Constraints from higher-spin symmetries in dS QFTs

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Motivation

QFTs are hard to solve.

Most soluble models have *enhanced symmetry*, and/or *simplified dynamics* relative to the Standard Model.

Definition

Higher-spin symmetries are symmetries which enhance the spacetime isometry group. Generators transform as tensor components under isometry group.

Classic results on how HS symmetries constrain QFTs:

- Minkowski S-matrix [Coleman Mandula '67]
- $D = 2$ S-matrix [Parke '80]
- $D = 2$ CFTs [Zamolodchikov '85]
- $D = 3$ CFTs [Maldacena-Zhiboedev '11]

Motivation

HS symmetries may also be incorporated into thys of quantum gravity. [Vasiliev . . .]

Higher-spin AdS/CFT

- Bulk 4D Vasiliev thys / 3D (critical) O(N) model [Klebanov Polyakov '02, Giombi Yin '11]
- 3D Chern-Simons gravity thys / 2D W-algebra CFTs

Proposed dS/CFT duality

4D Vasiliev thys / SP(N) CFT_3 [Anninos Hartman Strominger '11]
Provides a putative UV complete thy of quantum gravity admitting dS background.

HS QFTs obtained by breaking of local HS symmetry in HS gravity thys; HS gravity thys obtained from QFT by gauging HS symmetry.

Motivation

Soft thms in cosmology

- Maldacena consistency condition for single-field inflation [Maldacena '03]
- Infinite set of generalizations [Hinterbichler Hui Khoury '12]
- Understood as *Ward identities* applied to correlation functions near conformal boundary [Pimentel '13, McFadden '14]

Outline

In this talk:

- We study the consequences of HS symmetries in dS QFTs in $D = d + 1 > 2$.
- Examine Ward identities applied to correlation functions near conformal boundary.

Main result:

Consider a dS QFTs satisfying standard criteria (dS covariance, dS-invt vacua, flat-space limit). If thy admits HS symmetry, and satisfies a sparseness condition on the operator spectrum, then thy posses operators which become free near the conformal boundary – asymptotic gaussianity.

- Our result is an analogue of the Coleman Mandula thm for dS QFTs.
- Our analysis is more similar in spirit to [Maldacena-Zhiboedev '11].

Outline

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HS symmetries in free fields

Consider a $\mathbb{C}$ Klein-Gordon field on a Poincaré chart of $dS_{D=d+1}$:

$$ds^2 = \frac{\ell^2}{\eta^2}\left(-d\eta^2 + \delta_{ij}dx^i dx^j\right), \quad \eta \in (-\infty, 0),\ x^i \in \mathbb{R}^d,$$

$$S = \int d^D x \sqrt{-g}\left(-\frac{1}{2}\nabla_\mu \phi^\dagger \nabla^\mu \phi(x) - \frac{M^2}{2}\phi^\dagger \phi(x)\right), \quad M^2 \geq 0.$$

Spin-1,2 currents are familiar "Klein-Gordon" current, "stress tensor"

$$\begin{aligned} J_\mu(x) &= \phi^\dagger \overleftrightarrow{\nabla}_\mu \phi(x), \\ J_{\mu\nu}(x) &= \nabla_{(\mu}\phi^\dagger \nabla_{\nu)}\phi(x) - \frac{1}{2}g_{\mu\nu}\left(\nabla_\lambda \phi^\dagger \nabla^\lambda \phi(x) + M^2 \phi^\dagger \phi(x)\right) + \dots. \end{aligned}$$

These *normal-ordered* w.r.t. vacuum Ω. $J_{\mu\nu}(x)$ traceless at conformal coupling

$$J_\mu{}^\mu \propto (M^2 - M^2_{\mathrm{c.c.}}), \quad M^2_{\mathrm{c.c}}\ell^2 = \frac{d^2-1}{4}.$$

HS symmetries in free fields

Thy admits spin-s current $\forall\, s$:

$$J_{\mu_1\ldots\mu_s}(x) = \sum_{j=0}^{n} c_j \nabla_{(\mu_1}\ldots\nabla_{\mu_j}\phi^\dagger\nabla_{\mu_{j+1}}\ldots\nabla_{\mu_s)}\phi(x) + \text{trace terms},$$

$$J^{\nu}{}_{\nu\mu_2\ldots\mu_s}(x) \propto \left(M^2 - M^2_{\text{c.c.}}\right) J_{\mu_2\ldots\mu_s}(x).$$

E.g., spin-3:

$$J_{\mu\nu\lambda}(x) = \frac{1}{4(d+2)}\Big[(d-1)\left(\phi^\dagger\nabla_{(\mu}\nabla_\nu\nabla_{\lambda)}\phi(x) - \nabla_{(\mu}\nabla_\nu\nabla_{\lambda)}\phi^\dagger\phi(x)\right)$$

$$- 3(3+d)\nabla_{(\mu}\phi^\dagger\overleftrightarrow{\nabla}_\nu\nabla_{\lambda)}\phi(x) + 6g_{(\mu\nu}\nabla^\alpha\phi^\dagger\overleftrightarrow{\nabla}_\lambda\nabla_{\alpha)}\phi(x)$$

$$+ \left[6M^2 - (d-1)(3d+2)\ell^{-2}\right] g_{(\mu\nu}J_{\lambda)}(x)\Big],$$

$$J^{\mu}{}_{\mu\lambda}(x) = \left(M^2 - M^2_{\text{c.c.}}\right) J_\lambda(x).$$

HS symmetries in free fields

Using KVF p^μ tangent to $d\eta$ may construct charges

$$Q_p^{(s)} := \eta^{1-d} \int d^d x \, p^{\nu_1} \cdots p^{\nu_s} J_{\eta\nu_1\ldots\nu_s}(\eta, x)\Big|_{\eta=\text{const}},$$

$$\left[Q_p^{(s)}, \phi(x)\right] = i\partial_p^s \phi(x), \quad Q_p^{(s)} |\Omega\rangle = 0.$$

For each charge $\exists$ a charge conservation law (Ward identity)

$$0 = \left\langle \left[Q_p^{(s)}, \phi(x_1)\right] \phi(x_2)\ldots\phi(x_n) + \cdots + \phi(x_1)\ldots\phi(x_{n-1}) \left[Q_p^{(s)}, \phi(x_n)\right] \right\rangle_\Omega.$$

Letting $p^\mu \partial_\mu = \partial_1$, in Fourier space

$$0 = \sum_{i=1}^{n} (\pm)(k_{i1})^s.$$

For $s = 1$ this is usual momentum conservation. For $s > 1$ satisfied b.c. for transl.-invt *Gaussian states*, momenta form equal/opposite pairs.

HS symmetries in free fields

Why are HS symmetries so powerful?

1. $Q_p^{(s)}$ enlarge isometry subgroup $ISO(d) \subset SO(D,1)$

$$\left[P_i, Q_p^{(s)}\right] = 0, \quad \left[D, Q_p^{(s)}\right] = -sQ_p^{(s)}.$$

2. Finite action of $Q_p^{(s)}$ displaces wavepackets in position space by momentum-dependent amount.

$$\phi[f] := \int d^D x \sqrt{-g} f(x)\phi(x), \quad \hat{f}(k) = e^{-(k-k_0)^2/4w^2},$$

$$e^{i\alpha Q_p^{(s)}} \phi[f] \left|\Omega\right\rangle = \phi[f'] \left|\Omega\right\rangle, \quad \hat{f}'(k) = e^{-(k-k_0)^2/4w^2} e^{i\alpha(p\cdot k)^s}.$$

In $D > 2$ symmetry relates wavepackets that do not collide with those that do.

3. Frequently thys with one HS symmetry actually have *many*.

The Coleman Mandula thm

U is a symmetry of the S-matrix if:

i) U maps 1-particle states to 1-particle states,

ii) U maps many particle states as tensor product,

iii) U commutes with S,

U is an *internal symmetry* if it commutes with P_μ.

Theorem

Assume $S \neq 1$. If G is connected (Lie) symmetry group of S, then G is locally isomorphic to $ISO(D) \times I$, where I is an internal symmetry group.

Note:

- It is assumed that symmetry acts linearly on asympt. 1-particle Hilbert space.
- For any finite mass M, there exists a finite number of particle species with mass less than M.
- There are a few ugly technical assumptions about distributional nature of some objects which go into proof.

Outline

HS charges with linear action

Theorem

Consider a de Sitter QFT satisfying a few reasonable criteria which admits a local HS charge

$$Q_p^{(s)} := \eta^{1-d} \int d^d x \, p^{\nu_1} \cdots p^{\nu_s} J_{\eta\nu_1\ldots\nu_s}(\eta, x)\Big|_{\eta=\mathrm{const}}, \quad p^\mu \in \mathbb{R}^d.$$

If the action of $Q_p^{(s)}$ on a scalar operator $\phi(x)$ is linear then $\phi(x)$ is a generalized free field.

Linear action:

$$\left[Q_p^{(s)}, \phi(x)\right] = \sum_{k=0}^{\infty} c_{k-s}^{\mu_1\ldots\mu_k}(x)\nabla_{\mu_1}\ldots\nabla_{\mu_k}\phi(x).$$

HS charges with linear action

Reasonable criteria:

1. dS covariance: "thy does not have preferred direction"
2. flat-space limit: $\ell \to \infty$, all other dimensionful parameters fixed. All operator equations, OPE, etc., admit flat-space limit without assumptions about (expectation values of) operators.
3. $\exists$ $SO_0(D,1)$-invariant state(s) Ω_i.

$\Rightarrow$ correlation functions of Ω_i admit *generalized Mellin transform* [Marolf IAM '10, Hollands '10, Korai Tanaka '12]

$$\langle \Omega_1 | \, \phi(x_1) \ldots \phi(x_n) \, | \Omega_2 \rangle = \int_{\vec{\mu}} \mathcal{M}(\vec{\mu}) \prod_{i<j}^{n} X_{ij}^{\mu_{ij}},$$

$$\int_{\vec{\mu}} A(\vec{\mu}) := \left[\prod_{i<j}^{n} \int_{C_{ij}} \frac{d\mu_{ij}}{2\pi i} \right] A(\mu_{12}, \ldots, \mu_{n-1,n}), \quad X_{ij} = \frac{|\vec{x}_i - \vec{x}_j|^2 - (\eta_i - \eta_j)^2}{4\eta_i \eta_j}.$$

HS charges with linear action

Sketch of proof: consider $Q_p^{(2)}$:

Lemma

The most general linear action of $Q_p^{(2)}$ on $\phi(x)$ is

$$\left[Q_p^{(2)}, \phi(x)\right] = i\left(\frac{c_1}{\eta^2} + \frac{c_2}{\eta}\partial_\eta + c_3\partial_\eta^2 + c_4\partial_p^2 + c_5\Delta_s\right)\phi(x) =: \mathcal{D}_p(x)\phi(x),$$

where c_i are real constants, $\partial_p := p^\mu\partial_\mu$, and Δ_s is the scalar Laplacian compatible with the flat metric on $\mathbb{R}^d$.

Follows from considering: $C(x) := [Q_p^{(2)}, \phi(x)]$

- translations: $U(a)C(x)U^{-1}(a) = C(x-a)$
- dilations: $U(\lambda)C(x)U^{-1}(\lambda) = \lambda^{-2}C(\lambda^{-1}x)$
- length dimension & flat-space limit: $\ell \to \infty$
- $\mathbb{R}^d$ reflections $\partial_p \to -\partial_p$: $C(x) \to C(x)$

HS charges with linear action

Charge conservation law:

$$0 = \left(\sum_{i=1}^{n} \mathcal{D}_p(x_i) \right) \langle \Omega_1 | \, \phi(x_1) \ldots \phi(x_n) \, | \Omega_2 \rangle \,,$$

$$\mathcal{D}_p(x) = i \left(\frac{c_1}{\eta^2} + \frac{c_2}{\eta} \partial_\eta + c_3 \partial_\eta^2 + c_4 \partial_p^2 + c_5 \Delta_s \right) .$$

Analyze in "Mellin space":

- Each ∂_p^2 terms must vanish individually.
- Each term yields constraint on Mellin transform $\mathcal{M}(\vec{\mu})$.
- For $(n > 2)$-pt function: correlation function is *disconnected*.

$\Rightarrow \phi(x)$ is generalized free field. ■

Outline

HS charges with non-linear action

In general, action on $\phi(x)$ is *non-linear*:

$$\left[Q_p^{(s)}, \phi(x)\right] = i \sum_A \frac{1}{\eta^{s-k}} c_A^{\mu_1 \ldots \mu_k} \mathcal{O}^A_{\mu_1 \ldots \mu_k}(x).$$

Any operator w/ correct quantum numbers may enter RHS.

Modest goal:

Consider equal-time correlation functions in limit $\eta \to 0$,

$$F_n(\eta; \vec{x}_1, \ldots, \vec{x}_n) := \langle \Omega_1 | \, \phi(\eta, \vec{x}_1) \ldots \phi(\eta, \vec{x}_n) \, | \Omega_2 \rangle \, .$$

How do HS symmetries constrain asymptotic behavior near conformal boundary?

Definition

An operator $\phi(x)$ is asymptotically Gaussian if the leading behavior of all $\{F_n\}$ as $\eta \to 0$ is Gaussian (i.e., composed of connected 2-pt functions).

HS charges with non-linear action

Examine commutator in *Fefferman-Graham* expansion.

$$ds^2 = \frac{\ell^2}{\eta^2}\left(-d\eta^2 + \delta_{ij} dx^i dx^j\right).$$

Local operators have characteristic scaling set by EOM, Ward identity, etc.

$$\phi_\Delta(x) = O(\eta^\Delta), \quad \Delta > 0,$$

valid as *operator equation*. May regard Δ as one of the quantum numbers of $\phi(x)$.
E.g., for light KG fields,

$$\Delta_{KG} = \frac{d}{2} - \sqrt{\frac{d^2}{4} - M^2\ell^2}, \quad M^2\ell^2 < \frac{d^2}{4}.$$

For conserved currents, Ward identities require

$$J_\eta(x) = O(\eta^{d-1}), \quad T_{\eta i}(x) = O(\eta^{d-1}), \quad T_{\eta\eta}(x) = O(\eta^d), \quad \ldots.$$

HS charges with non-linear action

In FG expansion, asymptotic charge conservation identity simplifies:

$$\text{as } \eta \to 0: \quad \left[Q_p^{(s)}, \phi(x)\right] \to i \sum_{k=0}^{\infty} \frac{1}{\eta^{s-k}} c_A^{\mu_1 \ldots \mu_k} \mathcal{O}^A_{\mu_1 \ldots \mu_k}(x)\Big|_{O(\eta^\Delta)} .$$

Consider the following:

Sparseness condition:

All $O(\eta^\Delta)$ operators on RHS of commutator eqn. are linear in $\phi(x)$.

1. When SC satisfied, the asympt. action of $Q_p^{(s)}$ on $\phi(x)$ is linear. Thus $\phi(x)$ is asymptotically gaussian.
2. If SC not satisfied, thy contains another operator which
   - i) has quantum numbers to enter in $C(x) = [Q_p^{(s)}, \phi(x)]$,
   - ii) is $O(\eta^\Delta)$ as $\eta \to 0$,
   - iii) is not linear in $\phi(x)$.

HS charges with non-linear action

So far I have said nothing profound!

Consider light operators: $\Delta \in \mathbb{R},\ O(1)$. These of greatest interest because they feel background the most, and are most "visible" at late times. Consider the "minimal" operator algebra, $\mathrm{poly}(\phi)$+conserved currents:

1. For $\Delta \notin \mathbb{N}_0$, no obvious candidate to violate SC.
2. For $\Delta \in \mathbb{N}_0$, $\exists$ many candidates in minimal algebra which could violate SC.

Conjecture

Generic HS thys with non-integer Δ are asymptotically Gaussian, but there exist exceptional HS thys with operators of integer Δ which are not.

Resonates w/ perturbative global dS S-matrix [Marolf IAM Srednicki '12]. HS gravitational thys have integer Δ!

HS charges with non-linear action

The minimal operator algebra:

1. $\mathrm{poly}(\phi)$: $\phi(x)$, $\phi^2(x)$, $\phi^2\nabla_\mu\phi(x)$, $\ldots$.
   - WLOG may consider $\phi(x)$ unique scalar operator of scaling dimension Δ.
   - In GFF know exactly how all members scale at late times. Only $\phi(x)$ scales like η^Δ.
   - In generic thy don't know scaling dimension of composite operators, but do expect for very light fields that all polynomials to decay at least as fast as $\phi(x)$.
2. conserved currents
   - Ward id: scale like integer power: η^{d-1+n}, $n = 0, 1, \ldots,$
   - for $D > 2$ the "twist" $\Delta - s > 0$, so for most components higher spin currents decay more rapidly

Outline

Conclusions

Conclusions

For operators with "generic" quantum numbers, HS symmetries greatly constrain asymptotic form of correlation functions.

- scalars with $\Delta \notin \mathbb{N}_0$: expect *asymptotic gaussianity*
- scalars with $\Delta \in \mathbb{N}_0$: no statement

Remains to determine the generality of our analysis:

- extend discussion to tensor operators
- incorportate local gauge symmetries
- study concrete examples

Our analysis does not apply to $D = 2$, where $\exists$ interesting, interacting, *soluble* thys w/ HS symmetries (Ask me about non-linear sigma models in 2D dS)