Classical Solutions and Background Independence in OSFT

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Relevance of classical solutions in SFT

- SFT equation of motion can give new handles on exact (B)CFT's which are not easily accessible from the standard CFT approach: ***the "honeycomb" c=2 BCFT*** (*Kudrna-Schnabl-Vosmera*), ***RR backgrounds in RNS*** (*Cho-Collier-Yin*), etc…

- Some backgrounds just don't have a direct (B)CFT description (e.g. the ***tachyon condensate***) and to study their physics one needs field theory-like tools.

- In QFT's based on a path integral, classical solutions are the saddle points of the action and they account for ***non-perturbative contributions*** to amplitudes. If we want to have a non-perturbative understanding of string theory from SFT it is necessary to understand how string theory backgrounds appear as classical solutions.

Background independence in string field theory

- We can construct a string field theory on any given exact string background $(B)CFT_0$, with a dynamical field variable $\phi^{(0)}$ and an action $S\left[\phi^{(0)}\right]$
- Classical solutions $\phi^{(0)} = \Psi_*^{(0)}$, represent other consistent backgrounds $B(CFT)_*$
- Expand $\phi^{(0)} = \Psi_*^{(0)} + \phi_*^{(0)}$: dynamics of fluctuations around the solution

$$S\left[\Psi_*^{(0)} + \phi_*^{(0)}\right] = S\left[\Psi_*^{(0)}\right] + S_*\left[\phi_*^{(0)}\right]$$

- Background independence: the expanded action and the action directly formulated around $B(CFT)_*$ should be related by *field redefinition*

$$\phi_*^{(0)} = f\left(\phi^{(*)}\right)$$

$$S_*\left[\phi_*^{(0)}\right] = S\left[\phi^{(*)}\right]$$

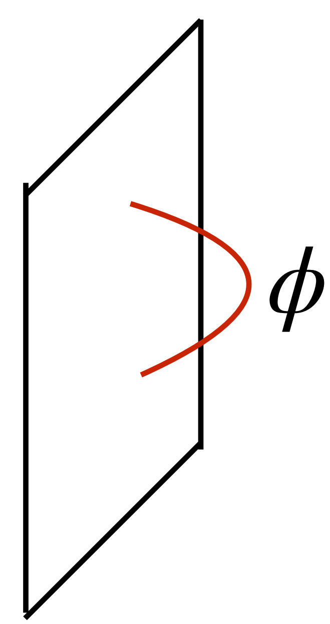
- A Jacobian is also generated from the field redefinition in the path integral measure (quantum effect)

$$\mathscr{D}\phi^{(0)} = \mathscr{D}\phi_*^{(0)} = \mathscr{D}\phi^{(*)}\left|\frac{\mathscr{D}\phi_*^{(0)}}{\mathscr{D}\phi^{(*)}}\right|$$

From now on let us focus on (classical) Witten bosonic Open String Field Theory

$$S\left[\phi\right] = -\frac{1}{g_s}\mathrm{Tr}\left[\frac{1}{2}\phi Q\phi + \frac{1}{3}\phi^3\right]$$

$$\phi \in H_{\mathrm{BCFT}_0}$$



- Solution to the equation of motion $Q\Psi + \Psi^2 = 0$ (Erler, C.M. 2014-2019)

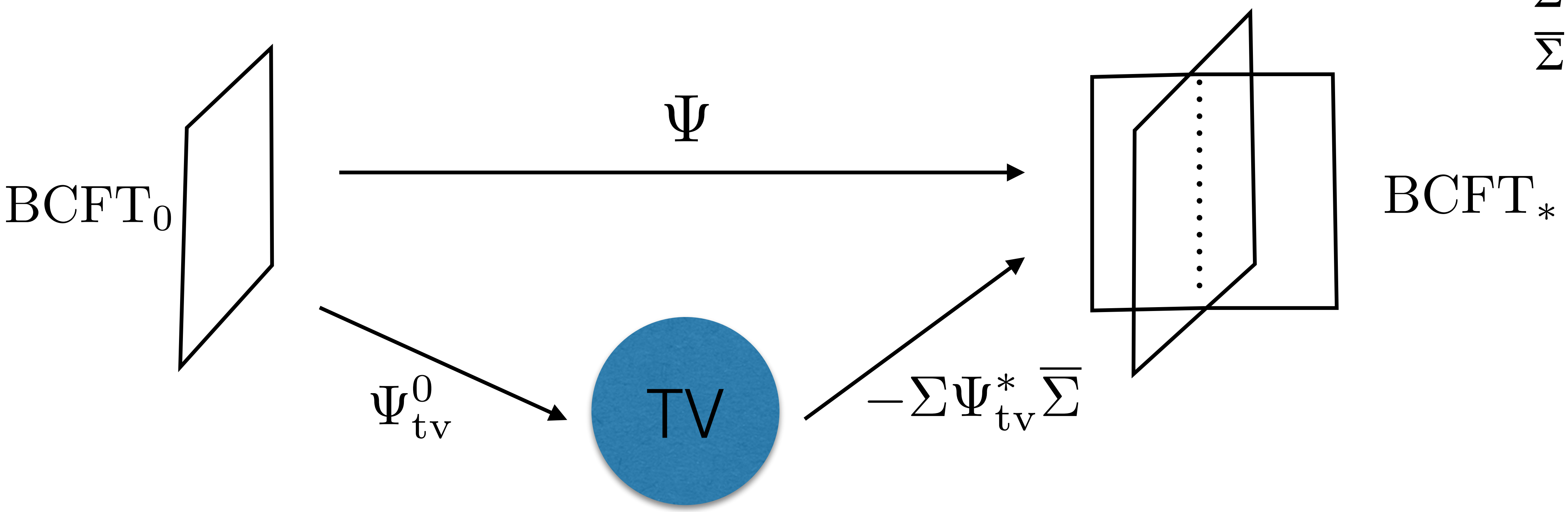
$$\Psi = \Psi^0_{\mathrm{tv}} - \Sigma\, \Psi^*_{\mathrm{tv}}\, \overline{\Sigma}$$

$\Psi^0_{\mathrm{tv}} \in H_{\mathrm{BCFT}_0}$

$\Psi^*_{\mathrm{tv}} \in H_{\mathrm{BCFT}_*}$

$\Sigma \in H_{0*}$ *stretched strings*

$\overline{\Sigma} \in H_{*0}$



- Equation of motion

$$Q_{\mathrm{tv}}\Sigma = Q\Sigma + \Psi^0_{\mathrm{tv}}\Sigma - \Sigma\Psi^*_{\mathrm{tv}} = 0$$

$$Q_{\mathrm{tv}}\overline{\Sigma} = Q\overline{\Sigma} + \Psi^*_{\mathrm{tv}}\overline{\Sigma} - \overline{\Sigma}\Psi^0_{\mathrm{tv}} = 0$$

$$\overline{\Sigma}\Sigma = 1$$

- In our construction we have

$$\overline{\Sigma}\Sigma = 1 \in H_{\mathrm{BCFT}_*}$$

$$\Sigma\overline{\Sigma} = P \in H_{\mathrm{BCFT}_0}$$

- The emerging star algebra projector $P^2 = P$ has an interesting role in the proof of background independence, as we will see.

- Other important properties which descend from the previous ones

$$Q_\Psi \left(\Sigma\phi^{(*)}\overline{\Sigma}\right) = \Sigma \left(Q_B\phi^{(*)}\right) \overline{\Sigma}$$

$$\phi^{(*)} \in H_{\mathrm{BCFT}_*}$$

$$\Sigma\phi^{(*)}\overline{\Sigma} \in H_{\mathrm{BCFT}_0}$$

$$\mathrm{Tr}_0[\Sigma\phi^{(*)}\overline{\Sigma}] = \mathrm{Tr}_*[\phi^{(*)}]$$

Observables

- **Action (it computes the energy for static solutions)**

$$E = -\frac{1}{6}\mathrm{Tr}[\Psi^3] = -\frac{1}{6}\mathrm{Tr}[(\Psi_{\mathrm{tv}})^3] + \frac{1}{6}\mathrm{Tr}[\Sigma(\Psi^*_{\mathrm{tv}})^3\overline{\Sigma}] \quad \text{(EOM)}$$

$$= -\frac{1}{6}\mathrm{Tr}[(\Psi_{\mathrm{tv}})^3] + \frac{1}{6}\mathrm{Tr}[(\Psi^*_{\mathrm{tv}})^3] = \frac{1}{2\pi^2}\left(-g_0 + g_*\right)$$

- **Ellwood Invariants (Boundary State of the new background)**

$$\langle I|V(i,-i)|\Psi\rangle \equiv \mathrm{Tr}_V[\Psi] = \mathrm{Tr}_V[\Psi_{\mathrm{tv}}] - \mathrm{Tr}_V[\Sigma\Psi^*_{\mathrm{tv}}\overline{\Sigma}]$$

$$= \mathrm{Tr}_V[\Psi_{\mathrm{tv}}] - \mathrm{Tr}_V[\Psi^*_{\mathrm{tv}}]$$

$$= \frac{1}{2\pi i}\left\langle c\bar{c}V^{(m)}(0)c(1)\right\rangle^{\mathrm{BCFT}_0}_{\mathrm{disk}} - \frac{1}{2\pi i}\left\langle c\bar{c}V^{(m)}(0)c(1)\right\rangle^{\mathrm{BCFT}_*}_{\mathrm{disk}}$$

Background Independence

- To study the physics around the solution we shift

$$\phi^{(0)} = \Psi_*^{(0)} + \phi_*^{(0)}$$

- Fluctuations $\phi_*^{(0)}$ can be decomposed according to P and $\bar{P} = 1 - P$

$$\phi_*^{(0)} = (P+\bar{P})\phi_*^{(0)}(P+\bar{P}) = \begin{pmatrix} \phi_{PP}^{(0)} & \phi_{P\bar{P}}^{(0)} \\ \phi_{\bar{P}P}^{(0)} & \phi_{\bar{P}\bar{P}}^{(0)} \end{pmatrix}$$

Matrix with ***constrained*** *entries*

- One-to-one linear field redefinition

$$\begin{pmatrix} \phi_{PP}^{(0)} & \phi_{P\bar{P}}^{(0)} \\ \phi_{\bar{P}P}^{(0)} & \phi_{\bar{P}\bar{P}}^{(0)} \end{pmatrix} = \begin{pmatrix} \Sigma & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \phi_{11}^{(*)} & \bar{\chi}_{1\bar{P}} \\ \chi_{\bar{P}1} & t_{\bar{P}\bar{P}}^{(0)} \end{pmatrix} \begin{pmatrix} \bar{\Sigma} & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \phi_{11}^{(*)} & \bar{\chi}_{1\bar{P}} \\ \chi_{\bar{P}1} & t_{\bar{P}\bar{P}}^{(0)} \end{pmatrix} = \begin{pmatrix} \bar{\Sigma} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \phi_{PP}^{(0)} & \phi_{P\bar{P}}^{(0)} \\ \phi_{\bar{P}P}^{(0)} & \phi_{\bar{P}\bar{P}}^{(0)} \end{pmatrix} \begin{pmatrix} \Sigma & 0 \\ 0 & 1 \end{pmatrix}$$

$\Phi_{11}^{(*)} \in H_{\mathrm{BCFT}_*}$ Expected new variables (unconstrained)

$t_{\bar{P}\bar{P}}^{(0)} \in H_{\mathrm{BCFT}_0}$
$\bar{\chi}_{1\bar{P}} \in H_{*0}$
$\chi_{\bar{P}1} \in H_{0*}$

Un-expected extra variables (constrained)

- The action can then be rewritten as (see also *Kishimoto, Masuda, Takahashi*)

$$S[\Psi+\phi_*^{(0)}]=\frac{1}{2\pi^2 g_s}(g^{(0)}-g^{(*)})-\frac{1}{g_s}\mathrm{Tr}_*\left[\frac{1}{2}\phi^{(*)}Q\phi^{(*)}+\frac{1}{3}(\phi^{(*)})^3\right]$$

$$-\frac{1}{g_s}\mathrm{Tr}_0\left[\frac{1}{2}tQ_{\mathrm{tv}}t+\frac{1}{3}t^3+\chi Q_{tv,*}\bar{\chi}+t\chi\bar{\chi}+\chi\phi^{(*)}\bar{\chi}\right]$$

Extra ***constrained*** *fluctuations at the TV*

$P\chi=\bar{\chi}P=Pt=tP=0$

- However the only perturbative solution around the TV is $\chi=\bar{\chi}=t=0$

- In the perturbative expansion around the saddle, the TV sector can be ***integrated out***, setting it to ***zero***. The action of the physical fluctuation $\phi^{(*)}$ ***DOES NOT*** change.

- This is very different from the integration out of the massive fields (Q_{tv} has no cohomology, “nothing is left”)

- Amplitudes in the background of the solution are mapped to amplitudes in the new BCFT background.

Background independence:

shift + field redefition + integration out of trivial fields

(expected) *(something genuinely new!)*

- The constrained TV sector would be absent if $\Sigma\bar{\Sigma} = 1$. Can this be possible?

- In this case the cohomologies of the two backgrounds would be strictly isomorphic but this is something we don't expect in general (except for special exactly marginal deformations)

- So, physically, $\Sigma\bar{\Sigma} \neq 1$ is there to allow to connect genuinely different backgrounds.

- Gauge equivalence in $H_{\rm BCFT_0}$ implies gauge equivalence in $H_{\rm BCFT_*}$ and viceversa. The extra pure gauge TV sector accounts for that.

Explicit realization: the flags

- The intertwining fields can be explicitly constructed from a TV solution and bcc operators ("twist fields")

$$\Psi_{\rm tv} = \sqrt{F(K)}\,c\,\frac{BK}{1-F^2(K)}\,c\sqrt{F(K)}$$

$$Bc + cB = 1 \qquad QB = K$$

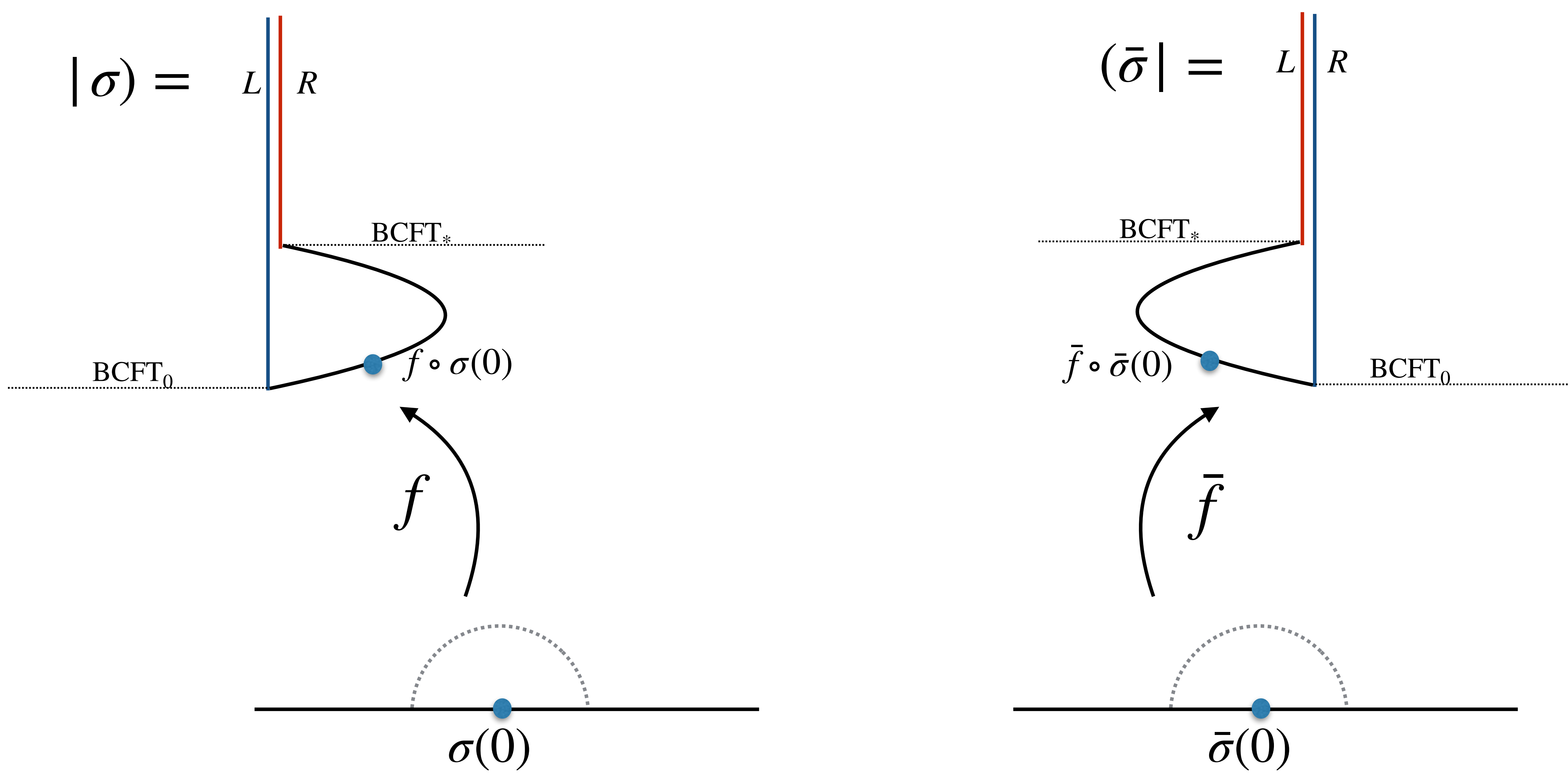
$$c^2 = B^2 = 0 \qquad Qc = cKc$$

$$\Sigma = Q_{\rm tv}\left(\sqrt{\frac{1-F^2}{K}}B\,|\sigma)\sqrt{\frac{1-F^2}{K}}\right) \qquad (\bar\sigma|\,|\sigma) = 1 \in H_{{\rm BCFT}_*}$$

$$\overline{\Sigma} = Q_{\rm tv}\left(\sqrt{\frac{1-F^2}{K}}(\bar\sigma|\,B\sqrt{\frac{1-F^2}{K}}\right) \qquad (\bar\sigma|\,B\,|\sigma) = B \in H_{{\rm BCFT}_*}$$

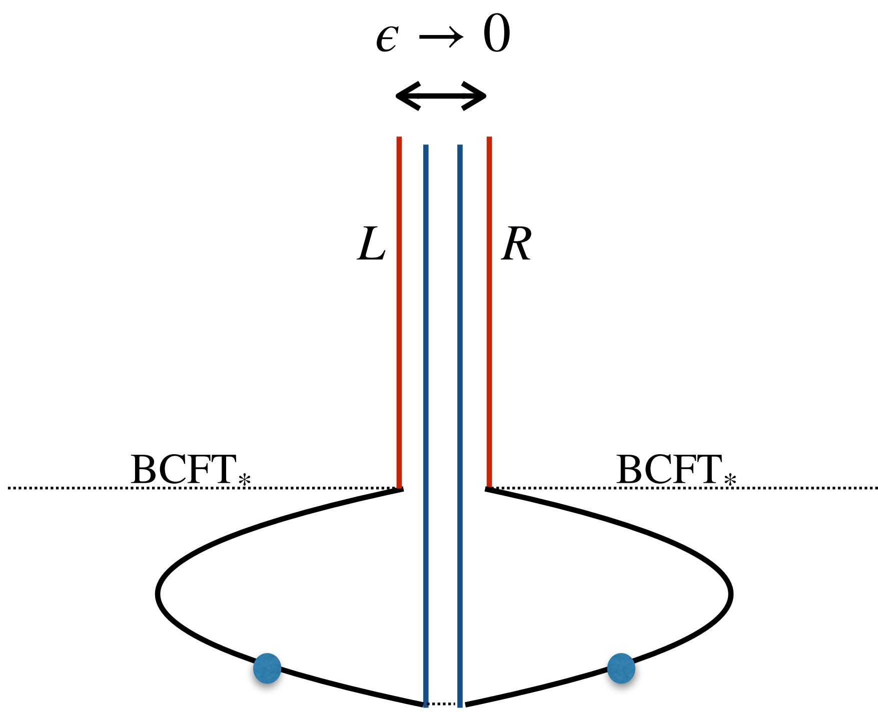
$$\overline{\Sigma}\Sigma = 1$$

The Flags as surface states with bcc insertion



It is possible to map the surfaces obtained by gluing flags and wedge states to the UHP: Schwarz-Christoffel map

- Multiplying in the order $(\bar{\sigma}|\star|\sigma)$: degenerating surface



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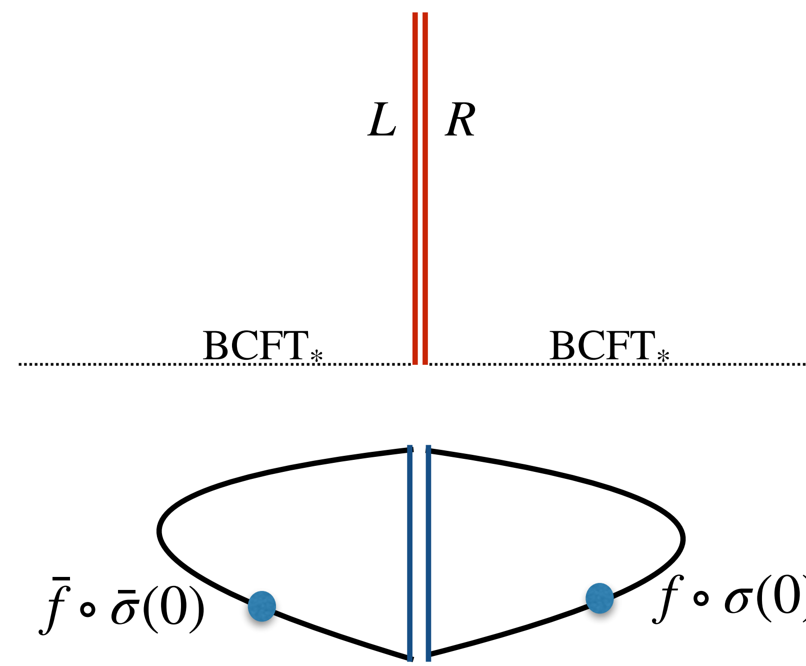
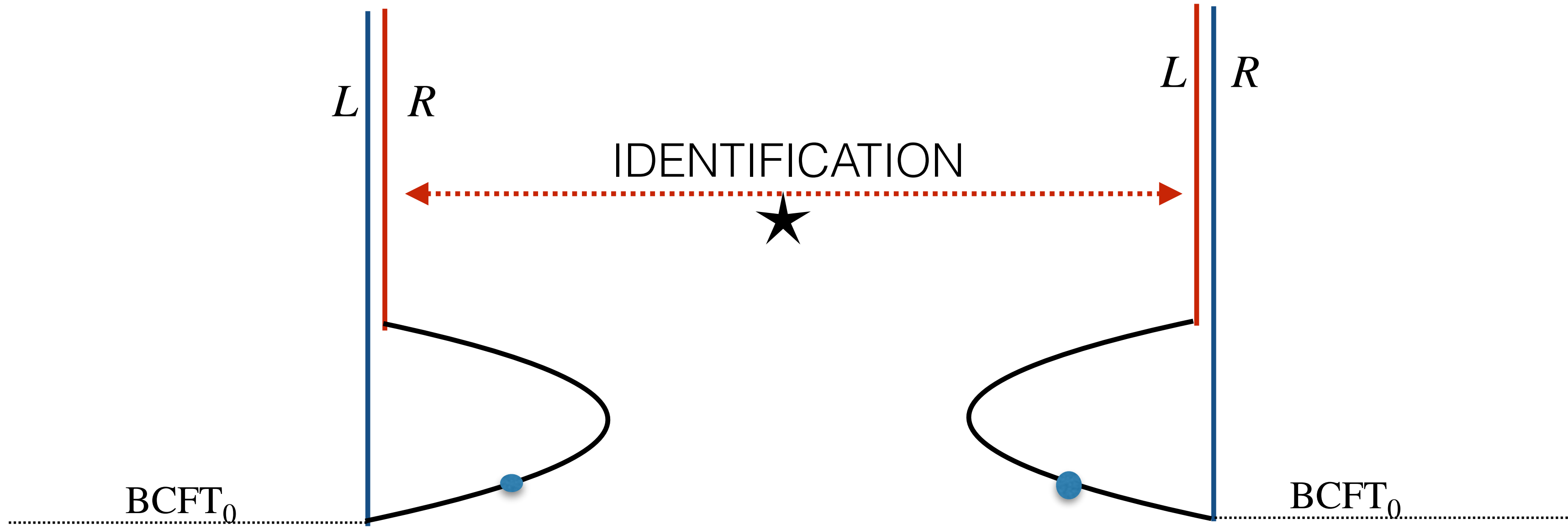


$$= \quad \frac{1}{g_*}\langle I\circ\bar{\sigma}(0)\,\sigma(0)\rangle \quad \times \quad 1^{(*)}$$

(normalization)

$$(\bar{\sigma}||\sigma) = 1$$

- Multiplying in the order $|\sigma) \star (\bar{\sigma}|$: ***new kind of surface state***



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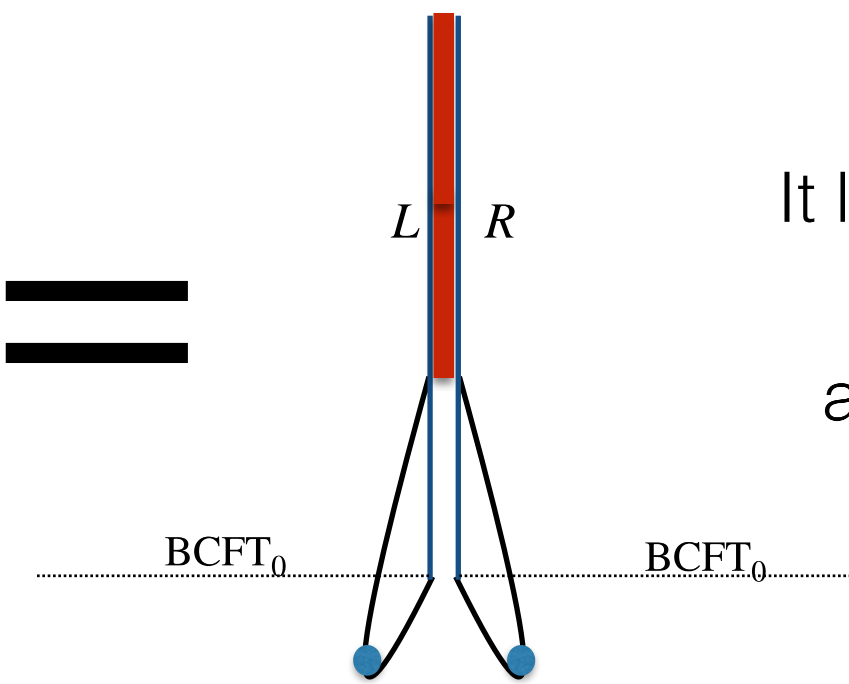


It looks like the identity string field towards the midpoint but it has a non degenerate boundary and it is "left/right" factorized towards the endpoints!

$$|\sigma)(\bar{\sigma}| = \text{new projector}$$

MULTIBRANES

$$\Psi = \Psi_{\text{tv}} - \sum_i \Sigma_i \Psi_{\text{tv}} \overline{\Sigma}_i$$

- Easily realized by orthogonal flags (generation of Chan-Paton's factors)

$$\overline{\Sigma}_i \Sigma_j = \delta_{ij}$$

- Obtained by choosing bcc operators as

$$\delta_{ij} = \langle I \circ \bar{\sigma}_i(0) \sigma_j(0) \rangle$$

- **Universal multibranes**: take (σ_i, σ_j) in the matter Verma module of the identity and diagonalize the Gram matrix.

Comments on non-perturbative amplitudes

- The ***multi-brane solution*** can be used to account for multi D-instantons contributions to closed string's scattering in two-dimensional string theory (*Sen, Baltazar-Rodriguez-Yin*) using ***Ellwood invariants*** and ***open string propagators*** (Witten vertex covers all bosonic moduli space). Open-closed amplitudes. Integration on moduli space from open string massless states (D-instanton moduli)

$$< V_1 \cdots V_n >^{(1D-Instanton)} \overset{2-brane}{\longrightarrow} < V_1 \cdots V_n >^{(2D-Instantons)} \overset{3-brane}{\longrightarrow} etc$$

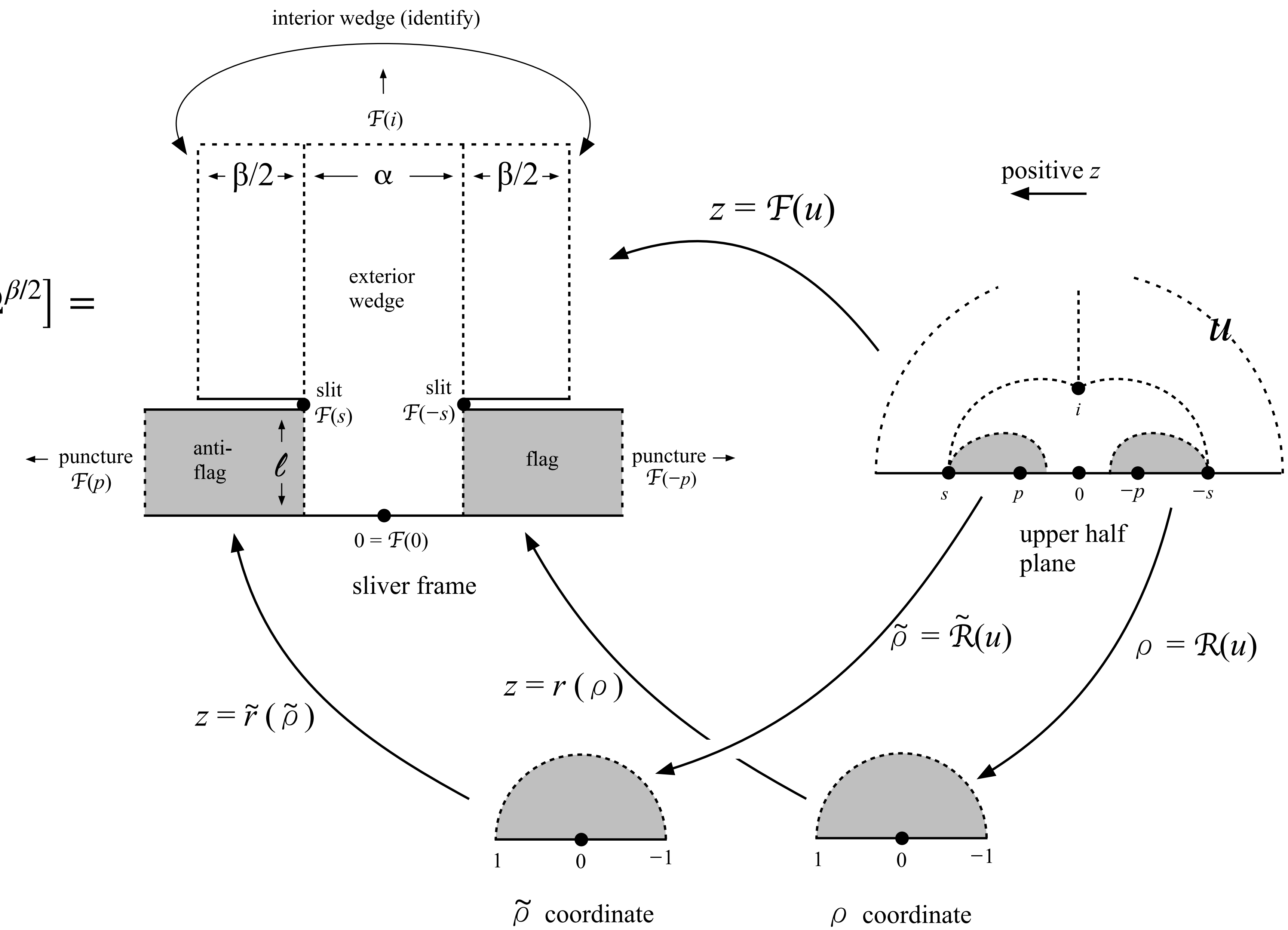
- The ***0-instanton*** sector should be the tachyon vacuum. Scattering of Ellwood invariants at the tachyon vacuum should give purely closed string amplitudes (*Gaiotto-Rastelli-Sen-Zwiebach* , *et al)*

$$< V_1 \cdots V_n >^{(0D-Instanton)} \overset{tv}{\longleftarrow} < V_1 \cdots V_n >^{(1D-Instantons)}$$

- Since bosonic two-dimensional string theory makes sense at the quantum level, this is a concrete arena to test the solution.

EXPLICIT REALIZATION OF THE FLAGS

$$\mathrm{Tr}\left[\Omega^{\beta/2}\,(\bar{\sigma}\,|\,\Omega^{\alpha}\,|\,\sigma)\,\Omega^{\beta/2}\right] =$$



$$z = \mathcal{F}(u) = \frac{2\ell}{\pi}\left(\frac{p(1+s^2)}{s^2-p^2}\tan^{-1}u + \tanh^{-1}\frac{u}{p}\right)$$

FOCK SPACE COEFFICIENTS

- The solution has a Fock space expansion

$$\Psi = \sum_i \psi_i \, c\phi^i(0)\,|\,0\rangle \; + \; (\,\ldots\,)$$

$$\psi_i = K(h_i, h_\sigma)\, C_{i\sigma\bar{\sigma}}$$

- ***K*** is a universal function which depends on the choice of tachyon vacuum and on the details of the Schwarz-Christoffel map

- $C_{i\sigma\bar{\sigma}}$ is the basic three-point function $\langle I \circ \bar{\sigma}(0)\, \hat{\phi}_i(1)\, \sigma(0) \rangle$

- We have analysed the gh=0 toy model (*gh=1 needs 7-dimensional integral on an implicitly defined region, the toy model "only" 3*)

$$\Gamma_* = 1 - \sqrt{1+K}\,|\,\sigma)\frac{1}{1+K}(\bar{\sigma}\,|\sqrt{1+K}$$

EXAMPLE: the Cosh Rolling Tachyon

- One of the advantage of this solution is the possibility to describe time dependent background (just as any other background)
- Sen's Rolling tachyon BCFT: exactly marginal deformation of Neumann bc

$$e^{\lambda \int_a^b ds\ \cosh(X^0)\,(s)} = \sigma_\lambda(b)\ \bar{\sigma}_\lambda(a)$$

- Periodic moduli space $\lambda \in [0,1)$,
- $\lambda = 0$: Neumann b.c. for X^0 (perturbative vacuum)
- $\lambda = \dfrac{1}{2}$: multiple Dirichlet b.c. at imaginary values $X^0 = 2\pi i(n + 1/2)$
- The boundary state vanishes at $\lambda = 1/2$ for real time (but non-trivial support at imaginary time) : Is it the Tachyon Vacuum?

OSFT solution for rolling tachyon

- The tachyon profile of Γ_* is given as

$$T^{\rm toy}(x^0,\lambda) = \sum_{n\in\mathbb{Z}} T_n^{\rm toy}(\lambda) e^{nx^0} = \langle \phi^{T_n^{\rm toy}(\lambda)} | \Gamma_* \rangle$$

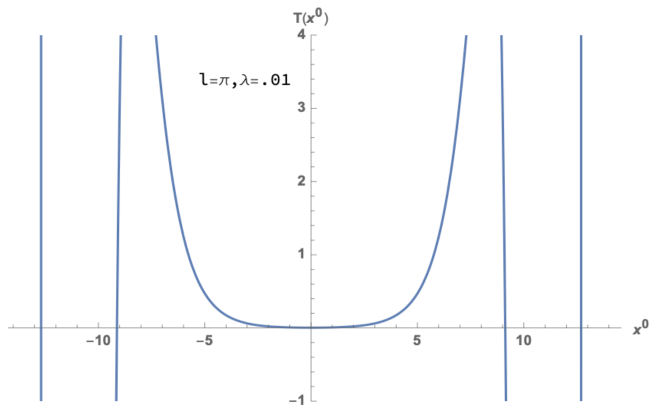
$$|\phi^{T_n^{\rm toy}(\lambda)}\rangle = -\frac{1}{2\langle 0|0\rangle_{\rm matter}} c\partial c \partial^2 c e^{-nX^0(0)}|0\rangle$$

- The needed input is the three-point function which we computed

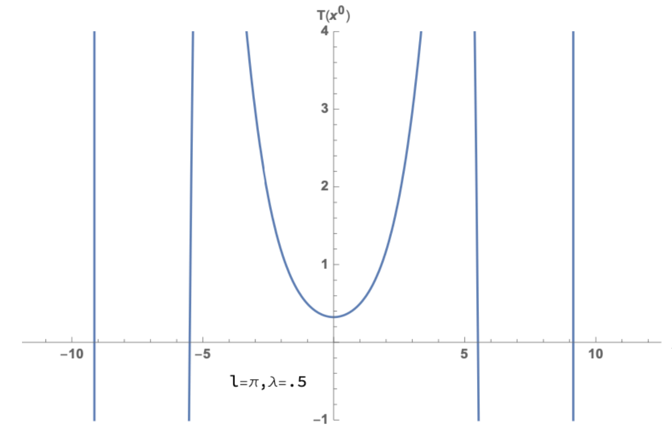
$$\Big\langle I \circ \big(\phi^{T_n^{\rm toy}(\lambda)}(0)\big)\sigma_\lambda(1)\overline{\sigma}_\lambda(0)\Big\rangle_{\rm UHP} = (-1)^n 4^{-n^2}\frac{\mathcal{P}_n(\lambda)}{\mathcal{P}_n(\frac{1}{2})}, \qquad (n\geq 0)$$

$$\mathcal{P}_n(\lambda) = \lambda^n \prod_{j=1}^{n-1}(j^2-\lambda^2)^{n-j}$$

- The obtained tachyon profile displays the well-know oscillations at late times



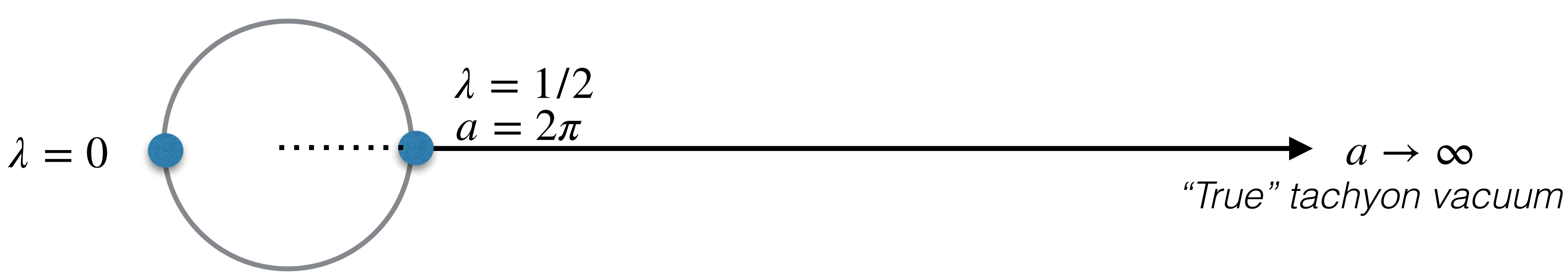
$\lambda = 0.01$



$\lambda = 1/2$

- It doesn't look that $\lambda = 1/2$ corresponds to the tachyon vacuum. Moreover the marginal current would be a physical state at the tachyon vacuum, which is not expected to happen

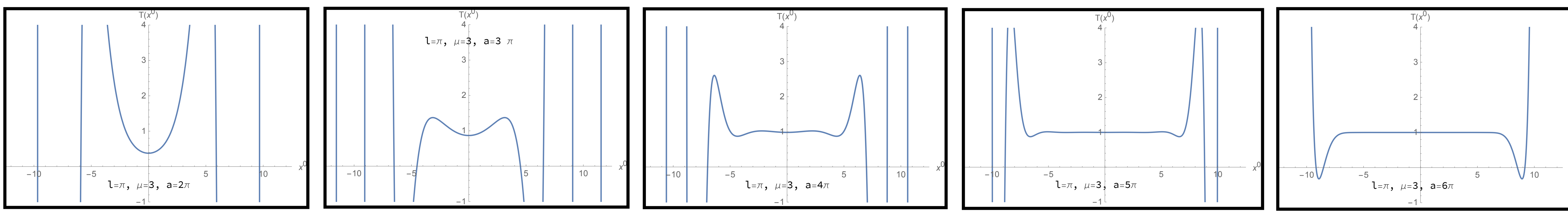
- In fact at $\lambda = 1/2$ a new branch of moduli space opens up, allowing to traslate the (imaginary) D-branes in imaginary time (*Gaiotto-Itzhaki-Rastelli*). The TV should correspond to pushing these D-branes at imaginary infinity.



- To give a new viewpoint on this problem, we can follow the new imaginary branch with our solution (multiple imaginary lumps at increasing separation)

- We constructed solutions for increasing value of imaginary separation

$$x^0 = 2\pi i \left(n + \frac{1}{2}\right) \quad \longrightarrow \quad x^0 = ia\left(n + \frac{1}{2}\right), \qquad n \in \mathbb{Z}.$$



$a = 2\pi$ $\quad$ $a = 3\pi$ $\quad$ $a = 4\pi$ $\quad$ $a = 5\pi$ $\quad$ $a = 6\pi$

- Now indeed the tachyon profile sits at the tachyon vacuum for larger and larger time

- Notice that these different backgrounds have all vanishing boundary state (in real time).

Conclusions

- For the first time we have a non perturbative realisation of background independence in bosonic open string field theory.
- Can we use this solution to account for non-perturbative contributions when the bosonic string makes sense at the quantum level?
- Can we better understand time-dependent backgrounds with vanishing boundary state in OSFT? (closed string radiation expressed in open string variables)
- It would be very desirable to have a similar solution for open-superstring field theory. Field theory understanding of RR charge? Instanton contributions etc… As of now we have it for cubic superstring field theory at picture zero (see Noris talk).
- Ideally we would like to be able to connect closed string backgrounds in a similar way.