Covariant graviton (and Faddeev-Popov ghost) propagators in de Sitter space

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Collaborators: Spyros Kouris, Yen Choeng Lee, Mir Faizal,
Jos Gibbons, William Couto Corrêa de Lima
Markus Fröb, Ian Morrison, Don Marolf

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- Useful gauge-invariant observables are notoriously difficult to evaluate, and all quantities secularly growing due to graviton loops found in the literature are suspected to be non gauge invariant. (Please raise your objections to this statement on Friday in one of the discussion sessions on observables.)
- Much of the secular growth comes from an explicit de Sitter breaking term in the propagator.

Main purposes of this talk

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- I describe our attempt to construct a Woodard standard (mode-sum, exact-gauge, Poincaré-patch) IR-finite de Sitter invariant propagator. We (Markus Fröb, William de Lima and AH) failed but came close.
- I'll convince most of the audience that I am not a "religious fundamentalist de Sitter fanatic" but a moderate de Sitter breaking sceptic.

Outline

1. History of IR divergences in graviton propagator in de Sitter space
2. The IR divergence of the graviton correlator in the TT-synchronous gauge
3. Covariant graviton propagator by mode sum

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2. The IR divergence of the graviton correlator in the TT-synchronous gauge
3. Covariant graviton propagator by mode sum
4. de Sitter invariant graviton vacuum state in the global patch
5. Removing the IR divergences in the Faddeev-Popov ghost propagator

Content

- Ford and Parker, "Quantized gravitation wave perturbations in Robertson-Walker universes," Phys. Rev. D16, 1601 (1977):
  "... The resulting [graviton] theory [in transverse-traceless-synchronous gauge] is equivalent to that for a pair of massless, minimally coupled scalar fields in [FLRW] space-time. As an application of the formalism, we calculate the spectrum of gravitons produced by a power-law expansion of the universe and show that it has no divergences."

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  The angular momentum $L = 0$ mode on the spatial section S^3 causes IR divergences of the minimally-coupled massless scalar field; the graviton modes starts at $L = 2$ [More later if time permits]

Covariant propagator: history

- Antoniadis, Illiopoulos and Tomaras, “Quantum instability of de Sitter space,” Phys. Rev. Lett. 56, 1319 (1986):
  “... The graviton propagator in a de Sitter background is found to be divergent. ... If we start from de Sitter space as a classical ground state, quantum corrections change it into flat Minkowski space.”

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  "... We find that there are no infrared divergences when [free gravitons are] coupled to conserved currents [which should have been "conserved stress-energy tensor"] [in the Poincaré patch].
  He showed that near a point source the IR divergence can be written in the form $\nabla_\mu\Lambda_\nu + \nabla_\nu\Lambda_\mu$.

Covariant propagator: history

- Allen, "The graviton propagator in de Sitter space," Phys. Rev. D34, 3670 (1986):
  The covariant graviton propagator in the (average) gauge corresponding to $\nabla^\mu[h_{\mu\nu} - (1+\beta^{-1})g_{\mu\nu}h] = 0$ is IR-divergent if $\beta = -L(L+3)/3$, $L = 1, 2, 3, \ldots$. The propagator of Antoniadis, Illiopoulos and Tomaras: $L = 1$. [More later]

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- Antoniadis and Mottola, "Graviton fluctuation in de Sitter space," J. Math. Phys. 3, 103 (1991):
  - Explicit construction of the propagator in the AIT gauge.
  - Claimed the retarded Green's function was singular.

The claim was that the retarded Green's function $\Delta^{ret}(x, x')$ for the spin-0 sector did not satisfy

$$2/3(\Box + 4H^2)\Delta^{ret}(x, x') = \delta^4(x, x') \qquad (A)$$

but satisfied instead

$$2/3(\Box + 4H^2)\Delta^{ret}(x, x') = \delta^4(x, x') - \sum_{i=1}^{5} \xi^{(i)}(x)\xi^{(i)}(x'),$$

where the $\xi^{(i)}(x)$ are the conformal Killing vectors on de Sitter space.

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Now the authors find that Eq. (A) is actually satisfied though the issue of quantum IR singularity is a different matter.

- Tsamis and Woodard, “Quantum gravity slows inflation,” hep-th/9602315, claimed that due to two-loop effects the effective Hubble constant behaved as a function of proper time t as

$$H_{\text{eff}}(t) = H\left\{1 - \left(\frac{\kappa H}{4\pi}\right)^4 \left[\frac{(Ht)^2}{6} + (\text{sub-leading}) + O(\kappa^2)\right]\right\}.$$

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The authors now believe that this result was entirely due to a non-gauge-invariant UV regularisation and that the two-loop effects will lead to

$$H_{\text{eff}}(t) = H\left\{1 - \left(\frac{\kappa H}{4\pi}\right)^4 \left(cHt + O(\kappa^2)\right)\right\},$$

c: some constant.

Covariant propagator: history

- Kouris and AH, "The covariant graviton propagator in de Sitter spacetime," gr-qc/0107036.
  - Generalisation of the Allen-Turyn and Antoniadis-Mottola propagators by the Euclidean method with the gauge-fixing term $-\frac{1}{2\alpha}[\nabla^\mu(h_{\mu\nu} - (1+\beta^{-1})g_{\mu\nu}h)]^2$. (This includes the exact gauge $\alpha = 0$!)
  - IR finite and de Sitter invariant if $\beta \neq -L(L+3)/3$, $L = 1, 2, 3, \ldots$.
  - If $\beta > 0$, the growth of the propagator as $Z = \cos(H\mu)$ (μ: geodesic distance for spacelike-separated points) the propagator behaves like $\alpha C_1 Z + C_2 \log Z$ for large $|Z|$.

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- Woodard, "De Sitter breaking in field theory," gr-qc/0408002. Among other things, the retarded Green's function obtained from the covariant propagator cannot generate the correct field in spacetime with spacelike past infinity due to causality.

Covariant propagator: history

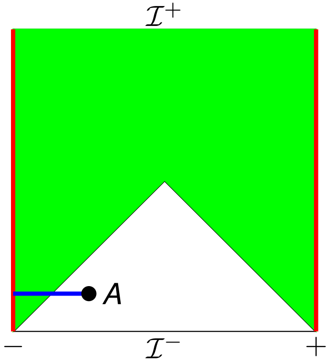

Figure: Carter-Penrose diagram for de Sitter space: field generated from the charges at antipodal points

The retarded Green's function obtained from the covariant propagator generates non-zero fields only in the shaded region due to causality. The electric field on the sphere at *A* vanishes, hence the Gauss' law is not satisfied.

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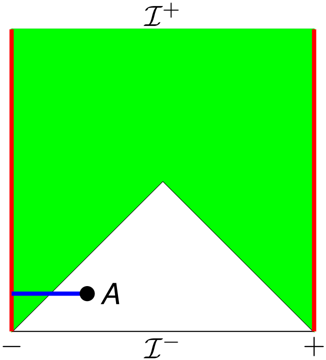

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Consensus: the retarded Green's function is correct but should be used differently (c.f. Lee and AH, "How to use retarded Green's functions in de Sitter spacetime," arXiv:0808.0642.)

Covariant propagator: history

- Miao, Tsamis and Woodard, "Transforming to Lorentz gauge on de Sitter," arXiv:0907.4930.
  - The average covariant gauge with gauge-fixing term $-(1/2\alpha)(\nabla_\mu A^\mu)^2$ for QED in flat D-torus space (and de Sitter space?) introduces space-independent $A_0(t)$, which should not be there according to the path-integral derivation.
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AH: In the Hamiltonian, $A_0(t)$ appears only as

$$H = \cdots + \frac{1}{2\alpha}[\dot{A}_0(t)]^2 + A_0(t)Q + \cdots$$

Q: conserved total charge. ($Q|\psi\rangle = 0$ to satisfy Gauss' law.) $A_0(t)$ is exactly given as $A_0(t) = \frac{1}{2}\alpha t^2 Q + tO_1 + O_2$ (O_1, O_2: constant operators) and it decouples from the rest. So, for QED there is no problem.

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Covariant propagator: history

- Miao, Tsamis and Woodard, "The graviton propagator in de Donder gauge on de Sitter background," arXiv:1106.0925,
- Mora, Tsamis and Woodard, "Graviton propagator in a general invariant gauge on de Sittter," arXiv:1205.4468.
  - Construction of the graviton propagator in the exact/Landau($\alpha = 0$) gauge, i.e. with the gauge condition $\nabla^{\mu}(h_{\mu\nu} - (1+\beta^{-1})g_{\mu\nu}h^{\alpha}{}_{\alpha}) = 0$.
  - Spin-0 sector has no IR divergences if $\beta > 0$.
  - Spin-2 sector is IR divergent and breaks de Sitter invariance.

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- Miao, Tsamis and Woodard, “Perils of analytic continuation,” arXiv:1107.4733. Rebuttal.

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- The 4th-order TT-projection operator:

$$P_{\mu\nu}{}^{\alpha\beta} h_{\alpha\beta} = \nabla^\gamma \nabla^\delta C^{(1)}_{\gamma\mu\delta\nu}(h),$$

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- The spin-2 sector of $T\langle 0|h_{\mu\nu}(x)h^{\overline{\mu\nu}}(x')|0\rangle$ ($H = 1$, $Z = \cos\mu$, $\mu(x, x')$: geodesic distance between x and x' if spacelike separated) is

$$\Delta^{TT\overline{\mu\nu}}_{\mu\nu}(Z) = P_{\mu\nu}{}^{\alpha\beta} P^{\overline{\mu\nu}}{}_{\overline{\alpha}\overline{\beta}} \left[A(Z)(\nabla_{(\alpha}\nabla^{(\overline{\alpha}} Z)(\nabla_{\beta)}\nabla^{\overline{\beta})} Z) \right],$$

$$\frac{1}{2}\Box A(Z) = U(Z),$$

$$\frac{(n-3)^2}{4(n-2)^2}\Box^2(\Box - (n-2))^2 U(Z) = a + bZ + i\frac{\delta^n(x, \overline{x})}{\sqrt{|g|}}.$$

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- Miao, Mora, Tsamis, Woodard,...: $a = 0$ ($b = 0$)
  $\Rightarrow$ de Sitter non-invariant solution.

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- Question: Which one do we find in the mode-sum construction?
  Fröb, de Lima and AH, work in progress. [More later]

Content

TT-synchronous gauge

- Allen, "The graviton propagator in homogeneous and isotropic spacetimes," Nucl. Phys. B287, 743 (1987),
- Kouris and AH, "Large distance behaviour of the graviton two-point function in de Sitter spacetime," gr-qc/0004079.
  - In the Poincaré patch $ds^2 = (H\eta)^{-2}(-d\eta^2 + d\mathbf{x}^2)$, $-\infty < \eta < 0$, the graviton correlator in the gauge $h_{0\mu} = 0$, $h^i{}_i = 0$, $\partial^j h_{ij} = 0$ is IR divergent (Ford and Parker).
  - However, the IR divergent part can be reproduced by a two-point function of the form $\langle 0|\nabla_{(\mu}A_{\nu)}(x)\nabla_{(\mu'}A_{\nu')}(x')|0\rangle$, i.e. as a 'pure gauge'.

TT-synchronous gauge

In the Poincaré patch and in the gauge $h_{0\mu} = 0$ (synchronous), $\partial^i h_{ij} = 0$ (transverse) $h^i{}_i = 0$ (traceless), the correlator is

$$\Delta^{(TT)}_{ijk\ell}(x, x') = \int d^{n-1}\mathbf{p} \sum_s \gamma^s_{ij}(\mathbf{p}, x)\overline{\gamma^s_{k\ell}(\mathbf{p}, x')},$$

where

$$\gamma^s_{ij}(\mathbf{p}, x) = C'\epsilon^s_{ij}(-\eta)^{(n-5)/2} H^{(2)}_{(n-1)/2}(p\eta)e^{i\mathbf{p}\cdot\mathbf{x}},$$

$$\epsilon^s_{ii} = p_i\epsilon^s_{ij} = 0, \quad \epsilon^s_{ij}\epsilon^{s'}_{ij} = 2\delta^{ss'}.$$

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For small p

$$\begin{aligned}
\gamma^s_{ij}(\mathbf{p}, x) &\approx C_T\epsilon^s_{ij}\eta^{(n-5)/2}\left[(p\eta)^{-(n-1)/2} + O((p\eta)^{-(n-5)/2}\right](1 + i\mathbf{p}\cdot\mathbf{x}) \\
&= C_T\epsilon^s_{ij}\eta^{-2}p^{-(n-1)/2} + O(p^{-(n-3)/2}).
\end{aligned}$$

The IR divergent contribution is space-independent.

TT-synchronous gauge

$$\Delta^{(TT)}_{ijk\ell}(x, x') \approx |C'_T|^2 (\eta\eta')^{-2} \int \frac{d^{n-1}\mathbf{p}}{p^{(n-1)}} \sum_s \epsilon^s_{ij}(\hat{\mathbf{p}}) \epsilon_{k\ell}(\hat{\mathbf{p}}),$$

$\hat{\mathbf{p}} \equiv \mathbf{p}/p$.

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$\hat{\mathbf{p}} \equiv \mathbf{p}/p$.
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$$\begin{aligned}\Delta^{(TT)}_{ijk\ell}(x,x') &\approx |C'_T|^2(\eta\eta')^{-2}\int_0^H \frac{dp}{p}\int d\Omega_{n-1}\sum_s \epsilon^s_{ij}(\hat{\mathbf{p}})\epsilon^s_{k\ell}(\hat{\mathbf{p}}) \\ &= |C_T|(\eta\eta')^{-2}\frac{n(n-3)}{(n+1)(n-2)}\delta_{ijk\ell}\int_0^H \frac{dp}{p},\end{aligned}$$

where

$$\delta_{ijk\ell} = \delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk} - \frac{2}{n-1}\delta_{ij}\delta_{k\ell}.$$

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Can this IR divergence be reproduced by $\nabla_\mu A_\nu + \nabla_\nu A_\mu$?

TT-synchronous gauge

Let $A_0 = 0$ and

$$\begin{aligned} A_i^s(\mathbf{p}, x) &= C_V'' \epsilon_i^s (-\eta)^{(n-3)/2} H^{(2)}_{(n+1)/2}(p\eta) e^{i\mathbf{p}\cdot\mathbf{x}} \\ &= C_V' \epsilon_i^s \left[\eta^{-2} p^{-(n+1)/2} + O(p^{-(n-3)/2}) \right] e^{i\mathbf{p}\cdot\mathbf{x}}, \end{aligned}$$

$\epsilon_i^s p_i = 0$, $\epsilon_i^s \epsilon_i^{s'} = \delta^{ss'}$.

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$\epsilon_i^s p_i = 0$, $\epsilon_i^s \epsilon_i^{s'} = \delta^{ss'}$. $\nabla_0 A_0 = 0$ and

$$\nabla_i A_0 + \nabla_0 A_i = \eta^{-2} \partial_\eta (\eta^2 A_i) = O(p^{-(n-3)/2}) \quad \text{(harmless)}.$$

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$$\begin{aligned} \gamma_{ij}^{s(V)}(\mathbf{p}, x) &\equiv \nabla_i A_j^s(\mathbf{p}, x) + \nabla_j A_i^s(\mathbf{p}, x) \\ &= i C_V' (\hat{p}_i \epsilon_j^s + \hat{p}_j \epsilon_i^s) \eta^{-2} p^{-(n-1)/2} + O(p^{-(n-2)/2}). \end{aligned}$$

Compare with $\gamma_{ij}^s(\mathbf{p}, x) = C_T' \epsilon_{ij}^s \eta^{-2} p^{-(n-1)/2} + O(p^{-(n-2)/2})$.

TT-synchronous gauge

$$
\begin{aligned}
&\Delta^{(V)}_{ijk\ell}(x,x') \\
&= \int d^{n-1}\mathbf{p} \sum_s \gamma^{s(V)}_{ij}(x)\overline{\gamma^{s(V)}_{k\ell}(x')} \\
&\approx |C'_V|^2(\eta\eta')^{-2}\int_0^H \frac{dp}{p}\int d\Omega_{n-1}\sum_s(\hat{p}_i\epsilon^s_j+\hat{p}_j\epsilon^s_i)(\hat{p}_k\epsilon^s_\ell+\hat{p}_\ell\epsilon^s_k) \\
&= |C_V|(\eta\eta')^{-2}\frac{2}{n+1}\delta_{ijk\ell}\int_0^H\frac{dp}{p}.
\end{aligned}
$$

$\delta_{ijk\ell} \equiv \delta_{ik}\delta_{j\ell}+\delta_{i\ell}\delta_{jk}-\frac{2}{n-1}\delta_{ij}\delta_{k\ell}$.

TT-synchronous gauge

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$$\Delta^{(TT)}_{ijk\ell}(x,x') \approx |C_T|(\eta\eta')^{-2}\frac{n(n-3)}{(n+1)(n-2)}\delta_{ijk\ell}\int_0^H\frac{dp}{p},$$

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$$
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$$

The IR divergence in the *TT*-synchronous gauge can be reproduced by a field of the form $\nabla_\mu A_\nu + \nabla_\nu A_\mu$.

TT-synchronous gauge

- Marolf, Morrison and AH, "de Sitter invariance of the dS graviton vacuum," arXiv:1107.2712.
  It is possible to choose the mode functions $h_{ij}^{(TT)}$ with $\partial^i h_{ij}^{(TT)} = h_i^{(TT)i} = 0$ such that the IR divergences are absent.

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IR-divergent contribution of $\gamma_{ij}^s(\mathbf{p}, x) \propto \eta^{-2}\epsilon_{ij}^s$.

$$
\begin{aligned}
ds^2 &= \eta^{-2}(-d\eta^2 + \delta_{ij}dx^i dx^j) \to \eta^{-2}\left[-d\eta^2 + (\delta_{ij} + q\epsilon_{ij}^s)dx^i dx^j)\right] \\
&= \eta^{-2}\left[-d\eta^2 + \delta_{ij}dX^i dX^j\right],
\end{aligned}
$$

where $X^i = x^i + q\epsilon^{s\,ij}x_j + O(q^2)$.

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where $X^i = x^i + q\epsilon^{s\,ij}x_j + O(q^2)$.
The subtraction of the leading IR divergence is a (large) gauge transformation:

$$\gamma_{ij}^s(p, \mathbf{x}) \to \gamma_{ij}^s(p, \mathbf{x}) - C_T' \epsilon_{ij}^s \eta^{-2} p^{-(n-1)/2} e^{-\rho p^2}.$$

TT-synchronous gauge

A rebuttal: Miao, Tsamis and Woodard, “Gauging away physics,” arXiv:1107.4733.

Content

1. A brief and slightly(?) biased history of IR divergences in graviton propagator in de Sitter space
2. The IR divergence of the graviton correlator in the TT-synchronous gauge
3. Covariant graviton propagator by mode sum (with Markus Fröb and William de Lima, work in progress)
4. de Sitter invariant graviton vacuum state in the global patch
5. Removing the IR divergences in the Faddeev-Popov ghost propagator

How to find commutators

$$L = \frac{1}{2}\mathcal{A}^{IJ}\dot{Q}_I\dot{Q}_J + \mathcal{B}^{IJ}\dot{Q}_I Q_J + \frac{1}{2}\mathcal{C}^{IJ}Q_I Q_J.$$

Define

$$P^I \equiv \frac{\partial L}{\partial \dot{Q}_I} = \mathcal{A}^{IJ}\dot{Q}_J + \mathcal{B}^{IJ}Q_J.$$

The canonical commutation relations:

$$[Q_I(t), P^J(t)] = i\delta_I^J, \quad [Q_I(t), Q_J(t)] = [P^I(t), P^J(t)] = 0. \qquad (C)$$

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Suppose that a full set of solutions is given by $\{q_I^{(\sigma)}(t), \overline{q_I^{(\sigma)}(t)}\}_\sigma$.

$q_I^{(\sigma)}(t)$: 'positive-frequency solutions'
$\overline{q_I^{(\sigma)}}(t)$: 'negative-frequency solutions'

$$p^{(\sigma)I}(t) \equiv \mathcal{A}^{IJ}\dot{q}_J^{(\sigma)}(t) + \mathcal{B}^{IJ}q_J^{(\sigma)}(t).$$

How to find commutators

The symplectic product:

$$\begin{aligned} S^{\sigma\sigma'} &= (q^{(\sigma)}, q^{(\sigma')})_{\text{symp}} \\ &\equiv i \sum_I \left[\overline{q_I^{(\sigma)}}(t) p^{(\sigma')I}(t) - \overline{p^{I(\sigma)}}(t) q_I^{(\sigma')}(t) \right]. \end{aligned}$$

(It can be shown that $(d/dt)S^{\sigma\sigma'} = 0$.)

Choose $\{q_I^{(\sigma)}(t)\}$ such that

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Expand

$$Q_I(t) = \sum_\sigma \left[a_\sigma q_I^{(\sigma)}(t) + a_\sigma^\dagger \overline{q_I^{(\sigma)}(t)} \right].$$

How to find commutators

The symplectic product:

$$\begin{aligned} S^{\sigma\sigma'} &= (q^{(\sigma)}, q^{(\sigma')})_{\text{symp}} \\ &\equiv i\sum_l \left[\overline{q_l^{(\sigma)}}(t) p^{(\sigma')l}(t) - \overline{p^{l(\sigma)}}(t) q_l^{(\sigma')}(t)\right]. \end{aligned}$$

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Expand

$$Q_l(t) = \sum_\sigma \left[a_\sigma q_l^{(\sigma)}(t) + a_\sigma^\dagger \overline{q_l^{(\sigma)}(t)}\right].$$

Then the canonical commutation relations (C) are equivalent to

$$\left[a_\sigma, a_{\sigma'}^\dagger\right] = (S^{-1})_{\sigma\sigma'}, \quad [a_\sigma, a_{\sigma'}] = \left[a_\sigma^\dagger, a_{\sigma'}^\dagger\right] = 0.$$

How to construct the two-point function

$$Q_I(t) = \sum_\sigma \left[a_\sigma q_I^{(\sigma)}(t) + a_\sigma^\dagger \overline{q_I^{(\sigma)}(t)} \right].$$

$$\left[a_\sigma, a_{\sigma'}^\dagger \right] = (S^{-1})_{\sigma\sigma'}, \quad [a_\sigma, a_{\sigma'}] = \left[a_\sigma^\dagger, a_{\sigma'}^\dagger \right] = 0.$$

$$\begin{aligned} S^{\sigma\sigma'} &\equiv i \sum_I \left[\overline{q_I^{(\sigma)}}(t) p^{(\sigma')I}(t) - \overline{p^{I(\sigma)}}(t) q_I^{(\sigma')}(t) \right], \\ S^{\overline{\sigma}\sigma'} &= 0. \end{aligned}$$

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$$S^{\bar{\sigma}\sigma'} = 0.$$

The two-point function for the state $|0\rangle$ defined by $a_\sigma|0\rangle = 0$ is

$$\langle 0|Q_I(t)Q_J(t)|0\rangle = \sum_{\sigma,\sigma'} q_I^{(\sigma)}(t)(S^{-1})_{\sigma\sigma'} \overline{q_J^{(\sigma')}(t)}.$$

The Lagrangian and field equations

$$(\sqrt{|g|})^{-1}\mathcal{L} = \mathrm{EH}^{(2)}(h) - \frac{1}{2\alpha}[\nabla^\alpha h_{\alpha\mu} - (1+\beta^{-1})\nabla_\mu h]^2 - \frac{1}{4}m^2(h^{\mu\nu}h_{\mu\nu} - h^2).$$

$$h_{\mu\nu} = h^{(TT)}_{\mu\nu} + \nabla_\mu A_\nu + \nabla_\nu A_\mu + \nabla_\mu\nabla_\nu B + g_{\mu\nu}\Psi.$$

$\nabla^\mu h^{(TT)}_{\mu\nu} = h^{(TT)} = 0$, $\nabla^\mu A_\mu = 0$.

$$\begin{aligned} (\Box - (n-2)H^2 - m^2)h^{(TT)}_{\mu\nu} &= 0, \\ (\Box + 2(n-1)H^2 - \alpha m^2)A_\mu &= 0. \end{aligned}$$

In the massless limit,

$$\begin{aligned} \left[\Box - (n-1)\beta H^2\right]\Psi &= 0, \\ \left[\Box - (n-1)\beta H^2\right]B &= -\left(n + \frac{\lambda\beta}{2n}\right)\Psi, \end{aligned}$$

$\lambda \equiv 2(n-1) - (n-2)\alpha$.

Gravition two-point function

$$\begin{aligned}\Delta_{\mu\nu\mu'\nu'}(x,x') &= \langle 0|h_{\mu\nu}(x)h_{\mu'\nu'}(x')|0\rangle \\ &= \Delta^{(T)}_{\mu\nu\mu'\nu'}(x,x') + \Delta^{(V)}_{\mu\nu\mu'\nu'}(x,x') + \Delta^{(S)}_{\mu\nu\mu'\nu'}(x,x').\end{aligned}$$

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$\Delta_{M^2}(x,x')$: the propagator for the scalar field with mass M,
$\Delta^{(1)}_{M^2}(x,x') \equiv \frac{\partial}{\partial M^2}\Delta_{M^2}(x,x')$.

$$\begin{aligned}&\Delta^{(S)}_{\mu\nu\mu'\nu'}(x,x') \\ &= \frac{\lambda}{(n-1)^2(n-2)H^4}\nabla_\mu\nabla_\nu\nabla_{\mu'}\nabla_{\nu'}\Delta_{(n-1)\beta H^2}(x,x') \\ &+\frac{2}{(n-1)(n-2)H^2}\left(n+\frac{\lambda\beta}{2}\right)\nabla_\mu\nabla_\nu\nabla_{\mu'}\nabla_{\nu'}\Delta^{(1)}_{(n-1)\beta H^2}(x,x') \\ &+\frac{2}{(n-1)(n-2)H^2}\left[g_{\mu\nu}(x)\nabla_{\mu'}\nabla_{\nu'}+g_{\mu'\nu'}(x')\nabla_\mu\nabla_\nu\right]\Delta_{(n-1)\beta H^2}(x,x').\end{aligned}$$

IR structure of the graviton two-point function

If we take the limit $m^2 \to 0$ before p-integration:

- $\Delta^{(S)}_{\mu\nu\mu'\nu'}(x, x')$ is basically a two-point function of scalar field with mass $(n-1)\beta H^2 \Rightarrow$ No IR problem or dS breaking if $\beta > 0$.

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- $\Delta^{(T)}_{\mu\nu\mu'\nu'}$ and $\Delta^{(V)}_{\mu\nu\mu'\nu'}$ have no IR problem except when μ, ν, μ', ν' are all space indices.

IR structure of the graviton two-point function

If we take the limit $m^2 \to 0$ before p-integration:

- $\Delta^{(S)}_{\mu\nu\mu'\nu'}(x, x')$ is basically a two-point function of scalar field with mass $(n-1)\beta H^2 \Rightarrow$ No IR problem or dS breaking if $\beta > 0$.
- $\Delta^{(T)}_{\mu\nu\mu'\nu'}$ and $\Delta^{(V)}_{\mu\nu\mu'\nu'}$ have no IR problem except when μ, ν, μ', ν' are all space indices.
- Write $\Delta^{(T)}_{iji'j'} = \Delta^{(TT-sync)}_{iji'j'} + \Delta^{(T,V+S)}_{iji'j'}$, where $\Delta^{(TT-sync)}_{iji'j'}$ is the transverse-traceless-synchronous (or 'physical') contribution.

- Then, in the infrared we have

$$
\begin{aligned}
&\Delta^{(TT-sync)}_{iji'j'} + \Delta^{(T,V+S)}_{iji'j'} + \Delta^{(V)}_{iji'j'} \\
&\approx |C_T|^2(\eta\eta')^{-2}\left(\delta_{ii'}\delta_{jj'} + \delta_{ij'}\delta_{ji'} - \tfrac{2}{n-1}\delta_{ij}\delta_{i'j'}\right)\int_0^H \frac{dp}{p} \\
&\times\left[\frac{n(n-3)}{(n+1)(n-2)} - \frac{2(n-3)(n-1)}{(n+1)(n-2)} + \alpha\frac{(n-1)(n-3)}{(n+1)^2}\right] \\
&= |C_T|^2(\eta\eta')^{-2}\left(\delta_{ii'}\delta_{jj'} + \delta_{ij'}\delta_{ji'} - \tfrac{2}{n-1}\delta_{ij}\delta_{i'j'}\right)\int_0^H \frac{dp}{p} \\
&\times\left[-\frac{n-3}{n+1} + \alpha\frac{(n-1)(n-3)}{(n+1)^2}\right]. \quad \left(\text{no IR div. if } \alpha = \frac{n+1}{n-1}\right)
\end{aligned}
$$

For the exact gauge ($\alpha = 0$) the gauge contribution over-compensates the ‘physical’ IR divergence and changes the sign of the IR divergence.

IR structure of the graviton two-point function

What happens if we perform the p-integration and then take the massless limit?

IR structure of the graviton two-point function

What happens if we perform the p-integration and then take the massless limit?

$$\Delta^{(TT-sync)}_{iji'j'} + \Delta^{(T,V+S)}_{iji'j'} + \Delta^{(V)}_{iji'j'}$$

$$\approx |C_T|^2(\eta\eta')^{-2}\left(\delta_{ii'}\delta_{jj'} + \delta_{ij'}\delta_{ji'} - \tfrac{2}{n-1}\delta_{ij}\delta_{i'j'}\right)$$

$$\times\left[-\frac{n-3}{n+1}\int_0^H \frac{dp}{p}p^{\frac{2m^2}{(n-1)H^2}} + \alpha\frac{(n-1)(n-3)}{(n+1)^2}\int_0^H \frac{dp}{p}p^{\frac{2\alpha m^2}{(n+1)H^2}}\right]$$

$$\approx |C_T|^2(\eta\eta')^{-2}\left(\delta_{ii'}\delta_{jj'} + \delta_{ij'}\delta_{ji'} - \tfrac{2}{n-1}\delta_{ij}\delta_{i'j'}\right)$$

$$\times\left[-\frac{n-3}{n+1}\frac{(n-1)H^2}{2m^2} + \alpha\frac{(n-1)(n-3)}{(n+1)^2}\frac{(n+1)H^2}{2\alpha m^2}\right]$$

$$= 0.$$

$\Delta^{(T)}_{iji'j'}(x, x') + \Delta^{(V)}_{iji'j'}(x, x')$ is IR-finite if we perform the p-integration and then take the massless limit.

IR structure of the graviton two-point function

$\Delta^{(T)}_{iji'j'}(x,x') + \Delta^{(V)}_{iji'j'}(x,x')$ is IR-finite if we perform the p-integration and then take the massless limit.

However...

IR structure of the graviton two-point function

$\Delta^{(T)}_{iji'j'}(x,x') + \Delta^{(V)}_{iji'j'}(x,x')$ is IR-finite if we perform the p-integration and then take the massless limit.

However... $\Delta^{(V)}_{0i0i'}(x,x')$ contains terms proportional to

$$m^2 \int_0^H \frac{dp}{p^3} p^{\frac{2\alpha m^2}{(n+1)H^2}},$$

which is IR-divergent unless m^2 is large enough.

Summary

- $\Delta_{\mu\nu\mu'\nu'}(x, x')$ is IR finite (only) for $\alpha = (n+1)/(n-1)$ if we take the massless limit and then perform the p-integration. Does the $\alpha = 0$ case agree with the MTW propagator?
- $\Delta_{iji'j'}(x, x')$ is IR finite for all α if we perform the p-integration and then take the massless limit, but then $\Delta^{(V)}_{0i0i'}(x, x')$ will be badly divergent though it is manifestly of 'pure-gauge' form.

Summary

- $\Delta_{\mu\nu\mu'\nu'}(x,x')$ is IR finite (only) for $\alpha = (n+1)/(n-1)$ if we take the massless limit and then perform the p-integration. Does the $\alpha = 0$ case agree with the MTW propagator?
- $\Delta_{iji'j'}(x,x')$ is IR finite for all α if we perform the p-integration and then take the massless limit, but then $\Delta^{(V)}_{0i0i'}(x,x')$ will be badly divergent though it is manifestly of 'pure-gauge' form.
- $\Delta_{\mu\nu\mu'\nu'}(x,x')$ is well-defined if αm^2 is large enough. This massive two-point function, as an analytic function of m^2, has a finite massless limit, which is expected to agree with the IR-finite covariant two-point function.

Content

AH, “Quantum linearization instabilties of de Sitter spacetime: I,” Class. Quant. Grav. 8, 1961 (1991); “Linearized gravity in de Sitter spacetime as a representation of $SO(4,1)$,” Class. Quant. Grav. 8, 12005 (1991).

We quantise the linearised gravity in the global patch ($H = 1$)

$$ds^2 = -dt^2 + \cosh^2 t\, d\Omega_3^2$$

with the gauge $N = 1$, $N^i = 0$ ($H = 1$) so that $h_{0\mu} = 0$.

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with the gauge $N = 1$, $N^i = 0$ ($H = 1$) so that $h_{0\mu} = 0$.

$h_{ij} = h_{ij}^{(TT)} + D_i A_j + D_j A_i + (D_i D_j + \delta_{ij})F.$

$$(\Box - 2) h_{\mu\nu}^{(TT)} = 0.$$

Unphysical modes:

$$\begin{aligned} \left(\frac{\partial^2}{\partial t^2} - \tanh t \frac{\partial}{\partial t} - 2 \right) A_i &= 0, \\ \left(\frac{\partial^2}{\partial t^2} - \tanh t \frac{\partial}{\partial t} - \frac{1}{\cosh^2 t} - 2 \right) F &= 0. \end{aligned}$$

The operator $\nabla_i \nabla^i$ does not appear!

$$A_i(t,\theta) = \sum_{L\geq 2}\sum_{\sigma}\left[b_{L\sigma}\cosh^2 t + \tilde{b}_{L\sigma} f_1(t)\right] V_i^{(L\sigma)}(\theta)$$
$$+\sum_{L\neq 0}\sum_{\sigma}\left[c_{L\sigma}\cosh^2 t + \tilde{c}_{L\sigma} f_1(t)\right] D_i Y^{(L\sigma)}(\theta),$$
$$F(t,\theta) = \sum_{L\neq 1}\left[s_{L\sigma}\sinh t\cosh t + \tilde{s}_{L\sigma} f_2(t)\right] Y^{(L\sigma)}(\theta).$$

$$f_1(t) = \frac{i\cosh^2 t}{2}\ln\frac{1+i\sinh t}{1-i\sinh t} - \sinh t,$$
$$f_2(t) = -\frac{d}{dt} f_1(t).$$

unphysical modes

- $[b_{L\sigma}, \tilde{b}_{L'\sigma'}]$, $[c_{L\sigma}, \tilde{c}_{L'\sigma'}]$ and $[s_{L\sigma}, \tilde{s}_{L'\sigma'}]$ are all proportional to $\delta_{LL'}\delta_{\sigma\sigma'}$.
- Classically, the linearised Hamiltonian and momentum constraints lead to $\tilde{b}_{L\sigma} = \tilde{c}_{L\sigma} = \tilde{s}_{L\sigma} = 0$. We incorporate these conditions quantum mechanically as $\tilde{b}_{L\sigma}|\psi\rangle = \tilde{c}_{L\sigma}|\psi\rangle = \tilde{s}_{L\sigma}|\psi\rangle = 0$.
- If we choose to represent $|\psi\rangle$ as a wave function that depends on $b_{L\sigma}$, $c_{L\sigma}$ and $s_{L\sigma}$, i.e.

$$|\psi\rangle = \Psi(\{b_{L\sigma}, c_{L\sigma}, s_{L\sigma}, \ldots\}),$$

  Then

$$\frac{\partial}{\partial b_{L\sigma}}\Psi = \frac{\partial}{\partial c_{L\sigma}}\Psi = \frac{\partial}{\partial s_{L\sigma}}\Psi = 0.$$

  That is, Ψ is independent of $b_{L\sigma}$, $c_{L\sigma}$ and $s_{L\sigma}$.
  The unphysical modes are irrelevant.

Physical modes

$$h_{ij}^{(TT)}(t,\theta) = \sum_{L=2}^{\infty}\sum_{\sigma}\left[a_{L\sigma}f_{ij}^{(L\sigma)}(t,\theta) + a_{L\sigma}^{\dagger}\overline{f_{ij}^{(L\sigma)}(t,\theta)}\right].$$

The vacuum $|0\rangle$ defined by $a_{L\sigma}|0\rangle = 0$ for all L and σ is $SO(4)$ invariant.
With a boost Killing vector X,

$$\begin{aligned}\mathcal{L}_X f_{\mu\nu}^{(L,\sigma)}(t,\theta) &= ic(L+1,\sigma)f_{\mu\nu}^{(L+1,\sigma)}(t,\theta) - ic(L,\sigma)f_{\mu\nu}^{(L-1,\sigma)}(t,\theta)\\ &\quad + \nabla_\mu\Lambda_\nu + \nabla_\nu\Lambda_\mu.\end{aligned}$$

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The ‘positive-frequency solutions’ transform among themselves modulo gauge transformations.
$\Rightarrow$ The annihilation operators $a_{L\sigma}$ transform among themselves under a boost as well as under $SO(4)$ rotations, i.e. they transform among themselves under $SO(4,1)$.
$\Rightarrow$ the condition $a_{L\sigma}|0\rangle = 0$ is $SO(4,1)$ invariant, i.e. the vacuum state is $SO(4,1)$ invariant.

Content

Yang-Mills Lagrangian (Nakanishi-Lautrup)

$$\frac{1}{\sqrt{|g|}}\mathcal{L} \;=\; -\frac{1}{4}F_{ab}\cdot F^{ab} - \nabla^a B\cdot A_a - \frac{\alpha}{2}B\cdot B + i\nabla^a\overline{c}\cdot D_a c$$

(B: The Nakanishi-Lautrup auxiliary field)

$$\begin{aligned} B\cdot B &\equiv B^A B^A, \\ (D_a c)^A &= D_a c^A + q f^{ABC} A_a^B c^C, \\ F_{ab}^A &= \nabla_a A_b^A - \nabla_b A_a^A + f^{ABC} A_a^B A_b^C. \end{aligned}$$

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f^{ABC} : totally anti-symmetric structure constant of the Lie algebra
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Define $f^{ABC}A_a^B A_b^C = (A_a \times A_b)^A$ etc.
We'll omit the gauge indices from now on, e.g. $A_a^A \to A_a$.

IR problem in the ghost sector

The free field ($q = 0$) equations :

$$\nabla_a \nabla^a c = \nabla_a \nabla^a \overline{c} = 0.$$

The ghost and anti-ghost are minimally-coupled massless scalar field. Hence the propagator is IR-divergent/dS non-invariant in de Sitter space. The ghosts in perturbative gravity also has similar IR divergences.

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The propagator on de Sitter space regularised with a small mass m

$$G_F(x, x') = \frac{H^n}{m^2 V_{S^n}} + \tilde{G}_F(x, x'),$$

V_{S^n} : the volume of the unit S^n; $\tilde{G}_F(x, x')$: de Sitter invariant & IR-finite.

The form of the FP-ghost coupling

The FP-ghost propagator in the Euclidean (dS invariant) vacuum (with small mass)

$$-iT\langle 0|c(x)\overline{c}(x')|0\rangle = \frac{H^n}{m^2 V_{S^n}} + \tilde{G}_F(x,x').$$

The coupling or the FP-ghosts and gauge field: $-iq\nabla^a\overline{c}\cdot(A_a\times c)$.

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- The derivative on $\overline{c}$ eliminates the IR-divergent constant in the propagator;
- The ghost propagator appears always in an internal loop.

Proposal : Use the dS-invariant effective propagator $\tilde{G}_F(x,x')$ instead of the IR-divergent one, $G_F(x,x')$. Faizal and AH, arXiv:0806.3735

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Question : Can we derive this without the IR-regularisation?
Yes! Jos Gibbons and AH, arXiV:1410.7830

Conserved charges

Field equations in the Landau gauge $\alpha = 0$:

$$\nabla_a A^a = \nabla_a D^a c = \nabla_a D^a \overline{c} = 0.$$

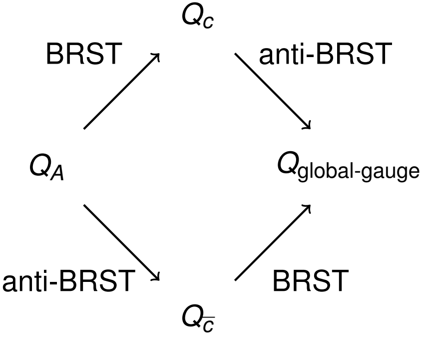
Conserved charges:

$$\begin{aligned} Q_A &= \int d\Sigma_a A^a, \\ Q_c &= \int d\Sigma_a D^a c, \\ Q_{\overline{c}} &= \int d\Sigma_a D^a \overline{c}. \end{aligned}$$

The proposal to use the effective IR-finite propagator is equivalent to imposing the following conditions on the states $|\psi\rangle$:

$$Q_A|\psi\rangle = Q_c|\psi\rangle = Q_{\overline{c}}|\psi\rangle = 0.$$

Consistency with the BRST (and anti-BRST) invariance



The consistency with the BRST (and anti-BRST) invariance leads to the condition $Q_{\text{global-gauge}}|\psi\rangle = 0$, i.e. the invariance of the states under the global gauge transformations.