RIEMANN CORRELATOR IN DE SITTER INCLUDING LOOPS OF CONFORMAL FIELDS

Enric Verdaguer

University of Barcelona

in collaboration with M. Fröb and A. Roura

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OUTLINE

1. Introduction
2. CTP (in-in) effective action for metric perturbations
3. One-loop metric correlators
4. Maximally symmetric bitensors in de Sitter
5. Riemann correlators
6. Conclusions and outlook

INTRODUCTION

- De Sitter plays central role in inflationary cosmology, potential of inflaton acts as effective Λ. Amplification of q vacuum fluctuations provides a natural mechanism for the generation of primordial cosmological inhomogeneities (*Mukhanov Chibisov, 81*)

- To explain CMB anisotropies quantize linear cosmological perturbations in nearly dS. At tree level almost scale invariant background of tensor perturbations. More recently one-loop radiative corrections are considered *(Weinberg 05):*

  i-test perturbation theory in dS,

  ii-see how significants effects are,

  iii-check predictions of different inflationary models.

INTRODUCTION

- Earlier work by *Tsamis and Woodard (96,97,98)* suggested that radiative corrections from gravitons at two-loop order could generated a secular screening of the cosmological constant. By interaction of IR graviton modes.

- More recently study of interacting theories in dS *(Marolf, Morrison, Higuchi, Hollands, Rajaraman,10,11)* led to a quantum “no hair” theorem:
  for any initial state correlation functions in a spacetime region of fixed size tend to those of dS invariant vacuum at late times (use Euclidian dS and define the HH vacuum)

OUR GOAL

- Consider the gravitational interaction with matter fields (conformal fields).
- Compute connected two-point correlators of Riemann tensor due to the metric perturbations.
- Include loops of matter but no graviton loops (large *N* expansion).
- Riemann characterizes the geometry of finite size region.
- Riemann tensor is *gauge invariant* so are the Riemann correlators to *1/N*.
- Riemann correlators are also *IR safe observables* at *1/N* *(Giddings, Marolf, Hartle, Urakawa,Tanaka, Senatore, Zaldarriaga, 06,10,12).*

STRATEGY

- Use *CTP formalism*: time evolution of expectation values and correlation functions.
- Use Poincaré patch of dS
- Define a dS invariant adiabatic initial interacting vacuum.
- Metric perturbations: scalar, vector, tensor perturbations.
- Integrate out matter degrees of freedom: effective action.
- Derive the two-point metric correlators.
- Derive Riemann correlators from metric correlators.
- Express results in maximally symmetric bitensors.
- Separate into Weyl, Weyl-Ricci and Ricci correlators.

CTP FORMALISM

IN-OUT vs IN-IN

- in-out generating functional

$$\boxed{Z[J] = \int D\phi e^{i(S[\phi]+\int J\phi d^n x)} = \langle out | T e^{i\int J\phi d^n x} | in \rangle}$$

$$\frac{\langle out | T\phi(x_1)\cdots | in \rangle}{\langle out | in \rangle} = \frac{1}{Z}\frac{\delta}{i\delta J(x_1)}\cdots Z[J]\Big|_{J=0}$$

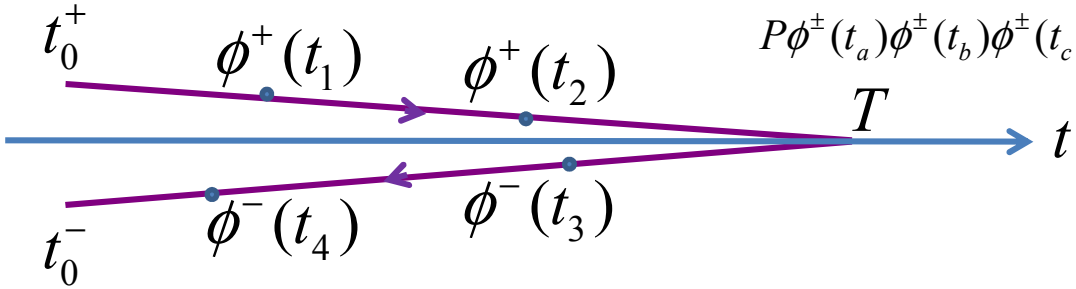
IN-OUT vs IN-IN

- in-in or CTP (closed-time-path)

$$\langle in|A|in\rangle = \sum_{\alpha}\langle in|\alpha T\rangle\langle \alpha T|A|in\rangle = \int D\phi^{+}D\phi^{-}D\alpha A(\phi^{+})e^{iS(\phi^{+})-iS(\phi^{-})}, \quad \hat{\phi}(T)|\alpha T\rangle = \alpha|\alpha T\rangle$$

- CTP generating functional

$$\boxed{\begin{aligned} Z\left[J^{\pm}\right] &= \sum_{\alpha}\langle \alpha T|Te^{i\int J^{-}\phi^{-}d^{n}x}|in\rangle^{*}\langle \alpha T|Te^{i\int J^{+}\phi^{+}d^{n}x}|in\rangle \\ &= \int D\phi^{+}D\phi^{-}e^{i(S(\phi^{+})+\int J^{+}\phi^{+}d^{n}x)-i(S(\phi^{-})+\int J^{-}\phi^{-}d^{n}x)} \end{aligned}}$$

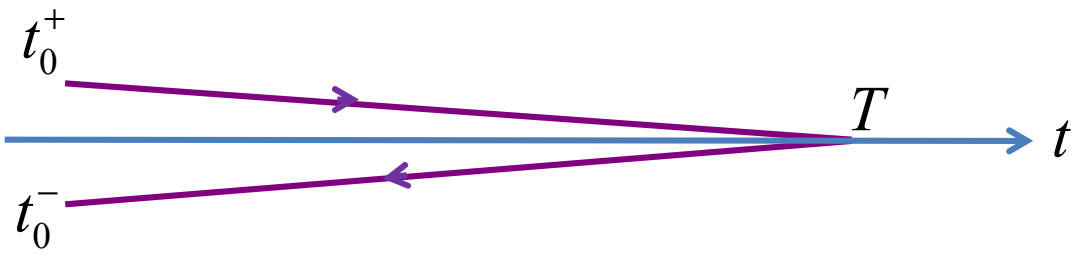


$$P\phi^{\pm}(t_a)\phi^{\pm}(t_b)\phi^{\pm}(t_c)\phi^{\pm}(t_d) = \phi^{-}(t_4)\phi^{-}(t_3)\phi^{+}(t_2)\phi^{+}(t_1)$$

$$P\phi^{+}(t_a)\phi^{+}(t_b) = T\phi^{+}(t_a)\phi^{+}(t_b)$$

INITIAL VACUUM

- For a time independent Hamiltonian the *ground state* of interacting theory is selected by the prescription



$$t_0^{\pm} \to t_\infty^{\pm}(1 \mp i\epsilon)$$

$$t_\infty \to -\infty$$

- In a spatially flat de Sitter (Poincaré patch) it defines an *adiabatic vacuum* of the interacting theory that generalizes the Bunch-Davies vaccum

GRAVITATIONAL AND MATTER FIELD INTERACTION

INTERACTION

- Quantize metric perturbations interacting with matter fields.

- QG as an Effective Field Theory (EFT) in which describe q phenomena at characteristic scale $L >> l_P$. High energy effects appear as local terms in the Lagrangian with $2n$ derivatives and powers of curvature i.e. expansion in powers of (l_P/L)

- Assume $\boxed{\tilde{g}_{ab} = g_{ab} + \kappa h_{ab}}$ $\kappa = \sqrt{16\pi G_N} = \sqrt{16\pi} l_P$

- Expand matter action:

$$\boxed{\begin{aligned} S_M[h,\phi] = \int L_M[\phi]\sqrt{-g}\,d^n x + \frac{\kappa}{2}\int h_{ab}T^{ab}[\phi]\sqrt{-g}\,d^n x + \\ + \frac{\kappa^2}{2}\int h_{ab}h_{cd}U^{abcd}[\phi]\sqrt{-g}\,d^n x \end{aligned}}$$

GRAVITATIONAL ACTION AND LARGE *N*

- Assume the gravitational perturbations interact with *N* (large) matter fields and redefine the gravitational constant

$$\bar{\kappa}^2 = N\kappa^2$$

$$S_G[\tilde{g}] = \frac{N}{\bar{\kappa}^2}\left(\int_M d^n x\sqrt{-\tilde{g}}\left(\tilde{R} - 2\Lambda\right) + S_g^c[\tilde{g}]\right)$$

$$S_G^c = a_1\bar{\kappa}^2\int d^n x\sqrt{-\tilde{g}}(\tilde{R}_{abcd}\tilde{R}^{abcd} - \tilde{R}_{ab}\tilde{R}^{ab}) + a_2\bar{\kappa}^2\int d^n x\sqrt{-\tilde{g}}\tilde{R}^2$$

$\bar{\kappa}$: fundamental cut off scale where *EFT* fails $(\bar{\kappa}/L)^{2n}$

κ : effective low energy scale (measured by exp as l_P)

$(\bar{\kappa}, \Lambda, a_1, a_2)$ are bare parameters

THE GENERATING FUNCTIONAL

- The CTP generating functional is

$$Z[J^{\pm}] = \int Dh^{\pm} D\phi^{\pm} e^{i(S_G[h^+]-S_G[h^-]+S_M[h^+,\phi^+]-S_M[h^+,\phi^+]\pm\int J^{\pm}h^{\pm})}$$

- Integrating out the matter fields becomes

$$Z[J^{\pm}] = \int Dh^{\pm} e^{i(S_G[h^+]-S_G[h^-]+\Sigma[h^{\pm}]\pm\int J^{\pm}h^{\pm})} = \int Dh^{\pm} e^{i(S_{eff}[h^{\pm}]\pm\int J^{\pm}h^{\pm})}$$

- The two-point metric correlation is then obtained from

$$\boxed{\langle in | P h_A(x) h_B(x') | in \rangle = \frac{1}{Z} \frac{\delta^2 Z[J^{\pm}]}{i\delta J^A(x) i\delta J^B(x')}\bigg|_{J=0} = iG_{AB}(x,x')}$$

G_{++}, G_{--} Feynman, Dyson propagators

G_{-+}, G_{+-} +ve, -ve Wightman functions

EFFECTIVE ACTION

EFFECTIVE ACTION

- Effective action $S_{eff}[h^{\pm}] = S_G[h^+] - S_G[h^-] + \Sigma[h^{\pm}]$
- The *UV divergencies* Σ_{div} are regularized with local counterterms in the gravitational action. Finally:

$$\boxed{\begin{aligned} S_{eff}[h^{\pm}] &= S_G^{ren}[h^+] - S_G^{ren}[h^-] + \Sigma^{ren}[h^{\pm}] \\ &= S_{eff}^{(0)} + \int \frac{\delta S_{eff}}{\delta h} h + \frac{1}{2}\int \frac{\delta^2 S_{eff}}{\delta h^2} hh \end{aligned}}$$

factorizes out

=0 semiclassical background

- Only quadratic terms in the metric perturbations needed in the large *N* approximation (no graviton loops)

TWO-POINT METRIC CORRELATIONS

- Perturbatively in the Planck length:

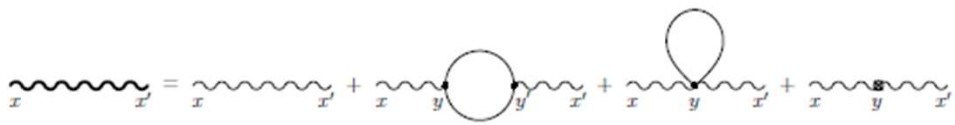
$$S_{eff}\left[h^{\pm}\right]=S_0\left[h^{\pm}\right]+\kappa^2 S_2\left[h^{\pm}\right]=\frac{1}{2}\iint h_M\left(A^0_{MN}+V_{MN}\right)h_N$$

$$S_{eff}\left[h\right]=S_G^{(2)}\left[h\right]+\frac{\kappa^2}{2}\int h_{ab}h_{cd}\left\langle U^{abcd}\right\rangle_{\phi}\sqrt{-g}\,d^4x$$

$$+\frac{i\kappa^2}{8}\iint h_{ab}h_{c'd'}\left\langle PT^{ab}T^{c'd'}\right\rangle_{c,\phi}\sqrt{-g}\sqrt{-g'}\,d^4x\,d^4x'$$

- From where the two-point metric correlation

$$\boxed{G_{AB}(x,x')=G^0_{AB}(x,x')-\kappa^2\iint G^0_{AM}(x,y)V_{MN}(y,z)G^0_{NB}(z,x')\sqrt{-g_y}\sqrt{-g_z}\,d^4y\,d^4z}$$



CONFORMAL FIELDS

- We will assume *N* massless conformally coupled scalar fields with action:

$$\boxed{S_M[\tilde{g},\phi] = -\frac{1}{2}\sum_1^N \int d^n x\sqrt{-\tilde{g}}\left(\tilde{g}^{ab}\nabla_a\phi\nabla_b\phi + \xi_c\tilde{R}\phi^2\right)} \qquad \xi_c = \frac{n-2}{4(n-1)}$$

- In this case in dim reg only one of the bare parameters needs renormalization:

$$Na_1 = Na_1^{ren}(\mu) + \alpha\left((n-4)^{-1} + \ln\mu + \cdots\right) \qquad \text{where} \qquad \alpha = \frac{N}{2880\pi^2}$$

$$Na_2 = (\alpha - \beta)/12$$

change renormalization scale: $Na_1^{ren}(\mu) = Na_1^{ren}(\mu_0) - \alpha\ln(\mu/\mu_0)$

we choose: $Na_1^{ren}(\bar{\mu}) = 0$

METRIC PERTURBATIONS

- On the Poincaré patch of de Sitter

$$\boxed{\tilde{g}_{ab} = a^2(\eta)(\eta_{ab} + h_{ab})} \quad a(\eta) = \frac{1}{-H\eta}, \quad -\infty < \eta \leq 0$$

- Separate into scalar,vector and tensor perturbations on the spatial sections of dS and *fix the gauge*

$$h_{ab} = \delta^0_a \delta^0_b \varphi + \delta^0_{(a} \nabla_{b)} \psi + \delta^0_{(a} v^T_{b)} + h^{TT}_{ab}$$

$$\nabla^a v^T_a = \nabla^a h^{TT}_{ab} = g^{ab} h^{TT}_{ab} = h^{TT}_{0a} = v_0 = 0$$

- *2* scalar, *2* vector, *2* tensorial

$h^{TT}_{ab} \rightarrow$ true dynamical degrees of freedom

$\varphi, \psi, v^T_a \rightarrow$ constrained in terms of the stress tensor

- They decouple in $S_{eff}[h] \rightarrow$ metric correlator can be calculated separately

ONE-LOOP METRIC CORRELATORS

TENSOR PERTURBATIONS

- Take $$h^{TT} = h_{0i}^{TT} = \nabla^a h_{ab}^{TT} = 0$$
- zero order $$\boxed{S_0[h] = -\frac{1}{4}\int a^2 (\partial^a h^{TTmn})(\partial_a h_{mn}^{TT}) d^4x}$$

  is the action for two massless minimally coupled scalars, free propagators (FT)

$$\boxed{\begin{aligned} \bar{G}^{0++} &= -i(\theta(\eta-\eta')f(\eta,\eta',\vec{p}) + \theta(\eta'-\eta)f^*(\eta,\eta',\vec{p})) \\ f(\eta,\eta',\vec{p}) &= \frac{H^2}{2|\vec{p}|^3}(|\vec{p}|\eta - i)(|\vec{p}|\eta' + i)e^{-i|\vec{p}|(\eta-\eta')} \end{aligned}}$$

$$G_{abcd}^{0AB} = G^{0AB} P_{abcd}$$

- It has a log *IR divergence* (but observables characterizing local geometry in a finite size region are finite)

TENSOR PERTURBATIONS

- The spatial Fourier transform to one loop order:

$$\bar{G}^{-+\mathrm{TT}}_{abcd}(\eta,\eta',\boldsymbol{p}) = -\mathrm{i}P_{abcd}\Big[f(\eta,\eta',\boldsymbol{p})\left(1+6\alpha\kappa^2H^2\ln\left(\frac{\bar{\mu}}{H}\right)-(5\alpha-2\beta)\kappa^2H^2\right) + \frac{3}{2}\alpha\kappa^2H^4g(\eta,\eta',\boldsymbol{p})\Big] + \mathcal{O}\left(\kappa^4\right),$$

$$g(\eta,\eta',\boldsymbol{p}) = I_1(\eta,\eta',\boldsymbol{p}) - I_2(\eta,\eta',\boldsymbol{p}) - I_2^*(\eta',\eta,\boldsymbol{p}) + I_3(\eta,\eta',\boldsymbol{p}) - I_4(\eta,\eta',\boldsymbol{p}) + I_5(\eta,\eta',\boldsymbol{p})$$

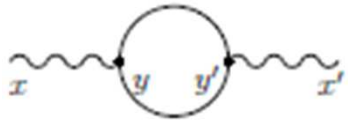
$$I_1(\eta,\eta',\boldsymbol{p}) = 2|\boldsymbol{p}|^{-1}\eta\eta'\mathrm{e}^{-\mathrm{i}|\boldsymbol{p}|(\eta-\eta')}$$

$$I_2(\eta,\eta',\boldsymbol{p}) = |\boldsymbol{p}|^{-3}\mathrm{e}^{\mathrm{i}|\boldsymbol{p}|(\eta+\eta')}\left(|\boldsymbol{p}|\eta+\mathrm{i}\right)\left(|\boldsymbol{p}|\eta'+\mathrm{i}\right)\left[\mathrm{Ein}\left(-2\mathrm{i}|\boldsymbol{p}|\eta\right)+\ln\left(2\mathrm{i}|\boldsymbol{p}|\eta\right)+\gamma\right] + |\boldsymbol{p}|^{-3}\mathrm{e}^{-\mathrm{i}|\boldsymbol{p}|(\eta-\eta')}\left(|\boldsymbol{p}|\eta-\mathrm{i}\right)\left(|\boldsymbol{p}|\eta'+\mathrm{i}\right)\ln\left(-2|\boldsymbol{p}|\eta\right)$$

$$I_3(\eta,\eta',\boldsymbol{p}) = |\boldsymbol{p}|^{-3}\mathrm{e}^{-\mathrm{i}|\boldsymbol{p}|(\eta-\eta')}\left(|\boldsymbol{p}|\eta-\mathrm{i}\right)\left(|\boldsymbol{p}|\eta'+\mathrm{i}\right)\left[\ln\left(2\mathrm{i}|\boldsymbol{p}|(\eta-\eta')\right)+\gamma\right]$$

$$I_4(\eta,\eta',\boldsymbol{p}) = |\boldsymbol{p}|^{-3}\mathrm{e}^{\mathrm{i}|\boldsymbol{p}|(\eta-\eta')}\left(|\boldsymbol{p}|\eta+\mathrm{i}\right)\left(|\boldsymbol{p}|\eta'-\mathrm{i}\right)\left[\mathrm{Ein}\left(-2\mathrm{i}|\boldsymbol{p}|(\eta-\eta')\right)+\ln\left(2\mathrm{i}|\boldsymbol{p}|(\eta-\eta')\right)+\gamma\right]$$

$$I_5(\eta,\eta',\boldsymbol{p}) = \eta^2(\eta')^2\bar{S}(\eta-\eta',\boldsymbol{p}).$$



$$\bar{S}(\eta-\eta',\boldsymbol{p}) = -\mathrm{i}\mathrm{e}^{-\mathrm{i}|\boldsymbol{p}|(\eta-\eta')}\mathcal{P}\frac{1}{\eta-\eta'} + \pi\delta(\eta-\eta')$$

- It is compatible with dS invariance:spatial *homogeneity*, *isotropy* and *rescaling of coordinates*.
- Obtained for arbitrary times
- Well defined distribution

POWER SPECTRUM

- Defined as FT at equal times is not well defined to one-loop (singular limit) Ignoring nonregular parts to compare with literature:

$$\delta^2(|\vec{p}|,\eta) = \frac{\kappa^2}{32\pi^3}|\vec{p}|^3 \eta_{am}\eta_{bn}\int \langle in|h^{ab}(\eta,\vec{p})h^{mn}(\eta,\vec{q})|in\rangle \frac{d^3q}{(2\pi)^3}$$

define $\quad p_{ph} = \frac{|\vec{p}|}{a} = -H|\vec{p}|\eta$

- *Sub-horizon* $(p_{ph} >> H)$

$$\boxed{\delta^2(p_{ph}) \to \frac{\kappa^2}{16\pi^3} p_{ph}^2 (1 - 2\alpha\kappa^2 p_{ph}^2 + O(\kappa^4))}$$

(power correction to Minkowski spectrum)

- *Super-horizon* $(p_{ph} << H)$

$$\boxed{\begin{aligned}\delta^2(p_{ph}) \to \frac{\kappa^2 H^2}{16\pi^3}(1 + 6\alpha\kappa^2 H^2(\ln(\bar{\mu}/H) + \gamma) \\ -(5\alpha - 2\beta)\kappa^2 H^2 + O(\kappa^4))\end{aligned}}$$

(small shift of scale inv spectrum)

SCALAR, VECTOR PERTURBATIONS

- Constrained degrees of freedom
- zero order

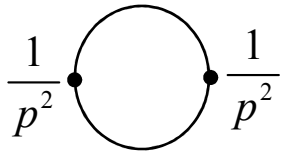
$$S_0^{SV}[h] = -\frac{1}{2}\int a^2 v_T^m \Delta v_m^T d^4x + 2H\int a^3 \varphi \Delta \psi d^4x - \frac{3}{2}H^2 \int a^4 \varphi^2 d^4x$$

free “propagators” (FT)

$$\bar{G}^{0++}_{v_+v_+} = \frac{H^2\eta^2}{2\vec{p}^2}\delta(\eta-\eta'),\ \ \bar{G}^{0++}_{\varphi\psi} = \frac{H^2\eta^3}{2\vec{p}^2}\delta(\eta-\eta'),\ \ \bar{G}^{0++}_{\psi\psi} = \frac{3H^2\eta^2}{4(\vec{p}^2)^2}\delta(\eta-\eta')$$

- contributions to the two-point function

$$\bar{G}^{-+V}_{abc'd'}(\eta,\eta',\vec{p}) = \kappa^2 \iint \bar{G}^{0++V}_{abmn}(\eta,\tau)\bar{V}^{mnpq}_{-+}(\tau,\tau')\bar{G}^{0++V}_{pqcd}(\tau',\eta')$$



it is related to the *stress tensor* correlator

METRIC PERTURBATIONS

- Finally the metric perturbations two-point function is:

$$-i\left\langle in\middle|h_{ab}(x)h_{c'd'}(x')\middle|in\right\rangle = G^{-+S}_{abc'd'}(x,x') + G^{-+V}_{abc'd'}(x,x') + G^{-+TT}_{abc'd'}(x,x')$$

- Acting with two 2nd order differential operators we get the Riemann correlator

MAXIMALLY SYMMETRIC BITENSORS

DE SITTER GEOMETRY

- De Sitter is an embedding in (d+1)-Minkowski:

$$\boxed{\eta_{AB} X^A(x) X^b(x) = H^{-2}, \quad A,B = 0,\cdots,d}$$

Inherits the symmetries of Minkowski that leave the hyperboloid unchanged. Maximally symmetric isotropy group is $O(d,1)$

- Define dS invariant biscalar: $\boxed{Z(x,x') = H^2 \eta_{AB} X^A(x) X^B(x')}$ related to Minkowski interval:

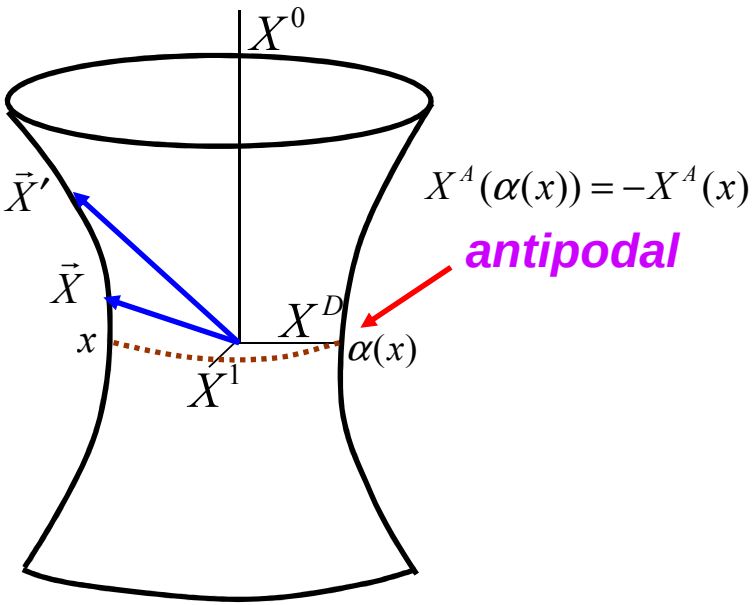
$$\eta_{AB}(X^A(x) - X^A(x'))(X^B(x) - X^B(x')) = 2H^{-2}(1 - Z(x,x'))$$

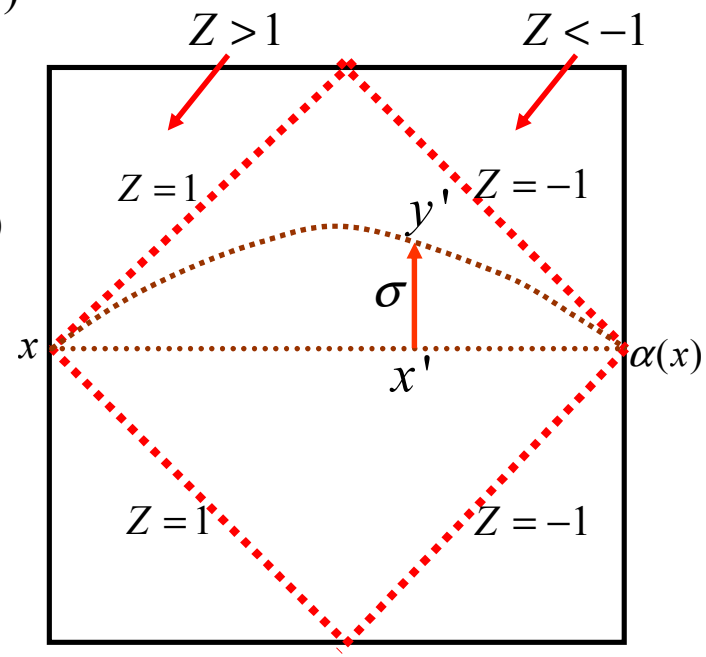
- Geodesic distance along shortest geodesic connecting *x* and *x'*: $\mu(x,x')$

$$\boxed{Z(x,x') = \cos(H\mu(x,x'))}$$

DE SITTER GEOMETRY

$$d^2 = 2H^{-2}(1-Z)$$





conformal diagram of dS

$Z(x,x') = 1$	light-like
$Z(x,x') > 1$	time-like
$Z(x,x') < 1$	space-like
$Z(x,x') < -1$	space-like not connected by geodesic

MAXIMALLY SYMMETRIC BITENSORS

- Functions, tensors inv under isometries $f(x) = cont,\ A_{ab} \propto g_{ab}$
- Biscalars $f(x,x') = f(Z(x,x'))$
- Allen-Jacobson *(86):* any maximally symmetric bitensor is a linear combination of products of the basic bitensors with coefficients that depend on $\mu(x,x')$ only:

$$\boxed{g_{ab}(x),\ n_a(x,x') = \nabla_a \mu(x,x'),\ n_{a'}(x,x') = \nabla_{a'} \mu(x,x'),\ g_{ab'}(x,x')}$$

$g_{ab'}$ parallel transports a vector along geodesic $w^a = g^a_{b'} w^{b'}$

- In terms of covariant derivatives of *Z(x,x')*:

$$n_a = \frac{-1}{H\sqrt{1-Z^2}} \nabla_a Z,\ H^2 g_{ab'} = \nabla_a \nabla_{b'} Z - \frac{1}{1+Z} \nabla_a Z \nabla_{b'} Z,\ \nabla_a \nabla_b Z = -ZH^2 g_{ab}$$

in Poincaré patch: $Z(x,x') = 1 - \dfrac{(\vec{x}-\vec{x}')^2 - (\eta-\eta')^2}{2\eta\eta'}$

RIEMANN CORRELATOR

RIEMANN CORRELATOR

$$\tilde{R}^{ab}{}_{cd} = 2H^2\delta^a_{[c}\delta^b_{d]} + 2\kappa H^4\eta^2 T^{ab}{}_{cd}{}^{mn} h_{mn} + O(\kappa^2)$$

- is gauge invariant $L_{\bar{\xi}} R^{(0)ab}_{cd} = 0$
- Connected Riemann correlator

$$\left\langle \tilde{R}^{ab}{}_{cd}(x)\tilde{R}^{m'n'}{}_{p'q'}(x')\right\rangle_c = 4\kappa^2 H^8\eta^2(\eta')^2 T^{[ab]}{}_{[cd]}{}^{kl} T^{[m'n']}{}_{[p'q']}{}^{s't'}(iG^{-+}_{kls't'}(x,x'))$$

after a long calculation all terms that are not dS invariant vanish and get

$$\left\langle \bar{R}^{ab}{}_{cd}(x)\bar{R}^{m'n'}{}_{p'q'}(x')\right\rangle_c =$$
$$\frac{4\kappa^2 H^6}{\pi^2}\sum_{k=1}^{9} {}^{(k)}\mathcal{N}^{[ab]}{}_{[cd]}{}^{[m'n']}{}_{[p'q']}\,\underset{\epsilon\to 0^+}{\text{d-lim}}\left[\frac{3}{2}\alpha\kappa^2 H^2\mathcal{S}^{(1,k)}(Z(x,x')-\mathrm{i}\epsilon^{\pm})\right.$$
$$\left. + \left(1+6\alpha\kappa^2 H^2\left(\ln\left(\frac{\bar{\mu}}{H}\right)+\gamma\right)-(5\alpha-2\beta)\kappa^2 H^2\right)\mathcal{S}^{(0,k)}(Z(x,x')-\mathrm{i}\epsilon^{\pm})\right]$$

${}^{(k)}N$ are 8 index *dS* inv bitensors. Well defined distribution, arbitrary *x* and *x'*. Defined in all *dS*

WEYL CORRELATOR

Riemann-Riemann= Weyl-Weyl+Weyl-Ricci+Ricci-Ricci

$$\tilde{C}^{ab}_{\ \ cd} = \left(\delta^a_k \delta^b_l \delta^s_c \delta^t_d + \cdots\right) \tilde{R}^{kl}_{\ \ st}$$

- Spacelike separations $\eta = \eta',\ d^2 = a^2(\eta)(\vec{x} - \vec{x}')^2 = 2H^{-2}(1 - Z)$
  - Super-horizon $(d >> H^{-1})$

$$\left\langle Weyl(x) Weyl(x') \right\rangle \approx \frac{\kappa^2 H^2}{d^4} + \frac{\kappa^2}{d^6} + \frac{\kappa^4 H^4}{d^4} + \frac{\kappa^4 H^2}{d^6} \ln(Hd) + \cdots \qquad \kappa \equiv \sqrt{16\pi} l_P$$

perturb expansion breaks for sufficiently large distances (need non perturb)

  - Sub-horizon $(d << H^{-1})$

$$\left\langle Weyl(x) Weyl(x') \right\rangle \approx \frac{\kappa^2}{d^6} + \frac{\kappa^4}{d^8} + \cdots$$

same result as in flat space

- Large timelike separations $\vec{x} = \vec{x}',\ |\eta - \eta'| \to \infty,\ \tau^2 = -\mu^2(x, x'),\ \tau = H^{-1} \ln Z$

$$\left\langle Weyl(x) Weyl(x') \right\rangle \approx \kappa^2 H^6 e^{-2H\tau} + \kappa^2 H^8 e^{-2H\tau} + \kappa^2 H^9 \tau e^{-3H\tau} + \cdots$$

fall off exponentially with proper time separation

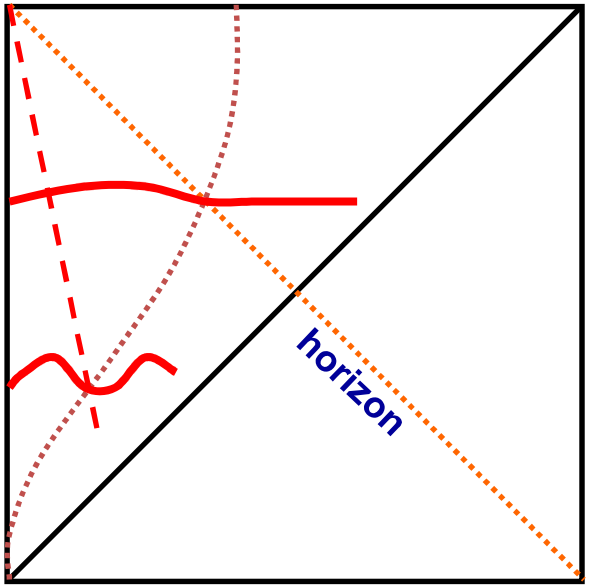
RICCI AND RICCI-WEYL CORRELATORS

$$\langle Ricci(x)Ricci(x')\rangle \approx \begin{cases} \dfrac{\kappa^4}{d^8}+\cdots & \text{super-horizon} \\ \kappa^4 H^8 e^{-4H\tau}+\cdots & \text{large timelike} \\ \dfrac{\kappa^4}{d^8}+\cdots & \text{sub-horizon (flat space)} \end{cases}$$

$$\langle Ricci(x)Weyl(x')\rangle \approx \begin{cases} \dfrac{\kappa^4 H^4}{d^4}+\cdots & \text{super-horizon} \\ \kappa^4 H^8 e^{-2H\tau}+\cdots & \text{large timelike} \\ \dfrac{\kappa^4}{d^8}+\cdots & \text{sub-horizon (flat space)} \end{cases}$$

- Weyl-Weyl at tree level agrees with *Kouris (01,12)*
- Ricci-Ricci agrees with stress tensor correlator $\boxed{\langle T^a_b(x)T^{c'}_{d'}(x')\rangle}$
- Symmetries and Bianchi identities satisfied
- Only singular at *Z =1*, as 2-point f of the scalar field

LOCALIZED vs NONLOCALIZED OBSERVABLES



- Riemann tensor provides a suitable characterization of local geometry
- Riemann correlator is *IR safe observable*

- Gauge invariant quantities as tensor metric correlators not determined by geometry of fixed size: depend on region whose proper size grows with expansion.

- Classical *no hair* theorems for dS do not apply to them and have IR divergencies.

- dS inv hold for observables depending on geometry of proper *finite size*:
  Riemann correlator is of this class.

CONCLUSIONS

- Found two-point *Riemann correlator* for metric perturbations on dS including radiative corrections of conformal fields

- Use an *adiabatic dS inv interacting* vacuum (generalize *BD*) in Poincaré patch.

- Obtain *explicit dS inv* results in terms of maximally symmetric bitensors, valid in all dS

- Riemann correlator is *gauge invariant* at order *1/N*, exclude graviton loops order *1/N^2*

- Riemann correlator is an *infrared (IR) safe* observable

OUTLOOK

- Need non preturbative calculation to capture full behavior at very large separations (terms such as *log(Hd))*

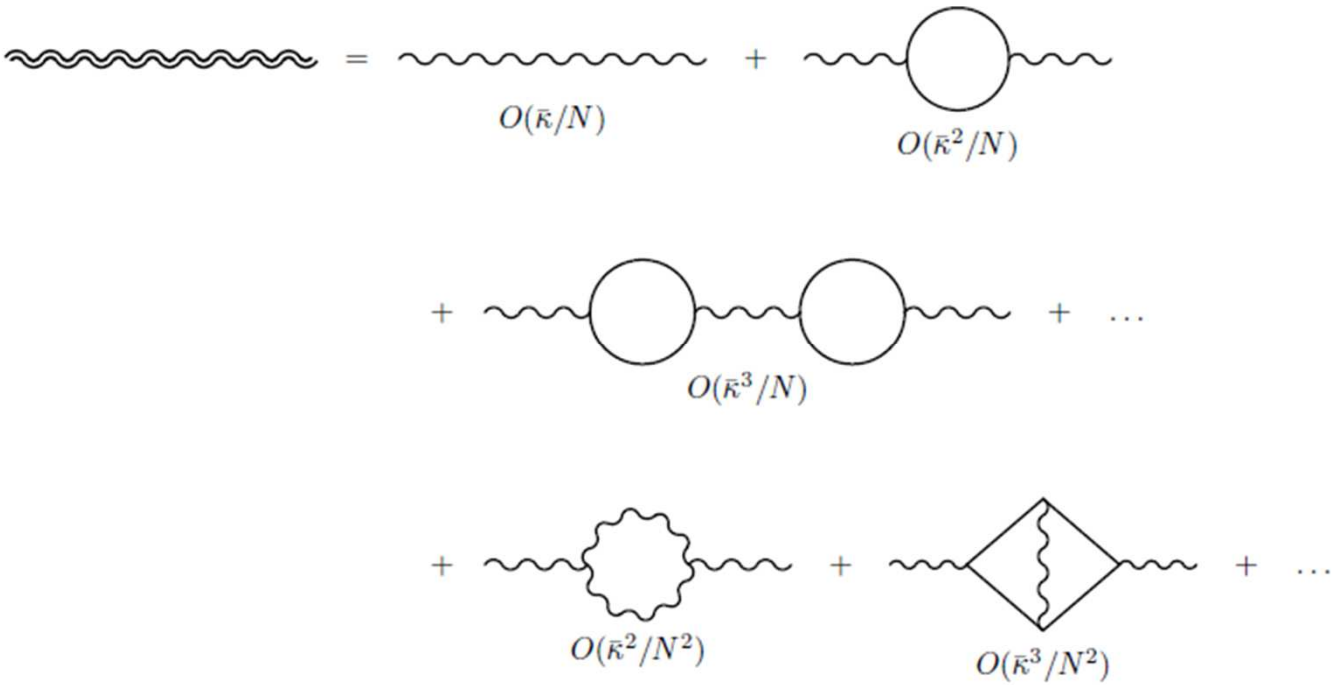
- Include graviton loops, need diffeo inv observables

- Extend result to other fields (ex minimally coupled).
  Result: Assuming the stress tensor correlator is dS inv the two-point *Riemann correlator is also dS inv*.
  It is given explicitly in terms of dS inv bitensors and it is determined by the matter fields except for a part which corresponds to the free graviton part of Weyl correlator, in terms of an undetermined constant.

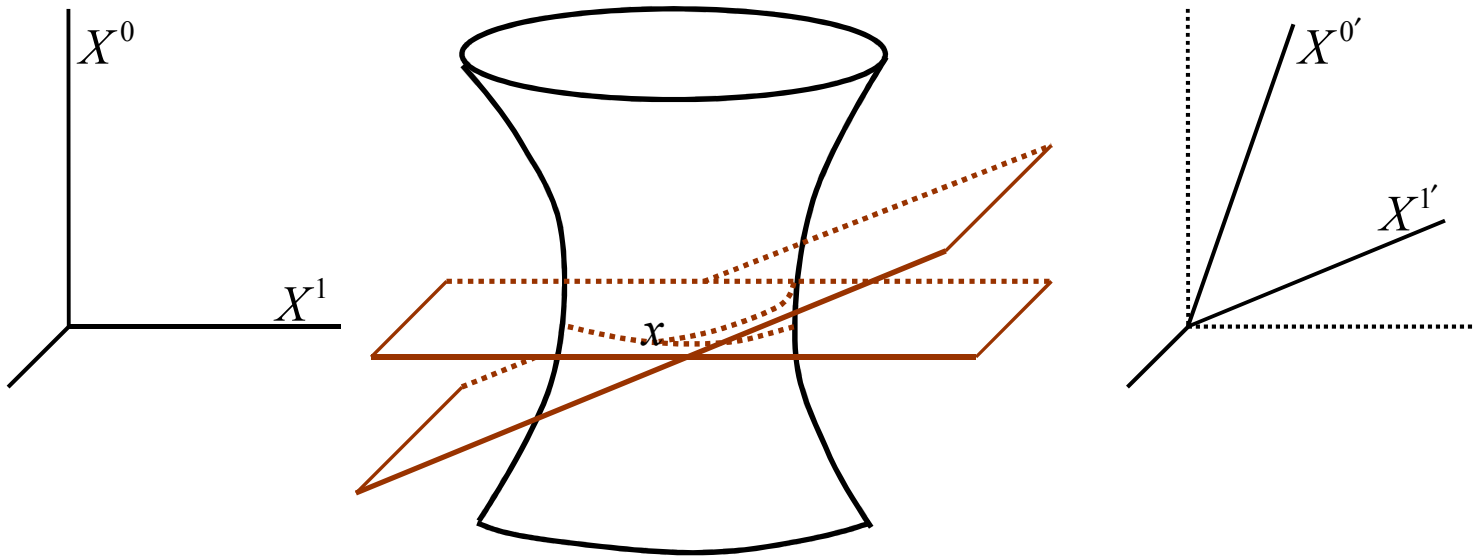
THANK YOU!

LARGE N EXPANSION

$$S = \frac{N}{\bar{\kappa}} \int d^4x \, (\partial_a h \partial^a h + h \, \partial_a h \partial^a h + \dots)$$
$$- \sum_j^N \int d^4x \, (\partial_a \phi_j \partial^a \phi_j + m^2 \phi^2) + \sum_j^N \int d^4x \, (h \, \partial_a \phi_j \partial^a \phi_j + \dots) \, .$$



LORENTZ BOOST



DE SITTER BITENSORS

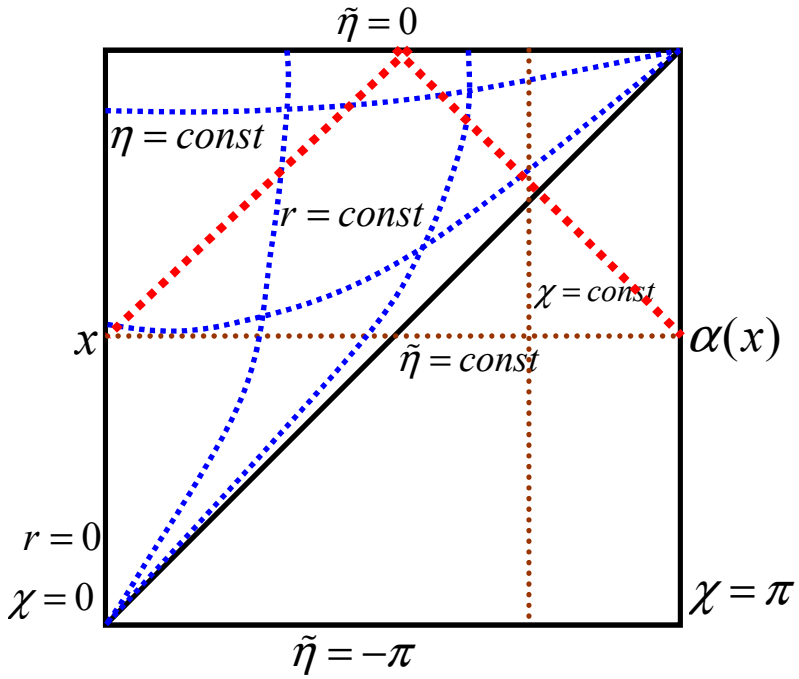
- physical distance in spatially flat coordinates is also:

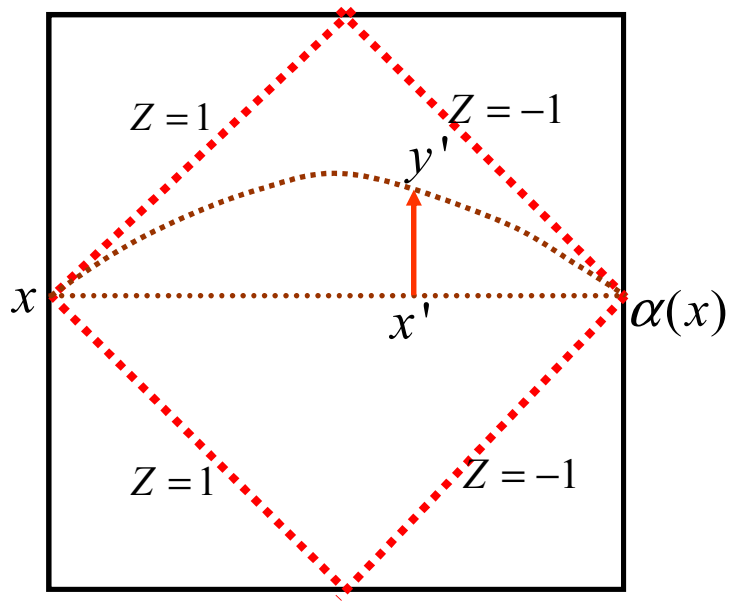
$$d^2(x,x') = e^{2Ht}\delta_{ij}\left(x^i(x)-x^i(x')\right)\left(x^j(x)-x^j(x')\right)$$

- **geodesic distance** *(x,x')*: $\mu(x,x') = \int_0^1 d\lambda \left(g_{ab}(x(\lambda))v^a(\lambda)v^b(\lambda)\right)^{1/2}$

- basic bitensors $n_a(x,x') = \nabla_a \mu, \quad n_{a'}(x,x') = \nabla_{a'}\mu, \quad g_{ab'}(x,x')$ parallel transp

  For maximally symmetric bitensors (dS inv)



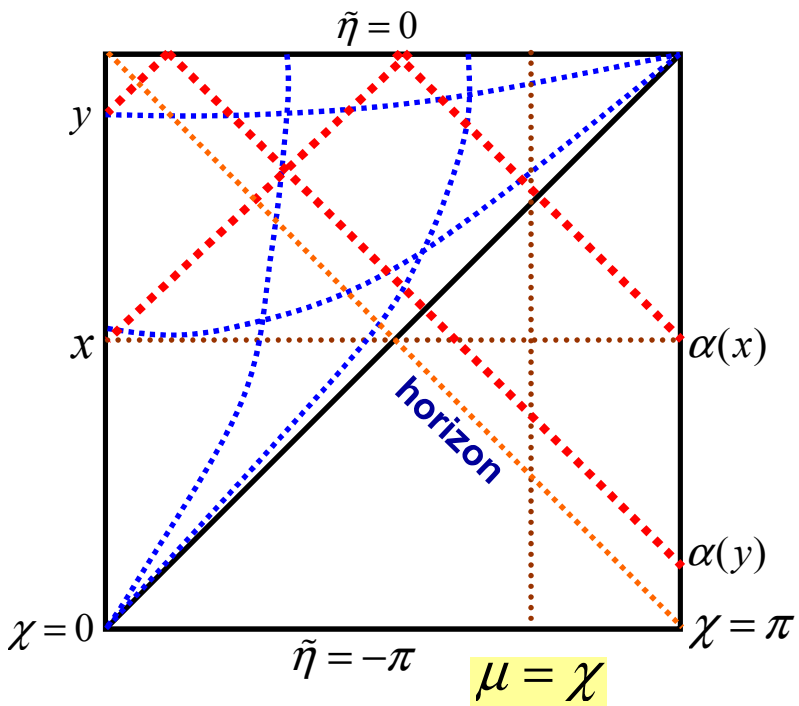
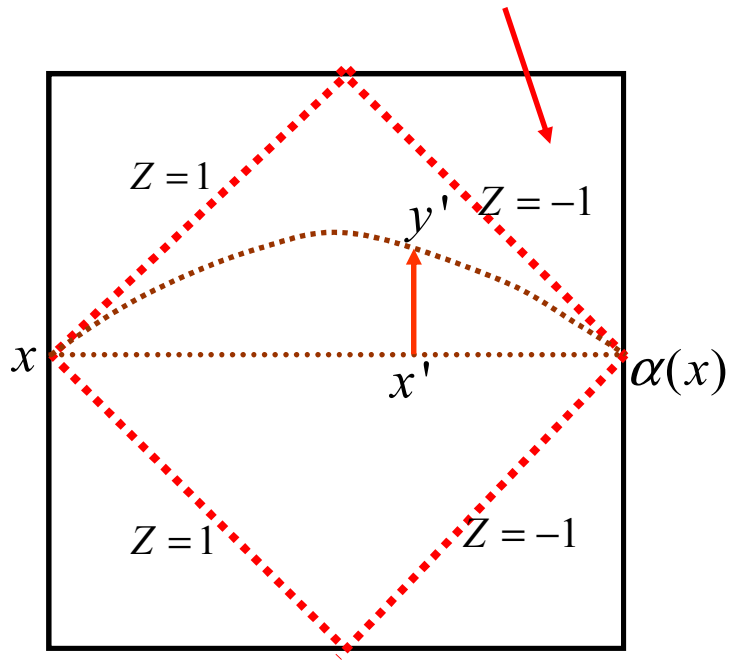


DE SITTER GEOMETRY

- covariant derivatives of basic bitensors in dS (th) *(Allen-Jacobson 86)*

$$\nabla_a n_b = \cot\mu (g_{ab} - n_a n_b),$$
$$\nabla_a n_{b'} = -\csc\mu (g_{ab'} + n_a n_{b'}),$$
$$\nabla_a g_{bc'} = (\csc\mu - \cot\mu)(g_{ab} n_{c'} + g_{ac'} n_b).$$

- geodesic dist for spacelike geod: can be def. for **points not connected by geod**

$1 \le Z < 1$

$Z < -1$