Infinite Derivative Gravity & Resolution of Curvature Singularities

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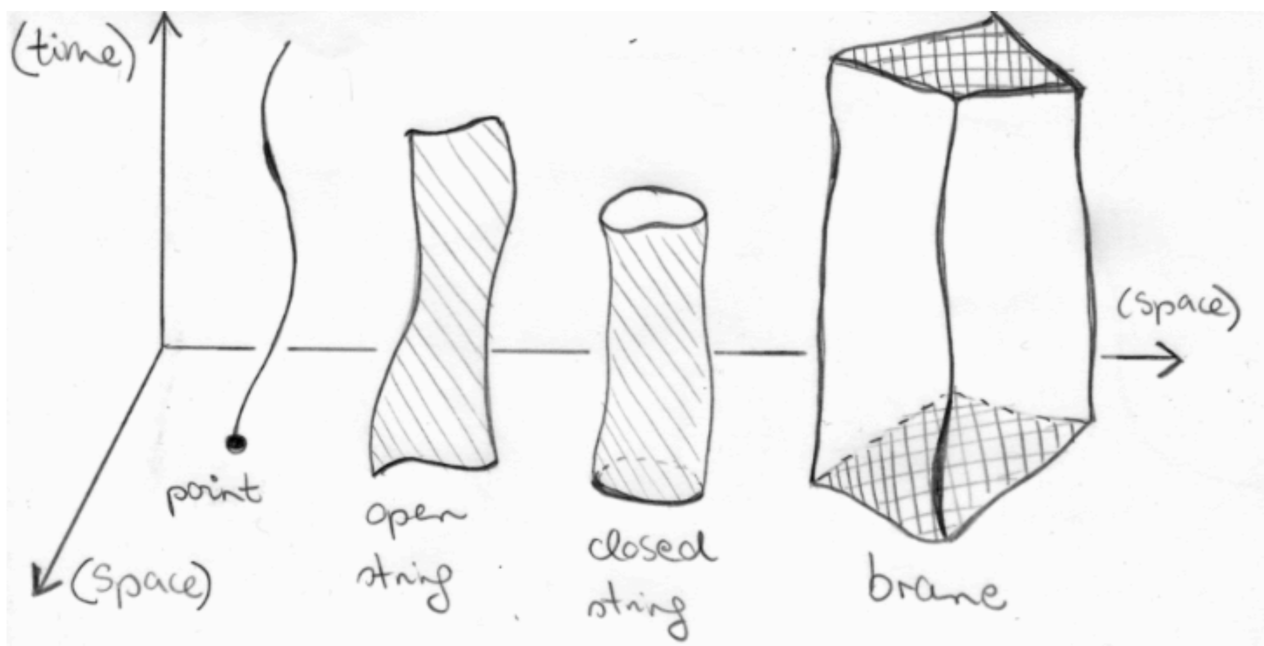
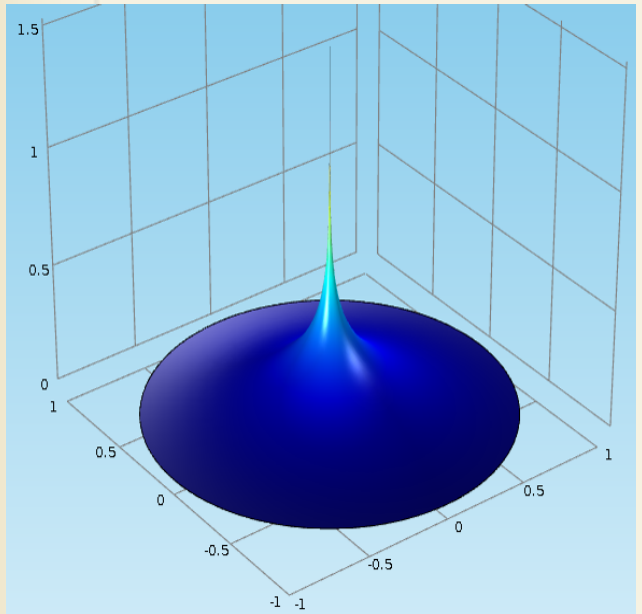


Aim: How do we mimic stringy features in Non-local gravity?

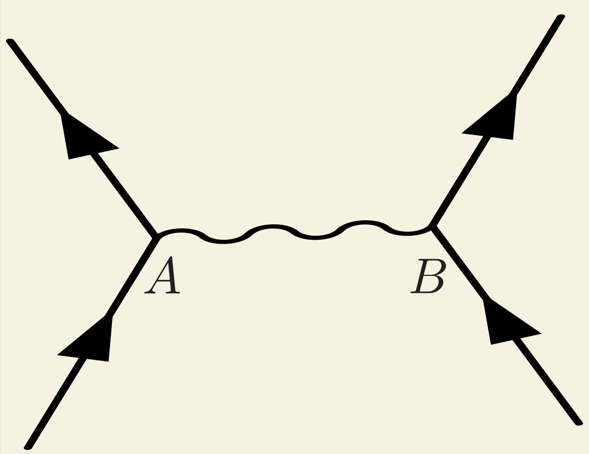
2020 Workshop on String Field Theory and Related Aspects, Sao Paolo.

Abel, Buoninfante, AM 1911.06697, Biswas, Gerwick, Koivisto, AM 1110.5249, Biswas, AM, Siegel, 0508194

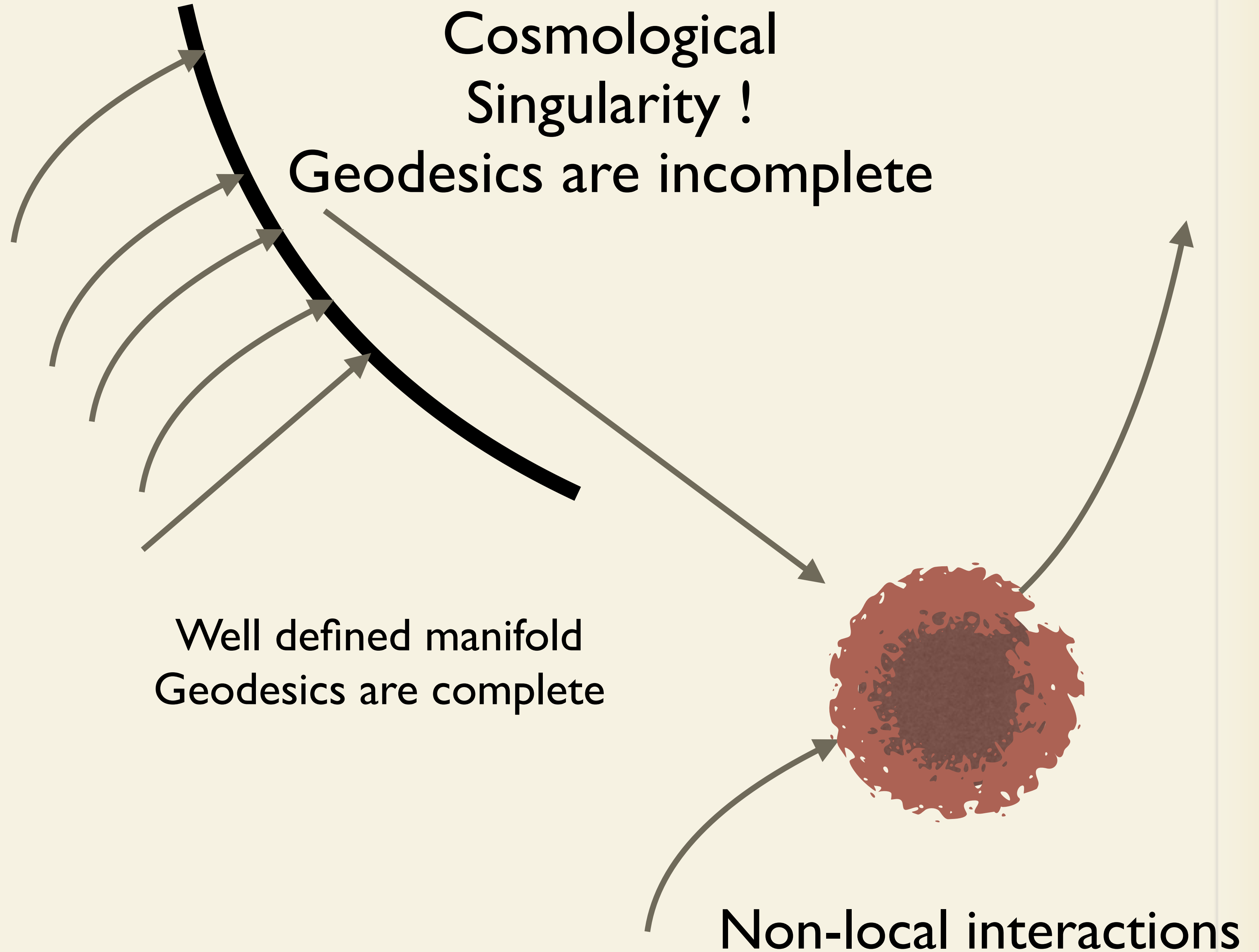
Locality in space & time : Blackhole to Cosmological Singularities



$$V \sim \frac{1}{r}$$



Graviton or Photon (mediator is massless)



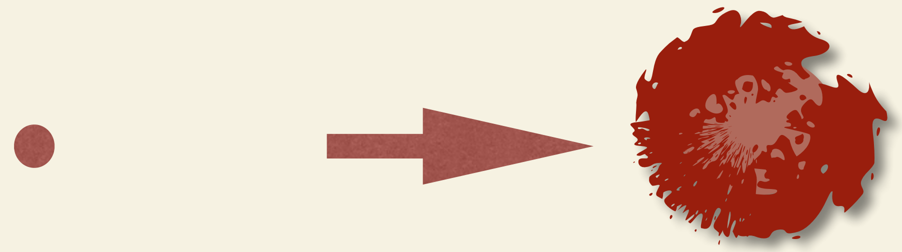
Motivation

Finite derivatives always have a point support

$$x^n \delta^n(x) = (-1)^n n! \delta(x)$$

Infinite derivatives acting on a delta source do not have any point support

$$e^{\alpha \nabla_x^2}\delta(x) = \frac{1}{\sqrt{2\pi}}\int dk e^{-\alpha k^2} e^{ik\cdot x} = \frac{1}{\sqrt{2\alpha}} e^{-x^2/4\alpha}$$

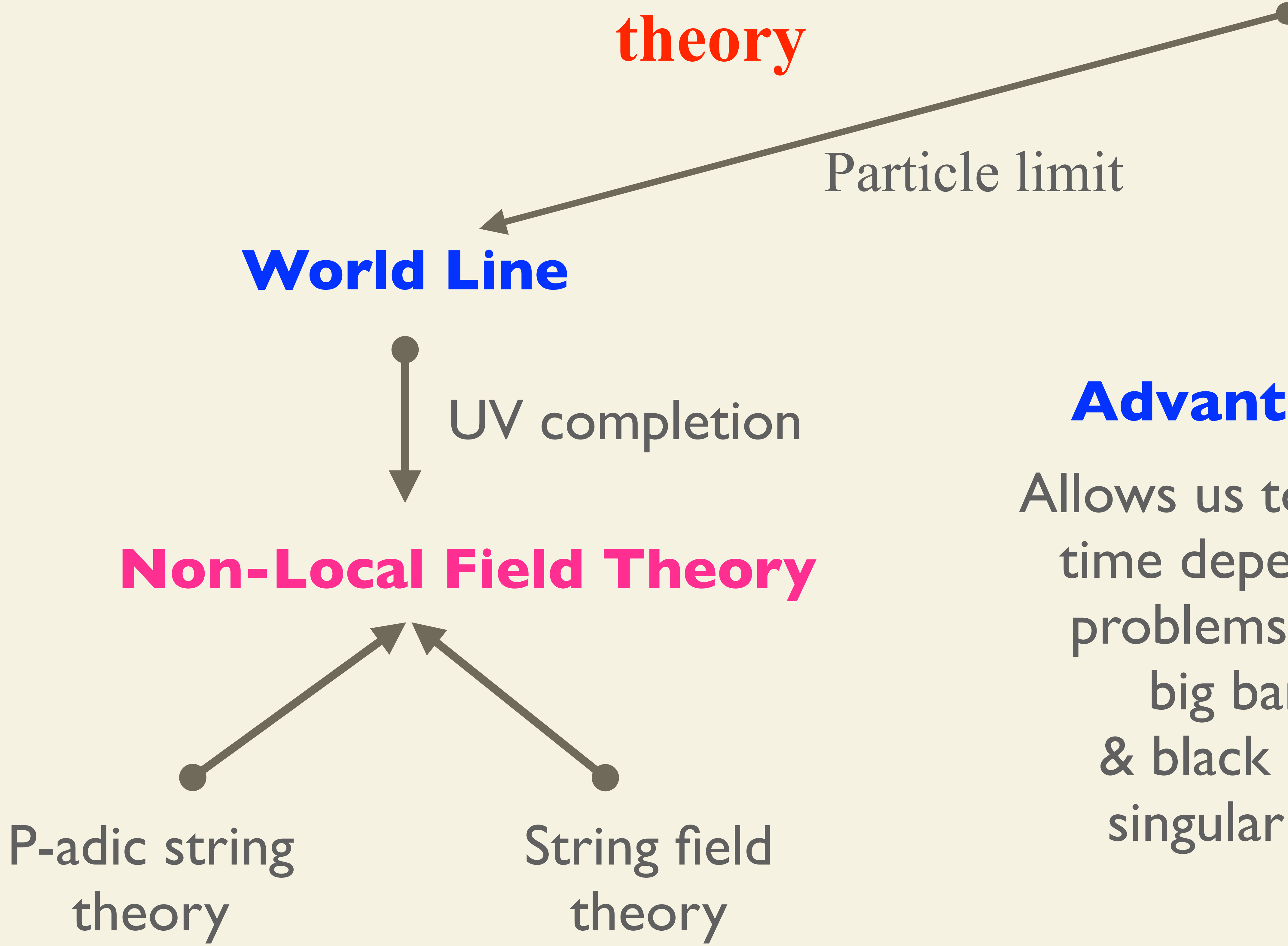


A point becomes a blob

Non-locality is one possible way for resolving singularities

hep-th/0508194, JCAP (2006), 1804.08195 [gr-qc]

Non-local theory is an appromation of a String theory

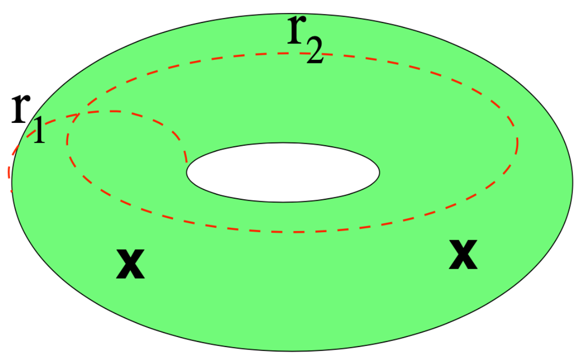


Advantages

Allows us to probe time dependent problems, such big bang & black hole singularities

Witten (1986), Freund, Olson (1987), Frampton, Okada (1988), Siegel (2001), Sen, Zwiebach (1994), Berkovits, Sen, Zwiebach (2000), Pius, Sen (2016), Sen (2002,2017, 2018)

What softens the UV behaviour?



Kaplunovsky, Dixon, Louis et al .

$$\mathcal{A} \sim \int_0^1 dxdy \int_0^1 d\tau_1 \int_{\sim 1}^{\infty} \frac{d\tau_2}{\tau_2^2} \mathcal{Z}(\tau)\, e^{-sx(1-x)\pi\alpha'\tau_2+\dots}$$

Partition function

External momenta is low

Minimal-length uncertainty

World sheet green function

External momenta is large

Gross-Mende (1988)

Particle limit

$$\mathcal{A} \sim \sum_{i=\text{physical}} \int_0^1 dx \int_{\sim\alpha'}^{\infty} \frac{dt}{t}\, e^{-(sx(1-x)+m_i^2)t+\dots}$$

$$\mathcal{A} \sim \sum_{i=\text{physical}} \int_0^1 dy \int_0^{\sim\alpha'} \frac{dt}{t}\, e^{-(sy(1-y)+m_i^2)\frac{1}{\mathcal{M}^4 t}+\dots}$$

Original action had a modular invariance

$$\tau \to -\frac{1}{\tau}$$

Scalar propagator: Resembling SFT

$$S = \frac{1}{2}\int d^4x \phi(x)\Pi^{-1}(-\Box)\phi(x) \qquad \Pi(p^2) = \int_0^\infty dt e^{-T(t)(p^2+m^2)}$$

Schwinger's proper time

$$T(t) = t + 1 \implies \Pi(p^2) = \frac{e^{-(p^2+m^2)/M_s^2}}{p^2+m^2}$$

$$T(t) = t + 1 + \frac{1}{t} \implies \Pi(p^2) = \frac{2}{Ms^2}K_1\left(\frac{2(p^2+m^2)}{M_s^2}\right) \overset{\text{UV}}{\implies} \frac{\sqrt{\pi}e^{-(p^2+m^2)}}{\sqrt{(p^2+m^2)}}$$

Infrared $\dfrac{1}{p^2+m^2}$

$$\lim_{x\to 0}\Pi(x) = \int \frac{dp^4}{(2\pi)^4}\Pi(p)e^{ip\cdot x} = \frac{M_s^2}{64\pi}$$

Aim: How do we mimic this feature in a non-local gravity?

Abel, Buoninfante, AM, 1911.06697.pdf

Higher Curvature Construction in Gravity

$$S_q = \int d^4x \sqrt{-g} R_{\mu_1\nu_1\lambda_1\sigma_1} \mathcal{O}^{\mu_1\nu_1\lambda_1\sigma_1}_{\mu_2\nu_2\lambda_2\sigma_2} R^{\mu_2\nu_2\lambda_2\sigma_2}$$

All possible terms allowed by symmetry

Unknown Infinite Functions of Covariant Derivatives

$$\begin{aligned} S_q = & \int d^4x \sqrt{-g} [R F_1(\Box) R + R F_2(\Box) \nabla_\mu \nabla_\nu R^{\mu\nu} + R_{\mu\nu} F_3(\Box) R^{\mu\nu} + R^\nu_\mu F_4(\Box) \nabla_\nu \nabla_\lambda R^{\mu\lambda} \\ + & R^{\lambda\sigma} F_5(\Box) \nabla_\mu \nabla_\sigma \nabla_\nu \nabla_\lambda R^{\mu\nu} + R F_6(\Box) \nabla_\mu \nabla_\nu \nabla_\lambda \nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\lambda} F_7(\Box) \nabla_\nu \nabla_\sigma R^{\mu\nu\lambda\sigma} \\ + & R^\rho_\lambda F_8(\Box) \nabla_\mu \nabla_\sigma \nabla_\nu \nabla_\rho R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1} F_9(\Box) \nabla_{\mu_1} \nabla_{\nu_1} \nabla_\mu \nabla_\nu \nabla_\lambda \nabla_\sigma R^{\mu\nu\lambda\sigma} \\ + & R_{\mu\nu\lambda\sigma} F_{10}(\Box) R^{\mu\nu\lambda\sigma} + R^\rho_{\mu\nu\lambda} F_{11}(\Box) \nabla_\rho \nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\rho_1\nu\sigma_1} F_{12}(\Box) \nabla^{\rho_1} \nabla^{\sigma_1} \nabla_\rho \nabla_\sigma R^{\mu\rho\nu\sigma} \\ + & R_\mu^{\nu_1\rho_1\sigma_1} F_{13}(\Box) \nabla_{\rho_1} \nabla_{\sigma_1} \nabla_{\nu_1} \nabla_\nu \nabla_\rho \nabla_\sigma R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1\rho_1\sigma_1} F_{14}(\Box) \nabla_{\rho_1} \nabla_{\sigma_1} \nabla_{\nu_1} \nabla_{\mu_1} \nabla_\mu \nabla_\nu \nabla_\rho \nabla_\sigma R^{\mu\nu\lambda\sigma}] \end{aligned}$$

1110.5249 [gr-qc]

Higher Curvature Action & Gravitational Form Factors

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + R\mathcal{F}_1\left(\frac{\Box}{M^2}\right)R + R_{\mu\nu}\mathcal{F}_2\left(\frac{\Box}{M^2}\right)R^{\mu\nu} + R_{\mu\nu\lambda\sigma}\mathcal{F}_3\left(\frac{\Box}{M^2}\right)R^{\mu\nu\lambda\sigma}\right]$$

Einstein-Hilbert Recovers IR

Ultra-violet modifications

$$\frac{\Box}{M^2} \qquad M \rightarrow \infty (\text{Theory reduces to GR})$$

Infinite Derivative Gravity (IDG)

Biswas, AM, Siegel, hep-th/0508194
Biswas, Gerwick, Koivisto, AM, gr-qc/1110.5249
Biswas, Koshelev, AM, (extension for de Sitter & Anti-deSitter), arXiv:1602.08475 , arXiv:1606.01250

Non-linear, Non-local Equations of Motion

$$
\begin{aligned}
P^{\alpha\beta} = {} & G^{\alpha\beta} + 4G^{\alpha\beta}\mathcal{F}_1(\Box)R + g^{\alpha\beta}R\mathcal{F}_1(\Box)R - 4\left(\nabla^\alpha\nabla^\beta - g^{\alpha\beta}\Box\right)\mathcal{F}_1(\Box)R \\
& - 2\Omega_1^{\alpha\beta} + g^{\alpha\beta}(\Omega_{1\sigma}^{\sigma} + \bar{\Omega}_1) + 4R_\mu^\alpha\mathcal{F}_2(\Box)R^{\mu\beta} \\
& - g^{\alpha\beta}R_\nu^\mu\mathcal{F}_2(\Box)R_\mu^\nu - 4\nabla_\mu\nabla^\beta(\mathcal{F}_2(\Box)R^{\mu\alpha}) + 2\Box(\mathcal{F}_2(\Box)R^{\alpha\beta}) \\
& + 2g^{\alpha\beta}\nabla_\mu\nabla_\nu(\mathcal{F}_2(\Box)R^{\mu\nu}) - 2\Omega_2^{\alpha\beta} + g^{\alpha\beta}(\Omega_{2\sigma}^{\sigma} + \bar{\Omega}_2) - 4\Delta_2^{\alpha\beta} \\
& - g^{\alpha\beta}C^{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box)C_{\mu\nu\lambda\sigma} + 4C^{\alpha}_{\ \mu\nu\sigma}\mathcal{F}_3(\Box)C^{\beta\mu\nu\sigma} \\
& - 4(R_{\mu\nu} + 2\nabla_\mu\nabla_\nu)(\mathcal{F}_3(\Box)C^{\beta\mu\nu\alpha}) - 2\Omega_3^{\alpha\beta} + g^{\alpha\beta}(\Omega_{3\gamma}^{\gamma} + \bar{\Omega}_3) - 8\Delta_3^{\alpha\beta} \\
& = T^{\alpha\beta},
\end{aligned}
$$

$$\Omega_1^{\alpha\beta} = \sum_{n=1}^{\infty} f_{1_n} \sum_{l=0}^{n-1} \nabla^\alpha R^{(l)} \nabla^\beta R^{(n-l-1)}, \quad \bar{\Omega}_1 = \sum_{n=1}^{\infty} f_{1_n} \sum_{l=0}^{n-1} R^{(l)} R^{(n-l)},$$

$$\Omega_2^{\alpha\beta} = \sum_{n=1}^{\infty} f_{2_n} \sum_{l=0}^{n-1} R_\nu^{\mu;\alpha(l)} R_\mu^{\nu;\beta(n-l-1)}, \quad \bar{\Omega}_2 = \sum_{n=1}^{\infty} f_{2_n} \sum_{l=0}^{n-1} R_\nu^{\mu(l)} R_\mu^{\nu(n-l)},$$

$$\Delta_2^{\alpha\beta} = \frac{1}{2}\sum_{n=1}^{\infty} f_{2_n} \sum_{l=0}^{n-1} [R_\sigma^{\nu(l)} R^{(\beta|\sigma|;\alpha)(n-l-1)} - R_\sigma^{\nu;(\alpha(l)} R^{\beta)\sigma(n-l-1)}]_{;\nu},$$

$$\Omega_3^{\alpha\beta} = \sum_{n=1}^{\infty} f_{3_n} \sum_{l=0}^{n-1} C^{\mu;\alpha(l)}_{\ \nu\lambda\sigma} C_\mu^{\ \nu\lambda\sigma;\beta(n-l-1)}, \ \bar{\Omega}_3 = \sum_{n=1}^{\infty} f_{3_n} \sum_{l=0}^{n-1} C^{\mu(l)}_{\ \nu\lambda\sigma} C_\mu^{\ \nu\lambda\sigma(n-l)},$$

$$\Delta_3^{\alpha\beta} = \frac{1}{2}\sum_{n=1}^{\infty} f_{3_n} \sum_{l=0}^{n-1} [C^{\lambda\nu(l)}_{\ \ \sigma\mu} C_\lambda^{\ (\beta|\sigma\mu|;\alpha)(n-l-1)} - C^{\lambda\nu\ \ ;(\alpha(l)}_{\ \ \sigma\mu} C_\lambda^{\ \beta)\sigma\mu(n-l-1)}]_{;\nu}.$$

$$
\begin{aligned}
P = {} & -R + 12\Box\mathcal{F}_1(\Box)R + 2\Box(\mathcal{F}_2(\Box)R) + 4\nabla_\mu\nabla_\nu(\mathcal{F}_2(\Box)R^{\mu\nu}) \\
& + 2(\Omega_{1\sigma}^{\sigma} + 2\bar{\Omega}_1) + 2(\Omega_{2\sigma}^{\sigma} + 2\bar{\Omega}_2) + 2(\Omega_{3\sigma}^{\sigma} + 2\bar{\Omega}_3) - 4\Delta_{2\sigma}^{\sigma} - 8\Delta_{3\sigma}^{\sigma} \\
& = T \equiv g_{\alpha\beta}T^{\alpha\beta}.
\end{aligned}
$$

Biswas, Conroy, Koshelev, AM. [arXiv:1308.2319 [hep-th]]

First solution of non-linear, non-local equations of motion: non-singular universe

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + R\mathcal{F}_1(\Box)R - \Lambda\right]$$

$\Box R = r_1 R + r_2$ $\qquad$ $\Box^n R = r_1^n\left(R + \frac{r_2}{r_1}\right)$

deSitter **No-Ghost criteria**	Anti deSitter **No-Ghost criteria**

deSitter

$$\mathcal{F}(\Box) = \frac{1}{M_s^6}(\Box - m^2)(\Box - r_1)^2 e^{\gamma(\Box)}$$

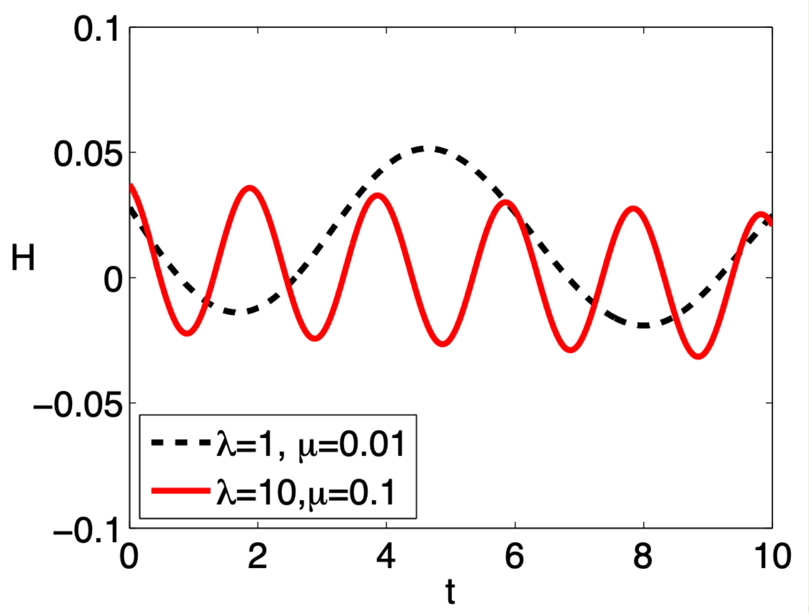
$$a(t) = a_0\cosh(\sqrt{r_1/2}\,t),\ a_0 e^{\lambda t^2}$$

Biswas, AM, Siegel, 0508194

Sravan-Kumar, Maheshwari, AM, Peng, 2005.01762

Anti deSitter

$$\mathcal{F}(\Box) = \frac{1}{M_s^4}(\Box - r_1)^2 e^{\gamma(\Box)}$$



Biswas, Koivisto, AM, 1005.0590

Perturbative unitarity around minkowski

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + R\mathcal{F}_1\left(\frac{\Box}{M^2}\right)R + R_{\mu\nu}\mathcal{F}_2\left(\frac{\Box}{M^2}\right)R^{\mu\nu} + R_{\mu\nu\lambda\sigma}\mathcal{F}_3\left(\frac{\Box}{M^2}\right)R^{\mu\nu\lambda\sigma}\right]$$

$$2\mathcal{F}_1 + \mathcal{F}_2 + 2\mathcal{F}_3 = 0 \qquad a(\Box) = 1 - \frac{1}{2}\mathcal{F}_2(\Box)\frac{\Box}{M_s^2} - 2\mathcal{F}_3(\Box)\frac{\Box}{M_s^2}$$

$$\Pi(k^2) = \frac{1}{a(k^2)}\left[\frac{P^{(2)}}{k^2} - \frac{P^0}{2k^2}\right]$$

Demand no extra poles other than massless graviton's, means:

$$a(k^2) = e^{\gamma(k^2)}$$

Entire Function

Simplest choice: $a(k^2) = e^{k^2/M_s^2}$

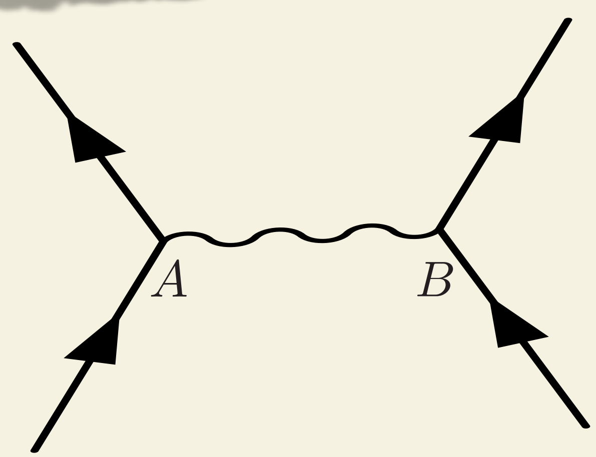
Pius, Sen (2016)

Infinite derivative Gravity action around Minkowski

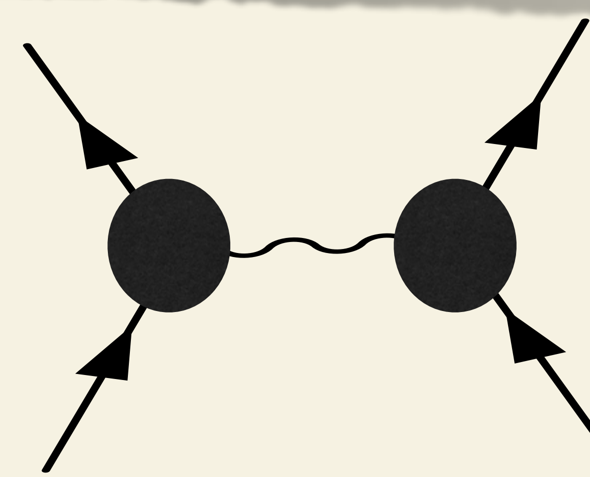
With the help of the earlier constraints:

$$S = \int d^4x \sqrt{-g} \left[M_p^2 \frac{R}{2} + R \left[\frac{e^{-\Box/M_s^2} - 1}{\Box} \right] R - 2R_{\mu\nu} \left[\frac{e^{-\Box/M_s^2} - 1}{\Box} \right] R^{\mu\nu} \right]$$

$k < M_s$



$k \geq M_s$



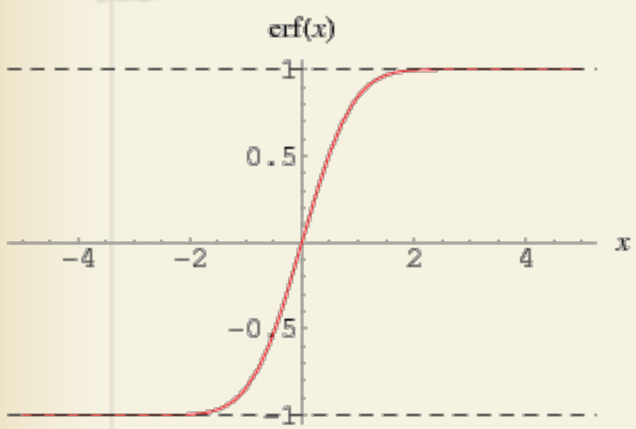
$$\Pi(k^2) = \frac{1}{a(k^2)} \left[\frac{P^{(2)}}{k^2} - \frac{P^0}{2k^2} \right] \qquad a(k^2) = e^{k^2/M_s^2}$$

Massless Graviton, massless spin-2 and spin-0 components propagate

Biswas, AM, Siegel (2006) JCAP, Biswas, Gerwick, Koivisto, AM (2012) Phy. Rev. Lett.

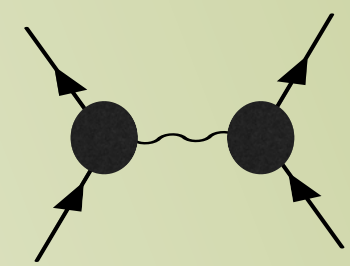
Non-Local Gravitational Potential

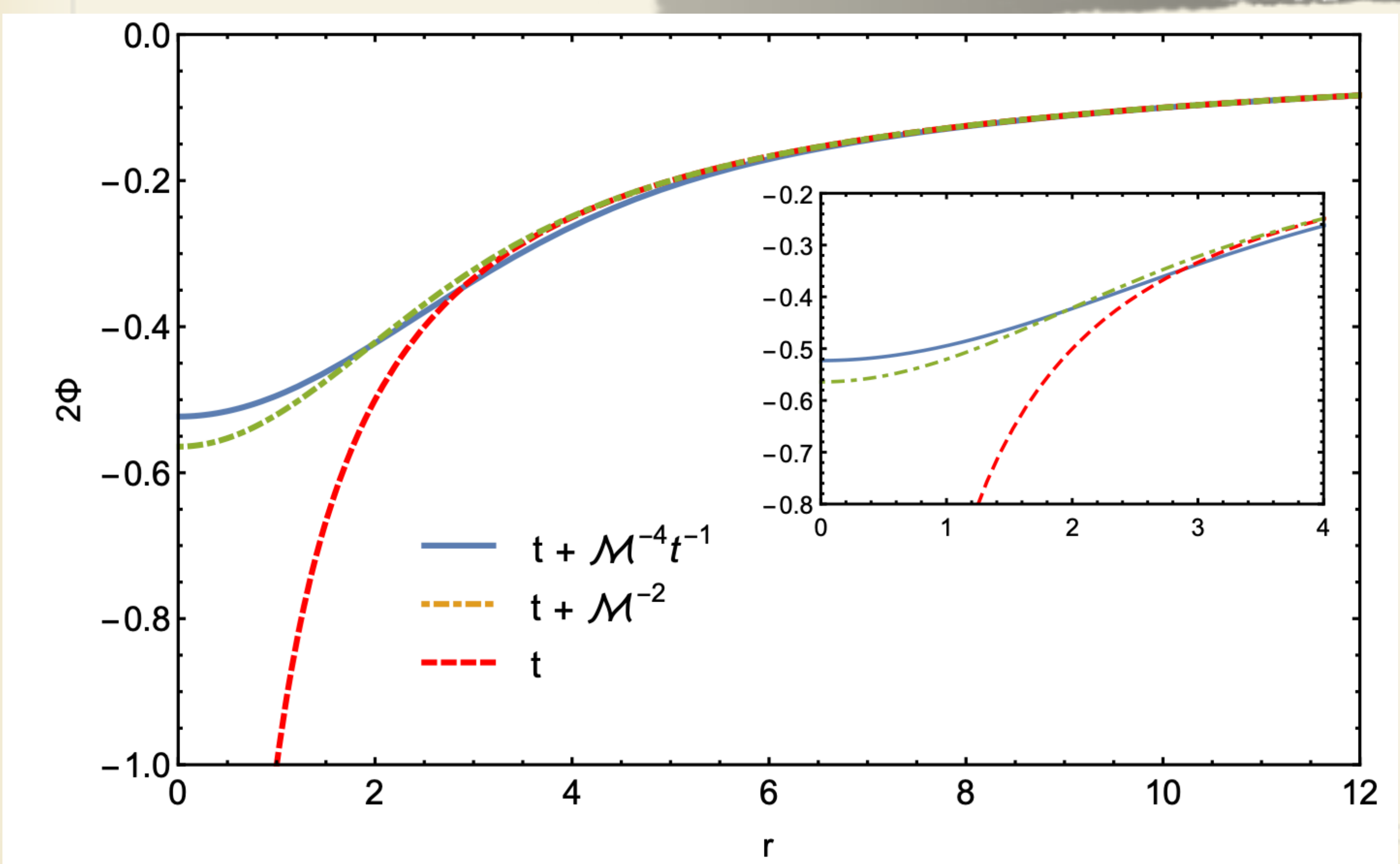
$$S = \int d^4x \sqrt{-g}\left[\frac{R}{2} + R\left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box}\right] R - 2R_{\mu\nu}\left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box}\right] R^{\mu\nu}\right]$$



$$ds^2 = -(1 - 2\Phi)dt^2 + (1 + 2\Psi)dr^2$$

$$\Phi = \Psi = \frac{Gm}{r}\mathbf{erf}\left(\frac{rM}{2}\right)$$

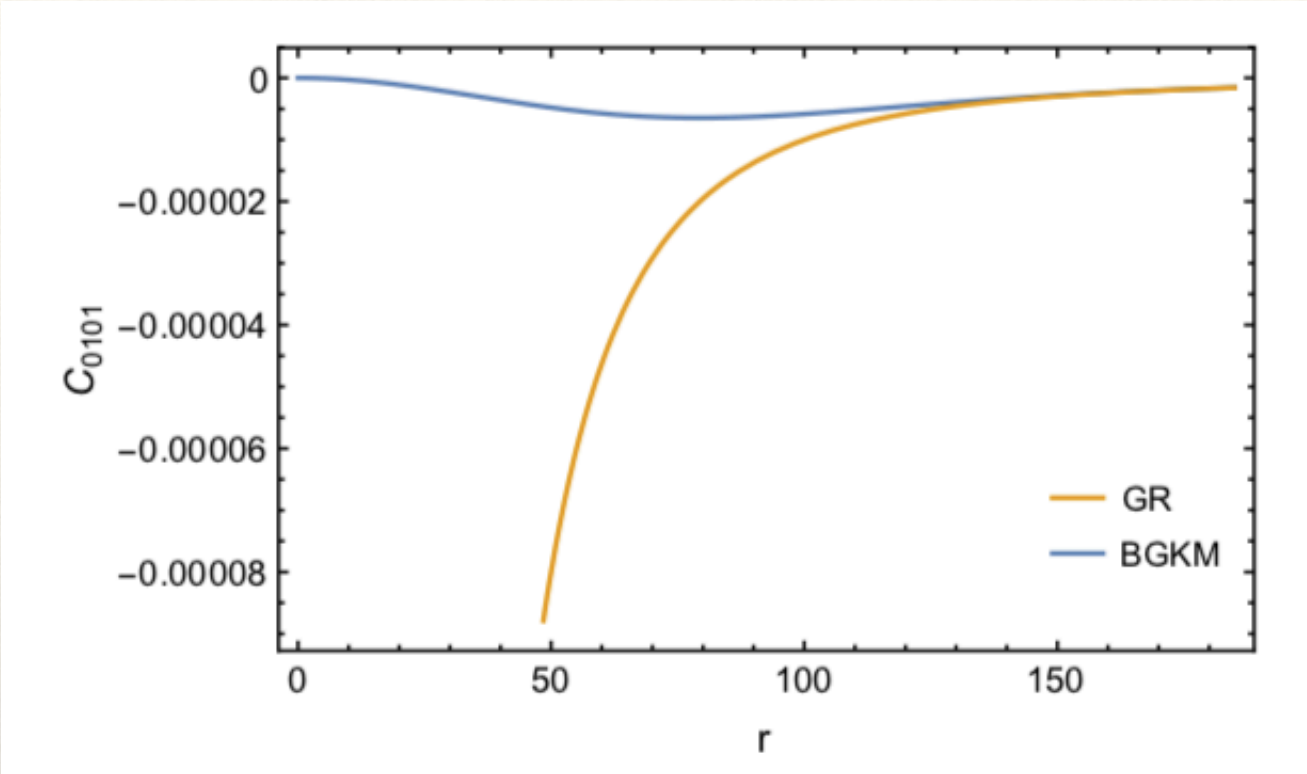


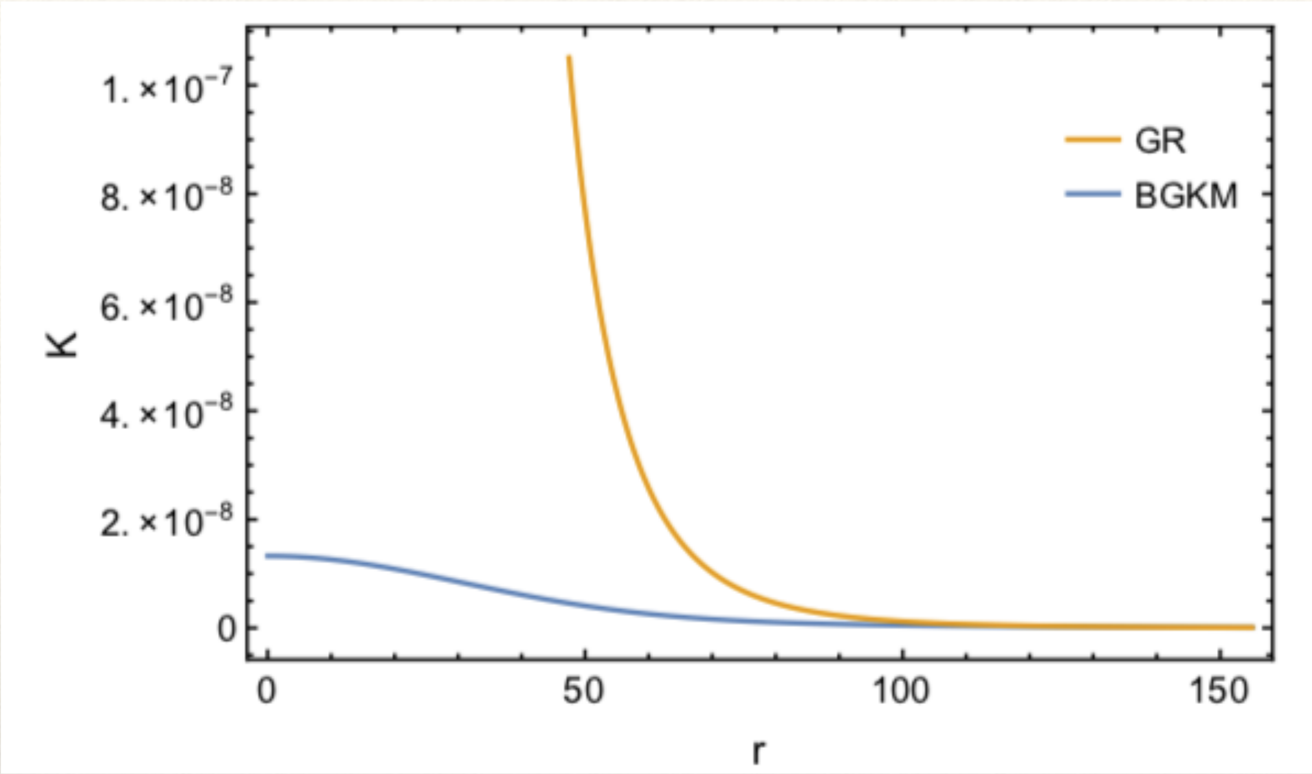


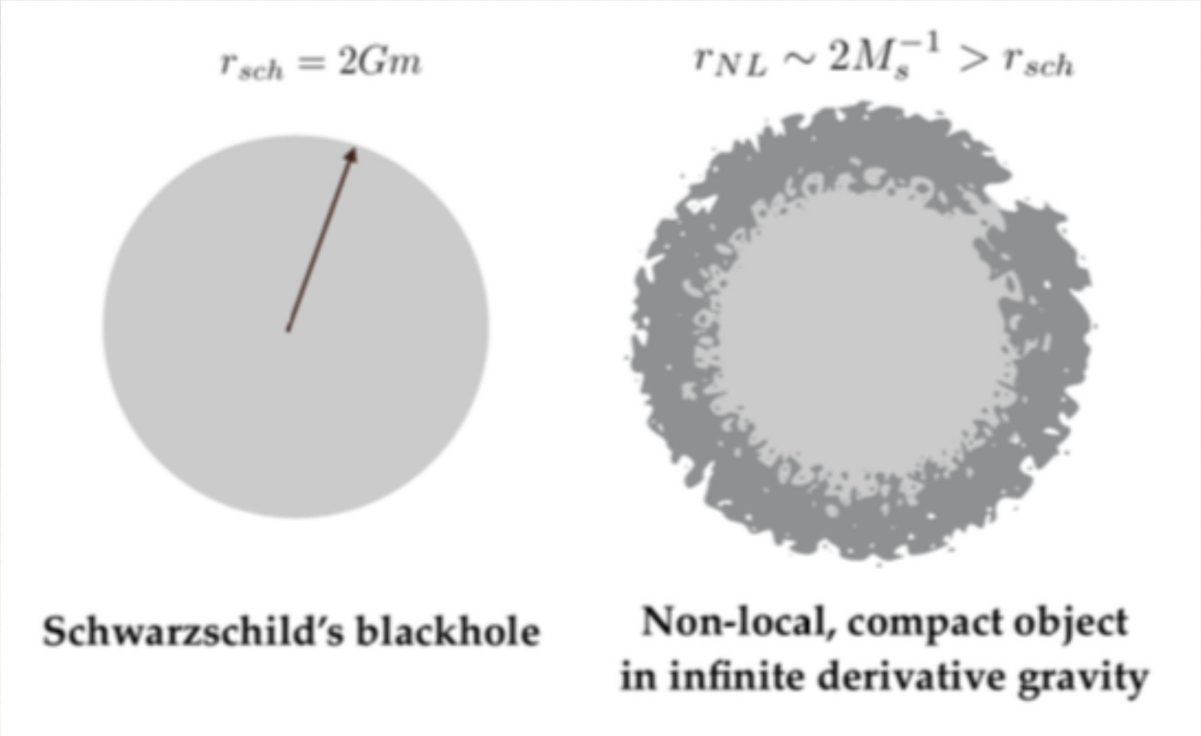
Interaction becomes Non-Local

Abel, Buoninfante, AM (2019), Biswas, Gerwick, Koivisto, AM,(gr-qc/1110.5249)

Conformally flat solution







Such non-local objects could be BHs provided linear solution is promoted all the way to non-linear level.

Buoninfante, Koshelev, Lambiase, AM
[arXiv:1802.00399 [gr-qc]]

Spherically symmetric non-linear, non-local metric

$$
\begin{aligned}
P^{\alpha\beta} \approx & \frac{\alpha_c}{8\pi G}\Big(4G^{\alpha\beta}\mathcal{F}_1(\Box_s)\mathcal{R} + g^{\alpha\beta}\mathcal{R}\mathcal{F}_1(\Box_s)\mathcal{R} - 4\left(\nabla^\alpha\nabla^\beta - g^{\alpha\beta}\Box\right)\mathcal{F}_1(\Box_s)\mathcal{R} \\
& - 2\Omega_1^{\alpha\beta} + g^{\alpha\beta}(\Omega_{1\sigma}^{\sigma} + \bar{\Omega}_1) + 4\mathcal{R}_\mu^\alpha\mathcal{F}_2(\Box_s)\mathcal{R}^{\mu\beta} \\
& - g^{\alpha\beta}\mathcal{R}_\nu^\mu\mathcal{F}_2(\Box_s)\mathcal{R}_\mu^\nu - 4\nabla_\mu\nabla^\beta(\mathcal{F}_2(\Box_s)\mathcal{R}^{\mu\alpha}) + 2\Box(\mathcal{F}_2(\Box_s)\mathcal{R}^{\alpha\beta}) \\
& + 2g^{\alpha\beta}\nabla_\mu\nabla_\nu(\mathcal{F}_2(\Box_s)\mathcal{R}^{\mu\nu}) - 2\Omega_2^{\alpha\beta} + g^{\alpha\beta}(\Omega_{2\sigma}^{\sigma} + \bar{\Omega}_2) - 4\Delta_2^{\alpha\beta}\Big) \\
= & T^{\alpha\beta} = 0\,,
\end{aligned}
$$

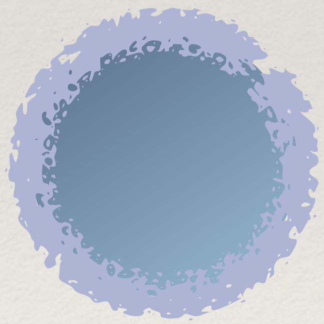
$$
\Omega_1^{\alpha\beta} = \sum_{n=1}^{\infty} f_{1_n} \sum_{l=0}^{n-1} \nabla^\alpha\mathcal{R}^{(l)}\nabla^\beta\mathcal{R}^{(n-l-1)}, \quad \bar{\Omega}_1 = \sum_{n=1}^{\infty} f_{1_n} \sum_{l=0}^{n-1} \mathcal{R}^{(l)}R^{(n-l)},
$$

$$
\Omega_2^{\alpha\beta} = \sum_{n=1}^{\infty} f_{2_n} \sum_{l=0}^{n-1} \mathcal{R}_\nu^{\mu;\alpha(l)}\mathcal{R}_\mu^{\nu;\beta(n-l-1)}, \quad \bar{\Omega}_2 = \sum_{n=1}^{\infty} f_{2_n} \sum_{l=0}^{n-1} \mathcal{R}_\nu^{\mu(l)}\mathcal{R}_\mu^{\nu(n-l)},
$$

$$
\Delta_2^{\alpha\beta} = \sum_{n=1}^{\infty} f_{2_n} \sum_{l=0}^{n-1} [\mathcal{R}_\sigma^{\nu(l)}\mathcal{R}^{(\beta\sigma;\alpha)(n-l-1)} - \mathcal{R}_\sigma^{\nu;\alpha(l)}\mathcal{R}^{\beta\sigma(n-l-1)}]_{;\nu}\,.
$$

[arXiv:1308.2319 [hep-th]]

$$
S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + R\mathcal{F}_1\left(\frac{\Box}{M^2}\right)R + R_{\mu\nu}\mathcal{F}_2\left(\frac{\Box}{M^2}\right)R^{\mu\nu} + R_{\mu\nu\lambda\sigma}\mathcal{F}_3\left(\frac{\Box}{M^2}\right)R^{\mu\nu\lambda\sigma}\right]
$$



$$
ds^2 = \left(\frac{2}{M_s r}\right)^2\left[-dt^2 + dr^2 + r^2 d\Omega^2\right]
$$

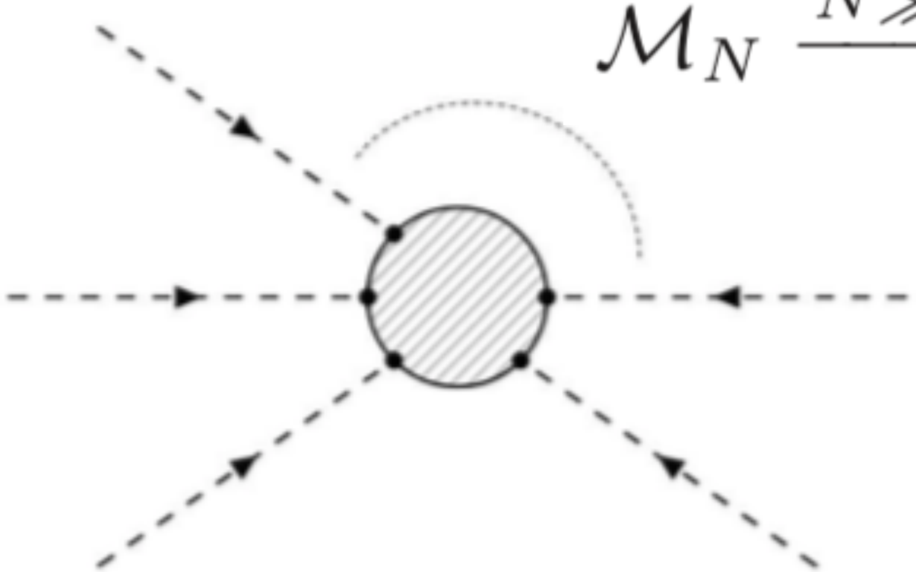
Buoninfante Koshelev, Lambiase, Marto, AM [arXiv:1803.00309 [gr-qc]]

Collective behavior : N scalar-gravitons interacting with non-local interaction

$$S = \int d^4x \, \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box} \right] R - 2R_{\mu\nu} \left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box} \right] R^{\mu\nu} \right]$$

$$S_{\texttt{free}} = \frac{1}{2} \int d^4x \, (\phi \Box a(\Box) \phi) \qquad a(\Box) = e^{-\Box/M^2}$$

$$S_{\texttt{int}} = \frac{1}{M_p} \int d^4x \left(\frac{1}{4} \phi \partial_\mu \phi \partial^\mu \phi + \frac{1}{4} \phi \Box \phi a(\Box) \phi - \frac{1}{4} \phi \partial_\mu \phi a(\Box) \partial^\mu \phi \right)$$



$$\mathcal{M}_N \xrightarrow{N \gg 1} \lambda^{3(N-2)} e^{-Np^2/M_s^2} = \lambda^{3(N-2)} e^{-p^2/M_{\text{eff}}^2}$$

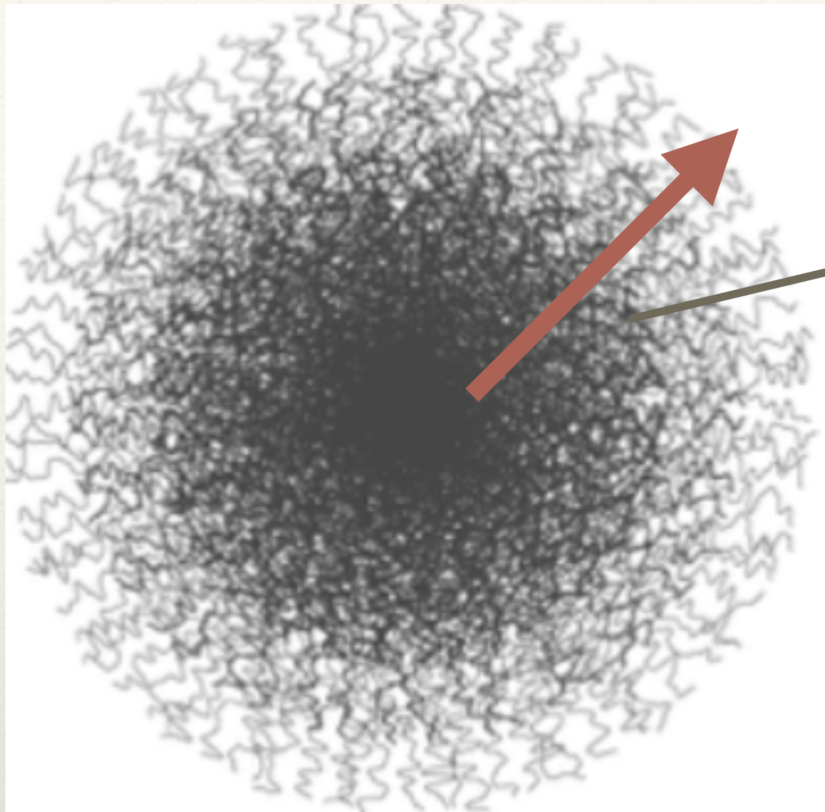
$$M_{\text{eff}} \sim \frac{M_s}{\sqrt{N}}$$

Persists with zero external momenta

N- gravitons behave like a condensate

Buoninfante, Ghosh, Lambiase, AM
arXiv:1812.01441 [hep-th]

Non-local star:
Coherent State of N Gravitons & a Black hole Mimicker



$$\lambda \sim M_{\rm eff}^{-1} = \sqrt{N} M_s^{-1}$$

$$E_g \sim M_{\rm eff} = M_s/\sqrt{N}$$

Individual graviton feels
Collective behavior

Mass of N gravitons interacting non – locally

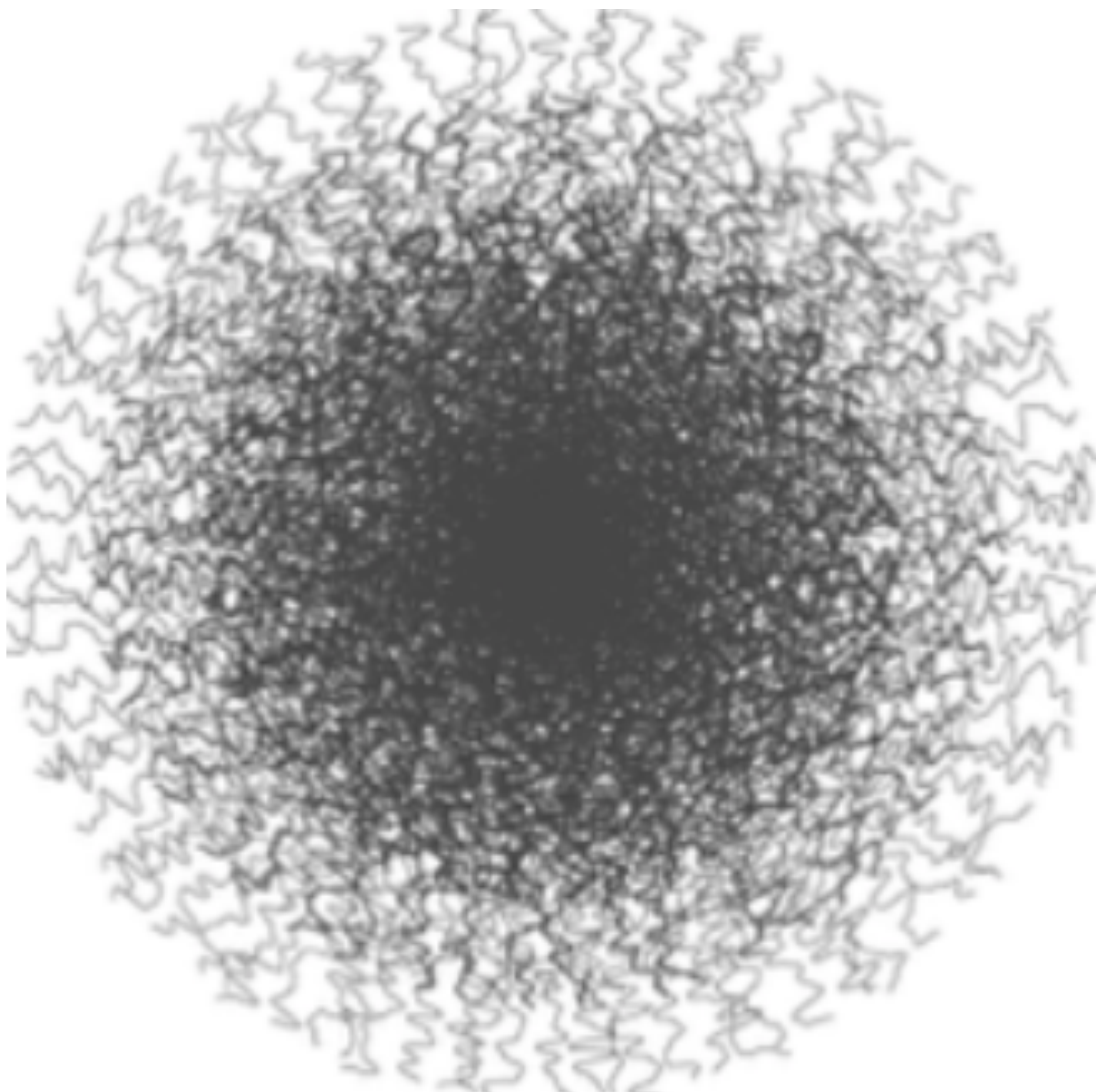
$$E_{\rm tot} = m_\circ = N M_{\rm eff} = N \frac{M_s}{\sqrt{N}} = \sqrt{N} M_s$$

For a solar mass object : $N \sim 10^{82}$

Forms a gravitationally bound system: a Non-local star!

Buoninfante, AM 1903.01542

Number of Bekenstein states



$$S \sim \hbar \left(\frac{4G^2 m_\circ^2}{L_p^2} + \frac{L_{\text{eff}}^2}{L_p^2} \right) \equiv \hbar\, s,$$

$$s \sim \frac{L_{\text{eff}}^2}{L_p^2} = N \frac{L_s^2}{L_p^2} = N \frac{M_p^2}{M_s^2}$$

$$\mathcal{N} \sim e^{N (L_s/L_p)^2} = e^{N (M_p/M_s)^2}$$

Bekenstein State

For a solar mass object : $\mathcal{N} = e^{10^{82} (M_p/M_s)^2}$

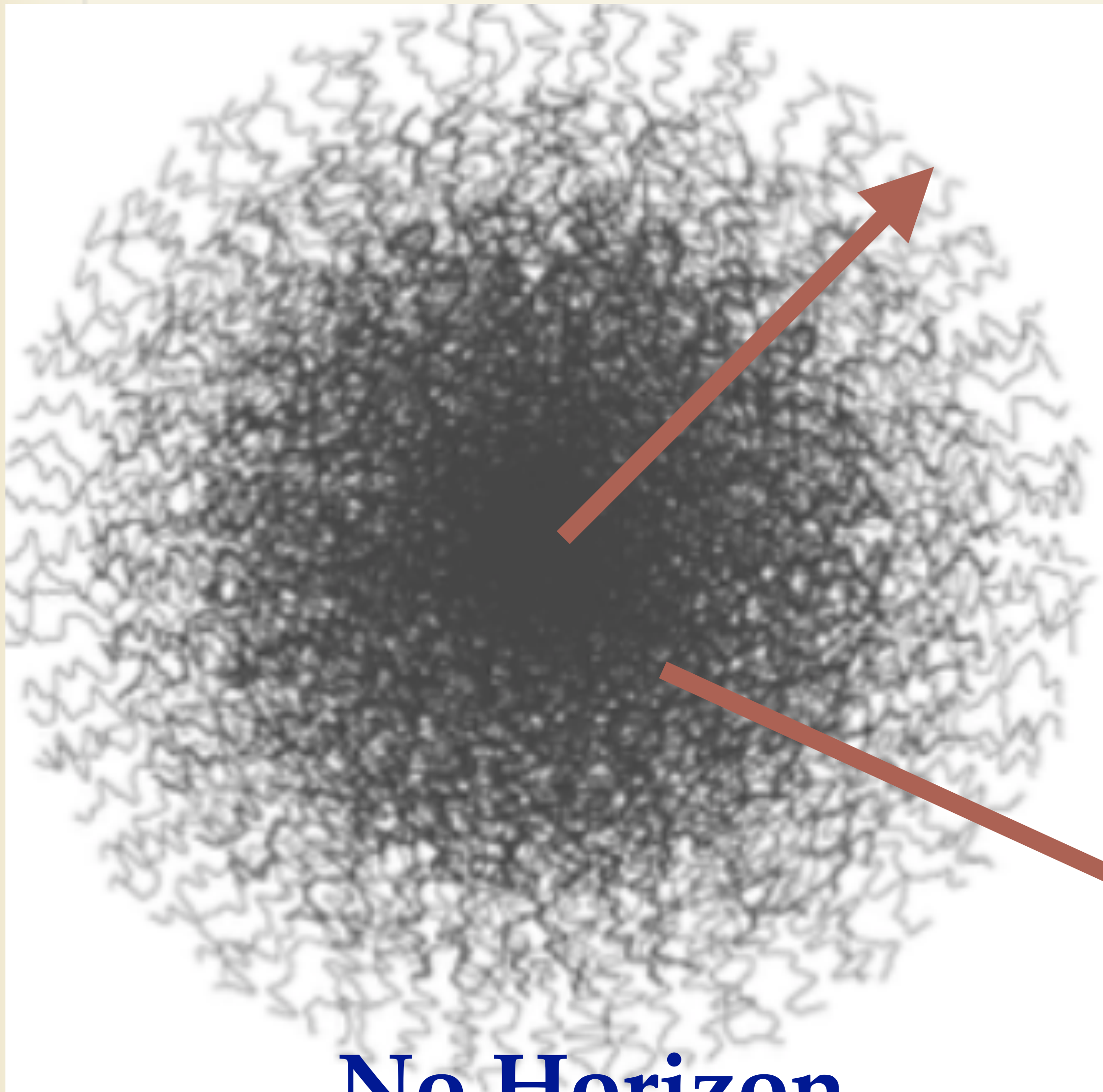
What happens when I throw a chalk, neutrino, …., anything…. inside?

$$\tau = \left(\frac{L_s}{L_p} \right)^9 \tau_{bh} = \left(\frac{M_p}{M_s} \right)^9 \tau_{bh}$$

Longer life time

The Non-local star absorbs everything, even better than a Blackhole!!!

Metric of a Non-Local Star with No-Horizon



No Horizon

$$r \sim \frac{2}{M_{\rm eff}} + \mathcal{O}(1/M_s)$$

$$r \sim 2.2Gm_{\circ} + \mathcal{O}(1/M_s)$$

$$\phi(r) \sim \frac{GmM_s}{\sqrt{\pi}} < 1 \qquad \mathcal{R} \sim \frac{GmM_s^3}{\sqrt{\pi}}$$

$$\mathcal{R}_{00} = \mathcal{R}_{11} \sim \frac{GmM_s^3}{2\sqrt{\pi}}, \qquad \mathcal{R}_{22} = \mathcal{R}_{33} \sim 0$$

$$\begin{aligned} ds^2 &= (1+2A)\left(-d\tau^2 + dr^2 + r^2 d\Omega^2\right) \\ &= F\eta, \end{aligned}$$

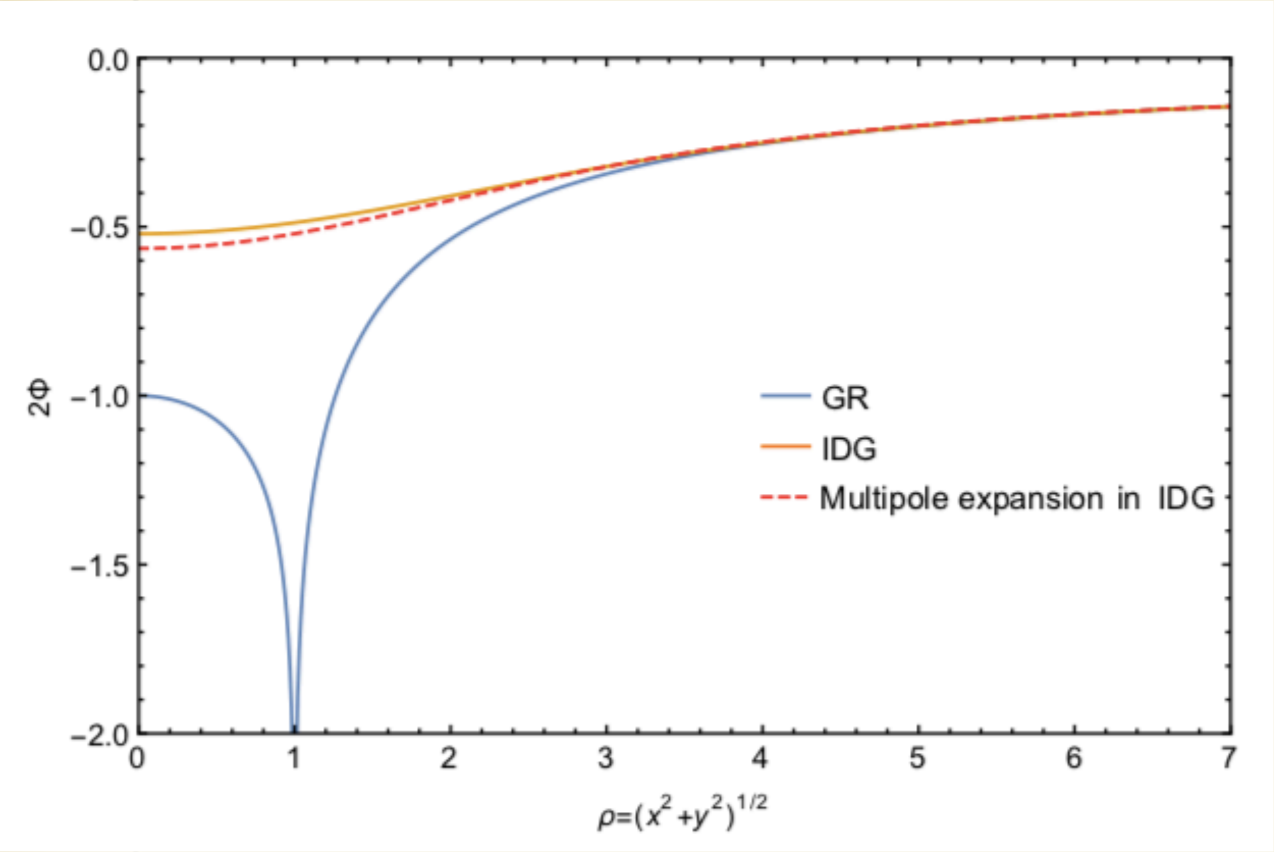
$$A \equiv \frac{GmM_s}{\sqrt{\pi}} < 1.$$

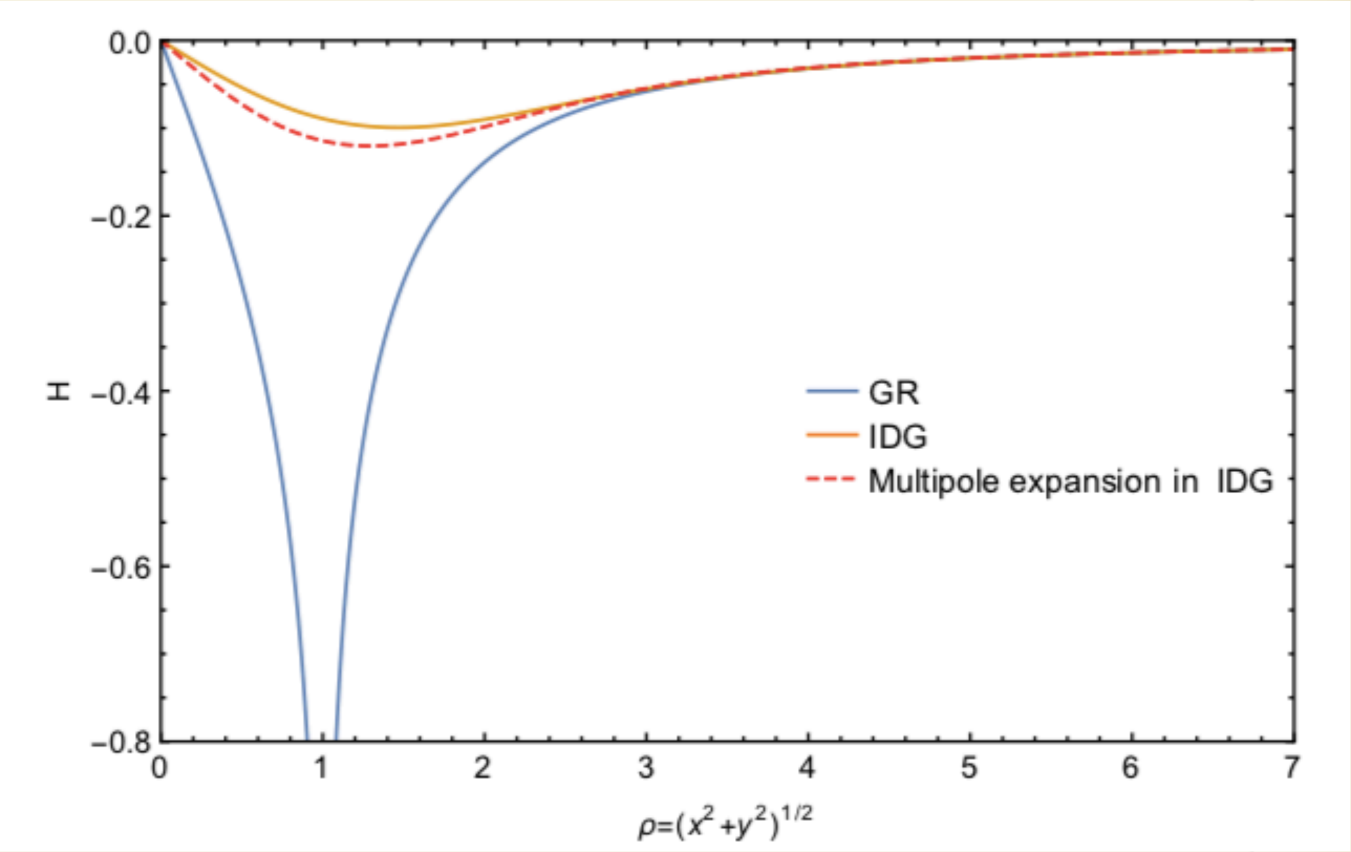
Inside Non-local Star

$$ds^2 = -(1 - [2.2Gm_{\circ} + \mathcal{O}(1/M_s)]/r)dt^2 + \frac{dr^2}{1 - [2.2Gm_{\circ} + \mathcal{O}(1/M_s)]/r} + r^2 d\Omega^2$$

Outside Non-local Star

Rotating solution with no ring singularity





$$ds^2 = -(1+2\Phi)dt^2 + 2\vec{h}\cdot d\vec{x}dt + (1-2\Psi)d\vec{x}^2,$$

$$\Phi(0) = -\frac{Gm}{a}\mathrm{Erf}\left(\frac{M_s a}{2}\right)$$

$$ds^2 = -\left(1-\frac{2Gm}{r}\mathrm{Erf}\left(\frac{M_s r}{2}\right)\right)dt^2 + \left(1+\frac{2Gm}{r}\mathrm{Erf}\left(\frac{M_s r}{2}\right)\right)(dr^2 + r^2 d\Omega^2)$$
$$-4GJ\left[\frac{1}{r}\mathrm{Erf}\left(\frac{M_s r}{2}\right) - \frac{M_s}{\sqrt{\pi}}e^{-\frac{M_s^2 r^2}{4}}\right]\sin^2\theta d\varphi dt.$$

$$a < \frac{2}{M_s} \quad (\text{radius of the ring} < \text{scale of non}-\text{locality})$$

At non-linear level only solution survives is a conformally flat metric

Buoninfante, et. al, [arXiv:1807.08896 [gr-qc]]

Non-Local, Infinite Derivative Gravity

~ Non-local graviton propagator motivated from the UV properties of string amplitude.

~ Non-locality resolves Curvature Singularities

~ Non-singular cosmology with no ghosts.

~ Non-local stars can mimic black hole without event horizon

~ Non-singular rotating compact objects & NUT-charge in linearised non-local gravity resolve curvature singularities

(see: Buoninfante, et,al. 1807.08896, Frolov, et,al. (2020), Kolar, AM 2004.07613)

Extra Slides

Extra degrees of freedom & Ghosts

$$\Pi(k^2) = \frac{1}{k^2}\left[P^{(2)} - \frac{P^{(0)}}{2}\right] - \frac{P^{(2)}}{k^2 - m_2^2} + \frac{P^{(0)}}{2(k^2 - m_0^2)}$$

$$m_2 = -(\beta/2)^{-1/2}, \quad m_0 = (\alpha + \beta)^{-1/2}$$

Massive Spin-0 & Massive Spin-2 (Ghost) Stelle (1977)

Utiyama (1960), De Witt (1961), Stelle (1977)

Modification of Einstein's GR

Modification of Graviton Propagator

Extra propagating degree of freedom

Challenge: How to get rid of the extra dof ?

Einstein & Weyl Gravity: Finite Derivative Theories

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} \right)$$

One loop pure gravitational action is renormalizable.
But it has a scale. The theory is not scale invariant

$$S = \int \sqrt{-g} d^4x \left[M_p^2 R + \alpha C^2 \right]$$

Weyl term does not introduce singularities

$$S = \int \sqrt{-g} d^4x \left[M_p^2 R + \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} \right]$$

Quadratic Curvature Gravity is renormalizable, but contains "Ghosts": Vacuum is Unstable

Utiyama (1961), De Witt (1961), Stelle (1977)
t'Hooft, Veltman (1974)

Potential resolution of Ghosts & Classical Instabilities

Higher derivative theories generically carry Ghosts (-ve Risidue)

$$S = \int d^4x \ \phi\Box(\Box + m^2)\phi \Rightarrow \Box(\Box + m^2)\phi = 0$$

$$\Delta(p^2) \sim \frac{1}{p^2} - \frac{1}{p^2 - m^2}$$

Propagator with first order poles

Ghosts cannot be cured order by order, finite terms in perturbative expansion will always lead to Ghosts !!

$$S = \int d^4x \ \phi e^{-\Box/M^2}(\Box + m^2)\phi \Rightarrow e^{-\Box/M^2}(\Box + m^2)\phi = 0$$

$$\Delta(p^2) = \frac{e^{-p^2/M^2}}{p^2 - m^2}$$

No extra states other than the original dof.

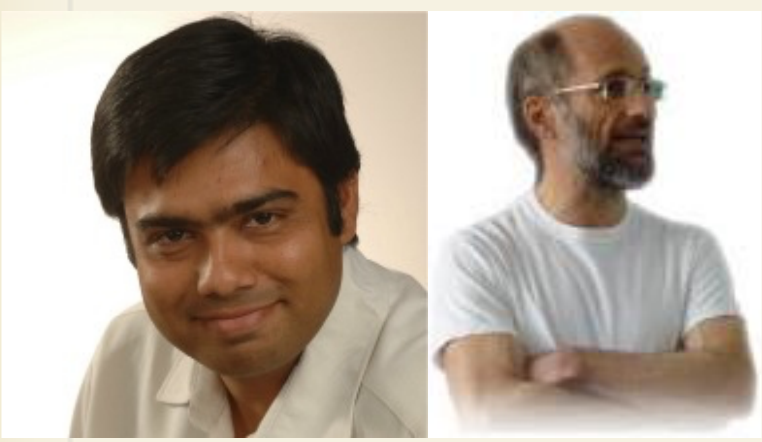
Woodard (1991), Moffat (1991), Tomboulis (1997), Tseytlin (1997), Siegel (2003), Biswas, Grisaru, Siegel (2004), Biswas, Mazumdar, Siegel (2006)

Infinite Derivative Gravity

GR in IR **Corrections in UV**

$0.001 \text{ eV} < M$ M_p

$M \to \infty$(Theory reduces to GR)



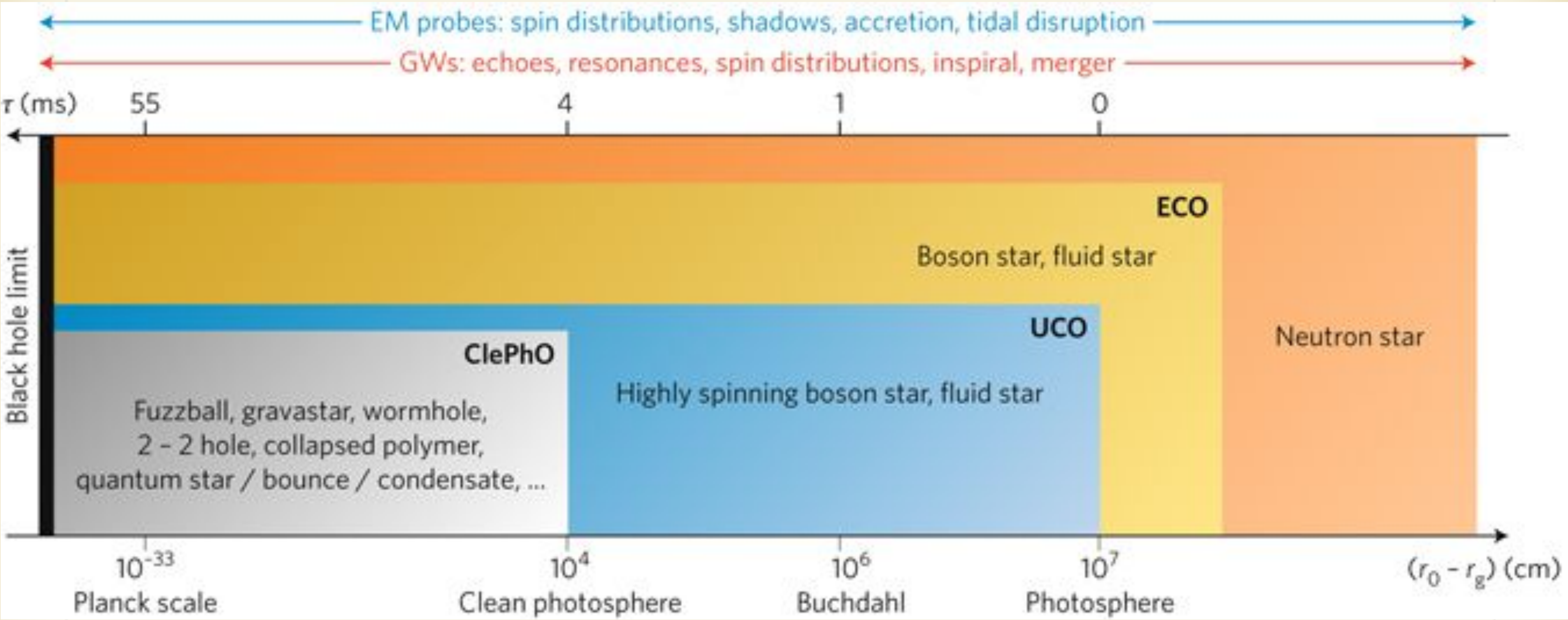
Biswas, AM, Siegel

Biswas, Gerwick, Koivisto, AM

Bouncing universes in string-inspired gravity, hep-th/0508194, JCAP (2006)

Towards singularity and ghost free theories of gravity, 1110.5249 [gr-qc], PRL (2012)

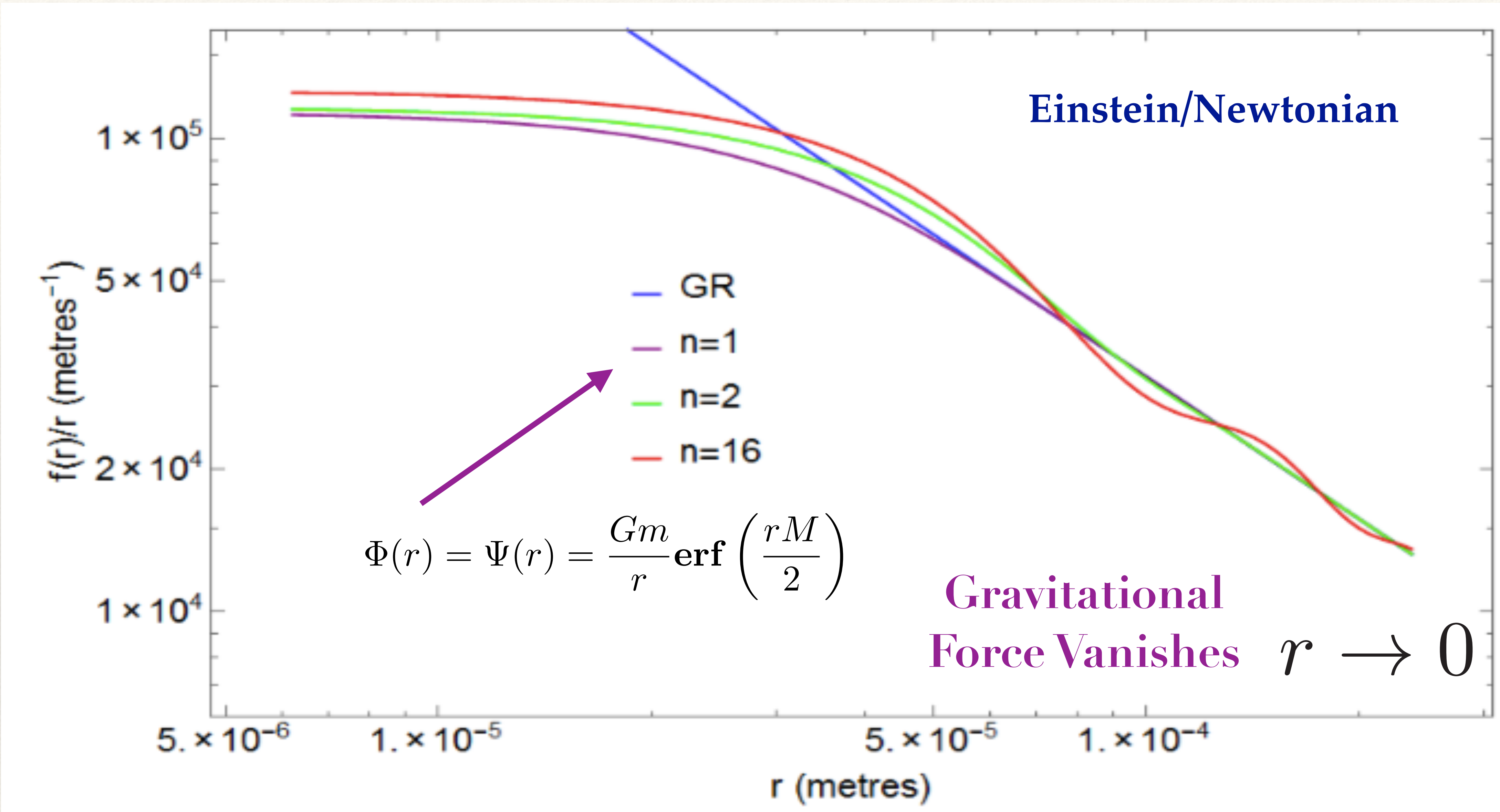
Non-Local Star as a ClePho



Resolution of Singularity at short distances

$$a(\Box) = e^{\gamma(\Box)}$$

Any Entire Function: $\gamma(\Box) = -\frac{\Box}{M^2} - \sum_N a_N \left(\frac{\Box}{M^2}\right)^N$

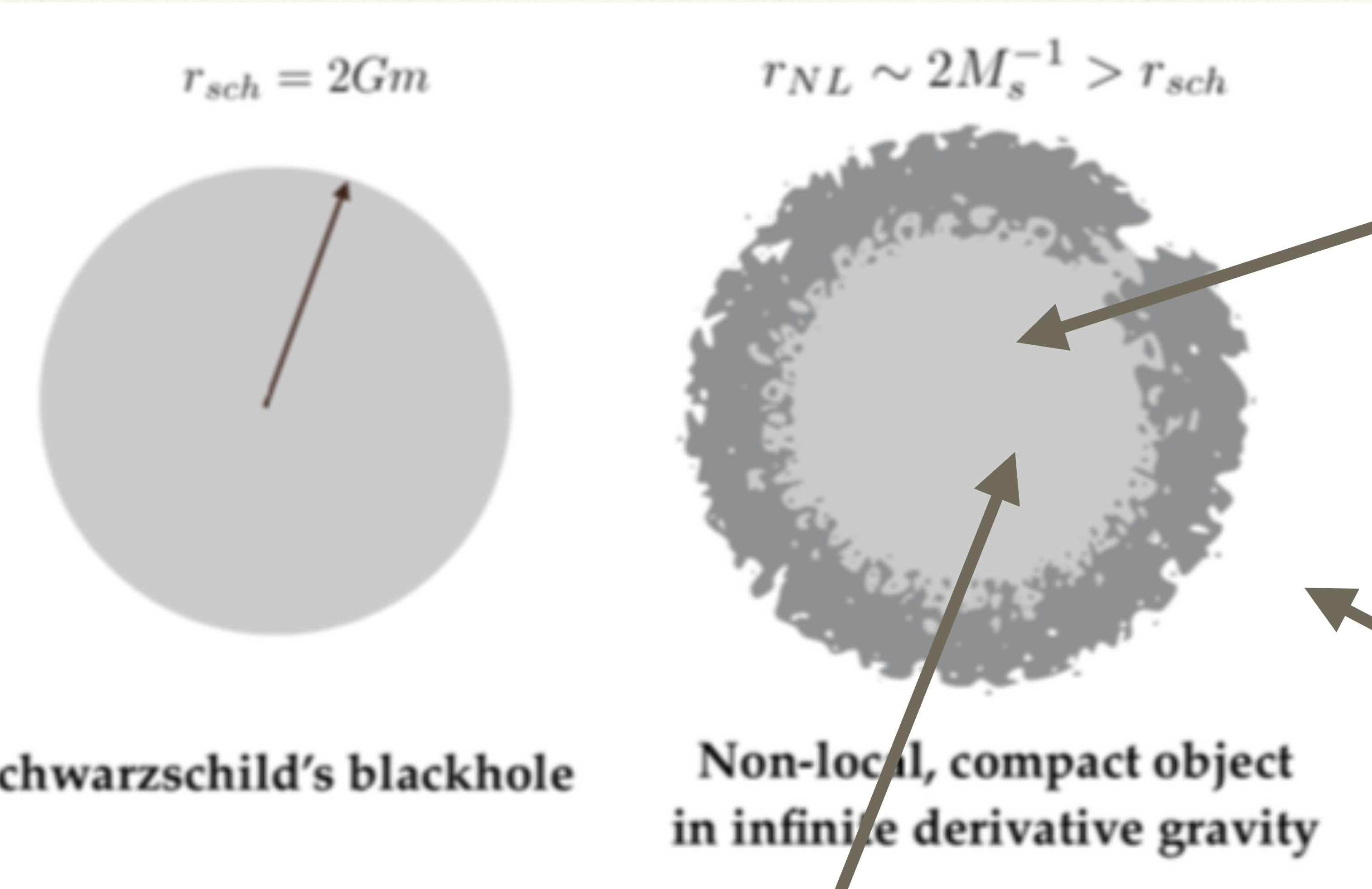


$$mM \ll M_p^2 \implies m \ll M_p$$

Current Bound : M > 0.01 eV $\quad m \leq 10^{25}$ grams

Biswas, Gerwick, Koivisto, AM (2012), Edholm, Koshelev, AM (2016), Frolov & Zelnikov (2015, 2016)

Conformally Flat metric, Non-Vacuum Solution, with no event horizon



$$\phi(r) \sim \frac{GmM_s}{\sqrt{\pi}} < 1$$

$$\mathcal{R}_{00} = \mathcal{R}_{11} \sim \frac{GmM_s^3}{2\sqrt{\pi}}, \qquad \mathcal{R}_{22} = \mathcal{R}_{33} \sim 0$$

$$\mathcal{R} \sim \frac{GmM_s^3}{\sqrt{\pi}}$$

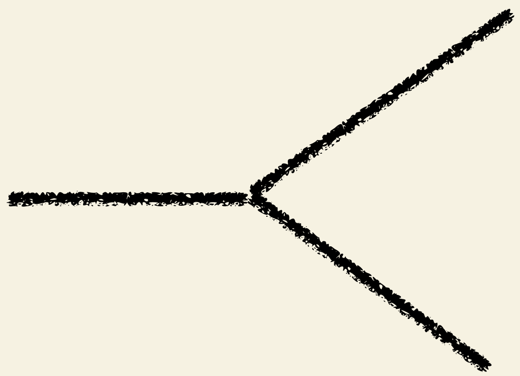
$$\phi(r) \sim \frac{Gm}{r} < 1$$

$$\begin{aligned} ds^2 &= (1 + 2A)\left(-d\tau^2 + dr^2 + r^2 d\Omega^2\right) \\ &= F\eta, \end{aligned} \qquad A \equiv \frac{GmM_s}{\sqrt{\pi}} < 1.$$

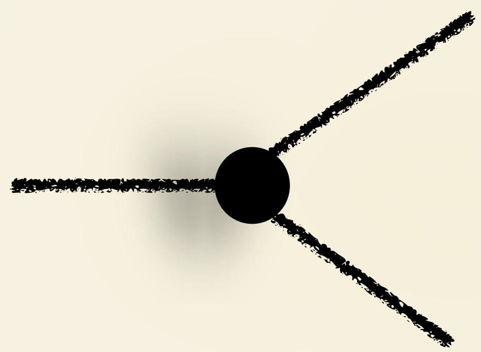
[arXiv:1802.00399 [gr-qc]]

Local vs Non-Local Field Theory

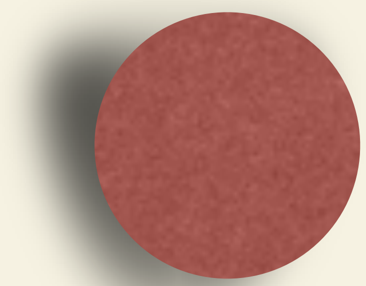
$$S = \int d^4x \left[-\frac{1}{2}\phi e^{\frac{\Box + m^2}{M^2}} (\Box + m^2)\phi - \frac{\lambda}{4!}\phi^4 \right] \qquad \Pi(p^2) = -\frac{ie^{-\frac{p^2+m^2}{M^2}}}{p^2+m^2}$$



$P^2 < M^2$



$P^2 \geq M^2$



$r \sim M^{-1}$

Scale of Non-Locality

$$\delta m^2 \sim \lambda M^2 \qquad \Gamma_4 \sim -\lambda^2 e^{-2m^2/M^2}\left[1 + \mathcal{O}(m^2/M^2)\right]$$

$$\sigma_{NL}(f\bar{f} \to f'\bar{f}') = e^{-s/M^2}\sigma_L(f\bar{f} \to f'\bar{f}')$$

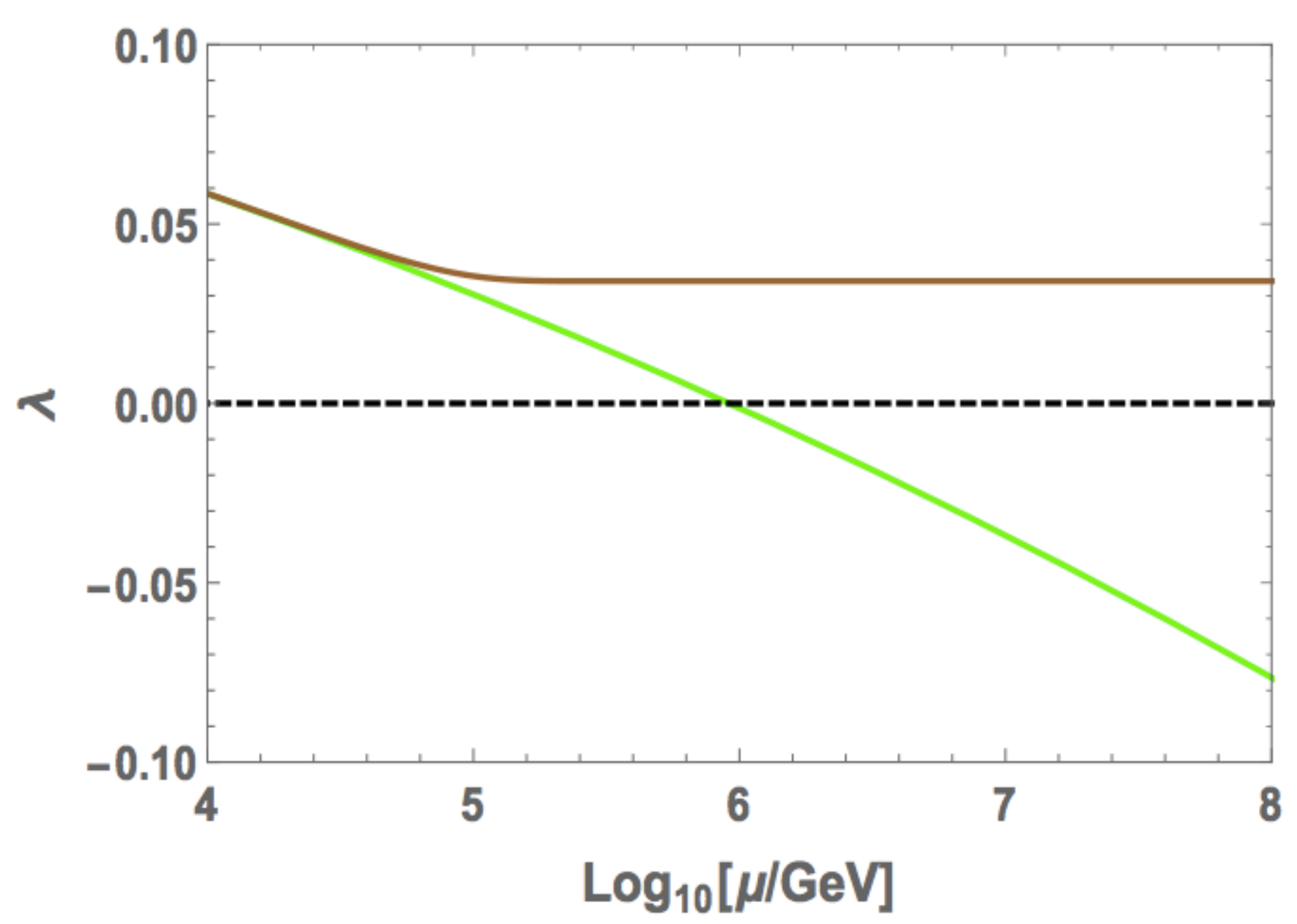
Moffat Phys.Rev.D (1990), Biswas + Okada, Nucl. Phys. B (2016)

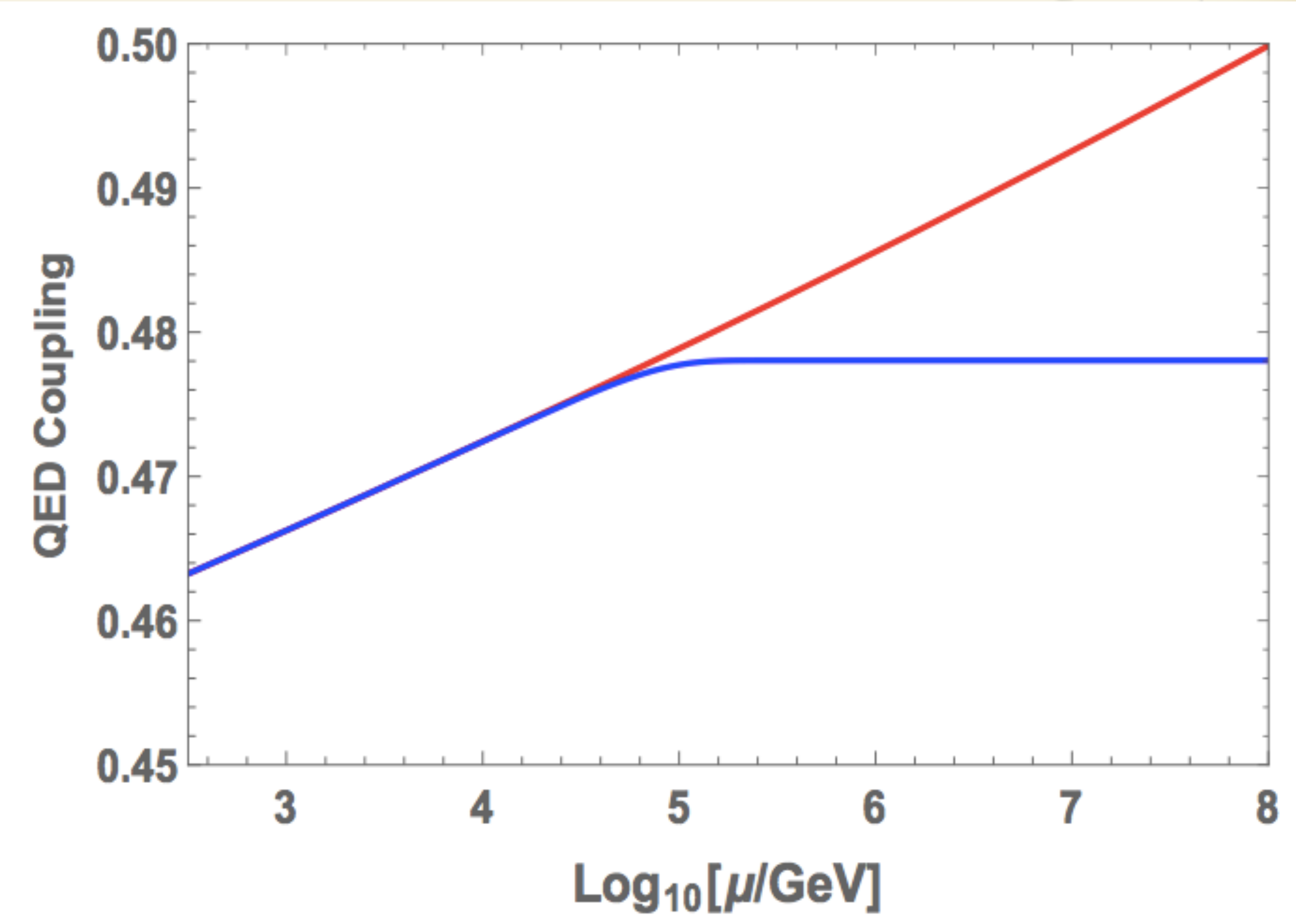
Scale-Free Abelian Higgs Interactions

$$\mathcal{L} = -\frac{1}{2}\phi e^{\frac{\Box + m_\phi^2}{M^2}} \left(\Box + m_\phi^2\right)\phi + i\bar{\psi} e^{\frac{\Box + m_\psi^2}{M^2}} \left(\gamma^\mu \partial_\mu - m_\psi\right)\psi$$

$$-\lambda\phi^4 - y\phi\bar{\psi}\psi + h.c. \qquad \text{Abelian Higgs} \qquad (1)$$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu} e^{\frac{\Box}{M^2}} F_{\mu\nu} + i\bar{\psi} e^{\frac{\nabla^2}{M^2}} \gamma^\mu D_\mu \psi + h.c.$$





Ghoshal, AM, Okada, Villalba (2017)