SFT for noncritical strings revisited

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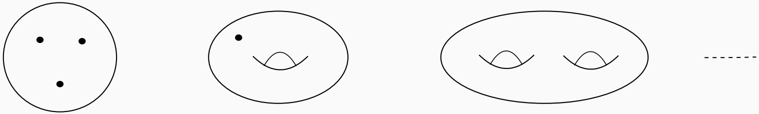
String Field Theory and Related Aspects 2020

June 9, 2020

Hyperbolic metric

- There exists a metric with $R = -2$ on a Riemann surface (genus g, n punctures) if $2g + n \geq 3$.



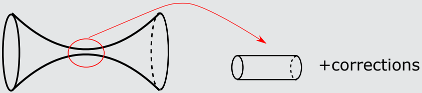
- The hyperbolic metric is useful in studying string theory (1980's)
  - multi-loop calculations D'Hoker-Phong '86 ...
  - superstrings Baranov-Manin-Frolov-Schwarz '87 ...

SFT with hyperbolic metric

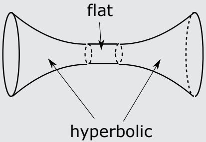
- Although we may not need the worldsheet metric to formulate SFT, it is better to have one for practical calculations.
- The hyperbolic metric, for which enormous results are available, would be the most convenient choice.
- However, the hyperbolic metric has not been used in SFT until recently.
  - In order to construct SFT, one should decompose the worldsheet into propagators and vertices.
  - Cutting out propagators from hyperbolic surfaces is not so straightforward.

Recent ideas

Moosavian-Pius '17



Costello-Zwiebach '19



String theory with hyperbolic metric

$(2,p)$ minimal strings

$(2,p)$ minimal model + Liouville

$$I_{\text{Liou.}} = \frac{p}{8\pi}\int d^2x\sqrt{\hat{g}}\left((\hat{\nabla}\phi)^2 + (1+\frac{2}{p})\phi\hat{R} + \mu e^{2\phi}\right)$$

- Realized as higher critical points of the one matrix model.
- Exactly solvable. (orthogonal polynomial, loop equation)

- In the limit $p \to \infty$ with $\mu = 1$, the worldsheet metric $\hat{g}e^{\phi}$ is fixed to satisfy $R = -2$.
  - Saad et al. have shown that the $p \to \infty$ limit is equivalent to the JT gravity.

SFT for minimal strings

We would like to construct an SFT for $(2, p)$ minimal strings in the limit $p \to \infty$, and get an SFT with hyperbolic metric.

- By doing so, we may be able to learn how to deal with the hyperbolic metric in the critical case.

In this talk, I will explain

- For $(2, 3)$ $(c = 0)$ minimal strings (closed), we formulated an SFT.
  H. Kawai and N.I. '93
- It is possible to generalize the $(2, 3)$ case, we construct SFT for $(2, p)$ minimal strings.
- Taking the limit $p \to \infty$, we get an SFT with hyperbolic metric.

Outline

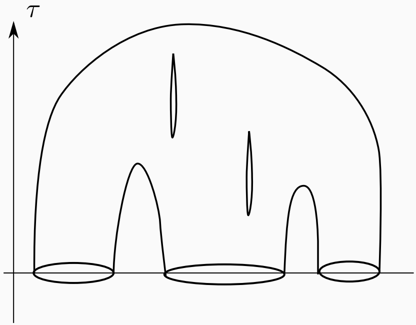
1. SFT for $(2, 3)$ strings
2. Stochastic quantization
3. SFT for $(2, p)$ strings
4. Outlook

1. SFT for $(2, 3)$ strings

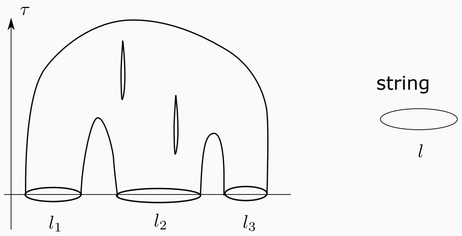
SFT for $(2,3)$ strings

Introduce a time coordinate τ on the worldsheet of $c = 0$ strings
Kawai-Kawamoto-Mogami-Watabiki '93

$$\tau(\mathrm{P}) = \min\{d(\mathrm{P},\mathrm{Q});\ \mathrm{Q} \in \text{boundary}\}$$



Time evolution for $c = 0$ strings



- string field $\hat{\phi}(l), \hat{\pi}(l)$, states $|0\rangle$, $\langle 0|$

$$[\hat{\pi}(l), \hat{\phi}(l')] = \delta(l - l')$$
$$\hat{\pi}(l) |0\rangle = \langle 0| \hat{\phi}(l) = 0$$

- Correlation functions

$$\lim_{\tau \to \infty} \langle 0| e^{-\tau \hat{H}} \hat{\phi}(l_1) \cdots \hat{\phi}(l_n) |0\rangle$$

Hamiltonian

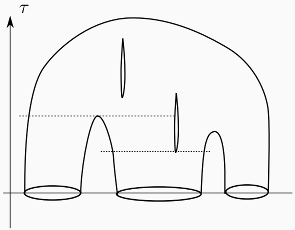
$$\lim_{\tau\to\infty} \langle 0| e^{-\tau \hat{H}} \hat{\phi}(l_1)\cdots\hat{\phi}(l_n)|0\rangle$$

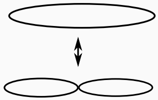
$$\begin{aligned}\hat{H} &= 2\int dl dl' \hat{\phi}(l) w(l') \hat{\pi}(l+l')(l+l') \\ &+ \int dl dl' w(l+l') \hat{\pi}(l) l \hat{\pi}(l') l' \\ &+ g_s \int dl dl' \hat{\phi}(l)\hat{\phi}(l')\hat{\pi}(l+l')(l+l') \\ &+ g_s \int dl dl' \hat{\phi}(l+l') \hat{\pi}(l) l \hat{\pi}(l') l'\end{aligned}$$

- $w(l) = (l^{-\frac{5}{2}} + \kappa l^{-\frac{3}{2}}) e^{-\kappa l}$ is the disk amplitude.

Hamiltonian: 3-string interaction

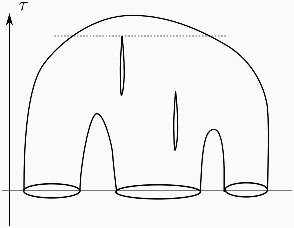
$$
\begin{aligned}
\hat{H} &= 2\int dl dl' \hat{\phi}(l) w(l') \hat{\pi}(l+l')(l+l') \\
&\quad + \int dl dl' w(l+l') \hat{\pi}(l) l \hat{\pi}(l') l' \\
&\quad + g_s \int dl dl' \hat{\phi}(l) \hat{\phi}(l') \hat{\pi}(l+l')(l+l') \\
&\quad + g_s \int dl dl' \hat{\phi}(l+l') \hat{\pi}(l) l \hat{\pi}(l') l'
\end{aligned}
$$

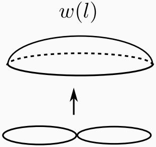




Hamiltonian: 2 string annihilation

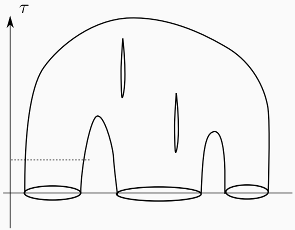
$$
\begin{aligned}
\hat{H} \;=\; & 2\int dl dl' \hat{\phi}(l) w(l') \hat{\pi}(l+l')(l+l') \\
& + \int dl dl' w(l+l') \hat{\pi}(l) l \hat{\pi}(l') l' \\
& + g_s \int dl dl' \hat{\phi}(l) \hat{\phi}(l') \hat{\pi}(l+l')(l+l') \\
& + g_s \int dl dl' \hat{\phi}(l+l') \hat{\pi}(l) l \hat{\pi}(l') l'
\end{aligned}
$$

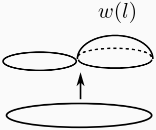




Hamiltonian: kinetic term

$$\begin{aligned}\hat{H} \;=\; & 2\int dl dl' \hat{\phi}(l) w(l') \hat{\pi}(l+l')(l+l') \\ & + \int dl dl' w(l+l') \hat{\pi}(l) l \hat{\pi}(l') l' \\ & + g_{\mathrm{s}} \int dl dl' \hat{\phi}(l) \hat{\phi}(l') \hat{\pi}(l+l')(l+l') \\ & + g_{\mathrm{s}} \int dl dl' \hat{\phi}(l+l') \hat{\pi}(l) l \hat{\pi}(l') l'\end{aligned}$$





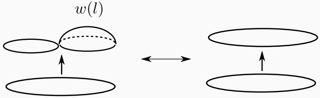
SFT for $(2,3)$ strings

With this Hamiltonian,

$$\lim_{\tau\to\infty} \langle 0| \, e^{-\tau \hat{H}} \hat{\phi}(l_1) \cdots \hat{\phi}(l_n) \, |0\rangle$$

coincide with the correlation functions of $(2,3)$ string theory.

- The propagator is a cylinder with bifurcating coordinates.



- The formulation looks quite like the stochastic quantization in ordinary QFT.

2. Stochastic quantization

Stochastic quantization (Parisi-Wu '81)

Correlation functions of a D dim. Euclidean field theory

$$\langle\phi(x_1)\cdots\phi(x_n)\rangle \equiv \frac{\int [d\phi]\, e^{-S[\phi]}\phi(x_1)\cdots\phi(x_n)}{\int [d\phi]\, e^{-S[\phi]}}$$

can be calculated by using a $D+1$ dim. one with fictitious time τ.

$$\langle\phi(x_1)\cdots\phi(x_n)\rangle = \lim_{\tau\to\infty} \langle 0|\, e^{-\tau\hat{H}}\hat{\phi}(x_1)\cdots\hat{\phi}(x_n)\,|0\rangle$$

$$[\hat{\pi}(x), \hat{\phi}(x')] = \delta(x-x')$$
$$\hat{\pi}(x)\,|0\rangle = \langle 0|\,\hat{\phi}(x) = 0$$
$$\hat{H} = -\int d^D x \left[\hat{\pi}(x) - \frac{\delta S[\hat{\phi}]}{\delta\hat{\phi}(x)}\right]\hat{\pi}(x)$$

$D+1 \to D$ reduction

$$\langle \phi(x_1)\cdots\phi(x_n)\rangle = \lim_{\tau\to\infty} \langle 0|\, e^{-\tau \hat{H}} \hat{\phi}(x_1)\cdots\hat{\phi}(x_n)\,|0\rangle$$

$$\hat{H} \;\; = \;\; -\int d^D x \left[\hat{\pi}(x) - \frac{\delta S[\hat{\phi}]}{\delta\hat{\phi}(x)}\right]\hat{\pi}(x)$$

- This system possesses a hidden supersymmetry. Gozzi '83

$$\{Q,\bar{Q}\} = \hat{H} + \text{fermions}$$

$$Q = \int dx \psi(x)\hat{\pi}(x),\;\; \bar{Q} = -\int dx \bar{\psi}(x)\left[\hat{\pi}(x) - \frac{\delta S[\hat{\phi}]}{\delta\hat{\phi}(x)}\right]$$

- Because of this supersymmetry, $D+1 \to D$ reduction occurs. Nakazato et al. '83 ... Kugo-Mitchard '90

Compared with SFT,

$$\langle \phi(x_1)\cdots\phi(x_n)\rangle = \lim_{\tau\to\infty} \langle 0| e^{-\tau\hat{H}}\hat{\phi}(x_1)\cdots\hat{\phi}(x_n)|0\rangle$$

$$\hat{H} \;=\; -\int d^D x \left[\hat{\pi}(x) - \frac{\delta S[\hat{\phi}]}{\delta\hat{\phi}(x)}\right]\hat{\pi}(x)$$

- The SFT Hamiltonian can be expressed as

$$\hat{H} = -\int dl dl' \left[\hat{\pi}(l) - \frac{\delta S_{\mathrm{st.}}[\hat{\phi}]}{\delta\hat{\phi}(l)}\right] K(l,l')\hat{\pi}(l')$$

- This type of Hamiltonian is called the one with kernel K. The correlation functions can be proved to be equal to those from the Euclidean action $S_{\mathrm{st.}}[\phi]$.

$S_{\mathrm{st.}}[\phi]$

$$\hat{H} = -\int dl dl' \left[\hat{\pi}(l) - \frac{\delta S_{\mathrm{st.}}[\hat{\phi}]}{\delta\hat{\phi}(l)}\right] K(l,l')\hat{\pi}(l')$$

$$K(l,l') = -\left(w(l+l') + g_{\mathrm{s}}\phi(l+l')\right) ll'$$

$$\int dl' K(l,l')\frac{\delta S_{\mathrm{st.}}[\phi]}{\delta\phi(l)} = l\int_0^l dl'(2w(l') + g_{\mathrm{s}}\phi(l'))\phi(l-l')$$

- In principle, the action $S_{\mathrm{st.}}$ can be obtained by solving the above equation.
- Schwinger-Dyson equation derived from $S_{\mathrm{st.}}$ coincides with the loop equation.
- Therefore the SFT results are equal to the correlation functions of $(2,3)$ string theory.

Remarks

- The stochastic quantization based on the Langevin equation

$$\frac{\partial\phi(x,\tau)}{\partial\tau} = -\left.\frac{\delta S[\phi]}{\delta\phi(x)}\right|_{\phi(x)=\phi(x,\tau)} + \eta(x,\tau)$$

  It is equivalent to the one given here.
- The SFT for the $(2,3)$ strings are derived from the stochastic quantization of the matrix model.
  Jevicki-Rodrigues '94

3. SFT for $(2, p)$ strings

SFT for $(2, p)$ strings

- We tried to construct SFT for higher critical points.
- Higher critical points are difficult because the disk amplitude is more singular for $l \sim 0$

$$w_p(l) \sim l^{-\frac{p}{2}-1}$$

- One such SFT was proposed by Ikehara '95, but it is impossible to take $p \to \infty$ limit in his Hamiltonian.
- Studying the formulation carefully, we obtain a Hamiltonian for which $p \to \infty$ can be taken.

SFT for $(2,p)$ strings

$$
\begin{aligned}
\hat{H} \;=\;& 2\left[\int dl dl' \hat{\phi}(l) w_p(l') \hat{\pi}(l+l')(l+l')\right]_{\mathrm{r}} \\
&+\left[\int dl dl' w_p(l+l') \hat{\pi}(l) l \hat{\pi}(l') l'\right]_{\mathrm{r}} \\
&+g_{\mathrm{s}} \int dl dl' \hat{\phi}(l)\hat{\phi}(l')\hat{\pi}(l+l')(l+l') \\
&+g_{\mathrm{s}} \int dl dl' \hat{\phi}(l+l')\hat{\pi}(l) l \hat{\pi}(l') l'
\end{aligned}
$$

- Ill defined kinetic terms can be made finite.
- With this Hamiltonian,

$$\lim_{\tau\to\infty} \langle 0| e^{-\tau\hat{H}} \hat{\phi}(l_1)\cdots\hat{\phi}(l_n)|0\rangle$$

coincide with the correlation functions of $(2,p)$ string theory.

$p \to \infty$ limit

- By taking the limit $p \to \infty$ with $\kappa = \frac{p}{4\pi}$ and taking

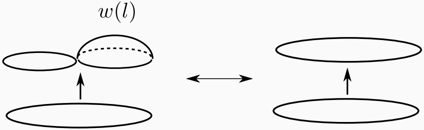
$$
\begin{aligned}
\phi(l) &\to e^{-\kappa l}\phi(l) \\
\pi(l) &\to e^{\kappa l}\pi(l) \\
l &\to \frac{p}{2\pi} l
\end{aligned}
$$

we obtain the SFT for JT gravity with the same form of the Hamiltonian.

4. Outlook

§4 Outlook

- The stochastic quantization like SFT for $(2, p)$ minimal strings can be constructed.
- The propagator is quite different from the usual ones.



- The formulation is a generalization of stochasitic quantization.

- It would be possible to do the same for superstrings.
- We would like to go on and consider similar formulation for critical strings.