Bosonic Tachyons from the Supersymmetric Point of View

arXiv:1905.09621

Lorenz Schlechter

June 10, 2020

Content

Tachyons in String Theory

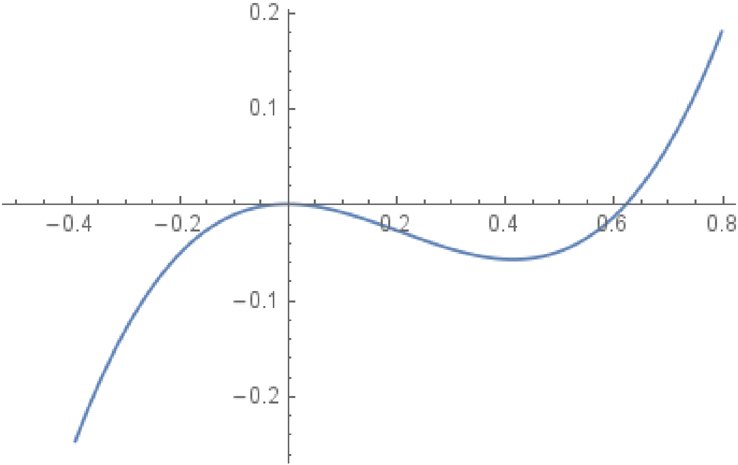
- Open tachyon condensation well understood.
- Local closed tachyons as well (decay of compact dimensions, orbifold defects)
- What about the bulk tachyon?

Results of the 90s and early 2000s

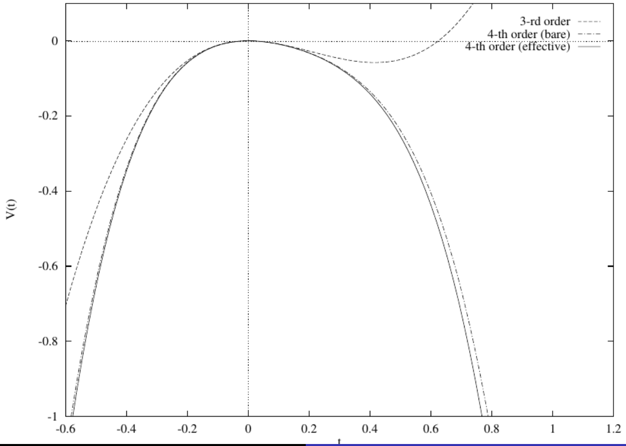
$$V(t) = -t^2 + \frac{6561}{4096} t^3 - 3.0172 t^4 + 9.924 t^5 \tag{1}$$

- Adding higher orders and higher level fields leads to oscillating behavior.
- Cubic order $\rightarrow$ minimum, Quartic order $\rightarrow$ run-away
- Quintic order $\rightarrow$ minimum ...
- Seams to converge to a minimum at $t \approx 0.05$ (Moeller, Yang 2006)

Potential of the Bosonic Closed String Tachyon (cubic order)



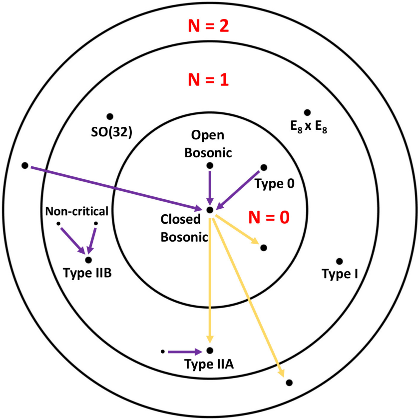
Potential of the Bosonic Closed String Tachyon (quartic order)



Idea

- The bosonic string can be embedded in the superstring (Berkovits, Vafa 1993)
- This adds additional d.o.f which exactly cancel at the bosonic point.
- What happens if one deforms the CFT away from the bosonic point?

The Theory Space of SFT



Steps in the Calculation

- Embed the bosonic string in the superstring.
- Use superstring field theory.
- Apply numerical methods (Rastelli, Zwiebach, Moeller).
- Calculate higher dimensional potential (in field space).
- Solve the resulting EOMs.

Berkovits' and Vafa's embedding (1993)

- Hidden $\mathcal{N} = 2$ SUSY in bosonic string (requires choice of current).
- Add spin-shifted fermionic b'c' ghost system with $h = (3/2, -1/2)$ to the matter system.
- Correct spin-statistic.

$$T = 26 \times T_X + T_{b'c'} + T_{bc} + T_{\beta\gamma} \tag{2}$$

$$T_{b'c'} = -3/2 b' \partial c' - 1/2 \partial b' c' + 1/2 \partial^2 (c' \partial c') \tag{3}$$

Berkovits' and Vafa's Embedding (1993)

$$T_{\mathcal{N}=1} = 26 \times T_X + T_{b'c'} + T_{bc} + T_{\beta\gamma} \tag{4}$$

- $\mathcal{N} = 1$ string, equivalent to the bosonic string.
- $\beta\gamma$ and $b'c'$ contributions to amplitudes cancel.
- Exact endpoint of the type 0 tachyon condensation. (S. Hellerman, I. Swanson 2008)
- Can be extended to $\mathcal{N} = 2$:

$$T_{\mathcal{N}=2} = 26 \times T_X + 2T_{b'c'} + T_\phi^{Q=1} + T_{bc} + 2T_{\beta\gamma} + T_{\eta\xi} \tag{5}$$

- $\mathcal{N} = 2$ has hidden $\mathcal{N} = 4$ symmetry $\rightarrow$ can in principle be continued

Steps in the computation

1. Construct level truncated Hilbert space of the CFT.
2. Rewrite the SFT potential as as sum of string functions (requires b-insertions, PCO prescriptions...).
3. Evaluate the string functions (conservation laws, ghost number conservations, conformal maps to the n-punctured sphere).
4. Solve the resulting EOMs.

1. Hilbert space

- $L_0 - \overline{L}_0 \left|\psi\right\rangle = 0$
- $b_0 \pm \overline{b}_0 \left|\psi\right\rangle = 0$
- picture -1 in NS, -3/2 or -1/2 in R
- no ghost number constraints
- usual treatment of $\beta\gamma$ system ($\eta\xi + \phi$)
- lowest lying state in NS sector:

$$\left|t\right\rangle = t(x) c_1 \overline{c}_1 c'_{1/2} \overline{c}'_{1/2} e^{-\phi-\overline{\phi}} \left|0\right\rangle , \quad h = -2 \tag{6}$$

- 16 additional tachyons B_i with weight $h = -1$
- 121 massless fields

Bosonic String Field

Applying the same rules to the usual bosonic string field one obtains:

$$\begin{aligned}\psi_{bos} = t(x)c_1\overline{c}_1\left|0\right\rangle + d_1(x)c_{-1}c_1\overline{c}_{-1}\overline{c}_1\left|0\right\rangle + d_2(x)c_{-1}c_1\left|0\right\rangle \\ +d_3(x)\overline{c}_{-1}\overline{c}_1\left|0\right\rangle + g_{\mu\nu}(x)\alpha^\mu_{-1}c_1\overline{\alpha}^\nu_{-1}\overline{c}_1\left|0\right\rangle \\ +A_{\mu,1}(x)\alpha^\mu_{-1}c_1\overline{c}_{-1}\overline{c}_1\left|0\right\rangle + A_{\mu,2}(x)c_{-1}c_1\overline{c}_1\overline{\alpha}^\mu_{-1}\left|0\right\rangle \\ +A_{\mu,3}(x)\alpha^\mu_{-1}c_1\left|0\right\rangle + A_{\mu,4}(x)\overline{c}_1\overline{\alpha}^\mu_{-1}\left|0\right\rangle \\ +I(x)\left|0\right\rangle\ .\end{aligned}$$

Demanding ghost number $g = 2$ would eliminate the additional massless fields.

Bosonic Action (Qubic order, level 2)

$$V(\psi) = -t^2 + \frac{27}{32} t d_2 d_3 + \frac{6561}{4096} t^3 + \frac{27}{32} I t d_1 + \frac{27}{16} g_{\mu\nu} g^{\mu\nu} t \,. \tag{7}$$

- Not in the usual twist symmetric basis.
- A field redefinition brings it to the usual form:

$$|d\rangle = |d_2\rangle - |d_3\rangle = (c_{-1}c_1 - \bar{c}_{-1}\bar{c}_1)\,|0\rangle \tag{8}$$

$$|d_g\rangle = |d_2\rangle + |d_3\rangle = (c_{-1}c_1 + \bar{c}_{-1}\bar{c}_1)\,|0\rangle \;, \tag{9}$$

$\mathcal{N} = 1$ Action, (EKS)

$$S_{NS} = -\frac{1}{2}\langle\psi| c_0^+ c_0^- L_0^+ |\psi\rangle + \frac{\omega\left(\Phi, L_2^{(1,1)}(\Phi,\Phi)\right)}{6} + \frac{\omega\left(\Phi, L_3^{(2,2)}(\Phi,\Phi,\Phi)\right)}{24}$$

$$\omega\left(\Phi, L_2^{(1,1)}(\Phi,\Phi,\Phi)\right) = \{f_1\circ\psi(0), f_2\circ\psi(0), f_3\circ X\overline{X}\psi(0)\} + 8 \text{ permutations}$$

$$\overline{X}XV(y) = \oint \frac{dz}{2\pi iz}\oint \frac{dw}{2\pi iw}\overline{X}(z)X(w)V(y)\,.$$

Quadratic terms

$$S_{kin,NS} = \frac{1}{2} \langle \Psi | \, c^+ c^- L_0 \, | \Psi \rangle = \frac{h_\Psi}{2} \langle \Psi | \, c^+ c^- \, | \Psi \rangle \ . \qquad (10)$$

These can be evaluated using the BPZ inner product. With normalization

$$\langle 0 | \, c_{-1} \bar{c}_{-1} c'_{-1/2} \bar{c}'_{-1/2} c_0 \bar{c}_0 c_1 \bar{c}_1 c'_{1/2} \bar{c}'_{1/2} e^{-2\Phi - 2\bar{\Phi}} \, | 0 \rangle = 2 \, , \qquad (11)$$

one obtains:

$$V_2^{(2)} = -t^2 + B_2 B_5 - B_4 B_7 + B_{10} B_{13} + B_{11} B_{16} \ . \qquad (12)$$

Diagonalizing the fields gives 4 additional tachyons, 8 massless fields and 4 massive fields.

Qubic terms

$$\{\{\Psi^3\}\} = \{f_1 \circ \psi(0), f_2 \circ \psi(0), X\overline{X} f_3 \circ \psi(0)\} + 8 \text{ permutations} \quad (13)$$

- Conformal transformations known. $(-\sqrt{3}, 0, \sqrt{3})$ convention.
- Eliminate all creation operators using conservation laws.

$$\langle V_3| \sum_{i=1}^{3} \oint_{\mathcal{C}_i} dz_i \phi^{(i)}(z_i) \mathcal{O}(z_i) = 0 \; . \quad (14)$$

- ϕ has dimenison $1 - h(\mathcal{O})$
- Only c_1 and $c'_{1/2}$ remain

Qubic terms

- Rewrite closed amplitudes into open.
- Signs form commuting operators cancel.

$$\langle 0|\, c_1^{(1)} c_1^{(2)} c_1^{(3)} \overline{c}_1^{(1)} \overline{c}_1^{(2)} \overline{c}_1^{(3)} c'^{(i)}_{1/2} c'^{(j)}_{1/2} \overline{c}'^{(l)}_{1/2} \overline{c}'^{(m)}_{1/2} e^{a_1 \Phi^{(1)} + a_2 \Phi^{(2)} + a_3 \Phi^{(3)} + b_1 \overline{\Phi}^{(1)} + b_2 \overline{\Phi}^{(2)} + b_3 \overline{\Phi}^{(3)}} \,|0\rangle =$$
$$2 \langle 0|\, c_1^{(1)} c_1^{(2)} c_1^{(3)} \,|0\rangle_o \langle 0|\, \overline{c}_1^{(1)} \overline{c}_1^{(2)} \overline{c}_1^{(3)} \,|0\rangle_o \langle 0|\, c'^{(i)}_{1/2} c'^{(j)}_{1/2} \,|0\rangle_o \langle 0|\, \overline{c}'^{(l)}_{1/2} \overline{c}'^{(m)}_{1/2} \,|0\rangle_o$$
$$\cdot \langle 0|\, e^{a_1 \Phi^{(1)} + a_2 \Phi^{(2)} + a_3 \Phi^{(3)} + b_1 \overline{\Phi}^{(1)} + b_2 \overline{\Phi}^{(2)} + b_3 \overline{\Phi}^{(3)}} \,|0\rangle_o$$

$$\langle 0|\, c_1(z_1) c_1(z_2) c_1(z_3) \,|0\rangle = (z_1 - z_2)(z_1 - z_3)(z_2 - z_3)\,, \tag{15}$$
$$\langle 0|\, c'_{1/2}(z_1) c'_{1/2}(z_2) \,|0\rangle = (z_1 - z_2) \tag{16}$$
$$\langle 0|\, e^{a_1 \Phi^{(1)} + a_2 \Phi^{(2)} + a_3 \Phi^{(3)}} \,|0\rangle = \delta(a_1 + a_2 + a_3 + 2) \prod_{i<j} (z_i - z_j)^{-a_i \cdot a_j} \tag{17}$$

Results

- Tachyonic part of the potential.
- Massless also evaluated, but too long to show here.

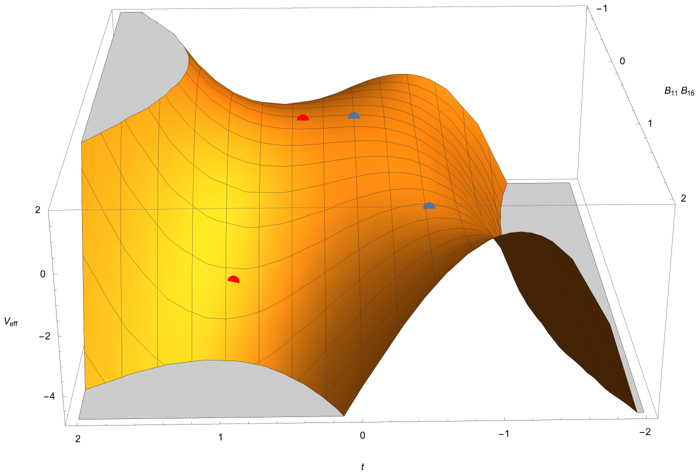
$$V_1^{(3)} = -\frac{243 B_1^2 t}{256} + \frac{243}{128} B_1 B_2 t + \frac{243}{128} B_1 B_5 t - \frac{243}{128} B_2 B_5 t - \frac{1215}{512} B_1 B_6 t - \frac{243}{128} B_{12} B_{15} t + \frac{243}{128} B_{11} B_{16} t + \frac{6561 t^3}{4096}$$

Solutions to EOMs

t	V	m	m	m	m	m	m	m	m	m	m	m	m
0	0	-2	-1	-1	-1	-1	0	0	0	0	1	1	1
0.42	-0.058	$-\frac{113}{16}$	-2	-1.09	-1	-1	-1	0	0	0.91	1	1	3.18
$+\frac{128}{243}$	-0.04	-2.18	-2	-1	-1	-1	0	0	1	1	1.18	2	$\frac{49}{16}$
$-\frac{128}{243}$	-0.51	-1.90	$-\frac{145}{81}$	-1	-1	$-\frac{64}{81}$	$-\frac{17}{81}$	0.09	$\frac{64}{81}$	1	1.23	$\frac{145}{81}$	2

Table: The values of the tachyon, the potential and the masses of the fields in the 4 different solutions at cubic order. This excludes the 4 massless fields which do not appear at cubic order.

Potential



Evaluation of Quartic Terms

- Much more complicated when qubic terms.
- All parts of the computation in principle known, but amount of terms to compute explodes.
- Conformal maps only known numerically. (but solved by Moeller)

EKS solution of L_∞-relations

$$L_{n+2}^{(p,q)} = \frac{1}{p+q}\sum_{k=0}^{n}\left(\sum_{r,s}[L_{n-k+1}^{(r,s)},\lambda_{k+2}^{(p-r,q-s)}] + \sum_{r,s}[L_{n-k+1}^{(r,s)},\overline{\lambda}_{k+2}^{(p-r,q-s)}]\right).$$

$$\lambda_{n+2}^{(p+1,q)} = \frac{n-p+1}{n+3}\Big(\xi_0 L_{n+2}^{(p,q)} - L_{n+2}^{(p,q)}(\xi_0\mathbb{I}_{N+1})\Big), \qquad (18)$$

$$\overline{\lambda}_{n+2}^{(p,q+1)} = \frac{n-q+1}{n+3}\Big(\overline{\xi}_0 L_{n+2}^{(p,q)} - L_{n+2}^{(p,q)}(\overline{\xi}_0\mathbb{I}_{N+1})\Big). \qquad (19)$$

- Express the superstring brackets as combinations of the bosonic brackets.
- Triple sum $\rightarrow$ terms add up fast.
- All parts of the computation in principle known, but explodes in number of term.

EKS solution of L_∞-relations

- Explicit form very long, $\mathcal{O}(200)$ terms for $L_3^{(2,2)}$.
- Includes terms of the form $L_2[L_2[\psi_1,\psi_2],\psi_2]$
- These turn out to be difficult to evaluate.

Explicit Evaluation of $L_2[L_2[\psi_1, \psi_2], \psi_2]$

Two possibilities:

- Use conservation laws and reflector state. Works but slow.
$$L_2[V_1, V_2] = \langle V_{123'}| \left(|V_1\rangle_{(1)} \bigotimes |R_{33'}\rangle \bigotimes |V_2\rangle_{(2)} \right) \quad (20)$$
- Explicit formula using surface states with insertions not at the origin. Problems with $e^{-\phi}$ terms.

Simplification for Special Ghost Structure

But if the ghost structure of the vertex operators is $|V\rangle = V(\alpha,\phi)c_1\bar{c}_1 c'_{1/2}\bar{c}'_{1/2}e^{-\Phi-\overline{\Phi}}|0\rangle$, drastic simplifications happen.
For heterotic string:

$$\frac{1}{4!}\omega\left(\Phi, L_3^{(2)}(\Phi,\Phi,\Phi)\right) =$$
$$\frac{5}{108}\omega_L\left(\xi_0 X_0\Phi, L_2^{(0)}(\Phi,\xi_0 L_2^{(0)}(\Phi,\Phi))\right) + \frac{1}{216}\omega_L\left(\xi_0 X_0\Phi, L_2^{(0)}(\Phi, L_2^{(0)}(\Phi,\xi_0\Phi))\right)$$
$$+\frac{1}{108}\omega_L\left(\xi_0\Phi, L_2^{(0)}(\Phi,\xi_0 X_0 L_2^{(0)}(\Phi,\Phi))\right) - \frac{1}{48}\omega_L\left(\xi_0 X_0\Phi, L_2^{(0)}(\xi_0\Phi, L_2^{(0)}(\Phi,\Phi))\right)$$
$$+\frac{1}{96}\omega_L\left(\xi_0 X_0^2\Phi, L_3^{(0)}(\Phi,\Phi,\Phi)\right) + \frac{1}{32}\omega_L\left(\xi_0 X_0\Phi, L_3^{(0)}(\Phi,\Phi,X_0\Phi)\right). \qquad (21)$$

Simplification for Special Ghost Structure

$L_2[L_2[\psi_1,\psi_2],\psi_2]$ comes always with 2 ξ_0 insertions and one PCO insertion.

$$X = \{Q,\xi\} = e^{\phi}(b' + c'(T_m + \partial(c')b') + \frac{5}{2}\partial^2 c') \\ +2\partial(\eta)e^{2\phi}b + \eta\partial(e^{2\phi}b) + c\partial\xi \ .$$

- Due to b'c' ghost number conservation only the first terms contribute.
- $L_2[L_2[\psi_1,\psi_2],\psi_2]$ terms vanish for ghost structure $c_1\bar{c}_1 c'_{1/2}\bar{c}'_{1/2}e^{-\Phi-\bar{\Phi}}$!
- This includes all fields of the bosonic theory.
- Truncated to this structure the SFT looks like bosonic SFT with 1 additional time.

Additional Problems at Higher Order/Level

- The additional tachyons do not have this structure, hard to compute.
- Beyond the massless level the b-insertions become much more complicated.
- Compared to the bosonic theory the number of fields increases much faster with the level.
- Work in Progress.

Outlook

- Further computations require better computational methods. (quartic perhaps already doable)
- Include torus contributions.
- Extension to the $\mathcal{N} = 2$ embedding.
- Application to open SFT, much easier computations.