STUDYING THE INTERSTELLAR MAGNETIC FIELD MEASURING THE ANISOTROPY IN VELOCITY

Alejandro Esquivel (ICN-UNAM, Mexico)

in collaboration with
David Hernández-Padilla (ICN-UNAM, Mexico)
Alex Lazarian (UW-Madison, USA), Jungyeon Cho (Chungnam National Univ., Korea)

Magnetic Fields in the Universe VI: From Laboratory and stars to the primordial structures (Natal, Brazil)
THE INTERSTELLAR MEDIUM (ISM) IS TURBULENT

- **Theory**
  - The Reynolds number in the ISM: $Re > 10^8$ (current computers can only reach $\sim 10^4$)

- **Observations**
  - Line widths show a “non-thermal” component.
  - Density/velocity/magnetic field fluctuations show a self-similar structure.

- **Important for:**
  - cosmic ray scattering and acceleration,
  - molecular cloud dynamics,
  - star formation,
  - mixing of elements,
  - magnetic field generation,
  - accretion processes,
  - … virtually any transport process in the ISM.

“The great power law in the sky”
Chepurnov & Lazarian (2010)
HOW DO WE STUDY TURBULENCE?

• Statistical tools, such as:
  
  • structure/correlation functions,
    
    \[ SF(r) = \left\langle \left[ v(x_1) - v(x_2) \right]^2 \right\rangle \]
    
    \[ r = |x_2 - x_1| \]
    
    \[ CF(r) = \left\langle v(x_1) \cdot v(x_2) \right\rangle \]
    
    \[ SF(r) = 2[ CF(0) - CF(r) ] \]
  
  • or power spectra

Fourier analysis
KOLMOGOROV MODEL OF TURBULENCE

• Model for incompressible HD turbulence
  • Energy injected at large scales, cascades without losses until dissipation takes over (inertial range).
  • Constant Energy transfer rate: $\dot{E} \approx \frac{\rho v_\ell^2}{\tau} \sim \frac{v_\ell^2}{\ell/v_\ell} \sim \frac{v_\ell^3}{\ell} \Rightarrow v_\ell \propto \ell^{1/3} \sim k^{-1/3}$
  • The energy power spectrum $P(k)$

$$\int P(k)dk \propto v_\ell^2 \Rightarrow P(k) \propto \frac{k^{-2/3}}{k} \sim k^{-5/3}$$

In 1D: $P(k) \propto k^{-5/3}$
In 2D: $P(k) \propto k^{-8/3}$
In 3D: $P(k) \propto k^{-11/3}$
MHD TURBULENCE IS ANISOTROPIC (AND SCALE DEPENDENT)

• Goldreich & Sridhar (1995) MHD turbulence model

  • Anisotropic cascade:
    • Motions perpendicular to B follow a Kolmogorov type cascade \( v_l \propto l_{\perp}^{1/3} \)
    • Parallel B are dominated by Alfvénic perturbations
    • A critical balance condition relate the two
      \[
      \frac{v_{\perp}}{l_{\perp}} \sim \frac{v_A}{l_{||}} \Rightarrow l_{||} \sim l_{\perp}^{2/3}
      \]
  • Later confirmed with numerical simulations (Cho & Vishniac 2000, Maron & Goldreich 2001)

  • The anisotropy should be measured with respect to the local magnetic field

\[\text{(b)}\]  
\[\text{Cho, Lazarian & Vishniac (2002)}\]
• Ideal 3D MHD simulations of fully developed (driven at large scales) turbulence.

• Isothermal, in a periodic Cartesian grid.

• The parameters that control the simulations are the Alfvén and the sonic Mach numbers.

\[ M_s \equiv \frac{v_L}{c_s}; \quad M_A = \frac{v_L}{v_A} \]

• where

\[ c_s = \sqrt{\frac{P}{\rho}}; \quad v_A = \frac{B}{\sqrt{4\pi \rho}} \]

• \( B_0 \) is in the \( x \) direction.

• We take the output of the simulations to create synthetic spectroscopic observations of optically thin media.

<table>
<thead>
<tr>
<th>Model</th>
<th>( v_A,0 )</th>
<th>( P_{\text{gas},0} )</th>
<th>( M_A )</th>
<th>( M_s )</th>
<th>( \beta )</th>
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<td>( \sim 11.9 )</td>
<td>( \sim 0.49 )</td>
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<td>( \sim 7.7 )</td>
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<td>( \sim 3.4 )</td>
<td>( \sim 5.0 )</td>
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<td>( \sim 0.2 )</td>
<td>( \sim 0.9 )</td>
<td>( \sim 0.04 )</td>
</tr>
</tbody>
</table>
SYNTHETIC OBSERVATIONS

Model M13 (with $M_S=2.7$ and $M_A=0.8$)

Simulation PPP $(x,y,z)$ space

Synthetic Observations

PPV $(x,y,v_z)$ space

Density cuts

Intensity $\propto \rho$
PPV DATA: THE EFFECT OF VARYING RESOLUTION

- Emissivity in PPV data depends on density and velocity at the same time.
- Lazarian & Pogosyan (2000) study the effect of varying the thickness in velocity channels (velocity resolution) to obtain the velocity spectral index from observations.
- As we lower the velocity resolution, the contribution of density becomes more prominent. In thinner velocity channels the velocity can dominate the spectrum.

**Velocity slice** $\delta v = \Delta v/120$

**Column density**
Column density

\[ N(x, y) = \int I(x, y, v_{\text{los}}) \, dv_{\text{los}}, \]

\[ N(x, y) = \int \rho(x, y, z) \, dz \quad \text{(Optically thin media, LOS}=z) \]

column density \( \text{LOS}=X \)  
column density \( \text{LOS}=Y \)  
column density \( \text{LOS}=Z \)
Velocity Centroids (unnormalized)

\[ C_{\text{los}}(x, y) = \int I(x, y, v_{\text{los}}) v_{\text{los}} \, dv_{\text{los}}, \]

\[ C_z(x, y) = \int \rho(x, y, z) v_z(x, y, z) \, dz \quad \text{(Optically thin media, LOS=z)} \]
The structure function of velocity centroids $SF(R) = \langle [C(X) - C(X + R)]^2 \rangle$,

Velocity centroids are a combination of density and velocity fluctuations. To isolate velocity one can use from the simulations maps of mean LOS velocity, e.g.

$$V_z(x, y) = \frac{1}{N_z} \int v(x, y, z) \, dz$$

The SFs are elongated in the direction of the mean magnetic field.

Esquivel & Lazarian (2011)
Velocity centroids sample the entire LOS at a given velocity, thus one probe the mean magnetic field (as opposed to the local one).

- Isotropy degree ($\ell$) = $\frac{SF(R_\parallel)}{SF(R_\perp)}$.

- Velocity centroids anisotropy is (mostly) SCALE INDEPENDENT.

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**Esquivel & Lazarian (2011)**

Isotropy degree ($\ell$) = $\frac{SF(R_\parallel)}{SF(R_\perp)}$.
Higher magnetization → more elongated structure functions, but...

Burkhart et al. (2014)
Recently Kandel, Lazarian & Pogosyan (MNRAS, 2016a,b) extended the VCA formalism to study the anisotropy in PPV and velocity centroids.

They study the level of anisotropy in the different velocity modes in the SF of velocity centroids (at a constant density)

- **Alfvén Mode:**
  anisotropic at low and high $\beta$, with more pronounced anisotropy at small $M_A$.

- **Slow mode:**
  Same general behavior as the Alfvén mode, but with vanishing signal at an angle perpendicular to $B_0$.

- **Fast mode:**
  Isotropic in high-$\beta$, but anisotropic in low-$\beta$
DIFFERENT MHD MODES ARE ALL PRESENT IN THE VELOCITY

- We decompose the velocity into Alfvén, Fast and Slow magneto-sonic modes.
- We take each of the velocity fields and compute the mean LOS velocity, and centroids combining such velocities with the original velocity.

*Cho & Lazarian 2002*
EXAMPLE OF DECOMPOSED MODES MAPS (PARALLEL TO $B_0$)

$$C_x(y, z) = \int \rho(x, y, z) v_x(x, y, z) \, dx$$
EXAMPLE OF DECOMPOSED MODES MAPS (PERPENDICULAR TO $B_0$)

$C_y(x, z) = \int \rho(x, y, z) v_y(x, y, z) dy$
AVERAGE ISOTROPY DEGREE FOR ALL MODES

LOS parallel to $B_0$

Centroids.

$C_A$

$C_S$

$C_F$

LOS perpendicular to $B_0$

Centroids.

$C_A$

$C_S$

$C_F$
SUMMARY/CONCLUSIONS

• Structure functions in velocity centroids are anisotropic.

• Such anisotropy points in the direction of the plane of the sky B field.

• The degree of anisotropy increases with the strength of $B_0$ (i.e. $\sim 1/M_A$), with a secondary dependence on the sonic Mach number ($M_S$).
  • Thus given an estimate of $M_S$ one can infer an upper bound on the Alfvénic Mach number.
  • With help of other techniques/measurements for the LOS component one could determine $M_A$.

• The Alfvén mode is the dominant contribution to the centroids map, and thus their structure function when the LOS is perpendicular to $B_0$ (maximum anisotropy).

• The slow mode dominates in the case of LOS parallel to $B_0$, but the SFs are isotropic from that point of view.

• These results are consistent with those previously obtained with velocity centroids (Esquivel & Lazarian 2011, Burkhart et al. 2014).

• Also consistent with analytical predictions for the Alfvén modes by Kandel et al. 2016.