How MMC Scalars and Gravitons Affect Gravity

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arXiv: 1508.xxxxx SP, Prokopec and Woodard
arXiv: 1403.0896 Leonard, SP, Prokopec and Woodard
arXiv: 1109.4187, 1101.5804, 1007.2662 SP and Woodard

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Quantum Fields and IR Issues in de Sitter Space, Brazil, Natal

Outline

- Reminder: Why we consider MMC scalars and Gravitons
- One loop contributions to the graviton self-energy from MMC scalar: covariant and noncovariant representations
- Quantum corrected linearized Einstein field eqn for dynamical gravitons and the force of gravity
- Summary and Discussion

Reminder: Why we consider MMC scalars and Gravitons

- Expansion of spacetime can lead to particle creation by delaying the annihilation of virtual pairs ripped out of the vacuum:

  Schrödinger, Physica 6 (1939) 899

- The effect is maximum if
  - the expansion is accelerated: de Sitter space (inflation)
  - the virtual particles are massless
  - no conformal symmetry.

  Parker 1968 - 1971, see the presentations of Miao and Tsamis

- Only two types of particles with no mass & no conformal symmetry:
  - gravitons
  - massless, minimally coupled (MMC) scalar

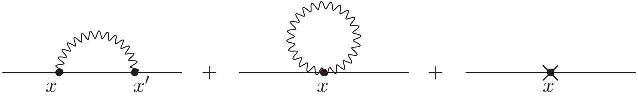
- Occupation number of these two grows with time:

  $N(k,t) = \left(\frac{Ha(t)}{2k}\right)^2$ see Woodard's presentation

Two models of GR + MMC scalar

- scalar self-mass-squared $-iM^2(x;x')$:

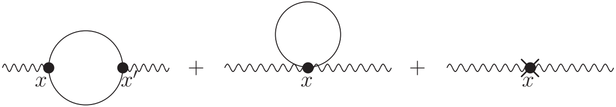
  Kahya and Woodard, arXiv: 0709.0536, 0710.5282



loop corrections to kinematics

- graviton self-energy $-i[^{\mu\nu}\Sigma^{\rho\sigma}](x;x')$:

  SP and Woodard, arXiv:1101.5804, 1109.4187 & Leonard, SP, Prokopec and Woodard, arXiv:1403.0896



loop corrections to kinematics and GR forces

Linearized effective field equations

- $\mathcal{L} = \frac{1}{16\pi G}\Big[R - 2\Lambda\Big]\sqrt{-g} - \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g}$
- Linearize: $g_{\mu\nu} = \overline{g}_{\mu\nu} + \kappa h_{\mu\nu}, \quad \overline{g}_{\mu\nu} = a^2\eta_{\mu\nu}, \quad a = -\frac{1}{H\eta}$
- Linearized quantum effective field equations for gravitons:

$$\sqrt{-\overline{g}}\,\mathcal{D}^{\mu\nu\rho\sigma}h_{\rho\sigma}(x) - \int d^4x'\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big](x;x')h_{\rho\sigma}(x') = \frac{1}{2}\kappa\sqrt{-\overline{g}}\,T^{\mu\nu}_{\rm lin}(x)\ ,$$

- Lichnerowicz operator: linearized Einstein tensor, $R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}(R - 2\Lambda)$ acting on $h_{\mu\nu}$

$$\mathcal{D}^{\mu\nu\rho\sigma} \equiv D^{(\rho}\overline{g}^{\sigma)(\mu}D^{\nu)} - \frac{1}{2}\Big[\overline{g}^{\rho\sigma}D^\mu D^\nu + \overline{g}^{\mu\nu}D^\rho D^\sigma\Big] + \frac{1}{2}\Big[\overline{g}^{\mu\nu}\overline{g}^{\rho\sigma} - \overline{g}^{\mu(\rho}\overline{g}^{\sigma)\nu}\Big]D^2 + (D-1)\Big[\frac{1}{2}\overline{g}^{\mu\nu}\overline{g}^{\rho\sigma} - \overline{g}^{\mu(\rho}\overline{g}^{\sigma)\nu}\Big]H^2\ ,$$

- $-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big](x;x')$: graviton self-energy = 1PI graviton 2-point function
  : quantum correction to the Lichnerowicz operator

1. Calculate the graviton self-energy
2. Solve the quantum corrected field eqn for dynamical gravitons and the force of gravity

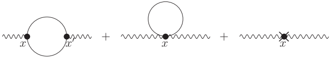
One Loop Scalar Contributions to Graviton Self-Energy

- Interaction between MMC scalars and gravitons:

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} \\ &= -\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi\overline{g}^{\mu\nu}\sqrt{-\overline{g}} - \frac{\kappa}{2}\partial_\mu\varphi\partial_\nu\varphi\Big(\frac{1}{2}h\overline{g}^{\mu\nu} - h^{\mu\nu}\Big)\sqrt{-\overline{g}} \\ &\quad -\frac{\kappa^2}{2}\partial_\mu\varphi\partial_\nu\varphi\bigg\{\Big[\frac{1}{8}h^2 - \frac{1}{4}h^{\rho\sigma}h_{\rho\sigma}\Big]\overline{g}^{\mu\nu} - \frac{1}{2}hh^{\mu\nu} + h^{\mu}_{\ \rho}h^{\rho\nu}\bigg\}\sqrt{-\overline{g}} + O(\kappa^3)\ .\end{aligned}$$

- One loop contribution to the graviton self-energy from MMC scalars on de Sitter background:

$$\begin{aligned}-i[^{\mu\nu}\Sigma^{\rho\sigma}](x;x') &= \frac{1}{2}\sum_{I=1}^{2}T_I^{\mu\nu\alpha\beta}(x)\sum_{J=1}^{2}T_J^{\rho\sigma\gamma\delta}(x') \times \partial_\alpha\partial'_\gamma i\triangle(x;x') \times \partial_\beta\partial'_\delta i\triangle(x;x') \\ &\quad + \frac{1}{2}\sum_{I=1}^{4}F_I^{\mu\nu\rho\sigma\alpha\beta}(x) \times \partial_\alpha\partial'_\beta i\triangle(x;x') \times \delta^D(x-x') \\ &\quad + 2\sum_{I=1}^{2}C_I^{\mu\nu\rho\sigma}(x) \times \delta^D(x-x')\ .\end{aligned}$$



MMC scalar propagator

- The MMC scalar propagator obeys $\partial_\mu\Big[\sqrt{-\bar{g}}\;\bar{g}^{\mu\nu}\partial_\nu\Big] i\triangle(x;x') = i\delta^D(x-x')$
- No de Sitter invariant solution for the propagator
- A solution preserving the homogeneity and isotropy:

$$\begin{aligned} i\triangle(x;x') &= A\Big(y(x;x')\Big) + \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\ln(aa') \\ &= \text{de Sitter invariant function of } y + \text{de Sitter breaking term} \end{aligned}$$

$$\text{where}\quad A(y) \equiv \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\Bigg\{\frac{\Gamma(\frac{D}{2})}{\frac{D}{2}-1}\Big(\frac{4}{y}\Big)^{\frac{D}{2}-1} + \frac{\Gamma(\frac{D}{2}+1)}{\frac{D}{2}-2}\Big(\frac{4}{y}\Big)^{\frac{D}{2}-2} - \pi\cot\Big(\frac{\pi D}{2}\Big)\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}$$
$$+\sum_{n=1}^{\infty}\Bigg[\frac{1}{n}\frac{\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2})}\Big(\frac{y}{4}\Big)^n - \frac{1}{n-\frac{D}{2}+2}\frac{\Gamma(n+\frac{D}{2}+1)}{\Gamma(n+2)}\Big(\frac{y}{4}\Big)^{n-\frac{D}{2}+2}\Bigg]\Bigg\}.$$

- The de Sitter breaking term drops differentiated by $\partial_\alpha\partial'_\beta$
- $y(x;x') \equiv aa'H^2\Delta x^2$: the de Sitter invariant length function
  $Z = \cos(\mu) = 1 - \frac{y}{2}$: Fröb and Verdaguer
  μ: the geodesic distance

Primitive Diagrams

- Contribution from 4-point vertices

$$-i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right]_{4\text{pt}}(x;x') \equiv \frac{1}{2}\sum_{I=1}^{4} F_I^{\mu\nu\rho\sigma\alpha\beta}(x) \times \partial_\alpha \partial'_\beta i\triangle(x;x') \times \delta^D(x-x')$$

$$= \Big(\frac{D-4}{4}\Big)\frac{i\kappa^2 H^D}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D)}{\Gamma(\frac{D}{2}+1)}\sqrt{-\overline{g}}\left\{\frac{1}{2}\overline{g}^{\mu\nu}\overline{g}^{\rho\sigma} - \overline{g}^{\mu(\rho}\overline{g}^{\sigma)\nu}\right\}\delta^D(x-x') = 0 \text{ for } D=4$$

- Contribution from 3-point vertices

$$-i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right]_{3\text{pt}}(x;x') = \sqrt{-\overline{g}}\sqrt{-\overline{g}'}\Bigg\{\frac{\partial^2 y}{\partial x_\mu \partial x'_{(\rho}}\frac{\partial^2 y}{\partial x'_{\sigma)}\partial x_\nu} \times \alpha(y) + \frac{\partial y}{\partial x_{(\mu}}\frac{\partial^2 y}{\partial x_{\nu)}\partial x'_{(\rho}}\frac{\partial y}{\partial x'_{\sigma)}} \times \beta(y)$$

$$+\frac{\partial y}{\partial x_\mu}\frac{\partial y}{\partial x_\nu}\frac{\partial y}{\partial x'_\rho}\frac{\partial y}{\partial x'_\sigma} \times \gamma(y) + \overline{g}^{\mu\nu}\overline{g}'^{\rho\sigma}H^4 \times \delta(y) + \left[\overline{g}^{\mu\nu}\frac{\partial y}{\partial x'_\rho}\frac{\partial y}{\partial x'_\sigma} + \frac{\partial y}{\partial x_\mu}\frac{\partial y}{\partial x_\nu}\overline{g}'^{\rho\sigma}\right]H^2 \times \epsilon(y)\Bigg\}$$

$$\propto \frac{1}{y^4} \sim \frac{1}{\Delta x^8} \text{ in } D=4 \quad \longrightarrow \int d^4x' \frac{1}{\Delta x^8}$$ quartically divergent

Correspondence with flat space limit

- Flat space limit $H \to 0$:
  $\Delta x^0 \longrightarrow t - t'$, $y(x;x') \longrightarrow H^2 \Delta x^2$
  $\frac{\partial y}{\partial x_\mu} \longrightarrow 2H^2 \Delta x^\mu$, $\frac{\partial y}{\partial x'_\nu} \longrightarrow -2H^2 \Delta x^\nu$, $\frac{\partial y^2}{\partial x_\mu \partial x'_\nu} \longrightarrow -2H^2 \eta^{\mu\nu}$ gives

$$-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big]_{\text{flat}}(x;x') = \frac{\kappa^2 \Gamma^2(\frac{D}{2})}{16\pi^D} \Big\{ \eta^{\mu(\rho}\eta^{\sigma)\nu} \times \Big[-\frac{2}{\Delta x^{2D}}\Big] + \Delta x^{(\mu}\eta^{\nu)(\rho}\Delta x^{\sigma)} \times \Big[\frac{4D}{\Delta x^{2D+2}}\Big]$$
$$+\Delta x^\mu \Delta x^\nu \Delta x^\rho \Delta x^\sigma \times \Big[-\frac{2D^2}{\Delta x^{2D+4}}\Big] + \eta^{\mu\nu}\eta^{\rho\sigma} \times \Big[-\frac{1}{2}\frac{(D^2-D-4)}{\Delta x^{2D}}\Big]$$
$$+\Big[\eta^{\mu\nu}\Delta x^\rho \Delta x^\sigma + \Delta x^\mu \Delta x^\nu \eta^{\rho\sigma}\Big] \times \Big[\frac{D(D-2)}{\Delta x^{2D+2}}\Big] \Big\}$$

- This agrees with G. 't Hooft and M. Veltman, Ann. Inst. Henri Poincaré **XX** (1974) 69.

Correspondence with stress tensor correlators

- The graviton self-energy is related to the 2-point correlator of the stress tensor as

$$-i\left[{}^{\mu\nu}\Delta^{\rho\sigma}\right](x;x') = -\frac{1}{4}\kappa^2\sqrt{-\overline{g}(x)}\,\sqrt{-\overline{g}(x')}\Big\langle\Omega\Big|\delta\mathcal{T}^{\mu\nu}(x)\delta\mathcal{T}^{\rho\sigma}(x')\Big|\Omega\Big\rangle + O(\kappa^4)$$

- The stress tensor correlator obtained by Perez-Nadal, Roura and Verdaguer (JCAP **1005** (2010) 036, arXiv:0911.4870) agrees with out result.

$$\Big\langle\Omega\Big|\delta\mathcal{T}^{\mu\nu}(x)\delta\mathcal{T}^{\rho\sigma}(x')\Big|\Omega\Big\rangle = F_{\mu\nu\rho\sigma} = P(\mu)n_\mu n_\nu n_\rho n_\sigma + Q(\mu)(n_\mu n_\nu \overline{g}_{\rho\sigma} + n_\rho n_\sigma \overline{g}_{\mu\nu})$$
$$+R(\mu)(n_\mu n_\rho \overline{g}_{\nu\sigma} + n_\nu n_\sigma \overline{g}_{\mu\rho} + n_\mu n_\sigma \overline{g}_{\nu\rho} + n_\nu n_\rho \overline{g}_{\mu\sigma}) + S(\mu)(\overline{g}_{\mu\rho}\overline{g}_{\nu\sigma} + \overline{g}_{\nu\rho}\overline{g}_{\mu\sigma}) + T(\mu)\overline{g}_{\mu\nu}\overline{g}_{\rho\sigma}$$

- Note: their 5 basis tensors are converted into ours as

$$\begin{aligned}
n_a n_b n_{c'} n_{d'} &= \frac{1}{H^4(4y-y^2)^2}\frac{\partial y}{\partial x^a}\frac{\partial y}{\partial x^b}\frac{\partial y}{\partial x'^{c'}}\frac{\partial y}{\partial x'^{d'}} \;, \\
n_a n_b \overline{g}_{c'd'} + n_{c'} n_{d'} \overline{g}_{ab} &= \frac{1}{H^2(4y-y^2)}\left[\overline{g}_{ab}\frac{\partial y}{\partial x'^{c'}}\frac{\partial y}{\partial x'^{d'}} + \frac{\partial y}{\partial x^a}\frac{\partial y}{\partial x^b}\overline{g}_{c'd'}\right] , \\
4n_{(a}\overline{g}_{b)(c'}n_{d')} &= -\frac{2}{H^4(4y-y^2)}\frac{\partial y}{\partial x^{(a}}\frac{\partial^2 y}{\partial x^{b)}\partial x'^{(c'}}\frac{\partial y}{\partial x'^{d')}} - \frac{2}{H^4(4y-y^2)(4-y)}\frac{\partial y}{\partial x^a}\frac{\partial y}{\partial x^b}\frac{\partial y}{\partial x'^{c'}}\frac{\partial y}{\partial x'^{d'}} \;, \\
2\overline{g}_{a(c'}\overline{g}_{d')b} &= \frac{1}{2H^4}\frac{\partial^2 y}{\partial x^a \partial x'^{(c'}}\frac{\partial^2 y}{\partial x^{d')}\partial x'^{b}} + \frac{1}{H^4(4-y)}\frac{\partial y}{\partial x^{(a}}\frac{\partial^2 y}{\partial x^{b)}\partial x'^{(c'}}\frac{\partial y}{\partial x'^{d')}} \\
&\quad + \frac{1}{2H^4}\frac{1}{(4-y)^2}\frac{\partial y}{\partial x^a}\frac{\partial y}{\partial x^b}\frac{\partial y}{\partial x'^{c'}}\frac{\partial y}{\partial x'^{d'}} \;, \\
\overline{g}_{ab}\overline{g}_{c'd'} &= \overline{g}_{ab}\overline{g}_{c'd'}
\end{aligned}$$

One Loop Counterterms

- For quantum gravity at one loop order the necessary counterterms are R^2 and C^2 first derived by t Hooft and Veltman, 1974

  S.D.D. $= 4 \rightarrow 4\partial$'s, with general coord. invariance 3 possibilities:

  $R^2, R^{\mu\nu}R_{\mu\nu}, R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$ with the Gauss-Bonet relation, only 2 are linearly indep.

- For calculational convenience, reorganize R^2 as

  $R^2 = \Big[R - D(D-1)H^2\Big]^2 + 2D(D-1)H^2R - D^2(D-1)^2H^4$
  So we employ four counterterms:

$$\Delta\mathcal{L}_1 \equiv c_1\Big[R - D(D-1)H^2\Big]^2\sqrt{-g}\ , \Delta\mathcal{L}_3 \equiv c_3H^2\Big[R - (D-1)(D-2)H^2\Big]\sqrt{-g}\ , \Delta\mathcal{L}_4 \equiv c_4H^4\sqrt{-g}\ .$$
$$\Delta\mathcal{L}_2 \equiv c_2C^{\alpha\beta\gamma\delta}C_{\alpha\beta\gamma\delta}\sqrt{-g}\ ,$$

  Note: the divergences can really be eliminated with just $\Delta\mathcal{L}_2$ and the particular linear combination of $\Delta\mathcal{L}_1$, $\Delta\mathcal{L}_3$ and $\Delta\mathcal{L}_4$ which is proportional to $R^2\sqrt{-g}$.

- We define two 2nd order differential operators by expanding the scalar and Weyl curvatures around de Sitter background

$$R - D(D-1)H^2 \equiv \mathcal{P}^{\mu\nu}\kappa h_{\mu\nu} + O(\kappa^2h^2)\ ,$$
$$C_{\alpha\beta\gamma\delta} \equiv \mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}\kappa h_{\mu\nu} + O(\kappa^2h^2)\ .$$

Counterterms in terms of two projection operators

- Spin zero projection operator:

$$\mathcal{P}^{\mu\nu} = D^{\mu} D^{\nu} - \overline{g}^{\mu\nu} \Big[D^2 + (D-1)H^2 \Big] \; ,$$

- Spin two projection operator

$$\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta} = \mathcal{D}^{\mu\nu}_{\alpha\beta\gamma\delta} + \frac{1}{D-2} \Big[\overline{g}_{\alpha\delta} \mathcal{D}^{\mu\nu}_{\beta\gamma} - \overline{g}_{\beta\delta} \mathcal{D}^{\mu\nu}_{\alpha\gamma} - \overline{g}_{\alpha\gamma} \mathcal{D}^{\mu\nu}_{\beta\delta} + \overline{g}_{\beta\gamma} \mathcal{D}^{\mu\nu}_{\alpha\delta} \Big] + \frac{1}{(D-1)(D-2)} \Big[\overline{g}_{\alpha\gamma} \overline{g}_{\beta\delta} - \overline{g}_{\alpha\delta} \overline{g}_{\beta\gamma} \Big] \mathcal{D}^{\mu\nu} \; ,$$

where we define,

$$\begin{aligned} \mathcal{D}^{\mu\nu}_{\alpha\beta\gamma\delta} &\equiv \frac{1}{2} \Big[\delta^{(\mu}_{\alpha} \delta^{\nu)}_{\delta} D_{\gamma} D_{\beta} - \delta^{(\mu}_{\beta} \delta^{\nu)}_{\delta} D_{\gamma} D_{\alpha} - \delta^{(\mu}_{\alpha} \delta^{\nu)}_{\gamma} D_{\delta} D_{\beta} + \delta^{(\mu}_{\beta} \delta^{\nu)}_{\gamma} D_{\delta} D_{\alpha} \Big] \; , \\ \mathcal{D}^{\mu\nu}_{\beta\delta} &\equiv \overline{g}^{\alpha\gamma} \mathcal{D}^{\mu\nu}_{\alpha\beta\gamma\delta} = \frac{1}{2} \Big[\delta^{(\mu}_{\delta} D^{\nu)} D_{\beta} - \delta^{(\mu}_{\beta} \delta^{\nu)}_{\delta} D^2 - \overline{g}^{\mu\nu} D_{\delta} D_{\beta} + \delta^{(\mu}_{\beta} D_{\delta} D^{\nu)} \Big] \; , \\ \mathcal{D}^{\mu\nu} &\equiv \overline{g}^{\alpha\gamma} \overline{g}^{\beta\delta} \mathcal{D}^{\mu\nu}_{\alpha\beta\gamma\delta} = D^{(\mu} D^{\nu)} - \overline{g}^{\mu\nu} D^2 \; . \end{aligned}$$

Counterterms in terms of two projection operators

- The counterterms are expressed in terms of these two operators:

$$\frac{i\delta\Delta S_1}{\delta h_{\mu\nu}(x)\delta h_{\rho\sigma}(x')}\Big|_{h=0} = 2c_1\kappa^2\sqrt{-\overline{g}}\,\mathcal{P}^{\mu\nu}\mathcal{P}^{\rho\sigma}i\delta^D(x-x') \longrightarrow 2c_1\kappa^2\Pi^{\mu\nu}\Pi^{\rho\sigma}i\delta^D(x-x'),$$

$$\frac{i\delta\Delta S_2}{\delta h_{\mu\nu}(x)\delta h_{\rho\sigma}(x')}\Big|_{h=0} = 2c_2\kappa^2\sqrt{-\overline{g}}\,\overline{g}^{\alpha\kappa}\overline{g}^{\beta\lambda}\overline{g}^{\gamma\theta}\overline{g}^{\delta\phi}\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}i\delta^D(x-x')$$

$$\longrightarrow 2c_2\kappa^2\Big(\frac{D-3}{D-2}\Big)\Big[\Pi^{\mu(\rho}\Pi^{\sigma)\nu} - \frac{\Pi^{\mu\nu}\Pi^{\rho\sigma}}{D-1}\Big]i\delta^D(x-x')$$

$$\frac{i\delta\Delta S_3}{\delta h_{\mu\nu}(x)\delta h_{\rho\sigma}(x')}\Big|_{h=0} = -c_3\kappa^2H^2\sqrt{-\overline{g}}\,\mathcal{D}^{\mu\nu\rho\sigma}i\delta^D(x-x') \longrightarrow 0$$

$$\frac{i\delta\Delta S_4}{\delta h_{\mu\nu}(x)\delta h_{\rho\sigma}(x')}\Big|_{h=0} = c_4\kappa^2H^4\sqrt{-\overline{g}}\Big[\frac{1}{4}\overline{g}^{\mu\nu}\overline{g}^{\rho\sigma} - \frac{1}{2}\overline{g}^{\mu(\rho}\overline{g}^{\sigma)\nu}\Big]i\delta^D(x-x') \longrightarrow 0$$

where we define $\Pi^{\mu\nu} \equiv \partial^\mu\partial^\nu - \eta^{\mu\nu}\partial^2$ in flat space limit.

Renormalizing the Flat Space Result: a guide for de Sitter

- Reorganize the primitive terms in the terms of two projection operators so as to be in the form of counterterms:

$$-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big]_{\rm flat}(x;x') = \Pi^{\mu\nu}\Pi^{\rho\sigma}F_0(\Delta x^2) + \Big[\Pi^{\mu(\rho}\Pi^{\sigma)\nu} - \frac{\Pi^{\mu\nu}\Pi^{\rho\sigma}}{D-1}\Big]F_2(\Delta x^2)\ .$$

- Find the structure functions F_0 and F_2 comparing this with the previous primitive result:

$$F_0(\Delta x^2) = \frac{\kappa^2\Gamma^2(\frac{D}{2})}{16\pi^D} \times -\frac{1}{8(D-1)^2}\Big(\frac{1}{\Delta x^2}\Big)^{D-2}$$

$$F_2(\Delta x^2) = \frac{\kappa^2\Gamma^2(\frac{D}{2})}{16\pi^D} \times -\frac{1}{4(D-2)^2(D-1)(D+1)}\Big(\frac{1}{\Delta x^2}\Big)^{D-2}$$

- Note: $\Pi^{\mu\nu}\Pi^{\rho\sigma} \sim \partial^4$ are w.r.t x Extract these outside the integral w.r.t x'. Now the factor of $1/\Delta x^{2D-4}$ is logarithmically divergent. Then extract one more d'Alembertian

$$\Big(\frac{1}{\Delta x^2}\Big)^{D-2} = \frac{\partial^2}{2(D-3)(D-4)}\Big(\frac{1}{\Delta x^2}\Big)^{D-3}\ .$$

- Now the integrand converges, however, we still cannot take the $D=4$ limit owing to the factor of $1/(D-4)$. The solution is to add zero in the form of the identity

$$\partial^2\Big(\frac{1}{\Delta x^2}\Big)^{\frac{D}{2}-1} - \frac{4\pi^{\frac{D}{2}}\, i\delta^D(x-x')}{\Gamma(\frac{D}{2}-1)} = 0\ .$$

Renormalizing the Flat Space Result: a guide for de Sitter

- Rewrite it by adding zero:

$$\Big(\frac{1}{\Delta x^2}\Big)^{D-2} = \frac{\partial^2}{2(D-3)(D-4)}\left\{\frac{1}{\Delta x^{2D-6}} - \frac{\mu^{D-4}}{\Delta x^{D-2}}\right\} + \frac{4\pi^{\frac{D}{2}}\mu^{D-4} i\delta^D(x-x')}{2(D-3)(D-4)\Gamma(\frac{D}{2}-1)}$$

$$= -\frac{1}{4}\partial^2\left\{\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2} + O(D-4)\right\} + \frac{4\pi^{\frac{D}{2}}\mu^{D-4} i\delta^D(x-x')}{2(D-3)(D-4)\Gamma(\frac{D}{2}-1)} .$$

: nonlocal finite term : local divergent term

- The divergence now segregated on the delta function: remove them with counterterms:

$$-i\Big[{}^{\mu\nu}\Delta\Sigma^{\rho\sigma}\Big]_{\rm flat}(x;x') = \Pi^{\mu\nu}\Pi^{\rho\sigma}\left\{2c_1\kappa^2 i\delta^D(x-x')\right\} + \Big[\Pi^{\mu(\rho}\Pi^{\sigma)\nu} - \frac{\Pi^{\mu\nu}\Pi^{\rho\sigma}}{D-1}\Big]\left\{2\Big(\frac{D-3}{D-2}\Big)c_2\kappa^2 i\delta^D(x-x')\right\}$$

- by choosing the constants c_1 and c_2 as ,

$$c_1 = \frac{\mu^{D-4}\Gamma(\frac{D}{2})}{2^8\pi^{\frac{D}{2}}}\frac{(D-2)}{(D-1)^2(D-3)(D-4)} , \quad c_2 = \frac{\mu^{D-4}\Gamma(\frac{D}{2})}{2^8\pi^{\frac{D}{2}}}\frac{2}{(D+1)(D-1)(D-3)^2(D-4)} .$$

- The fully renormalized graviton self-energy for flat space background is,

$$-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big]_{\substack{\rm ren\\ \rm flat}} = \lim_{D\to4}\left\{-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big]_{\rm flat}(x;x') - i\Big[{}^{\mu\nu}\Delta\Sigma^{\rho\sigma}\Big]_{\rm flat}(x;x')\right\} ,$$

$$= \Pi^{\mu\nu}\Pi^{\rho\sigma}\partial^2\left\{\frac{\kappa^2}{2^9 3^2\pi^4}\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\right\} + \Big[\Pi^{\mu(\rho}\Pi^{\sigma)\nu} - \frac{1}{3}\Pi^{\mu\nu}\Pi^{\rho\sigma}\Big]\partial^2\left\{\frac{\kappa^2}{2^{10}3^1 5^1\pi^4}\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\right\}$$

Again, this agrees with 't Hooft and Veltman

Renormalizing de Sitter result

- Reorganize the primitive result in terms of the projection operators as for flat space:

$$-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big](x;x') = \sqrt{-\overline{g}(x)}\,\mathcal{P}^{\mu\nu}(x)\sqrt{-\overline{g}(x')}\,\mathcal{P}^{\rho\sigma}(x')\Big\{\mathcal{F}_0(y)\Big\}$$
$$+\sqrt{-\overline{g}(x)}\,\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}(x)\sqrt{-\overline{g}(x')}\,\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}(x')\Big\{\mathcal{T}^{\alpha\kappa}\mathcal{T}^{\beta\lambda}\mathcal{T}^{\gamma\theta}\mathcal{T}^{\delta\phi}\Big(\frac{D-2}{D-3}\Big)\mathcal{F}_2(y)\Big\},$$

where the bitensor is $\mathcal{T}^{\alpha\kappa}(x;x') \equiv -\frac{1}{2H^2}\frac{\partial^2 y(x;x')}{\partial x_\alpha \partial x'_\kappa}$. Note: $\mathcal{T}^{\alpha\kappa}(x;x') \;\; \leftarrow \;\; \eta^{\alpha\kappa}$ in flat space

- Find the structure functions F_0 and F_2 comparing this with the previous primitive result:

$$\mathcal{F}_0(y) = \frac{\kappa^2 H^{2D-4}\Gamma^2(\frac{D}{2})}{(4\pi)^D}\Big\{\frac{-1}{8(D-1)^2}\Big(\frac{4}{y}\Big)^{D-2}+\dots\Big\}$$
$$\mathcal{F}_2(y) = \frac{\kappa^2 H^{2D-4}\Gamma^2(\frac{D}{2})}{(4\pi)^D}\Big\{\frac{-1}{4(D-3)(D-2)(D-1)(D+1)}\Big(\frac{4}{y}\Big)^{D-2}+\dots\Big\}$$

- Add zero in the form of the identity

$$\Big[\square - \frac{D}{2}\Big(\frac{D}{2}-1\Big)H^2\Big]\Big(\frac{4}{y}\Big)^{\frac{D}{2}-1} - \frac{(4\pi)^{\frac{D}{2}}\, i\delta^D(x-x')}{\Gamma(\frac{D}{2}-1)H^{D-2}\sqrt{-\overline{g}}} = 0\,.$$

- Then

$$\Big(\frac{4}{y}\Big)^{D-2} = -\Big[\frac{\square}{H^2}-2\Big]\Big\{\frac{4}{y}\ln\Big(\frac{y}{4}\Big)\Big\} - \frac{4}{y} + O(D-4) + \frac{2(4\pi)^{\frac{D}{2}}\, i\delta^D(x-x')/\sqrt{-\overline{g}}}{(D-4)(D-3)\Gamma(\frac{D}{2}-1)H^D}$$

nonlocal finite term — local divergent term

Renormalizing de Sitter result

- Add the counterterms to subtract the divergences off:

$$-i\Big[{}^{\mu\nu}\Delta\Sigma^{\rho\sigma}\Big](x;x') = \sqrt{-\overline{g}}\Big[2c_1\kappa^2\mathcal{P}^{\mu\nu}\mathcal{P}^{\rho\sigma} + 2c_2\kappa^2\overline{g}^{\alpha\kappa}\overline{g}^{\beta\lambda}\overline{g}^{\gamma\theta}\overline{g}^{\delta\phi}\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}$$
$$-c_3\kappa^2H^2\mathcal{D}^{\mu\nu\rho\sigma} + c_4\kappa^2H^4\sqrt{-\overline{g}}\left[\frac{1}{4}\overline{g}^{\mu\nu}\overline{g}^{\rho\sigma} - \frac{1}{2}\overline{g}^{\mu(\rho}\overline{g}^{\sigma)\nu}\right]\Big]i\delta^D(x-x') \,.$$

- The fully renormalized graviton self-energy for de Sitter is :

$$-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}_{\rm ren}\Big](x;x') = \lim_{D\to4}\Big\{-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big](x;x') - i\Big[{}^{\mu\nu}\Delta\Sigma^{\rho\sigma}\Big](x;x')\Big\} \,,$$
$$= \sqrt{-\overline{g}(x)}\,\mathcal{P}^{\mu\nu}(x)\sqrt{-\overline{g}(x')}\,\mathcal{P}^{\rho\sigma}(x')\Big[\mathcal{F}_{0R}(y)\Big]$$
$$+2\sqrt{-\overline{g}(x)}\,\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}(x)\sqrt{-\overline{g}(x')}\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}(x')\Big[\mathcal{T}^{\alpha\kappa}\mathcal{T}^{\beta\lambda}\mathcal{T}^{\gamma\theta}\mathcal{T}^{\delta\phi}\mathcal{F}_{2R}(y)\Big] \,.$$

where $\mathcal{F}_{0R} = \frac{\kappa^2H^4}{(4\pi)^4}\left\{\frac{\square}{H^2}\left[\frac{1}{72}\times\frac{4}{y}\ln\left(\frac{y}{4}\right)\right]+\cdots\right\}$, $\mathcal{F}_{2R} = \frac{\kappa^2H^4}{(4\pi)^4}\left\{\frac{\square}{H^2}\left[\frac{1}{240}\times\frac{4}{y}\ln(\left(\frac{y}{4}\right)\right]+\cdots\right\}$

Note 1: The leading terms agree with the corresponding flat results.

Note 2: $\mathcal{F}_{0R}$ and $\mathcal{F}_{2R}$ are the first fully renormalized results for the graviton structure functions on de Sitter.

Spin zero structure function

$$
\begin{aligned}
\mathcal{F}_{0R} = \frac{\kappa^2 H^4}{(4\pi)^4} \Bigg\{ & \frac{\square}{H^2} \left[\frac{1}{72} \times \frac{4}{y} \ln\Big(\frac{y}{4}\Big) \right] - \frac{1}{12} \times \frac{4}{y} \ln\Big(\frac{y}{4}\Big) + \frac{1}{72} \times \frac{4}{y} + \frac{1}{6} \ln^2\Big(\frac{y}{4}\Big) \\
& + \frac{1}{45} \times \frac{4}{4-y} \ln(\frac{y}{4}) - \frac{1}{45} \ln(\frac{y}{4}) + \frac{43}{216} \times \frac{4}{4-y} - \frac{5}{6} \times \frac{y}{4} \ln(1 - \frac{y}{4}) \\
& + \frac{7}{90} \times \frac{4}{y} \ln(1 - \frac{y}{4}) - \frac{1}{20} \ln(1 - \frac{y}{4}) - \frac{7(12\pi^2 + 265)}{540} \times \frac{y}{4} \\
& + \frac{84\pi^2 - 131}{1080} - \frac{1}{3} \times \frac{y}{4} \ln^2\Big(\frac{y}{4}\Big) + \frac{4}{9} \times \frac{y}{4} \ln\Big(\frac{y}{4}\Big) \\
& - \frac{1}{30}(2 - y)\left[7\mathrm{Li}_2(1 - \frac{y}{4}) - 2\mathrm{Li}_2(\frac{y}{4}) + 5\ln(1 - \frac{y}{4}) \ln(\frac{y}{4}) \right] \Bigg\} .
\end{aligned}
$$

Spin two structure function

$$
\begin{aligned}
\mathcal{F}_{2R} = \frac{\kappa^2 H^4}{(4\pi)^4}\Bigg\{ & \frac{\Box}{H^2}\left[\frac{1}{240}\times\frac{4}{y}\ln\left(\frac{y}{4}\right)\right] + \frac{3}{40}\times\frac{4}{y}\ln\left(\frac{y}{4}\right) - \frac{11}{48}\times\frac{4}{y} + \frac{1}{4}\ln^2\left(\frac{y}{4}\right) - \frac{119}{60}\ln\left(\frac{y}{4}\right) \\
& + \frac{4096}{(4y-y^2-8)^4}\Bigg[\Bigg[-\frac{47}{15}\left(\frac{y}{4}\right)^8 + \frac{141}{10}\left(\frac{y}{4}\right)^7 - \frac{2471}{90}\left(\frac{y}{4}\right)^6 + \frac{34523}{720}\left(\frac{y}{4}\right)^5 \\
& - \frac{132749}{1440}\left(\frac{y}{4}\right)^4 + \frac{38927}{320}\left(\frac{y}{4}\right)^3 - \frac{10607}{120}\left(\frac{y}{4}\right)^2 + \frac{22399}{720}\left(\frac{y}{4}\right) - \frac{3779}{960}\Bigg]\frac{4}{4-y} \\
& + \left[\frac{193}{30}\left(\frac{y}{4}\right)^4 - \frac{131}{10}\left(\frac{y}{4}\right)^3 + \frac{7}{20}\left(\frac{y}{4}\right)^2 + \frac{379}{60}\left(\frac{y}{4}\right) - \frac{193}{120}\right]\ln(2-\frac{y}{2}) \\
& + \left[-\frac{14}{15}\left(\frac{y}{4}\right)^5 - \frac{1}{5}\left(\frac{y}{4}\right)^4 + \frac{19}{2}\left(\frac{y}{4}\right)^3 - \frac{889}{60}\left(\frac{y}{4}\right)^2 + \frac{143}{20}\left(\frac{y}{4}\right) - \frac{13}{20} - \frac{7}{60}\left(\frac{4}{y}\right)\right]\ln(1-\frac{y}{4}) \\
& + \Bigg[-\frac{476}{15}\left(\frac{y}{4}\right)^9 + 160\left(\frac{y}{4}\right)^8 - \frac{5812}{15}\left(\frac{y}{4}\right)^7 + \frac{8794}{15}\left(\frac{y}{4}\right)^6 - \frac{18271}{30}\left(\frac{y}{4}\right)^5 + \frac{54499}{120}\left(\frac{y}{4}\right)^4 \\
& \quad - \frac{59219}{240}\left(\frac{y}{4}\right)^3 + \frac{1917}{20}\left(\frac{y}{4}\right)^2 - \frac{1951}{80}\left(\frac{y}{4}\right) + \frac{367}{120}\Bigg]\frac{4}{4-y}\ln(\frac{y}{4}) \\
& + \left[4\left(\frac{y}{4}\right)^7 - 12\left(\frac{y}{4}\right)^6 + 20\left(\frac{y}{4}\right)^5 - 20\left(\frac{y}{4}\right)^4 + 15\left(\frac{y}{4}\right)^3 - 7\left(\frac{y}{4}\right)^2 + \left(\frac{y}{4}\right)\right]\frac{4-y}{4}\ln^2(\frac{y}{4}) \\
& + \left[\frac{367}{30}\left(\frac{y}{4}\right)^4 - \frac{4121}{120}\left(\frac{y}{4}\right)^3 + \frac{237}{16}\left(\frac{y}{4}\right)^2 + \frac{1751}{240}\left(\frac{y}{4}\right) - \frac{367}{120}\right]\ln(\frac{y}{2}) \\
& + \frac{1}{64}(y^2-8)\left[4(2-y)-(4y-y^2)\right]\left[\frac{1}{5}\mathrm{Li}_2(1-\frac{y}{4}) + \frac{7}{10}\mathrm{Li}_2(\frac{y}{4})\right]\Bigg]\Bigg\}\,.
\end{aligned}
$$

Solving the quantum-corrected linearized Einstein equation

- Use the renormalized self-energy for the quantum correction term:

$$\sqrt{-g}\,\mathcal{D}^{\mu\nu\rho\sigma}h_{\rho\sigma}(x)-\int d^4x'\left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{\rm ren}\right](x;x')h_{\rho\sigma}(x')=\frac{1}{2}\kappa\sqrt{-g}\,T^{\mu\nu}_{\rm lin}(x)\;,$$

- Only know the self-energy at one loop order (at order $\kappa^2=16\pi G$), solve it perturbatively:

$$h_{\mu\nu}(x)=h^{(0)}_{\mu\nu}(x)+\kappa^2 h^{(1)}_{\mu\nu}(x)+O(\kappa^4)\;,\quad \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{\rm ren}\right](x;x')=\kappa^2\left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{1}\right](x;x')+O(\kappa^4)\;.$$

- The corresponding one loop correction is

$$\int d^4x'\left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{1}\right](x;x')h^{(0)}_{\rho\sigma}(x')=i\int d^4x'\sqrt{-g(x)}\,\mathcal{P}^{\mu\nu}(x)\sqrt{-g(x')}\,\mathcal{P}^{\rho\sigma}(x')\Big\{\mathcal{F}_0\Big\}h^{(0)}_{\rho\sigma}(x')$$
$$+2i\int d^4x'\sqrt{-g(x)}\,\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}(x)\sqrt{-g(x')}\,\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}(x')\Big\{\mathcal{T}^{\alpha\kappa}\mathcal{T}^{\beta\lambda}\mathcal{T}^{\gamma\theta}\mathcal{T}^{\delta\phi}\mathcal{F}_2\Big\}h^{(0)}_{\rho\sigma}(x')\;.$$

- Simplification stratedy: partial integration

  Step 1: pull the projectors $\mathcal{P}^{\mu\nu}(x)$ and $\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}(x)$, which act on a function of x^μ outside the integration over x'^μ

  Step 2: partially integrate the projectors $\mathcal{P}^{\rho\sigma}(x')$ and $\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}(x')$ on $h^{(0)}_{\rho\sigma}(x')$.

$$\int d^4x'\left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{1}\right](x;x')h^{(0)}_{\rho\sigma}(x')=i\sqrt{-g(x)}\,\mathcal{P}^{\mu\nu}(x)\int d^4x'\sqrt{-g(x')}\,\mathcal{F}_0\Big\{\mathcal{P}^{\rho\sigma}(x')h^{(0)}_{\rho\sigma}(x')\Big\}$$
$$+2i\sqrt{-g(x)}\,\mathcal{P}^{\mu\nu}_{\alpha\beta\gamma\delta}(x)\int d^4x'\sqrt{-g(x')}\,\mathcal{T}^{\alpha\kappa}\mathcal{T}^{\beta\lambda}\mathcal{T}^{\gamma\theta}\mathcal{T}^{\delta\phi}\mathcal{F}_2\Big\{\mathcal{P}^{\rho\sigma}_{\kappa\lambda\theta\phi}(x')h^{(0)}_{\rho\sigma}(x')\Big\}.$$

Solving the quantum-corrected linearized field eqn for dynamical gravitons

- For dynamical gravitons, that is for zero stress-energy $T^{\mu\nu}_{\text{lin}}(x) = 0$:

$$h^{(0)}_{\rho\sigma}(x) = \epsilon_{\rho\sigma}(\vec{k}) a^2 u(\eta, k) e^{i\vec{k}\cdot\vec{x}} \;, \quad u(\eta, k) = \frac{H}{\sqrt{2k^3}} \Big[1 - \frac{ik}{Ha} \Big] \exp\Big[\frac{ik}{Ha} \Big], \quad 0 = \epsilon_{0\mu} = k_i \epsilon_{ij} = \epsilon_{jj} \quad \text{and} \quad \epsilon_{ij}\epsilon^*_{ij} = 1$$

- The result is zero!

$$\int d^4x' \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_1 \right](x; x') h^{(0)}_{\rho\sigma}(x') = 0$$

"Inflationary Scalars Don't Affect Gravitons at One Loop," SP and Woodard, arXiv: 1109.4187

- Gravitons interact with MMC scalar only through their kinetic energies which are redshifted. (Gravitons couple minimally only to differentiated scalars.)
- A little Doubt about this result: ignoring surface terms is really legitimate?
  - Ignored surface terms based on the assumption that either they fall off or can be absorbed into corrections of the initial state.
  - Found a counterexample to this assumption: A certain surface terms cannot be ignored.

    Leonard, Prokopec and Woodard, arXiv: 1210.6968

Noncovariant representation of the graviton self-energy

- A noncovariant representation of the conformally rescaled graviton field
  - Noncovariant Rep includes de Sitter breaking basis vectors in terms of $u(x;x') \equiv \ln(aa')$
  - Covariant Rep: $g_{\mu\nu} = \overline{g}_{\mu\nu} + \kappa \chi_{\mu\nu}$ vs Noncovariant Rep: $g_{\mu\nu} = a^2(\eta_{\mu\nu} + \kappa h_{\mu\nu})$
  - Covariant Rep: 5 basis tensors - 3 relations = 2 structure ftns vs Noncovariant Rep: 14 - 10 = 4

$$-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}\Big](x;x') = \mathcal{F}^{\mu\nu}(x) \times \mathcal{F}^{\rho\sigma}(x')\Big[F_0(x;x')\Big] + \mathcal{G}^{\mu\nu}(x) \times \mathcal{G}^{\rho\sigma}(x')\Big[G_0(x;x')\Big] + \mathcal{F}^{\mu\nu\rho\sigma}\Big[F_2(x;x')\Big] + \mathcal{G}^{\mu\nu\rho\sigma}\Big[G_2(x;x')\Big]$$

Leonard, SP, Prokopec and Woodard, arXiv: 1403.0896

- Checked that surface terms really fall off like powers of the scale factor
- Confirmed the previous result: no effect on dynamical gravitons from MMC scalars
- Much simpler than the de Sitter covariant representation, so can be easily employed to study the force of gravity

Structure functions in the noncovariant representation

$$F_{0R}(x;x') = \frac{\kappa^2(aa'H^2)^2}{2304\pi^4}\left\{\frac{\partial^2}{2(aa'H^2)^2}\left[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\right] - \frac{6}{y} + 6 + \left[-\frac{2}{y} + 6 - \frac{2}{4-y}\right]\ln\left(\frac{y}{4}\right) + \frac{3}{2}(2-y)\Psi(y)\right\}$$

$$F_{2R}(x;x') = \frac{\kappa^2(H^2aa')^2}{(4\pi)^4}\left\{\frac{\partial^2}{30(H^2aa')^2}\left[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\right] + \frac{2}{3}\left[\frac{1}{y} - \frac{1}{4-y}\right]\ln\left(\frac{y}{4}\right) - \frac{1}{3}\Psi(y)\right\}$$

$$G_0(x;x') = 0$$

$$G_2(x;x') = \frac{\kappa^2(H^2aa')^2}{(4\pi)^4}\left\{-2 + \frac{8}{3}\frac{\ln(\frac{y}{4})}{(4-y)} + \frac{2}{3}\Psi(y)\right\}$$

where

$$\Psi(y) \equiv \frac{1}{2}\ln^2\left(\frac{y}{4}\right) - \ln\left(1-\frac{y}{4}\right)\ln\left(\frac{y}{4}\right) - \mathrm{Li}_2\left(\frac{y}{4}\right)$$

One loop corrections from MMC scalar to the Newtonian potential

- For the linearized response to a stationary point mass M

$$h^{(0)}_{00}(x) = \frac{2GM}{a\|\vec{x}\|} = -2\Phi^{(0)} \ , \ \ h^{(0)}_{0i}(x) = 0 \ , \ \ h^{(0)}_{ij}(x) = \frac{2GM}{a\|\vec{x}\|}\delta_{ij} = -2\Psi^{(0)}\delta_{ij} \ , \quad T_{\mu\nu} = \frac{M}{a}\delta^3(\vec{x})\delta^0_\mu\delta^0_\nu$$

in flat space $h^{(0)}_{00}(x) = \frac{2GM}{\|\vec{x}\|}$, $h^{(0)}_{0i}(x) = 0$, $h^{(0)}_{ij}(x) = \frac{2GM}{\|\vec{x}\|}\delta_{ij}$, $\quad T_{\mu\nu} = M\delta^3(\vec{x})\delta^0_\mu\delta^0_\nu$

- One loop corrections

$$h^{(1)}_{00}(x) \equiv f_1 \ , \ \ h^{(1)}_{0i}(x) = 0 \ , \ \ h^{(1)}_{ij}(x) \equiv f_3\delta_{ij}$$

The solutions are

$$\begin{aligned} f_1(x) &= -\frac{\kappa^2 M}{2a^2}S^1_0(x) + \frac{\kappa^2 M}{a^2}\Big[-\frac{2}{3} + \nabla^{-2}(\partial_0^2 - aH\partial_0)\Big]S^1_2(x) = -2\Phi^{(1)} \\ f_3(x) &= \frac{\kappa^2 M}{2a^2}S^1_0(x) + \frac{\kappa^2 M}{a^2}\Big[-\frac{1}{3} - \nabla^{-2}aH\partial_0\Big]S^1_2(x) = -2\Psi^{(1)} \end{aligned}$$

where $\nabla^{-2}f(\eta,\vec{x}) = -[1/(4\pi)]\int d^3x' f(\eta,\vec{x}')/\|\vec{x}-\vec{x}'\|$

$$\begin{aligned} S^1_0(x) &= \int \frac{d\eta'}{a(\eta')}[iF^1_0(x,x')]_{\vec{x}'=0} \ , \\ S^1_2(x) &= \int \frac{d\eta'}{a(\eta')}\Big[F^1_2(x;x') + \frac{1}{2}G^1_2(x;x')\Big]_{\vec{x}'=0} \end{aligned}$$

One loop corrections from MMC scalar to the Newtonian potential

- In flat space

$$\Phi_{flat} = -\frac{GM}{r}\left\{1 + \frac{\hbar}{20\pi c^3}\frac{G}{r^2} + O\left(G^2\right)\right\}$$
$$\Psi_{flat} = -\frac{GM}{r}\left\{1 - \frac{\hbar}{60\pi c^3}\frac{G}{r^2} + O\left(G^2\right)\right\}$$

SP and Woodard, arXiv:1007.2662, Marunovic and Prokopec, arXiv: 1101.5059

Not the first for this result, but the first to solve the effective field eqns using in-in formalism.

- In de Sitter space

$$\Phi_{dS} = -\frac{GM}{ar}\left\{1 + \frac{\hbar}{20\pi c^3}\frac{G}{r^2} + \cdots + O\left(G^2\right)\right\}$$
$$\Psi_{dS} = -\frac{GM}{ar}\left\{1 - \frac{\hbar}{60\pi c^3}\frac{G}{r^2} + \cdots + O\left(G^2\right)\right\}$$

Confirmed the de Sitterized version of the flat space result; found some corrections but no secular growth

SP, Prokopec and Woodard, arXiv:1508.xxxxx

- One loop correction to the gravitational slip differs from zero in both flat and de Sitter space: $\Phi^{(1)} - \Psi^{(1)} \neq 0$

Summary and Discussion

- Derived one loop contributions to the graviton self-energy from MMC scalar on de Sitter background: covariant and noncovariant representations for the tensor structure.
- Used these representations to quantum correct the linearized Einstein field equations for dynamical gravitons and the force of gravity.
- The noncovariant representation is much easier to use in the effective field equations than the covariant one.
- Inflationary production of MMC scalars has no effect on dynamical gravitons.
- Inflationary production of MMC scalars has some effect on the force of gravity, though no secular growth.
- Future projects (having possibility of secular growth):
  - One loop contributions to the graviton self-energy from gravitons:
    Done in hep-ph/9602317 Tsamis and Woodard, but needs to be redone using dimensional regularization;
    1307.1422 Mora, Tsamis and Woodard the effect of gravitons to other gravitons using Hartree approximation
  - One loop corrections to the non-minimally coupled scalar self-mass-squared from gravitons:

    1409.7753 Boran, Kahya and SP work in progress for the corrections to MCC scalar from gravitons

THE END

- Thank you for your attention!