g_{\mu\nu}$, $A_\mu \to A_\mu$, $\psi_b \to \Omega^{-\frac{3}{2}}\, \psi_b$, $\phi \to \Omega^{-1}\, \phi$)

  ▷ particles with *all* these properties are: *graviton*, massless minimally-coupled scalar, ?

- **inflationary graviton production**



Short wavelength ($\lambda_{\text{phys}} < H^{-1}$) graviton pairs *(violet)* recombine.

Long wavelength ($\lambda_{\text{phys}} > H^{-1}$) graviton pairs (*red)* cannot recombine.

Gravitational Back-Reaction

- **classical back-reaction**
  - ▷ gravitation couples to *any* stress-energy source (classical or quantum)
  - ▷ $\pi.\chi.$ infrared graviton production out of the vacuum is a *quantum* process while the gravitational response to its presence is essentially *classical*
  - ▷ the non-linearity of gravitation is a hindrance for the analytical description of the response to the presence of the source via the gravitational field equations
  - ▷ it is however possible to obtain non-perturbative results for some static sources
  - ▷ it is also possible to obtain non-perturbative information on an initial value surface (IVS) for arbitrary initial value initial value data (IVD)

- **gravitational response to static sources**

  ▷ *simple newtonian example:*

  the mass of the Earth is a little less than the sum M_{bare} of the masses of its constituents owing to their negative gravitational interaction energy:

$$M_{\text{tot}} = M_{\text{bare}} + M_{\text{int}} \approx M_{\text{bare}} - \frac{3GM_{\text{bare}}^2}{5R}$$

(We assume the constituents are distributed uniformly through a sphere of radius R)

  the decrease works out to over 2×10^{15} kilograms

  ▷ *static point particle of bare mass M_0 in GR:* (ADM)

$$\begin{aligned} mass: &\quad M_0 \quad \longrightarrow \quad M = 0 \\ geometry: &\quad \text{Schwarzschild} \quad \longrightarrow \quad \text{flat}^* \end{aligned}$$

  * with a "glitch" at the origin

  gravity has the degrees of freedom to screen mass

▷ *static point particle of bare mass M_0 and charge e in GR:*

$$mass: \quad M_0 \quad \longrightarrow \quad M \approx \sqrt{\frac{e^2}{G}}$$
$$geometry: \quad \text{R}-\text{N} \quad \longrightarrow \quad \text{extremal R}-\text{N}$$

(newtonian argument gives the same answer: $\lim_{\varepsilon=0}\left\{M = M_0 + \frac{e^2}{\varepsilon} - \frac{GM^2}{\varepsilon}\right\}$)

no gravitational degrees of freedom to screen charge

▷ there is an *upper* limit to the screening: the complete elimination of the source

▷ there is *change* in the background geometry

▷ these are *non-perturbative* results

- **generic gravitational response on the IVS**

  ▷ the gravitational field equations are:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0 \qquad (\Lambda = 3H^2 > 0)$$

  ▷ they are supplemented by arbitrary initial value data

  ▷ we cannot determine the full response of the system under time evolution

  ▷ thus, we are interested in the full response of the system under infinitesimal time evolution

  ▷ **Question:** which observable can quantify the back-reaction in a useful way?

- **a measure of the back-reaction**

  ▷ an appropriate measure is the invariant expansion rate observable

  ▷ its local definition:

  $$\mathcal{H}[g](x) = \frac{1}{3} D^{\mu} V_{\mu}[g](x) = \frac{1}{3} \frac{1}{\sqrt{-g}} \partial_{\mu}[\sqrt{-g}\, g^{\mu\nu}\, V_{\nu}]$$

  is in terms of a timelike 4-velocity field V_{μ}:

  $$g^{\mu\nu}(x)\, V_{\mu}(x)\, V_{\nu}(x) = -1$$

  ▷ for our purposes we shall construct V_{μ} from a scalar functional Φ of the metric:

  $$\Box \Phi[g](x) = \frac{1}{\sqrt{-g}} \partial_{\mu}[\sqrt{-g}\, g^{\mu\nu}\, \partial_{\nu}\Phi] = 3H$$

  where H is the Hubble parameter

  ▷ On the IVS:

$$\Phi(t, \mathrm{x})|_{\mathrm{IVS}} = 0 \quad , \quad -g^{\alpha\beta}(t, \mathrm{x})\, \partial_{\alpha}\Phi(t, \mathrm{x})\, \partial_{\beta}\Phi(t, \mathrm{x})|_{\mathrm{IVS}} = 1$$

- **contrast with the 4-velocity field of the inflaton φ**

  ▷ the presence of φ automatically provides a clock in terms of the 4-velocity u_μ:

$$u_\mu \equiv -\frac{\partial_\mu \varphi}{\sqrt{-g^{\alpha\beta}\,\partial_\alpha\varphi\,\partial_\beta\varphi}}$$

  ▷ u_μ is timelike while φ is rolling down its potential

  ▷ by expanding about the classical inflaton $\bar{\varphi}(t)$ in a FRW background geometry and by fixing the time:

$$\varphi(t,\mathbf{x}) = \bar{\varphi}(t) + \delta\varphi(t,\mathbf{x}) \quad , \quad \delta\varphi(t,\mathbf{x}) = 0$$

  u_μ corresponds to the field of observers co-moving with the inflaton

- ▷ the resulting 4-velocity V_μ equals:

$$V_\mu[g](x) \equiv +\frac{\partial_\mu \Phi[g](x)}{\sqrt{-g^{\alpha\beta}(x)\,\partial_\alpha \Phi[g](x)\,\partial_\beta \Phi[g](x)}}$$

- ▷ under coordinate transformations that preserve the IVS:

$$\mathcal{H}[g'](x) = \mathcal{H}[g](x'^{\,-1}(x))$$

- ▷ **Question:** what is the value of $\mathcal{H}$ and its first time derivative $\partial_0 \mathcal{H}$ on the IVS?

- ▷ the natural coordinate system to use is the "3+1" (ADM)

- **the electromagnetic analogy**
  - ▷ the dynamical variable is $A_\mu(x)$ with 4 degrees of freedom
  - ▷ there is a $U(1)$ local invariance which when gauge fixed, eg $A_0(x) = 0$, reduces the physical degrees of fredom to 3
  - ▷ there is a constrained equation of motion which the IVD must obey, $\nabla \cdot \mathbf{E} = 0$ (Gauss's law), which further reduces the physical degrees of fredom to 2

- **the "3+1" decomposition**

  ▷ the "3+1" line element:

$$ds^2 = -g_{00}dt^2 + 2g_{0i}\,dtdx^i + g_{ij}\,dx^i dx^j$$
$$= -N^2 dt^2 + \gamma_{ij}\,(dx^i + N^i dt)(dx^j + N^j dt)$$

(N is the lapse, N^i is the shift vector, γ_{ij} is the spatial metric)

  ▷ the dynamical equations take the "3+1" form:

$$\partial_0 \gamma_{ij} = -2NK_{ij} + \bar{D}_i N_j + \bar{D}_j N_i$$
$$\partial_0 K_{ij} = -\bar{D}_i \bar{D}_j N + N^k \bar{D}_k K_{ij} + K_{ik}\bar{D}_j N^k + K_{jk}\bar{D}_i N^k$$
$$+N[\bar{R}_{ij} - 2K_{ik}K^k_j + KK_{ij} - 3H^2\gamma_{ij}]$$

(K_{ij} is the extrinsic curvature, $K \equiv \gamma^{ij}K_{ij}$ is its trace)

  ▷ the constraint equations take the "3+1" form:

$$\bar{R} + K^2 - K_{ij}K^{ij} = 6H^2$$
$$\bar{D}_j(K^{ij} - \gamma^{ij}K) = 0$$

- there are $12 = 6 + 6$ degrees of freedom that γ_{ij} and K_{ij} contain

- of these, only $4 = 2 + 2$ are dynamical and correspond to the two polarization states of the graviton

- the other are the 4 constrained degrees of freedom from the $1+3$ constraint equations, and the 4 gauge degrees of freedom from the initial coordinate system choices

- **the elements of $\mathcal{H}$ on the IVS**
  - ▷ the scalar functional Φ:

$$\Phi\,|_{\text{IVS}} = 0 \quad , \quad \Phi_{,\mu}\,|_{\text{IVS}} = -N\,\delta^0_\mu$$

  - ▷ the 4-velocity field V_μ:

$$V_\mu\,|_{\text{IVS}} = -N\,\delta^0_\mu \quad , \quad V^\mu\,|_{\text{IVS}} = +\frac{1}{N}\,\delta^\mu_0$$

- **$\mathcal{H}$ on the IVS**
  - ▷ in general:

$$\mathcal{H} = \frac{1}{3}D_\mu V^\mu = \frac{1}{3}\,\frac{1}{\sqrt{-g}}\,\partial_\mu\left(\frac{\sqrt{-g}g^{\mu\nu}\Phi_{,\nu}}{\sqrt{-g^{\alpha\beta}\Phi_{,\alpha}\Phi_{,\beta}}}\right)$$

$$= \frac{1}{3}\,\frac{\Box\Phi}{\sqrt{-g^{\alpha\beta}\Phi_{,\alpha}\Phi_{,\beta}}} + \frac{g^{\mu\nu}\Phi_{,\mu}\,g^{\rho\sigma}\Phi_{,\rho}\,D_\nu D_\sigma\Phi}{3\,(-g^{\alpha\beta}\Phi_{,\alpha}\Phi_{,\beta})^{\frac{3}{2}}}$$

- $\mathcal{H}$ **on the IVS**

▷ when restricted to the IVS:

$$
\begin{aligned}
\mathcal{H}\,|_{\text{IVS}} &= \frac{1}{3}(3H + g^{\mu\nu}\Phi_{,\mu}\, g^{\rho\sigma}\Phi_{,\rho}\, D_\nu D_\sigma \Phi)|_{\text{IVS}} \\
&= \frac{1}{3}(3H + N^2 g^{0\nu} g^{0\sigma}\, D_\nu D_\sigma \Phi)|_{\text{IVS}} \\
&= \frac{1}{3}\{3H + N^2[g^{00}g^{00}\, D_0 D_0 \Phi + 2g^{00}g^{0i}\, D_0 D_i \Phi \\
&\quad + g^{0i}g^{0j}\, D_i D_j \Phi]\}|_{\text{IVS}} \\
&= \frac{1}{3}\{3H + N^2[-N^{-2}(3H - g^{ij} D_i D_j \Phi) + g^{0i}g^{0j}\, D_i D_j \Phi]\}|_{\text{IVS}} \\
&= \frac{1}{3}\gamma^{ij} D_i D_j \Phi\,|_{\text{IVS}} = -\frac{1}{3}\gamma^{ij} K_{ij}\,|_{\text{IVS}} = -\frac{1}{3} K\,|_{\text{IVS}}
\end{aligned}
$$

▷ since K is a pure gauge degree of freedom we conclude that $\mathcal{H}$ can take *any* initial value of our choice

▷ a natural choice is $K = 0$, so that $\mathcal{H}\,|_{\text{IVS}} = 0$

- $\partial_0\mathcal{H}$ **on the IVS**

  ▷ to investigate the behaviour of $\mathcal{H}$ under infinitesimal time evolution, consider its first time derivative

  ▷ in general:

$$D_\mu\mathcal{H} = \frac{Hg^{\kappa\lambda}\Phi_{,\kappa}\, D_\mu D_\lambda\Phi}{(-g^{\alpha\beta}\Phi_{,\alpha}\Phi_{,\beta})^{\frac{3}{2}}} + \frac{Hg^{\kappa\lambda}\Phi_{,\kappa}\, g^{\rho\sigma}\Phi_{,\rho}\,(D_\lambda D_\sigma\Phi)\, g^{\gamma\delta}\Phi_{,\gamma}\, D_\mu D_\delta\Phi}{(-g^{\alpha\beta}\Phi_{,\alpha}\Phi_{,\beta})^{\frac{5}{2}}} + \frac{\frac{2}{3}g^{\kappa\lambda}\Phi_{,\kappa}\, g^{\rho\sigma}(D_\mu D_\rho\Phi)D_\lambda D_\sigma\Phi + \frac{1}{3}g^{\kappa\lambda}\Phi_{,\kappa}\, g^{\rho\sigma}\Phi_{,\rho}\, D_\mu D_\lambda D_\sigma\Phi}{(-g^{\alpha\beta}\Phi_{,\alpha}\Phi_{,\beta})^{\frac{3}{2}}}$$

  ▷ on the IVS:

$$D_\mu\mathcal{H}\,|_{\text{IVS}} = Hg^{0\lambda}(-ND_\mu D_\lambda\Phi) + N^2 g^{0\lambda}g^{0\sigma}D_\lambda D_\sigma(-Ng^{0\delta}D_\mu D_\delta\Phi) - \frac{2}{3}Ng^{0\lambda}g^{\rho\sigma}(D_\mu D_\rho\Phi)D_\lambda D_\sigma\Phi + \frac{1}{3}N^2 g^{0\lambda}g^{0\sigma}D_\mu D_\lambda D_\sigma\Phi$$

  ▷ we are interested in the $\mu = 0$ component

- $\partial_0 \mathcal{H}$ **on the IVS**

  ▷ a tedious but straightforward computation (in $K = 0$ gauge) gives:

$$\partial_0 \mathcal{H}\,|_{\text{IVS}} = N(H^2 - \frac{1}{3} K_{ij} K^{ij})$$

  ▷ the lapse function N sets the choice of physical time as opposed to the coordinate time t

  ▷ because $K_{ij} K^{ij}$ is positive the expansion rate *can* indeed diminish

  ▷ there seems to be no restriction on K_{ij} besides that its covariant divergence vanishes (in $K = 0$ gauge)

  ▷ of special interest are the correspondence limits of de Sitter spacetime, the case of $\Lambda = 0$, and flat spacetime

- **correspondence limit: de Sitter**

  ▷ *de Sitter in open coordinates* – the cosmological patch:

$$N = 1 \quad , \quad N^i = 0 \quad , \quad \gamma_{ij} = e^{2Ht}\,\delta_{ij}$$

  so that:

$$K_{ij} = -H\gamma_{ij} \quad , \quad K = -3H$$

  implying:

$$\mathcal{H}\,\big|_{\rm IVS} = H \quad , \quad \partial_0\mathcal{H}\,\big|_{\rm IVS} = 0$$

  ▷ the expansion rate started at H and stays at H

- **correspondence limit: de Sitter**

  ▷ *de Sitter in closed coordinates* – the full manifold:

$$N = 1 \quad , \quad N^i = 0 \quad , \quad \gamma_{ij} = H^{-2} \cosh^2(H\tau)\, \Omega_{ij}$$

  (Ω_{ij} is the angular line element)

  ▷ hence: $$K_{ij} = -H \tanh(H\tau)\, \gamma_{ij}$$

  ▷ the choice of $\tau = 0$ (corresponding to the throat of the hyperboloid) as the IVS implies that $K_{ij}\,|_{\mathrm{IVS}} = 0$

  ▷ therefore, the physical system started with no expansion and instantaneously began accelerating:

$$\mathcal{H}\,|_{\mathrm{IVS}} = 0 \quad , \quad \partial_0 \mathcal{H}\,|_{\mathrm{IVS}} = H^2$$

- **correspondence limit:** $\Lambda = 0$

  ▷ *The* $\Lambda = 3H^2 = 0$ *limit:*

$$\mathcal{H}\,|_{\text{IVS}} = 0 \quad , \quad \partial_0 \mathcal{H}\,|_{\text{IVS}} = -\frac{N}{3} K_{ij} K^{ij}$$

  ▷ contraction when $K_{ij} K^{ij} \neq 0$

- **correspondence limit: flat**

  ▷ *The flat spacetime limit:*

$$N = 1 \quad , \quad N^i = 0 \quad , \quad \gamma_{ij} = \delta_{ij}$$

  ▷ the expansion rate $\mathcal{H}$ vanishes for all time

- **upper bound**

  ▷ **Question:** does an upper bound to $K_{ij}K^{ij}$ exist?

  ▷ the dimensionality of K_{ij} is mass and only two mass scales are present: the Hubble parameter H and the Planck mass M_{PL}

  ▷ an upper bound on $K_{ij}K^{ij}$ cannot vanish with H vanishing since – in direct contradiction – there exist configurations with $H = 0$ & $K_{ij} > 0$

  ▷ thus, the upper bound must involve M_{Pl}, a situation which still allows cancellation of the H^2 term because $M_{\mathrm{Pl}}^2 \gg H^2$ ($\pi.\chi.$ $K_{ij}K^{ij} = M_{\mathrm{Pl}}^2 + H^2$)

  ▷ however, a definitive answer requires precision numerical analysis (SpEC, work in progress)

- **summary**

  in the presence of a positive cosmological constant:

  ▷ the initial value of the expansion rate can be gauged to zero

  ▷ the presence of initial gravitational waves with $K_{ij}K^{ij} \neq 0$ makes the initial time derivative of the expansion less than its value in de Sitter

  ▷ it seems that nothing precludes initial value data which make the initial first derivative of the expansion rate vanish

  ▷ there is a classical alternative to constant H expansion

  (all these results are non-perturbative)

Effective Theory

- **lagrangian**

$$\mathcal{L}_{\mathsf{GR}} = \frac{1}{16\pi G}(-2\Lambda + R)\sqrt{-g} + \text{(counterterms)}$$

- **2-parameter theory**
  - ▷ Newton's constant G
  - ▷ Cosmological constant Λ: take it to be "large" and positive

(Here "large" means a Λ induced by a matter scale M which can be as high as $10^{18}\,GeV$)

- **perturbation theory**
  - ▷ the *dimensionless* coupling constant is $G\Lambda$
  - ▷ even for $M = 10^{18}\,GeV$ it is *very* small:

$$G\Lambda = \left(\frac{M^4}{M_{\mathsf{Pl}}^4}\right) \sim 10^{-4}$$

- background geometry:

$$ds^2 = -dt^2 + a^2(t)\, d\mathbf{x} \cdot d\mathbf{x} \;=\; a^2(\eta)\, (-d\eta^2 + d\mathbf{x} \cdot d\mathbf{x})$$

- quantum-induced stress tensor:

$$8\pi G\, T_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda$$

- quantum-induced expansion rate:

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)} = \frac{a'(\eta)}{a^2(\eta)} = \sqrt{\frac{\Lambda}{3} + \frac{8\pi G}{3} \rho(t)}$$

- the de Sitter background

(the maximally symmetric solution of this theory)

$$a_{dS}(t) = e^{H_0\, t} = a_{dS}(\eta) = -\frac{1}{H_0\, \eta} \quad , \quad H_0^2 \equiv \frac{1}{3} \Lambda > 0$$

- **perturbative results for the de Sitter background**

  ▷ for large observation times *(infrared limit)*:

$$\rho_{dS}(t) = -H_0^4 \Big\{ \# \, (G\Lambda \, H_0 \, t) \, + \, O\Big[(G\Lambda \, H_0 \, t)^2\Big] \Big\}$$

$$p_{dS}(t) = +H_0^4 \Big\{ \# \, (G\Lambda \, H_0 \, t) \, + \, O\Big[(G\Lambda \, H_0 \, t)^2\Big] \Big\}$$

$$H_{dS}(t) = H_0 \Big\{ 1 - G\Lambda \Big\{ \#'(G\Lambda \, H_0 \, t) + O\Big[(G\Lambda \, H_0 \, t)^2\Big] \Big\}\Big\}$$

  ▷ the rate of expansion decreases by an amount which becomes *non-perturbatively* large at late times

  ▷ the perturbation theory breakdown occurs when the effective coupling constant becomes of order *one* :

$$G\Lambda \, H_0 \, t_1 \sim 1 \quad \Rightarrow \quad N_1 \equiv H_0 \, t_1 \sim \left(\frac{M_{\rm Pl}}{M}\right)^4 \gg 60$$

  (more than adequate # of inflationary e-foldings)

  ▷ the 2-loop effect becomes unreliable just when it starts to get interesting

▷ *all* loops become comparable when the *effective* coupling constant $\Phi = G\Lambda\, N \sim O(1)$ $\Rightarrow$ perturbative *breakdown*

(The breakdown occurs not because any single graviton-graviton interaction gets strong but rather because there are so many of them)

- **possible resummation?**
  - ▷ Starobinskiĩ developed a stochastic technique to sum the leading infrared behaviour of the $\lambda\varphi^4$ theory
  - ▷ in gravity, the general form of the infrared corrections is:

$$H(t)\big|_{\mathrm{IR}} \;=\; H_0 \left\{ 1 \,-\, \sum_{\ell=2}^{\infty} (G\Lambda)^{\ell} \sum_{k=0}^{\ell-1} c_{\ell k}\, (H_0\, t)^{k} \right\}$$

  - ▷ the leading infrared sum is:

$$H(t)\big|_{\text{leading IR}} \;=\; H_0 \left\{ 1 \,-\, G\Lambda \sum_{\ell=2}^{\infty} c_{\ell,\,\ell-1}\, (G\Lambda\, H_0\, t)^{\ell-1} \right\}$$

  - ▷ it is *unknown* how to implement the stochastic technique in gravity: derivative interactions, local invariance, etc

- **known resummation results**

  (in non-dynamical de Sitter background geometry)

  ▷ $\lambda\varphi^4$ theory shows a small, constant *increase* of the vacuum energy

  ▷ a scalar-fermion theory with a Yukawa interaction shows an unbounded *decrease* of the vacuum energy

  ▷ scalar QED shows a small, constant *decrease* of the vacuum energy

- **the physics of perturbative screening**

  ▷ *graviton degrees of freedom:* wave number **k** and polarization

  ▷ *graviton dynamics:* same as massless minimally-coupled scalar (up to $O(1)$)

  ▷ any mode **k** evolves *independently* as a SHO with time-dependent mass $m(t)$ and frequency $\omega(t)$:

$$L = \frac{1}{2} m \dot{q}^2 - \frac{1}{2} m \omega^2 q^2 \quad , \quad m(t) = a^3(t) \quad \& \quad \omega(t) = \frac{k}{a(t)}$$

  ▷ the exact solution is:

$$\ddot{q} + 3H\,\dot{q} + \frac{k^2}{a^2}\,q = 0 \quad \Rightarrow \quad q(t) = u(t,k)\,\alpha + u^*(t,k)\,\alpha^\dagger$$

$$u(t,k) = \frac{H}{\sqrt{2k^3}}\Big[1 - \frac{ik}{H\,a(t)}\Big] \exp\left(\frac{ik}{H\,a(t)}\right) \quad , \quad [\,\alpha\,,\,\alpha^\dagger\,] = 1$$

▷ the *number of gravitons* of wave vector $\mathbf{k}$ produced after N e-foldings of inflation is: $\mathcal{N}(\mathbf{k}) = \frac{\Lambda}{6k^2}\, e^{2N}$

▷ $\mathcal{N}(\mathbf{k})$ only reaches unity after λ_{phys} has redshifted to horizon scale H_0^{-1}:

$$\lambda_{\text{phys}} = \frac{2\pi}{k_{\text{phys}}} = \frac{2\pi}{k}\, e^N \;\geq\; H_0^{-1}$$

$\Rightarrow$ these particles are very infrared.

▷ the *total kinetic energy density* of IR gravitons is *constant*:

$$\rho_{\text{IR}} = e^{-3N} \int \frac{d^3k}{(2\pi)^3}\, \theta(\mathcal{N}(\mathbf{k}) - 1) \times \mathcal{N}(\mathbf{k}) \times k e^{-N} = \frac{\Lambda^2}{144\pi^2}$$

▷ the kinetic energy density ρ_{IR} sources a gravitational field

▷ as each newly-created graviton pair recedes, the intervening space is filled by their long-range gravitational potentials

- ▷ these potentials persist *even after* the gravitons that caused them have reached cosmological separations

- ▷ as more pairs are ripped apart, their potentials add to those already present $\Rightarrow$ the total potential Φ grows

- ▷ IR gravitons *potential* : $\Phi \approx -\hbar c^{-3}(G\Lambda)(H_0 t)$

- ▷ IR gravitons *interaction energy* : $\rho_{\text{int}} \sim \rho_{\text{IR}} \times \Phi$

- ▷ therefore screening occurs

- ▷ particle production is a 1-loop effect, gravitational response to its presence is a 2-loop effect *(QFT result)*

Non-Perturbative Screening

- **the problem**

  ▷ the perturbative screening mechanism becomes unreliable when it starts to get interesting

  ▷ *all* loops become comparable when the effective coupling constant Φ becomes of order one, and perturbation theory breaks down

  ▷ how big *should* the screening get and how big *can* it get?

- **the question**
  - ▷ the total energy density consists of 3 parts:

$$\rho_{\text{tot}} = \rho_\Lambda + \rho_{\text{IR}} + \rho_{\text{int}}$$

  - ▷ ρ_Λ is much bigger than ρ_{IR}:

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} \approx M^2 M_{\text{Pl}}^2 \quad , \quad \rho_{\text{IR}} \approx \Lambda^2 \approx M^4 \quad \Rightarrow \quad \rho_\Lambda \gg \rho_{\text{IR}}$$

  - ▷ *if* we want $\rho_{\text{tot}} \approx 0$, we *must* have:

$$|\rho_{\text{int}}| \gg \rho_{\text{IR}}$$

  - ▷ **Question:** is this possible within gravitation?
  - ▷ a few remarks are in order before attempting to answer

- **1st remark: small is not zero**

  ▷ super-horizon gravitons have kinetic energy ke^{-N} which redshifts towards zero as the universe expands

  ▷ however, this is balanced by the fact that a *lot* of them are produced:

$$total\,\# \;=\; (\frac{3}{\Lambda})^{\frac{3}{2}} \int \frac{d^3k}{(2\pi)^3}\,\theta(\mathcal{N}(\mathbf{k})-1)\times\mathcal{N}(\mathbf{k}) \;=\; \frac{e^{3N}}{2^{\frac{5}{2}}\pi^2}$$

  for $N \sim (G\Lambda)^{-1} \gtrsim 10^4$ this number is *staggering*

- **2nd remark: gravitational interactions screen their sources**

  (discussed earlier)

- **3rd remark: big volume can beat small density**

  ▷ consider the total energy density ρ_{tot} produced by a static energy density ρ_{bare} distributed throughout a sphere of radius R

  ▷ use the Newtonian formula assuming it is the total mass $\frac{4}{3}\pi R^3\,\rho_{\text{tot}}$ that gravitates *(ADM)*:

$$\rho_{\text{tot}} \approx \rho_{\text{bare}} - \frac{4\pi R^2\,G\rho_{\text{tot}}^2}{5} \quad \Rightarrow$$

$$\rho_{\text{tot}} \approx \frac{5}{8\pi G R^2}\left[\sqrt{1+\frac{16\pi R^2\,G\rho_{\text{bare}}}{5}}-1\right]$$

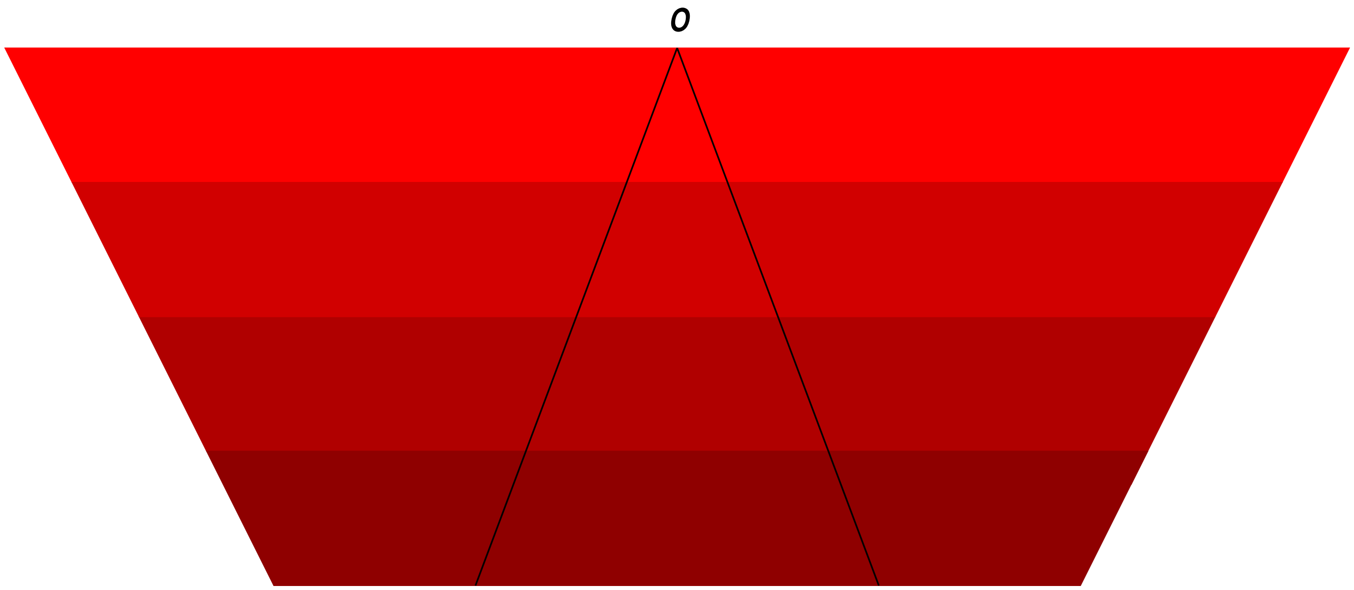
  as $R \to +\infty$ the screening becomes *total:* $\rho_{\text{tot}} \to 0$, *independent* of how small ρ_{bare} is

  ▷ even a small energy density can experience *total* screening if it interacts over a sufficiently large volume

- **4th remark: causality accesses early times**
  - ▷ the cosmological case requires *much more* than the static "source screening" upper limit of: $|\rho_{\rm int}| = \rho_{\rm IR}$
    how can $|\rho_{\rm int}| \gg \rho_{\rm IR}$ occur?
  - ▷ cosmology is *not* static, the universe evolves *causally*
  - ▷ $|\rho_{\rm int}| \gg \rho_{\rm IR}$ *can* occur at late times by means of gravitational potentials which were sourced far back in the past light-cone, when screening was still insignificant
  - ▷ instead of the effect being too weak, it is actually prone to grow too strong because the past light-cone opens up as the expansion rate slows down
  - ▷ to see this compare the volume of the past light-cone – in synchronous gauge – for inflation and for flat space:

$$V_{\rm infl} = \frac{4\pi}{\sqrt{3\Lambda^3}}\, ct + O(1) \qquad , \qquad V_{\rm flat} = \frac{\pi}{3}\,(ct)^4$$

- **local observer**



The past light-cone of the local observer $\mathcal{O}$ accesses the diffuse energy density of super-horizon gravitons of progressively smaller wavelengths.

- **the mechanism**

  ▷ Λ-driven inflation seems to be natural *iff* some mechanism could be found to eventually halt it

  ▷ quantum gravity can provide such a mechanism in the form of the back-reaction to infrared virtual gravitons which are continually ripped out of the vacuum during inflation

  ▷ these gravitons possess a *negative* gravitational potential energy (ρ_{int}) and a *positive* kinetic energy (ρ_{IR}) both of which contribute to the total vacuum energy

  ▷ the kinetic energy is present immediately whereas the potential energy must build up causally as more and more infrared gravitons come into contact with one another

- ▷ although the kinetic energy density is small, the potential energy can be large because it derives from interactions over the enormous volume of the past light-cone

- ▷ because screening was small in the distant past, the negative potential energy can vastly exceed the positive kinetic energy which sourced it

- ▷ a large bare Λ can be screened by the vacuum polarization of a sea of infrared gravitons produced during primordial inflation

Epilogue

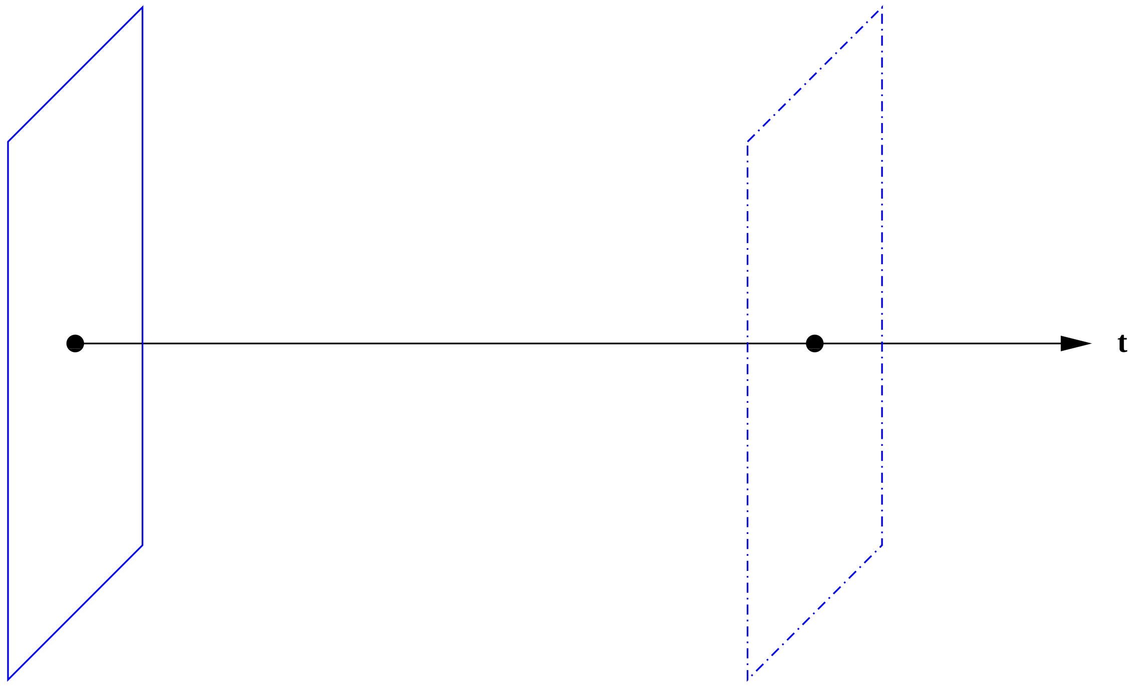
▷ **(classical is enough)** For the physical situation at hand, once inflationary gravitons are produced their effect on cosmological evolution can be understood in completely classical terms.

▷ **(gravity is attractive)** Since gravitational waves attract each other and act to diminish expansion, when enough of them are present they *can* completely stop it and even reverse the trend leading to collapse.

▷ **(classical state)** It should be possible to find a classical configuration of gravitational waves such that the universe holds itself together, against the tendency for de Sitter expansion.

▷ **(initial value data)** We do not know what initial value data describe this classical configuration of gravitons. We do however know from our present *non-perturbative* analysis that initial value data *exist* for which the universe does not succumb to accelerated expansion.

▷ it would be very significant to explicitly verify that inflationary graviton production eventually forms a state of the kind that stops inflation



The initial value surface and its cousin after inflationary evolution

▷ **(stability)** Such a classical state will almost certainly *not* be completely stable but if it is formed from the steady production of infrared gravitons over a prolonged period of inflation, the decay time would almost certainly be *longer* than the lifetime of the universe.

▷ **(in one sentence)** The cosmological evolution of the universe can be a *sustained gravitational collapse*.