Stochastic Dynamics of Infrared Fluctuations in Accelerating Universe

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Introduction

- In the presence of massless and minimally coupled scalar fields in accelerating universes, quantum fluctuations at super-horizon scales make physical quantities growing with time

- From a semiclassical view point, it was proposed that such IR effects are well-described by a Langevin equation

- In de Sitter space, the stochastic approach has been proved to be equivalent to resummation of leading powers of the growing time dependences

- We extend these investigations in a general accelerating universe

Outline

- IR effects in de Sitter space
  - Resummation of leading IR effects
  - Semiclassical description of fields
- IR effects in Accelerating universe where $\epsilon \equiv \frac{-\dot{H}}{H^2}, \dot{\epsilon} = 0$
  - Resummation of leading IR effects
  - Semiclassical description of fields
- IR effects in Accelerating universe where $\dot{\epsilon} \neq 0$
  - Resummation of leading IR effects
  - Semiclassical description of fields

Review

New results, Improvements

Free scalar field in dS space

$$dS_4: \; ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$
$$= a^2(\eta)(-d\eta^2 + d\mathbf{x}^2)$$

scale invariance
$\eta \to C\eta, \; \mathbf{x} \to C\mathbf{x}$

$$H \equiv \frac{\dot{a}}{a} = H_0 \; : \; \text{const.}$$

$\dot{} \equiv \partial_t$

$$a = e^{H_0 t} = \frac{1}{-H_0\eta}$$

massless,
minimally coupled

$$S_2 = -\frac{1}{2}\int \sqrt{-g}d^4x \; [g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \; + m^2\varphi^2 + \xi R\varphi^2]$$

$$\varphi_0(x) = \int \frac{d^3p}{(2\pi)^3} \; [a_{\mathbf{p}}\phi_{\mathbf{p}}(x) + a^\dagger_{\mathbf{p}}\phi^*_{\mathbf{p}}(x)]$$

$$\phi_{\mathbf{p}}(x) = \frac{-H_0\eta}{\sqrt{2p}}(1 - \frac{i}{p\eta})e^{-ip\eta + i\mathbf{p}\cdot\mathbf{x}}$$

Scale invariance breaking

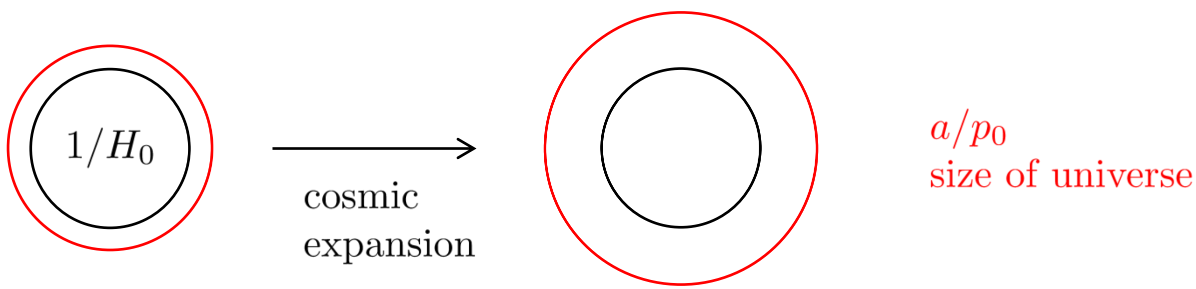
'82 A. Vilenkin, L. H. Ford,
A. D. Linde,
A. A. Starobinsky

At $P \equiv p/a \ll H_0 \;\Leftrightarrow\; -p\eta \ll 1,$

$$\phi_{\mathbf{p}} \simeq i\frac{H_0}{\sqrt{2p^3}} e^{+i\mathbf{p}\cdot\mathbf{x}} \;\Rightarrow\; \langle \varphi_0^2(x) \rangle \simeq \int \frac{d^3p}{(2\pi)^3} \frac{H_0^2}{2p^3}$$

IR divergence

We introduce an IR cut-off $p_{\min} = p_0$: $\displaystyle\int_{p_0/a}^{H_0} dP$



$$\langle \varphi_0^2(x) \rangle \simeq \frac{H_0^2}{4\pi^2} \int_{-p_0\eta}^{1} \frac{d(-p\eta)}{(-p\eta)} = \frac{H_0^2}{4\pi^2} \log(a/a_0)$$

$a_0 \equiv p_0/H_0$

Not inv. under
$\eta \to C\eta,\; \mathbf{p} \to C^{-1}\mathbf{p}$

In interacting field theories (i)

At each vertex integral,

$$\begin{cases} \text{one of propagators is retarded: } G^R(x,x') = \theta(\eta-\eta')[\varphi_0(x),\varphi_0(x')] \\ \text{the other are the Wightman functions: } \langle\varphi_0(x)\varphi_0(x')\rangle,\ \langle\varphi_0(x')\varphi_0(x)\rangle \end{cases}$$

due to Causality

Secular growths of the Wightman functions originate in

$$\varphi_0(x) \simeq \int \frac{d^3p}{(2\pi)^3}\ \theta(H_0 a - p)\big[i\underline{\frac{H_0}{\sqrt{2p^3}}}e^{+i\mathbf{p}\cdot\mathbf{x}}a_{\mathbf{p}} + (\text{h.c.})\big]$$

const. (spectrum is frozen
at super-horizon scales)

In interacting field theories (ii)

$$\phi_{\mathbf{p}}(x) \simeq i\frac{H_0}{\sqrt{2p^3}}\{1+\frac{1}{2}(-p\eta)^2+\frac{i}{3}(-p\eta)^3\}e^{+i\mathbf{p}\cdot\mathbf{x}}$$

Retarded propagator has no secular growth

$$G^R(x,x') \simeq \theta(\eta-\eta')\int\frac{d^3p}{(2\pi)^3}\;\frac{-i}{3}H_0^2\left[(-\eta')^3-(-\eta)^3\right]e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')}$$
$$= \theta(\eta-\eta')\times\frac{-i}{3}H_0^2\left[(-\eta')^3-(-\eta)^3\right]\underline{\delta^{(3)}(\mathbf{x}-\mathbf{x}')}$$

spatially local

but integral of it induces a secular growth as

$$\int\sqrt{-g'}d^4x'\;G^R(x,x') \simeq \frac{-i}{3H_0^2}\int^{\eta}\frac{d\eta}{(-\eta')^4}\left[\underline{(-\eta')^3}-(-\eta)^3\right]$$
$$\simeq \frac{-i}{3H_0^2}\int^{a}\underline{d(\log a')}$$

Leading IR effects

For example in φ^4 theory, as loop level is increased by one, quantum corrections are multipled by up to the factor:

$$\lambda \log^2(a/a_0)$$

λ: coupling constant

Even if $\lambda \ll 1$, perturbation theory is eventually broken

(after $\lambda \log^2(a/a_0) \sim 1$, in φ^4 theory)

$$\Downarrow$$

Resummation formula for leading powers of IR logs. is necessary to evaluate them nonperturbatively

Resummation formula

'05 N. C. Tsamis,
R. P. Woodard

Yang-Feldman formalism is reduced to the stochastic equation up to leading powers of IR logarithms:

$$\varphi(x) = \varphi_0(x) - i\int \sqrt{-g'}d^4x'\ G^R(x,x')\frac{\partial}{\partial\varphi}V(\varphi(x'))$$

Up to leading logs.

$$\varphi_0(x) \simeq \bar{\varphi}_0(x) = \int \frac{d^3p}{(2\pi)^3}\ \theta(H_0 a - p)\left[i\frac{H_0}{\sqrt{2p^3}}e^{+i\mathbf{p}\cdot\mathbf{x}}a_{\mathbf{p}} + (\mathrm{h.c.})\right]$$

$$\int \sqrt{-g'}d^4x'\ G^R(x,x') \simeq \frac{-i}{3H_0}\int^t dt'$$

$$\varphi(x) = \bar{\varphi}_0(x) - \frac{1}{3H_0}\int^t dt'\ \frac{\partial}{\partial\varphi}V(\varphi(t',\mathbf{x}))$$

$$\Downarrow$$

Langevin eq.: $\dot{\varphi}(x) = \dot{\bar{\varphi}}_0(x) - \frac{1}{3H_0}\frac{\partial}{\partial\varphi}V(\varphi(x))$

$$\langle\dot{\bar{\varphi}}_0(t,\mathbf{x})\dot{\bar{\varphi}}_0(t',\mathbf{x})\rangle = \frac{H_0^3}{4\pi^2}\delta(t-t')$$

Fokker-Planck eq.

Langevin eq. can be translated to the equation of the probability density ρ:

$$\dot{\rho}(t,\phi) = \frac{H_0^3}{8\pi^2}\frac{\partial^2}{\partial\phi^2}\rho(t,\phi) + \frac{1}{3H_0}\frac{\partial}{\partial\phi}\big(\rho(t,\phi)\frac{\partial}{\partial\phi}V(\phi)\big)$$

$$\langle F(\varphi(x))\rangle = \int_{-\infty}^{\infty} d\phi\ \rho(t,\phi)F(\phi), \quad F\text{: any function}$$

An equilibrium state is eventually established: $\rho(t,\phi) \to \rho_\infty(\phi),\ t \to \infty$

The solution of the equilibrium state is given by

$$\rho_\infty(\phi) = N\exp\big(-\frac{8\pi^2}{3H_0^4}V(\phi)\big)$$

e.g. in φ^4 theory, $\langle V(\varphi(x))\rangle_\infty = \frac{3H_0^4}{32\pi^2}$

Not suppressed by λ

Scale invariance is recovered

Semiclassical description

’94 J. Yokoyama, A. A. Starobinsky

$$\left(\frac{\partial^2}{\partial t^2} + 3H_0\frac{\partial}{\partial t} - \frac{1}{a^2}\frac{\partial^2}{\partial \mathbf{x}^2}\right)\varphi(x) = -\frac{\partial}{\partial\varphi}V(\varphi(x))$$

Extracting IR dynamics: $\varphi = \bar{\varphi} + \varphi_{\rm UV}$

↓

Neglecting second derivatives

Identifying $\varphi_{\rm UV}$ as a source of $\bar{\varphi}$

$$3H_0\frac{\partial}{\partial t}\{\bar{\varphi}(x) + \varphi_{\rm UV}(x)\} = -\frac{\partial}{\partial\bar{\varphi}}V(\bar{\varphi}(x))$$

$$\varphi_{\rm UV}(x) = \int \frac{d^3p}{(2\pi)^3}\ \theta(p - H_0 a)\left[i\frac{H_0}{\sqrt{2p^3}}e^{+i\mathbf{p}\cdot\mathbf{x}}a_{\mathbf{p}} + ({\rm h.c.})\right]$$

Since $\dot{\varphi}_{\rm UV} = -\dot{\bar{\varphi}}_0$, the reduced equation is

$$\dot{\bar{\varphi}}(x) = \dot{\bar{\varphi}}_0(x) - \frac{1}{3H_0}\frac{\partial}{\partial\bar{\varphi}}V(\bar{\varphi}(x))$$

Same result with Resummation formula

Free scalar field in Accelerating universe where $\dot{\epsilon} = 0$

$$\epsilon \equiv \frac{-\dot{H}}{H^2} = \epsilon_0 \; : \; \text{const.}$$

$$0 \leq \epsilon_0 < 1 \;\; \Rightarrow \;\; \text{Accelerated expansion}$$

$$H = H_0 a^{-\epsilon_0}$$

$$a = (1 + \epsilon_0 H_0 t)^{\frac{1}{\epsilon_0}} = \left\{ \frac{1}{-(1-\epsilon_0) H_0 \eta} \right\}^{\frac{1}{1-\epsilon_0}}$$

On the background, the wave function is given by Bessel functions

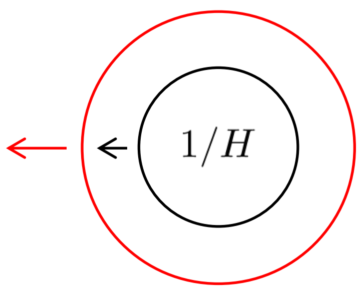
massless, minimally coupled

$$\phi_{\mathbf{p}}(x) = a^{-1} \times \frac{\sqrt{\pi}}{2} (-\eta)^{\frac{1}{2}} H^{(1)}_{\nu_0}(-p\eta) e^{+i\mathbf{p}\cdot\mathbf{x}} \qquad \nu_0 = \frac{3}{2} + \frac{\epsilon_0}{1-\epsilon_0}$$

$$\sim \begin{cases} a^{-1} \times \dfrac{1}{\sqrt{2p}} e^{-ip\eta + i\mathbf{p}\cdot\mathbf{x} - i\frac{2\nu_0+1}{4}\pi} & \text{at } P \gg H \Leftrightarrow -p\eta \gg \frac{1}{1-\epsilon_0} \\ a^{-1} \times -i(-\eta)^{\frac{1}{2}} \dfrac{2^{\nu_0 - 1}\Gamma(\nu_0)}{\sqrt{\pi}} \dfrac{1}{(-p\eta)^{\nu_0}} e^{+i\mathbf{p}\cdot\mathbf{x}} & \text{at } -p\eta \ll \frac{1}{1-\epsilon_0} \end{cases}$$

Scaling law breaking

Accelerated expansion:



size of universe: a/p_0

We focus on fluctuations at super-horizon scales:

$$\langle \varphi_0^2(x) \rangle \simeq \frac{2^{2\nu_0-2}\Gamma^2(\nu_0)}{\pi} a^{-2}(-\eta) \int \frac{d^3p}{(2\pi)^3} \frac{1}{(-p\eta)^{2\nu_0}} \qquad \nu_0 \geq 3/2$$

$$= \frac{2^{2\nu_0-3}\Gamma^2(\nu_0)}{\pi^3} a^{-2}(-\eta)^{-2} \int_{-p_0\eta}^{\frac{1}{1-\epsilon_0}} \frac{d(-p\eta)}{(-p\eta)^{2\nu_0-2}} \qquad p_{\min} \text{ is fixed}$$

$$= \frac{(2-2\epsilon_0)^{2\nu_0-3}\Gamma^2(\nu_0)}{\pi^3} (1-\epsilon_0)^2 H_0^2 a^{-2\epsilon_0} \times \frac{1-\epsilon_0}{2\epsilon_0}\{(a/a_0)^{2\epsilon_0} - 1\}$$

$\propto H^2$ scaling law — growing with time

$$a_0 \equiv (p_0/H_0)^{\frac{1}{1-\epsilon_0}}$$

Rewritten as

$$\langle \varphi_0^2(x) \rangle \simeq \frac{(2-2\epsilon_0)^{2\nu_0-3}\Gamma^2(\nu_0)}{\pi^3} (1-\epsilon_0)^3 \int_{a_0}^{a} d(\log a')\ H'^2$$

In interacting field theories (i)

For resummation of leading IR effects, we extract dominant terms of the Wightman functions and the retarded propagator

Secular growths of the Wightman functions originate in

$$\varphi_0(x) \simeq \int \frac{d^3p}{(2\pi)^3}\ \theta(Ha-p)\Big[-i\frac{2^{\nu_0-1}\Gamma(\nu_0)}{\sqrt{\pi}}\frac{a^{-1}(-\eta)^{\frac{1}{2}}}{(-p\eta)^{\nu_0}}e^{+i\mathbf{p}\cdot\mathbf{x}}a_{\mathbf{p}}+(\text{h.c.})\Big]$$

$$= \int \frac{d^3p}{(2\pi)^3}\ \theta(Ha-p)\Big[-i\frac{2^{\nu_0-1}\Gamma(\nu_0)}{\sqrt{\pi}}\underline{\frac{\{(1-\epsilon_0)H_0\}^{\frac{1}{1-\epsilon_0}}}{p^{\nu_0}}}e^{+i\mathbf{p}\cdot\mathbf{x}}a_{\mathbf{p}}+(\text{h.c.})\Big]$$

const. (spectrum is frozen
at super-horizon scales)

In interacting field theories (ii)

$$\phi_{\mathbf{p}}(x) \simeq -i\big\{(1-\epsilon_0)H_0\big\}^{\frac{1}{1-\epsilon_0}}\Big[\frac{2^{\nu_0-1}\Gamma(\nu_0)}{\sqrt{\pi}}\frac{1}{p^{\nu_0}} + i\frac{\sqrt{\pi}}{2^{\nu_0+1}\nu_0\Gamma(\nu_0)}p^{\nu_0}(-\eta)^{2\nu_0}\Big]e^{+i\mathbf{p}\cdot\mathbf{x}}$$

Retarded propagator has no secular growth

$$G^R(x,x') \simeq \theta(\eta-\eta')\int\frac{d^3p}{(2\pi)^3}\ \frac{-i}{2\nu_0}\big\{(1-\epsilon_0)H_0\big\}^{\frac{2}{1-\epsilon_0}}\Big[(-\eta')^{2\nu_0}-(-\eta)^{2\nu_0}\Big]e^{+i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')}$$
$$= \theta(\eta-\eta')\times\frac{-i}{2\nu_0}\big\{(1-\epsilon_0)H_0\big\}^{\frac{2}{1-\epsilon_0}}\Big[(-\eta')^{2\nu_0}-(-\eta)^{2\nu_0}\Big]\underline{\delta^{(3)}(\mathbf{x}-\mathbf{x}')}$$

spatially local

but integral of it induces a secular growth as

$$\int\sqrt{-g'}d^4x'\ G^R(x,x') \simeq \frac{-i}{2\nu_0}\big\{(1-\epsilon_0)H_0\big\}^{-\frac{2}{1-\epsilon_0}}\int^{\eta}\frac{d\eta'}{(-\eta')^{\frac{4}{1-\epsilon_0}}}\Big[\underline{(-\eta')^{\frac{3-\epsilon_0}{1-\epsilon_0}}}-(-\eta)^{\frac{3-\epsilon_0}{1-\epsilon_0}}\Big]$$
$$\simeq \frac{-i}{3-\epsilon_0}\int^{a}\underline{d(\log a')}\ H'^{-2}$$

Leading IR effects

For example in φ^4 theory, as loop level is increased by one, quantum corrections are multipled by up to the factor:

$$\lambda \int d(\log a')\ H'^{-2} \int d(\log a'')\ H''^{2}$$

$$\rightarrow \quad \lambda \log^2(a/a_0) \quad \text{at dS limit}$$

Even if $\lambda \ll 1$, perturbation theory is broken after an enough time passed

In a similar way in dS space, resummation formula for leading IR effects is derived from reduction of Yang-Feldman formalism

Resummation formula

$$\varphi(x) = \varphi_0(x) - i\int \sqrt{-g'}d^4x'\ G^R(x,x')\frac{\partial}{\partial\varphi}V(\varphi(x'))$$

Up to leading IR effects

$$\varphi_0(x) \simeq \bar{\varphi}_0(x) = \int \frac{d^3p}{(2\pi)^3}\ \theta(Ha-p)\big[\underline{(\text{const. spectrum})}\big]$$

$$\int \sqrt{-g'}d^4x'\ G^R(x,x') \simeq \frac{-i}{3-\epsilon_0}\int^t dt'\ H'^{-1}$$

$$\varphi(x) = \bar{\varphi}_0(x) - \frac{1}{3-\epsilon_0}\int^t dt'\ H'^{-1}\frac{\partial}{\partial\varphi}V(\varphi(t',\mathbf{x}))$$

$$\Downarrow$$

Langevin eq.: $$\dot{\varphi}(x) = \dot{\bar{\varphi}}_0(x) - \frac{1}{(3-\epsilon_0)H}\frac{\partial}{\partial\varphi}V(\varphi(x))$$

$$\langle\dot{\bar{\varphi}}_0(t,\mathbf{x})\dot{\bar{\varphi}}_0(t',\mathbf{x})\rangle = \frac{(2-2\epsilon_0)^{2\nu_0-3}\Gamma^2(\nu_0)}{\pi^3}\big\{(1-\epsilon_0)H\big\}^3\underline{\delta(t-t')}$$

Fokker-Planck eq.

$$\dot{\rho}(t,\phi) = \frac{1}{2}A\frac{\partial^2}{\partial\phi}\rho(t,\phi) + \frac{1}{(3-\epsilon_0)H}\frac{\partial}{\partial\phi}\big(\rho(t,\phi)\frac{\partial}{\partial\phi}V(\phi)\big)$$

$$A = \frac{(2-2\epsilon_0)^{2\nu_0-3}\Gamma^2(\nu_0)}{\pi^3}\big\{(1-\epsilon_0)H\big\}^3$$

In free field theory, $\langle\varphi_0^2(x)\rangle = \int_{t_0}^{t} dt'\ A'$

For example in φ^4 theory, if the scaling of H is negligibly slow compared with the growth of the IR effects ($\epsilon_0 \ll \lambda^{\frac{1}{2}}$),

Eventually, $\rho(t,\phi) \to N\exp\Big(-\frac{\pi^3}{(2-2\epsilon_0)^{2\nu_0-3}\nu_0\Gamma^2(\nu_0)}\frac{V(\phi)}{\big\{(1-\epsilon_0)H\big\}^4}\Big)$

$$\langle V(\varphi(x))\rangle \to \underline{\frac{(2-2\epsilon_0)^{2\nu_0-3}\nu_0\Gamma^2(\nu_0)}{4\pi^3}\big\{(1-\epsilon_0)H\big\}^4}$$

Not suppressed by λ

Scaling law is recovered

Naive extension of semiclassical description

$$(\frac{\partial^2}{\partial t^2} + 3H\frac{\partial}{\partial t} - \frac{1}{a^2}\frac{\partial^2}{\partial \mathbf{x}^2})\varphi(x) = -\frac{\partial}{\partial \varphi}V(\varphi(x))$$

Extracting IR dynamics: $\varphi = \bar{\varphi} + \varphi_{\mathrm{UV}}$

Neglecting ∂_t^2, $\partial_{\mathbf{x}}^2$

Identifying φ_{UV} as a source of $\bar{\varphi}$

$$3H\frac{\partial}{\partial t}\{\bar{\varphi}(x) + \varphi_{\mathrm{UV}}(x)\} = -\frac{\partial}{\partial \bar{\varphi}}V(\bar{\varphi}(x))$$

$$\varphi_{\mathrm{UV}}(x) = \int \frac{d^3p}{(2\pi)^3}\ \theta(p - Ha)\big[(\text{const. spectrum})\big]$$

Since $\dot{\varphi}_{\mathrm{UV}} = -\dot{\bar{\varphi}}_0$, the reduced equation is

$$\dot{\bar{\varphi}}(x) = \dot{\bar{\varphi}}_0(x) - \frac{1}{3H}\frac{\partial}{\partial \bar{\varphi}}V(\bar{\varphi}(x))$$

Inconsistent with Resummation formula

Improved semiclassical description

It is distinguished by e-folding number whether time variations are at early times ($N < 1$) or at late times ($N > 1$)

$$H\,dt = d(\log a) \equiv dN$$

Neglecting second derivative with respect to N rather than t:

$$\frac{\partial}{\partial t} = H \underline{\frac{\partial}{\partial N}}$$

$$\frac{\partial^2}{\partial t^2} = H^2 \cancel{\frac{\partial^2}{\partial N^2}} - H^2 \epsilon_0 \underline{\frac{\partial}{\partial N}}$$

Semiclassical description is consistent with Resummation formula including all coefficients

$$\underline{(3-\epsilon_0)} H \frac{\partial}{\partial t}\{\bar{\varphi}(x) + \varphi_{\rm UV}(x)\} = -\frac{\partial}{\partial \bar{\varphi}} V(\bar{\varphi}(x))$$

Free scalar field in Accelerating universe where $\dot{\epsilon} \neq 0$

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At $P \ll H$, the wave function of a masslsess, minimally coupled scalar field behaves as

$$\phi_{\mathbf{p}}(x) \simeq -i\frac{2^{\nu_*-1}\Gamma(\nu_*)}{\sqrt{\pi}}\left\{(1-\epsilon_*)H_* a_*^{\epsilon_*}\right\}^{\frac{1}{1-\epsilon_*}}\frac{1}{p^{\nu_*}}e^{+i\mathbf{p}\cdot\mathbf{x}} \qquad \nu = \frac{3}{2}+\frac{\epsilon}{1-\epsilon}$$

$*$: horizon crossing

The IR behavior of the corresponding propagator is

$$\langle\varphi_0^2(x)\rangle \simeq \int_{p_0}^{Ha} dp\, \frac{2^{2\nu_*-3}\Gamma^2(\nu_*)}{\pi^3}\frac{\left\{(1-\epsilon_*)H_* a_*^{\epsilon_*}\right\}^{\frac{2}{1-\epsilon_*}}}{p^{2\nu_*}}$$

$$= \int_{a_0}^{a} d(\log a')\ \underline{(1-\epsilon')} \times \frac{(2-2\epsilon')^{2\nu'-3}\Gamma^2(\nu')}{\pi^3}\left\{(1-\epsilon')H'\right\}^2$$

We changed coordinates as $p = H'a'$ $(dp = (1-\epsilon')p\,d(\log a'))$

Integration at each vertex

Leading IR behavior of the retarded propagator is given by

$$G^R(x,x') \simeq -i\theta(t-t')\Big(\int_{t'}^{t} dt''\ a''^{-3}\Big)\underline{\delta^{(3)}(\mathbf{x}-\mathbf{x}')}$$

spatially local

Integration at each vertex is evaluated as

$$\begin{aligned}\int \sqrt{-g'}d^4x'\ G^R(x,x') &\simeq -i\int^{t} dt'\ a'^3\Big(\int_{t'}^{t} dt''\ a''^{-3}\Big)\\ &= -i\int^{t} dt'\ a'^3\Big[\underline{\int_{t'}^{\infty} dt''\ a''^{-3}} - \int_{t}^{\infty} dt''\ a''^{-3}\Big]\\ &\simeq -i\int^{t} dt'\ \Big(a'^3\int_{t'}^{\infty} dt''\ a''^{-3}\Big)\end{aligned}$$

Resummation formula

Substituting the leading IR behaviors of $\varphi_0(x)$ and $G^R(x,x')$, Yang-Feldman formalism is reduced to the Langevin eq.:

$$\dot{\varphi}(x) = \dot{\bar{\varphi}}_0(x) - \underline{\left(a^3 \int_t^\infty dt'\ a'^{-3}\right)} \frac{\partial}{\partial \varphi} V(\varphi(x))$$

$\neq \frac{1}{(3-\epsilon)H}$ for $\dot{\epsilon} \neq 0$

$$\langle \dot{\bar{\varphi}}_0(t,\mathbf{x}) \dot{\bar{\varphi}}_0(t',\mathbf{x}) \rangle = \frac{(2-2\epsilon)^{2\nu-3}\Gamma^2(\nu)}{\pi^3} \big\{(1-\epsilon)H\big\}^3 \delta(t-t')$$

The corresponding Fokker-Planck eq. is given by

$$\dot{\rho}(t,\phi) = \frac{1}{2} A \frac{\partial^2}{\partial \phi^2} \rho(t,\phi) + \left(a^3 \int_t^\infty dt'\ a'^{-3}\right) \left(\rho(t,\phi) \frac{\partial}{\partial \phi} V(\phi)\right)$$

$$A = \frac{(2-2\epsilon)^{2\nu-3}\Gamma^2(\nu)}{\pi^3} \big\{(1-\epsilon)H\big\}^3$$

If $\dot{\epsilon} \neq 0$, e-folding number is not a good choice for semiclassical description

Generalized semiclassical description

For comparison between first and second derivatives, we choose a time coordinate T as its friction coefficient μ is constant:

$$\frac{\partial^2}{\partial t^2} + 3H\frac{\partial}{\partial t} = \mathcal{H}^2(T)\Big(\frac{\partial^2}{\partial T^2} + \mu\frac{\partial}{\partial T}\Big), \quad \frac{\partial \mu}{\partial T} = 0$$

$$\text{e.g.} \quad \mu = 1, \quad \mathcal{H} = \Big(a^3\int_t^\infty dt'\ a'^{-3}\Big)^{-1}$$

$$T = -\ln\Big(\int_t^\infty dt'\ a'^{-3}\Big)$$

Extracting the first derivative with respect to T,

$$\mathcal{H}^2\mu\frac{\partial}{\partial T}\{\bar{\varphi} + \varphi_{\mathrm{UV}}\} = \Big(a^3\int_t^\infty dt'\ a'^{-3}\Big)^{-1}\frac{\partial}{\partial t}\{\bar{\varphi} + \varphi_{\mathrm{UV}}\}$$

Consistent with Resummation formula

Summary

- In accelerating universes, the increase of d. o. f. at super-horizon scales makes physical quantities growing with time through the propagator of a massless, minimally coupled scalar field

- In order to evaluate the IR effects nonperturbatively, we extended the resummation formula of the leading IR effects in a general accelerating universe

- The resulting equation is given by a Langevin eq. with a white noise, each coefficient of which is modified by the slow-roll parameter

- If we adopt a time coordinate as its friction coefficient is constant, the semiclassical description of the scalar field leads to the same equation