Open-Closed Super String Field Theory

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(Work with Faroogh Moosavian and Ashoke Sen)
(ArXiv: 1907.10632 + Ongoing work)

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Brief Outline

1. Introduction
2. Construction of open-closed string field theory
3. Generalization to unoriented strings

Introduction

- String Field theory is the field theoretic formulation of string theory.
  (Kaku, Kikkawa; Witten; Zwiebach; Hata, Zwiebach; Sen; Berkovits; $\cdots$)
- The usual string theory based on Polyakov prescription relies very heavily on world sheet conformal field theory.
- The world sheet CFT gives powerful tools allowing us to explore the theory deeply. However, it also puts some restrictions on how far one can go.
- The world sheet calculations typically give IR divergences. One can trace these divergences to be coming from boundary regions of the moduli space.

Introduction

- Some of these divergences can be cured within the framework of world-sheet theory by making an analytic continuation in the external momenta. (Berera; Witten)
- However, for some of the problems such as
  - Mass renormalization of generic string state
  - Tadpole divergences due to incorrect vacuum
  - Contribution of D instantons to perturbation theory,

  the analytic continuation is not enough.
- During the last few years, string field theory has given systematic tools to deal with these IR issues.

  (Rudra, Pius, Sen; Sen)

Introduction

- String field theory has also given a deeper understanding of the issues of unitarity and analyticity in string theory.
  (Pius, Sen; Sen; Lacroix, Erbin, Sen)
- The string field theory has also been used to study the superstrings in RR background. (Cho, Collier, Yin)
- Apart from perturbative questions, the string field theory has also given insights into the non perturbative aspects of string theory.
- Following Ashoke Sen's work on Tachyon condensation, a lot of results have been obtained both anlytically and numerically regarding the open strings in D-brane backgrounds.
  (Sen; Sen, Zwiebach; Gaiotto, Rastelli; Schnabl; Erler, Maccaferri; $\cdots$)

Introduction

- The bosonic closed string field theory was developed in early 90s and provides the language we shall be using. (Zwiebach; Sen, Zweibach)
- The closed superstring field theory has been constructed recently. (Sen)
- The open super string field theory was first constructed by Edward Witten. However, it was soon found that it contained divergences which correspond to propagation of closed string states. (Freedman, Giddings, Shapiro, Thorn)

Introduction

- There have been various attempts to reformulate the open string field theory so that these divergences disappear.
  (Berkovits; Gaberdiel, Zwiebach; Kroyter, Okawa, Schnabl, Torii, Zwiebach; Earler, Konopka, Sachs; Kunitomo, Okawa; $\cdots$)

- One way to deal with these divergences is to explicitly add the closed string sector to the open string field theory which leads to combined open closed string field theory. Such systems natuarally arise in the presence of D branes.

- For the bosonic case, the combined open closed string field theory was constructed by Zwiebach.

- Our goal will be to describe the super symmetric version of the combined open closed theory. (Moosavian, Sen, MV)

How to Construct String Field Action ?

- Before proceeding further, we describe the schematic way in which the string field theories are typically constructed.

- A string field is a general linear combination of all the string states which appear in first quantized world-sheet theory

$$\Psi = \sum_r \psi_r |\phi_r\rangle$$

  where $|\phi_r\rangle$ denote the basis states which are in one to one correspondence with the states in the world-sheet theory and ψ_r turn out to be the space-time fields.

How to Construct String Field Action ?

- The usual string theory, based on Polyakov prescription, gives a formal expression for the on-shell amplitudes given as integrals over the moduli spaces.

$$\mathcal{A}_{g,n} = \int_{\mathcal{M}_{g,n}} [dm_i] \left\langle \mathcal{V}_1 \cdots \mathcal{V}_n \right\rangle_{g,n}$$

  where $\{m_i\}$ denote the coordinates on the moduli space and $\left\langle \mathcal{V}_1 \cdots \mathcal{V}_n \right\rangle_{g,n}$ denotes a correlation function on the genus g Riemann surface with n external string states.

- One generalizes this to construct the off shell amplitudes. This involves making the amplitudes depend upon some off shell data (such as local coordinates around the punctures).

How to Construct String Field Action ?

- One now divides the integration regions in ''1PI and 1PR regions"

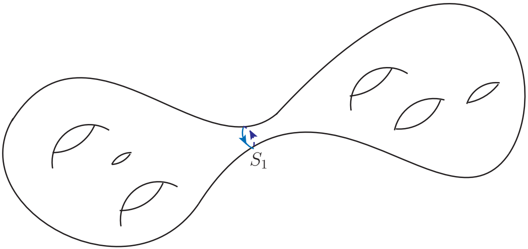
$$\mathcal{A}_{g,n} = \left(\int_{\text{1PI}} + \int_{\text{1PR}} \right) [dm_i] \left\langle \mathcal{V}_1 \cdots \mathcal{V}_n \right\rangle_{g,n}$$

- The 1PR part includes those regions in moduli space which correspond to separating type degeneration of Riemann surfaces.

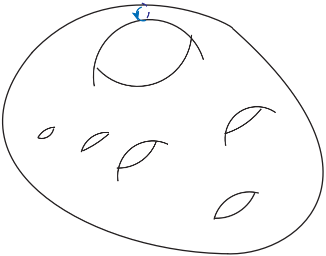
- On the other hand, the 1PI part excludes the separating type degeneration.

- Identifying the 1PI and 1PR parts of amplitudes, we can construct the vertices of string field theory to be used in the action.

How to Construct String Field Action ?



$Separating\ \ type\ \ degeneration$



$non-separating\ \ type\ \ degeneration$

Construction of Open Closed String Field Theory

- Ingredients for constructing open closed SFT
- 1PI effective action
- Section Independence

Ingredients for Constructing Open Closed SFT

- We start by introducing the vertices which shall be used in the construction of the 1PI action.
- For this, we introduce some specific vector spaces in which string fields live

$$
\begin{aligned}
\mathcal{H}^c &\equiv \mathcal{H}_{-1,-1} \oplus \mathcal{H}_{-1/2,-1} \oplus \mathcal{H}_{-1,-1/2} \oplus \mathcal{H}_{-1/2,-1/2}, \\
\widetilde{\mathcal{H}}^c &\equiv \mathcal{H}_{-1,-1} \oplus \mathcal{H}_{-3/2,-1} \oplus \mathcal{H}_{-1,-3/2} \oplus \mathcal{H}_{-3/2,-3/2}, \\
\mathcal{H}^o &\equiv \mathcal{H}_{-1} \oplus \mathcal{H}_{-1/2}, \\
\widetilde{\mathcal{H}}^o &\equiv \mathcal{H}_{-1} \oplus \mathcal{H}_{-3/2}
\end{aligned}
$$

$\mathcal{H}_{m,n}$ denotes the vector space of closed string states with picture number (m,n) and $\mathcal{H}_m$ denotes the vector space of open string states with picture number m.

Ingredients for Constructing Open Closed SFT

- For fields $\Psi_i^c \in \mathcal{H}^c$ and $\Psi_i^o \in \mathcal{H}^o$, we denote the off shell 1PI amplitude/vertex by

$$\{\Psi_1^c \cdots \Psi_N^c; \Psi_1^o \cdots \Psi_M^o\} \equiv \sum_{\substack{g,b\geq 0 \\ (g,b,N,M)\neq(0,1,1,0)}} (g_s)^{2g-2+b} \int_{\mathcal{R}_{g,b,N,M}} \Omega_p(\Psi_1^c, \cdots, \Psi_N^c; \Psi_1^o, \cdots, \Psi_M^o)$$

- $\mathcal{R}_{g,b,N,M}$ is the part of moduli space of punctured Riemann surfaces which includes information about the local coordinates around the punctures and PCO locations.

- Ω_p is a correlation function on the Riemann surface and $p = 6g - 6 + 3b + 2N + M$ denotes its dimension.

Ingredients for Constructing Open Closed SFT

- The disc one point function is not included in the definition of $\{\Psi_1^c \cdots \Psi_N^c; \Psi_1^o \cdots \Psi_M^o\}$. We need to treat this separately.

- The reason for this is that in the RR sector with picture number $(-\frac{1}{2}, -\frac{1}{2})$, the quantity $\{\Psi^c;\}$ vanishes since it can not satisfy the picture number conservation (we need the total picture number to be -2 on the disc).

- However, for elements in $\widetilde{\mathcal{H}}^c$, there is no problem. In the RR sector, they carry the picture number $(-\frac{3}{2}, -\frac{3}{2})$ and we can construct non zero disc one point function using them (by inserting a picture changing operator).

Ingredients for Constructing Open Closed SFT

- Hence, for $\tilde{\Psi}^c \in \widetilde{\mathcal{H}}^c$, we define $\{\tilde{\Psi}^c\}_D$ to be the contribution from one point function on the disc.

- For the NS-NS sector, we can include the disc one point contribution either in $\{\Psi^c;\}$ or $\{\tilde{\Psi}^c\}_D$. For uniformity, we include it in the definition of $\{\tilde{\Psi}^c\}_D$.

- The basic reason that the disc one point function needs the states from $\widetilde{\mathcal{H}}^c$ is that the D branes carry the RR charge and couple to the RR potential which is contained in $\tilde{\Psi}^c$ (the Ψ^c contain the RR field strengths).

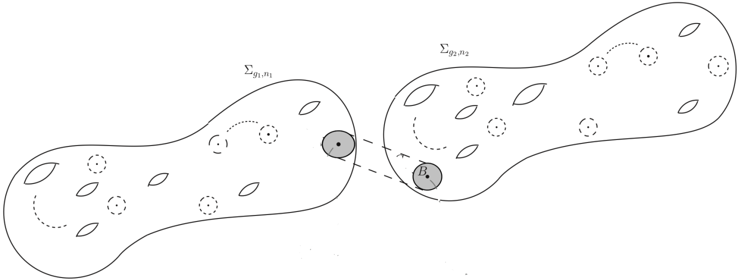
Ingredients for Constructing Open Closed SFT

- Before proceeding further, we note some features of the section segments $\mathcal{R}_{g,b,N,M}$.
- The section segments must not contain any separating type degeneration.
- Given two section segments, we can generate new section segments by repeated applications of following two operations

  1. Plumbing fixture (sewing)
  2. Hole creation

Ingredients for Constructing Open Closed SFT

In the plumbing fixture, we take two section segments $\mathcal{R}_{g_1,b_1,N_1,M_1}$ and $\mathcal{R}_{g_2,b_2,N_2,M_2}$. We now pick a Riemann surface from one family and sew one of its puncture to a puncture on the Riemann surface from the other family by identifying the local coordinates around the punctures.

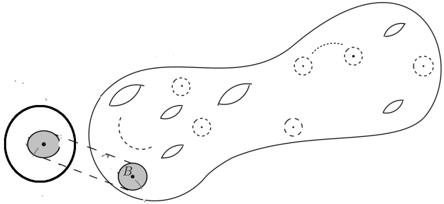


$$\text{Closed String :} \qquad w_1 w_2 = e^{-s-i\theta}, \quad 0 \le s \le \infty$$

$$\text{Open String :} \qquad w_1 w_2 = -e^{-s}$$

Ingredients for Constructing Open Closed SFT

In the hole creation operation, the Riemann surfaces from the section segment $\mathcal{R}_{g,b,N,M}$ are glued to a disc with one closed string puncture



$$w_1 w_2 = e^{-s}$$

This needs to be specified separately since the disc with one bulk puncture is special in the combined open closed super string field theory.

Ingredients for Constructing Open Closed SFT

A very useful identity satisfied by $\{\cdots\}$ and $\{\tilde{\Psi}^c\}_D$ is the so called 'main identity'

$$\{(Q_B\widetilde{A}^c)\}_D = 0$$

and,

$$\begin{aligned}
&\sum_{i=1}^{N}\{A_1^c\cdots A_{i-1}^c(Q_BA_i^c)A_{i+1}^c\cdots A_N^c;A_1^o\cdots A_M^o\} \\
&\quad+\sum_{j=1}^{M}\{A_1^c\cdots A_N^c;A_1^o\cdots A_{j-1}^o(Q_BA_j^o)A_{j+1}^o\cdots A_M^o\}(-1)^{j-1} \\
= \quad & -\frac{1}{2}\sum_{k=0}^{N}\sum_{\{i_1\cdots,i_k\}\subset\{1,\cdots,N\}}\sum_{\ell=0}^{M}\sum_{\{j_1,\cdots,j_\ell\}\subset\{1,\cdots,M\}}\Big(g_s^2\,\{A_{i_1}^c\cdots A_{i_k}^c\mathcal{B}^c;A_{j_1}^o\cdots A_{j_\ell}^o\} \\
&\qquad\qquad +g_s\,\{A_{i_1}^c\cdots A_{i_k}^c;\mathcal{B}^oA_{j_1}^o\cdots A_{j_\ell}^o\}\Big) \\
&\quad -g_s^2\{[A_c^1\cdots A_N^c;A_1^o\cdots A_M^o]^c\}_D\,,
\end{aligned}$$

Ingredients for Constructing Open Closed SFT

where

$$\mathcal{B}^c \equiv \mathcal{G}[A^c_{\bar{i}_1} \cdots A^c_{\bar{i}_{N-k}}; A^o_{\bar{j}_1} \cdots A^o_{\bar{j}_{M-\ell}}]^c, \quad \mathcal{B}^o \equiv \mathcal{G}[A^c_{\bar{i}_1} \cdots A^c_{\bar{i}_{N-k}}; A^o_{\bar{j}_1} \cdots A^o_{\bar{j}_{M-\ell}}]^o$$

with the conditions

$$\{i_1, \cdots, i_k\} \cup \{\bar{i}_1, \cdots, \bar{i}_{N-k}\} = \{1, \cdots, N\}$$

$$\{j_1, \cdots, j_\ell\} \cup \{\bar{j}_1, \cdots, \bar{j}_{M-\ell}\} = \{1, \cdots, M\}$$

The states $[;]^c$ and $[;]^o$ are defined via the inner products

$$\begin{aligned} \langle \Psi^c_0 | c^-_0 | [\Psi^c_1 \cdots \Psi^c_N; \Psi^o_1 \cdots \Psi^o_M]^c \rangle &= \{\Psi^c_0 \Psi^c_1 \cdots \Psi^c_N; \Psi^o_1 \cdots \Psi^o_M\}, \\ \langle \Psi^o_0 | [\Psi^c_1 \cdots \Psi^c_N; \Psi^o_1 \cdots \Psi^o_M]^o \rangle &= \{\Psi^c_1 \cdots \Psi^c_N; \Psi^o_0 \Psi^o_1 \cdots \Psi^o_M\} \end{aligned}$$

where $\langle \Psi |$ denotes the BPZ conjugate of $| \Psi \rangle$

Ingredients for Constructing Open Closed SFT

The operator $\mathcal{G}$ is given by

$$\mathcal{G}|s^o\rangle = \begin{cases} |s^o\rangle & \text{if } |s^o\rangle \in \mathcal{H}_{-1} \\ \frac{1}{2}(\mathcal{X}_0 + \bar{\mathcal{X}}_0)\,|s^o\rangle & \text{if } |s^o\rangle \in \mathcal{H}_{-3/2} \end{cases}$$

$$\mathcal{G}|s^c\rangle = \begin{cases} |s^c\rangle & \text{if } |s^c\rangle \in \mathcal{H}_{-1,-1} \\ \mathcal{X}_0\,|s^c\rangle & \text{if } |s^c\rangle \in \mathcal{H}_{-1,-3/2} \\ \bar{\mathcal{X}}_0\,|s^c\rangle & \text{if } |s^c\rangle \in \mathcal{H}_{-3/2,-1} \\ \mathcal{X}_0\bar{\mathcal{X}}_0\,|s^c\rangle & \text{if } |s^c\rangle \in \mathcal{H}_{-3/2,-3/2} \end{cases}$$

where $\mathcal{X}_0$ and $\bar{\mathcal{X}}_0$ denote the zero modes of holomorphic and anti-holomorphic PCOs

$$\mathcal{X}_0 \equiv \oint \frac{dz}{z}\,\mathcal{X}(z), \quad \bar{\mathcal{X}}_0 \equiv \oint \frac{d\bar{z}}{\bar{z}}\,\bar{\mathcal{X}}(\bar{z}),$$

1PI Effective Action

- Having described the 1PI vertices, we can now write down the 1PI effective action for the open-closed SFT

$$\begin{aligned} S_{1PI} \quad = \quad & -\frac{1}{2\,g_s^2}\langle\widetilde{\Psi}^c|c_0^- Q_B\mathcal{G}|\widetilde{\Psi}^c\rangle + \frac{1}{g_s^2}\langle\widetilde{\Psi}^c|c_0^- Q_B|\Psi^c\rangle \\ & -\frac{1}{2\,g_s}\langle\widetilde{\Psi}^o|Q_B\mathcal{G}|\widetilde{\Psi}^o\rangle + \frac{1}{g_s}\langle\widetilde{\Psi}^o|Q_B|\Psi^o\rangle \\ & +\langle\widetilde{\Psi}^c|c_0^-|[]_D\rangle + \sum_{N\geq 0}\sum_{M\geq 0}\frac{1}{N!\,M!}\,\{(\Psi^c)^N;(\Psi^o)^M\} \end{aligned}$$

where, $\langle\widetilde{\Psi}^c|c_0^-|[]_D\rangle = \{\widetilde{\Psi}^c\}_D$.

- Even though $\tilde{\Psi}^c$ and $\tilde{\Psi}^o$ fields appear in the action, they decouple from the physical processes.

1PI Effective Action

- The equations of motion following from the 1PI effective action are

$$|\tilde\Psi^c\rangle \quad : \quad Q_B\big(|\Psi^c\rangle - \mathcal{G}|\tilde\Psi^c\rangle\big) + g_s^2 |[\,]_D\rangle \; = \; 0$$

$$|\Psi^c\rangle \quad : \quad Q_B|\tilde\Psi^c\rangle + g_s^2 \sum_{N=1}^{\infty}\sum_{M=0}^{\infty} \frac{1}{(N-1)!M!} [(\Psi^c)^{N-1};(\Psi^o)^M]^c \; = \; 0$$

$$|\tilde\Psi^o\rangle \quad : \quad Q_B\big(|\Psi^o\rangle - \mathcal{G}|\tilde\Psi^o\rangle\big) = \; 0$$

$$|\Psi^o\rangle \quad : \quad Q_B|\tilde\Psi^o\rangle + g_s \sum_{N=0}^{\infty}\sum_{M=0}^{\infty} \frac{1}{N!(M-1)!} [(\Psi^c)^{N};(\Psi^o)^{M-1}]^o \; = \; 0$$

1PI Effective Action

- By appropriately combining the two closed and two open equations, one gets

$$Q_B|\Psi^c\rangle \;+\; g_s^2 \sum_{N=1}^{\infty}\sum_{M=0}^{\infty} \frac{1}{(N-1)!M!}\mathcal{G}[(\Psi^c)^{N-1};(\Psi^o)^M]^c \;+\; g_s^2|[\,]_D\rangle \;=\; 0$$

$$Q_B|\Psi^o\rangle \;+\; g_s \sum_{N=0}^{\infty}\sum_{M=1}^{\infty} \frac{1}{N!(M-1)!}\mathcal{G}[(\Psi^c)^N;(\Psi^o)^{M-1}]^o \;=\; 0$$

- These give the interacting equations of motion for the physical string fields Ψ^c and Ψ^o. The $\widetilde{\Psi}^c$ and $\widetilde{\Psi}^o$ can be determined after fixing a solution to Ψ^c and Ψ^o upto the freedom to add solutions to $Q_B|\widetilde{\Psi}^c\rangle = 0 = Q_B|\widetilde{\Psi}^o\rangle$.

- This implies that $\widetilde{\Psi}^c$ and $\widetilde{\Psi}^o$ represent the free field degrees of freedom and completely decouple from the interacting part of the theory.

1PI Effective Action

- This theory has a natural BV structure and it can be shown that the 1PI effective action satisfies the classical BV master equation (since we are supposed to work at tree level using the 1PI action).
- For this, we identify the fields and anti fields of the theory in the following manner: We first divide the Hilbert space of the theory as

$$\mathcal{H}^c \quad \Longrightarrow \quad \{\varphi_s^-, \varphi_+^r\} \quad ; \quad \tilde{\mathcal{H}}^c \quad \Longrightarrow \quad \{\widetilde{\varphi}_s^-, \widetilde{\varphi}_+^r\}$$

$$\mathcal{H}^o \quad \Longrightarrow \quad \{\phi_s^-, \phi_+^r\} \quad ; \quad \tilde{\mathcal{H}}^o \quad \Longrightarrow \quad \{\widetilde{\phi}_s^-, \widetilde{\phi}_+^r\}$$

$$\langle \widetilde{\varphi}_+^r | c_0^- | \varphi_s^- \rangle = \delta_s^r = \langle \widetilde{\varphi}_r^- | c_0^- | \varphi_+^s \rangle \quad , \quad \langle \widetilde{\phi}_+^r | \phi_s^- \rangle = \delta_s^r = \langle \widetilde{\phi}_r^- | \phi_+^s \rangle$$

1PI Effective Action

- Using this division, we can expand the string fields as

$$|\widetilde{\Psi}^c\rangle = g_s \sum_r (-1)^{\widetilde{\varphi}_r^-} (\widetilde{\psi}^c)^r |\widetilde{\varphi}_r^-\rangle - g_s \sum_r (\psi^c)^*_r |\widetilde{\varphi}_+^r\rangle$$

$$|\Psi^c\rangle - \frac{1}{2}\mathcal{G}|\widetilde{\Psi}^c\rangle = g_s \sum_r (-1)^{\varphi_r^-} (\psi^c)^r |\varphi_r^-\rangle - g_s \sum_r (\widetilde{\psi}^c)^*_r |\varphi_+^r\rangle$$

$$|\widetilde{\Psi}^o\rangle = \sqrt{g_s} \sum_r (\widetilde{\psi}^o)^r |\widetilde{\phi}_r^-\rangle + \sqrt{g_s} \sum_r (-1)^{\widetilde{\varphi}_+^r + 1} (\psi^o)^*_r |\widetilde{\phi}_+^r\rangle$$

$$|\Psi^o\rangle - \frac{1}{2}\mathcal{G}|\widetilde{\Psi}^o\rangle = \sqrt{g_s} \sum_r (\psi^o)^r |\phi_r^-\rangle + \sqrt{g_s} \sum_r (-1)^{\phi_+^r + 1} (\widetilde{\psi}^o)^*_r |\phi_+^r\rangle$$

- We can consistently assign positive (≥ 0) ghost numbers with the coefficient fields $(\psi^c)^r, (\widetilde{\psi}^c)^r, (\psi^o)^r, (\widetilde{\psi}^o)^r$ and negative (≤ -1) with the coefficient fields $(\psi^c)^*_r, (\widetilde{\psi}^c)^*_r, (\psi^o)^*_r, (\widetilde{\psi}^o)^*_r$.

1PI Effective Action

- Consequently, for the BV quantization, $(\psi^c)^r, (\tilde{\psi}^c)^r, (\psi^o)^r, (\tilde{\psi}^o)^r$ are interpreted as fields whereas $(\psi^c)^*_r, (\tilde{\psi}^c)^*_r, (\psi^o)^*_r, (\tilde{\psi}^o)^*_r$ are interpreted as anti-fields.

- Using the Main identity, the field expansion and the properties of the basis states, we find that the 1PI effective action satisfies the classical BV master equation

$$(S_{1PI}, S_{1PI}) = 2\frac{\partial_R S_{1PI}}{\partial \psi^r}\frac{\partial_L S_{1PI}}{\partial \psi^*_r} = 0$$

1PI Effective Action

The main identity also implies that the 1PI effective action is invariant under the gauge transformation

$$
\begin{aligned}
|\delta\Psi^c\rangle &= Q_B|\Lambda^c\rangle + g_s^2 \sum_{N,M} \frac{1}{N!\,M!} \Big(\mathcal{G}[\Lambda^c(\Psi^c)^N;(\Psi^o)^M]^c + \mathcal{G}[(\Psi^c)^N;\Lambda^o(\Psi^o)^M]^c \Big) \\
|\delta\Psi^o\rangle &= Q_B|\Lambda^o\rangle - g_s \sum_{N,M} \frac{1}{N!\,M!} \Big(\mathcal{G}[\Lambda^c(\Psi^c)^N;(\Psi^o)^M]^o + \mathcal{G}[(\Psi^c)^N;\Lambda^o(\Psi^o)^M]^o \Big) \\
|\delta\widetilde{\Psi}^c\rangle &= Q_B|\widetilde{\Lambda}^c\rangle + g_s^2 \sum_{N,M} \frac{1}{N!\,M!} \Big([\Lambda^c(\Psi^c)^N;(\Psi^o)^M]^c + [(\Psi^c)^N;\Lambda^o(\Psi^o)^M]^c \Big) \\
|\delta\widetilde{\Psi}^o\rangle &= Q_B|\widetilde{\Lambda}^o\rangle - g_s \sum_{N,M} \frac{1}{N!\,M!} \Big([\Lambda^c(\Psi^c)^N;(\Psi^o)^M]^o + [(\Psi^c)^N;\Lambda^o(\Psi^o)^M]^o \Big)
\end{aligned}
$$

where $|\Lambda^c\rangle \in \mathcal{H}^c$, $|\Lambda^o\rangle \in \mathcal{H}^o$, $|\widetilde{\Lambda}^c\rangle \in \widetilde{\mathcal{H}}^c$, $|\widetilde{\Lambda}^o\rangle \in \widetilde{\mathcal{H}}^o$, are the gauge transformation parameters.

Section Independence

- In defining the interaction vertices $\{(\Psi^c)^N; (\Psi^o)^M\}$ and $\{\widetilde{\Psi}^c\}_D$, we need to make a choice of PCO locations and/or the local coordinate system around the punctures.
- These different choices of sections apparently define different string field theories.
- However, it can be shown that all the string field theories defined this way are related by an appropriate field redefinition.
- More specifically, consider two SFTs defined using nearby sections. Also, let $|\mathcal{E}^o\rangle$ and $|\mathcal{E}^c\rangle$ denote the open and closed string field equations of motion.

Section Independence

- Let the variation in these equations due to the change in the section be $|\widehat{\delta}\mathcal{E}^o\rangle$ and $|\widehat{\delta}\mathcal{E}^c\rangle$ respectively.

- Now, under a string field redefinition

$$|\Psi^o\rangle \longrightarrow |\Psi^o\rangle + |\widetilde{\delta}\Psi^o\rangle, \qquad |\Psi^c\rangle \longrightarrow |\Psi^c\rangle + |\widetilde{\delta}\Psi^c\rangle$$

  we denote the changes in $|\mathcal{E}^o\rangle$ and $|\mathcal{E}^c\rangle$ by $|\widetilde{\delta}\mathcal{E}^o\rangle$ and $|\widetilde{\delta}\mathcal{E}^c\rangle$, respectively.

- The independence from the choice of integration subspace means the following equalities

$$|\widehat{\delta}\mathcal{E}^o\rangle - |\widetilde{\delta}\mathcal{E}^o\rangle = 0, \qquad |\widehat{\delta}\mathcal{E}^c\rangle - |\widetilde{\delta}\mathcal{E}^c\rangle = 0$$

  upon using the equations of motion $|\mathcal{E}^o\rangle = 0 = |\mathcal{E}^c\rangle$.

Section Independence

- One can show that using a string field redefinition of the form (Moosavian, MV)

$$\langle\Phi|\widetilde{\delta}\Psi^o\rangle = g_s \sum \frac{g_s^{2g-2+b}}{M!N!} \int_{\mathcal{R}_{g,b,N,M+1}} \Omega[U]((\Psi^c)^N; \mathcal{G}\Phi(\Psi^o)^M)$$

$$\begin{aligned}\langle\Psi|c_0^-|\widetilde{\delta}\Psi^c\rangle &= g_s^2 \sum \frac{g_s^{2g-2+b}}{M!N!} \int_{\mathcal{R}_{g,b,N+1,M}} \Omega[U]\Big(\mathcal{G}\Psi(\Psi^c)^N; (\Psi^o)^M\Big) \\ &\quad + g_s \int_{\mathcal{R}_{0,1,1,0}} \Omega[U](\mathcal{G}\Psi)\end{aligned}$$

  the section independence follows (U denotes an infinitesimal vector field connecting the nearby sections).

Generalization to Unoriented Strings

Some General Remarks

- There are essentially two types of open closed superstring theories : Type I and type II with D branes.

- The open closed superstring field theory, we have constructed, describes the type II superstring theory with D branes (We have considered only oriented strings).

- The type I theory is obtained by orientifold of type IIB theory. We now describe the essential differences which one encounters while considering this case.

Unoriented Open Closed String Field Theory

- The type I theory includes the oriented as well as unoriented strings.
- The unoriented strings are obtained by making an orientifold operation. This corresponds to taking a projection by the world-sheet parity operator possibly accompanied by some symmetry transformation acting on world-sheet SCFT.
- This means that in the vector space in which string fields live, we need to introduce orientifold projection. All the sewing now should be done with this projection. This automatically gives rise to unoriented surfaces like mobius strip and Klein bottle.

Unoriented Open Closed String Field Theory

- Denoting the world-sheet orientation reversing operator by W, the sewing should include the projection operator

$$P = \frac{1+W}{2}$$

- Due to the inclusion of this operator in the sewing operation, one needs to include some numerical factors in the definitions of $\{\Psi_1^c \cdots \Psi_N^c; \Psi_1^o \cdots \Psi_M^o\}$ and $\{\tilde{\Psi}^c\}_D$ to ensure correct factorization near degenerations.

- The $\{\cdots\}$ do not refer to any specific CFT. Hence, apart from the above change, essentially the same construction as in the oriented case, can also be applied to the unoriented theories

Unoriented Open Closed String Field Theory

- The main new features in the case of unoriented string field theory are

  1. The definition of $\mathcal{H}^c, \widetilde{\mathcal{H}}^c, \mathcal{H}^o$ and $\widetilde{\mathcal{H}}^o$ include the appropriate orientifold projection operator.

  2. The off-shell amplitudes include the sum over moduli spaces of oriented as well as non orientable surfaces.

  3. The $\{\tilde{\Psi}^c\}_D$ now includes not only the one point function on the disc but also on RP^2 (i.e. sphere with one crosscap).

Unoriented Open Closed String Field Theory

4. The definition of $\{\Psi_1^c \cdots \Psi_N^c; \Psi_1^o \cdots \Psi_M^o\}$ gets modified to

$$
\begin{aligned}
& \{\Psi_1^c \cdots \Psi_N^c; \Psi_1^o \cdots \Psi_M^o\} \\
\equiv \; & \sum_{\substack{g,b,c\geq 0 \\ (g,b,c,N,M)\neq(0,1,0,1,0),(0,0,1,1,0)}} (g_s)^{2g-2+b+c}\, 2^{-g-(b+c)/2+M/4} \\
& \int_{\mathcal{R}_{g,b,c,N,M}} \Omega^{g,b,c,N,M}_{6g-6+3b+3c+2N+M}(\Psi_1^c \cdots \Psi_N^c; \Psi_1^o \cdots \Psi_M^o)
\end{aligned}
$$

The coefficient involving the factor of 2 is needed to ensure that the amplitude factorizes correctly near the degeneration limit.

Thank You