UNIVERSITY OF AMSTERDAM

Many-body strategies for multi-qubit gates

Kareljan Schoutens

Exactly Solvable Quantum Chains

IIP Natal, 19 June 2018

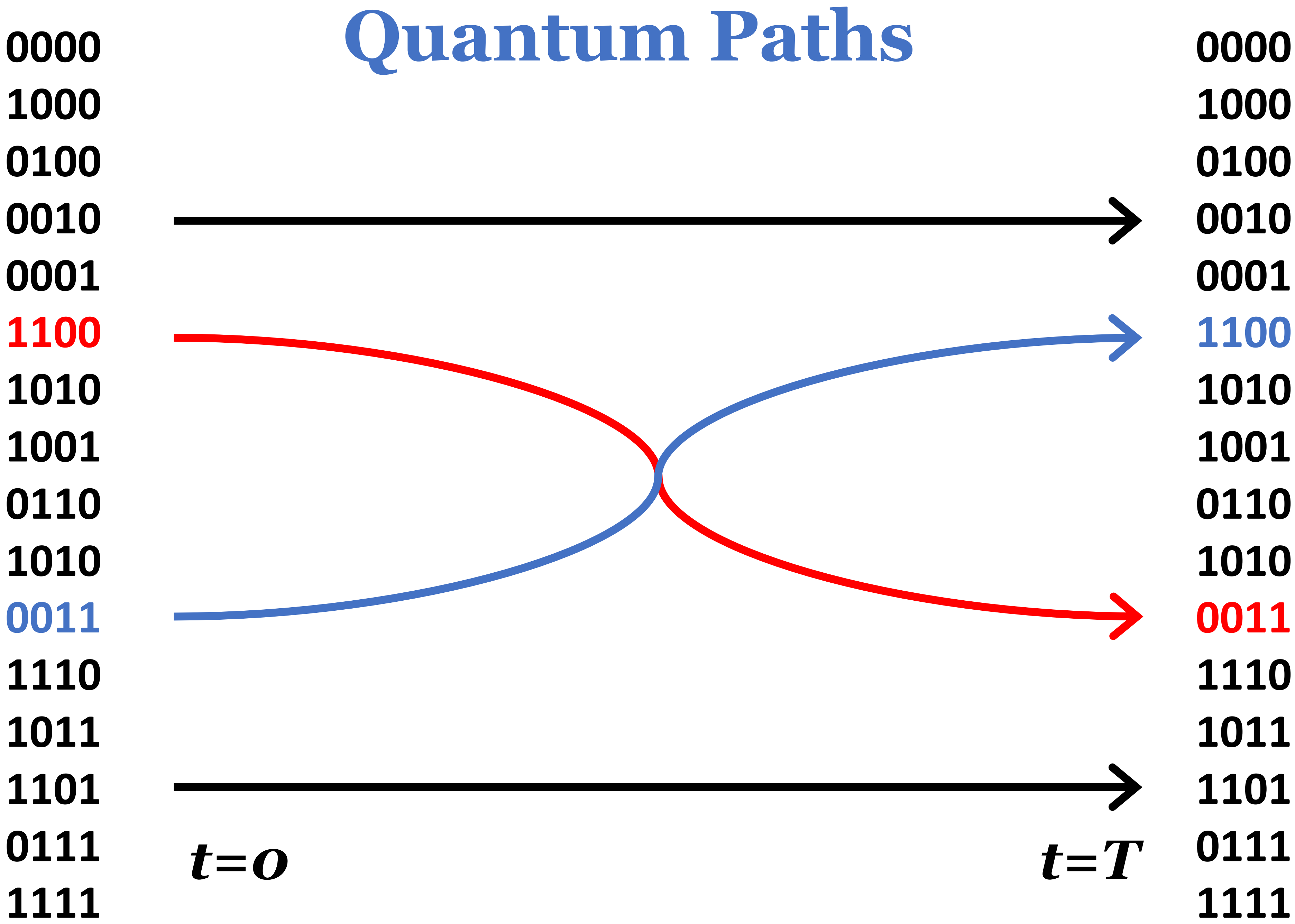
QuSoft

Delta Institute for Theoretical Physics

Quantum Paths (ESI)

&

Quantum Chains (IIP)


Quantum Paths
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Quantum Paths

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quantum paths realize **multi-qubit** quantum gates on N-qubit registers

we realize multi-qubit gates via driven dynamics of N **coupled qubits**

example will be the resonant coupling of eigenstates of **Krawtchouk qubit chain**

$$H^K = -\frac{J}{2}\sum_{x=0}^{n}\sqrt{(x+1)(n-x)}\left[X_x X_{x+1} + Y_x Y_{x+1}\right]$$

Quantum Chains

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Many-body strategies for multiqubit gates: Quantum control through Krawtchouk-chain dynamics

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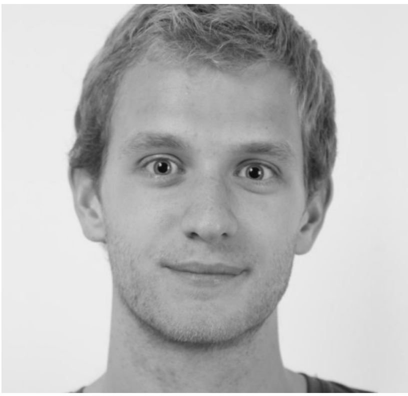
We propose a strategy for engineering multiqubit quantum gates. As a first step, it employs an *eigengate* to map states in the computational basis to eigenstates of a suitable many-body Hamiltonian. The second step employs resonant driving to enforce a transition between a single pair of eigenstates, leaving all others unchanged. The procedure is completed by mapping back to the computational basis. We demonstrate the strategy for the case of a linear array with an even number N of qubits, with specific $XX + YY$ couplings between nearest neighbors. For this so-called Krawtchouk chain, a two-body driving term leads to the iSWAP$_N$ gate, which we numerically test for $N = 4$ and 6.



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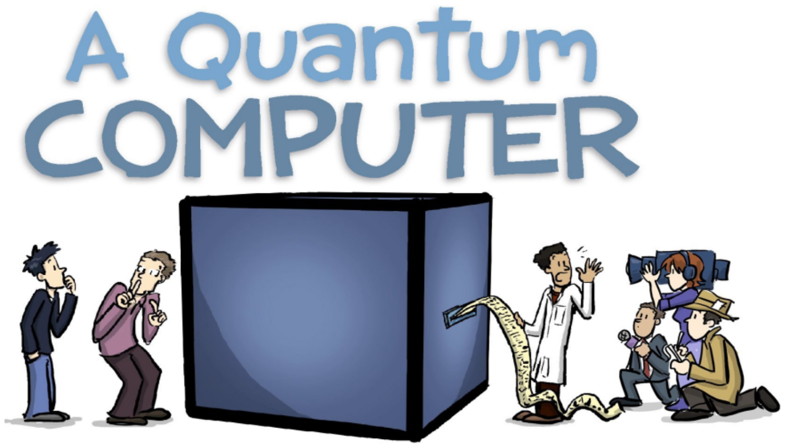
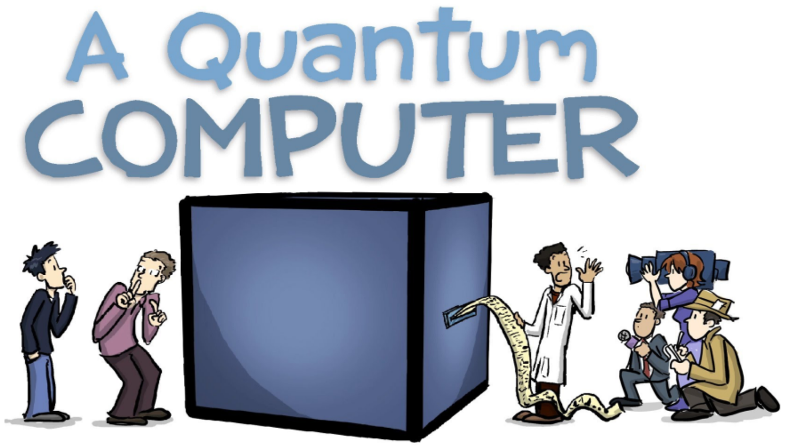


outline

- background and motivation
- many-body strategies for multi-qubit gates
- quantum control on the Krawtchouk chain



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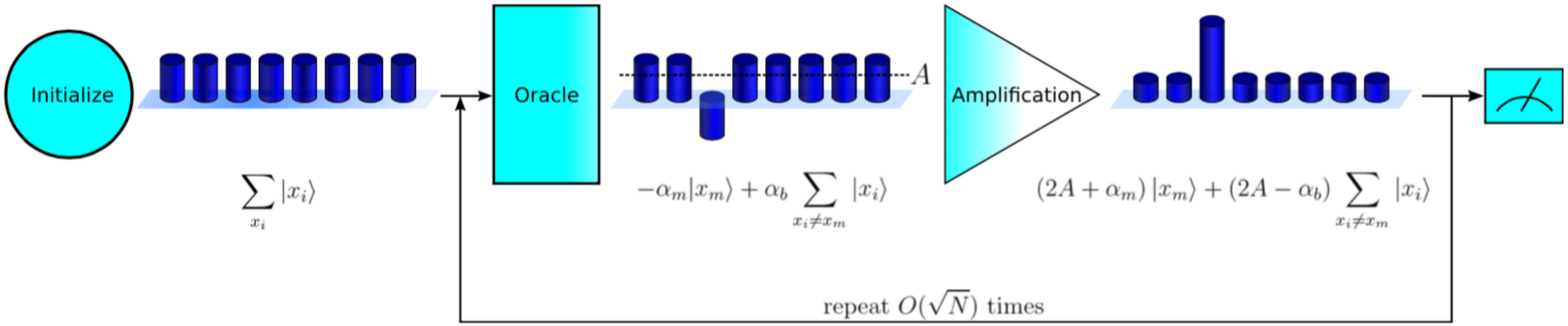
quantum algorithms

For specific problems quantum algorithms can be made to outperform classical computers by cunningly combining quantum parallelism with interference.

Grover search algorithm:

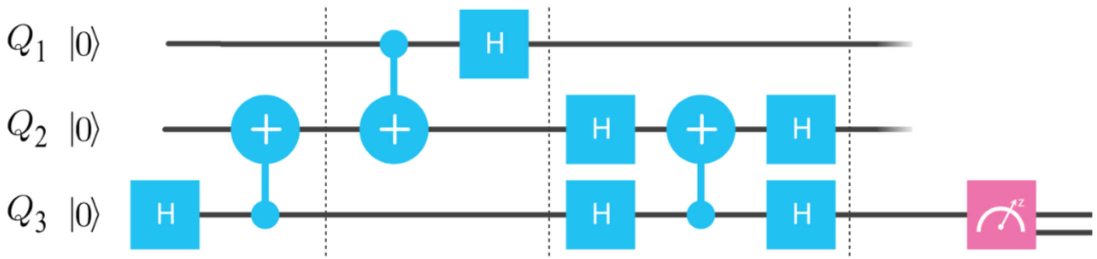
finding tagged element in size-*N* database in $O(\sqrt{N})$ steps

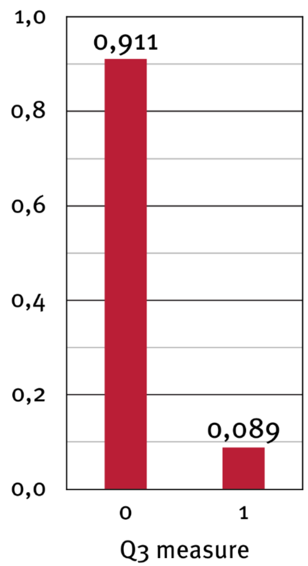


quantum circuit

3-step implementation of quantum algorithm on
N-qubit quantum register

- **initialization**
- **unitary evolution** via quantum gates
- read-out through **measurement**

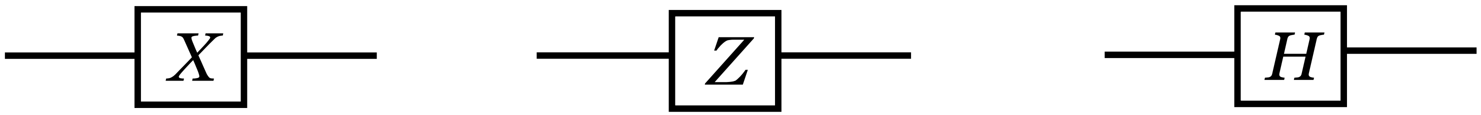




IBM Quantum Experience

quantum gates

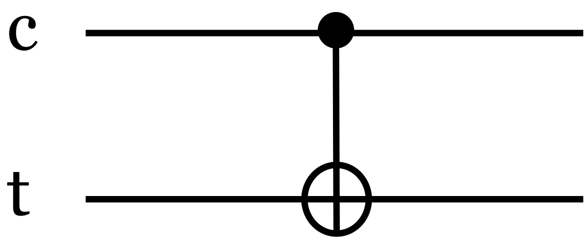
- **1-qubit gates:** *X*, *Z*, *H*, …



$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

- **2-qubit gates:** CNOT, *XX(θ)*, SWAP, …

CNOT:



flips target qubit t
iff control qubit c is
in state |1>

universal gate sets

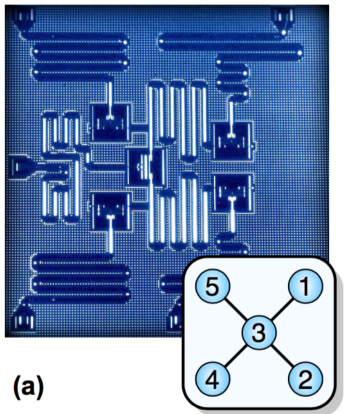
strong universality:

all N-qubit unitaries can be built from CNOTs
plus sufficiently many 1-qubit gates

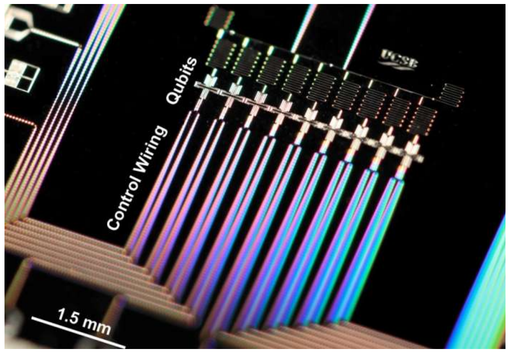
all quantum paths can be realized via quantum circuits with 1- and 2-qubit gates

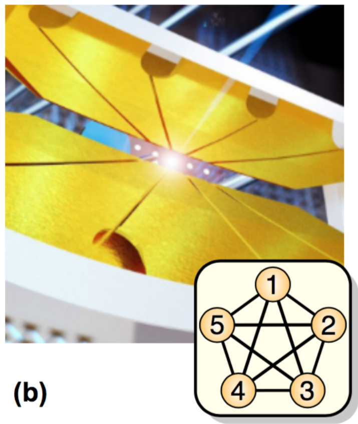
state of the art

quantum hardware has progressed to the point that programmable qubit platforms with up to some 20 qubits are available → real-world testing of few-qubit quantum algorithms!



(a)



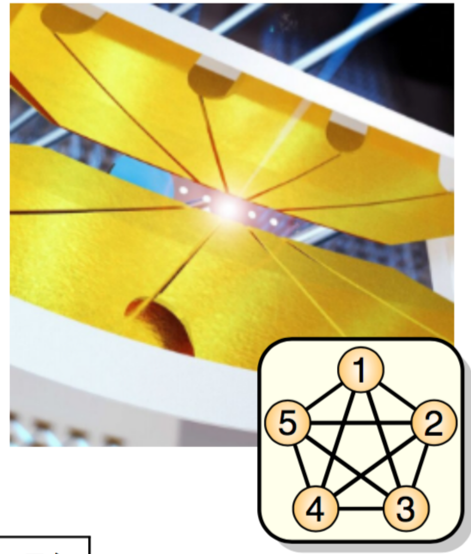


(b)

native gates and quantum compiling

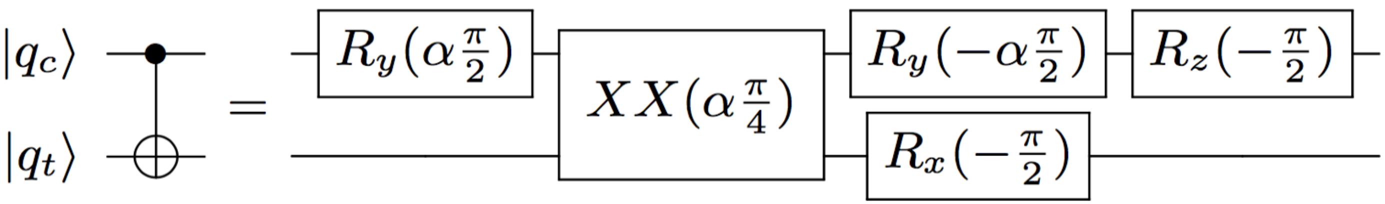
- **native gate libraries**
  the 1-qubit and 2-qubit interactions that are natural for a given qubit platform lead to a `native gate library'.

- **quantum compiling**
  expressing universal gates in native gates



example: native gate library for trapped ions

- all 1-qubit rotations $R_\alpha(\theta)$
- 2-qubit gates $X_i X_j\,(\theta)$



Complete 3-Qubit Grover Search on a Programmable Quantum Computer

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[2] *National Science Foundation, Arlington, VA 22230, USA*

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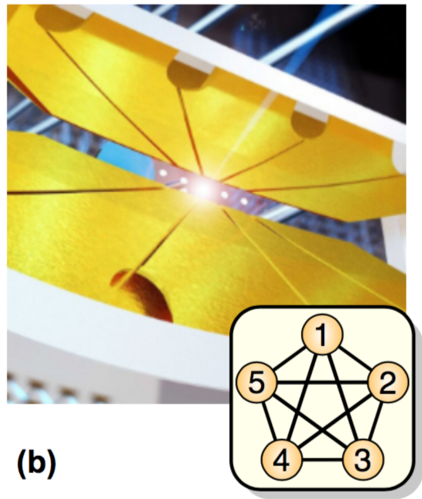
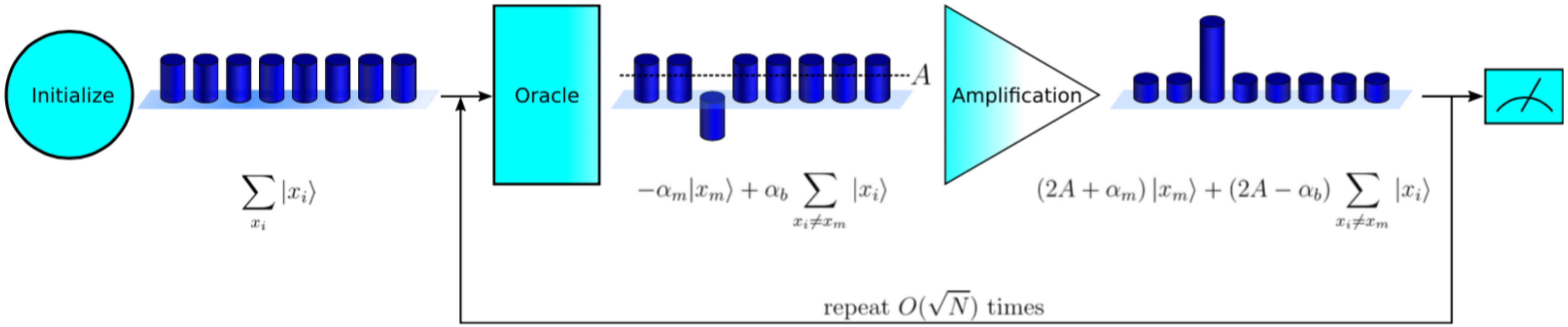
(Dated: March 31, 2017)



(b)

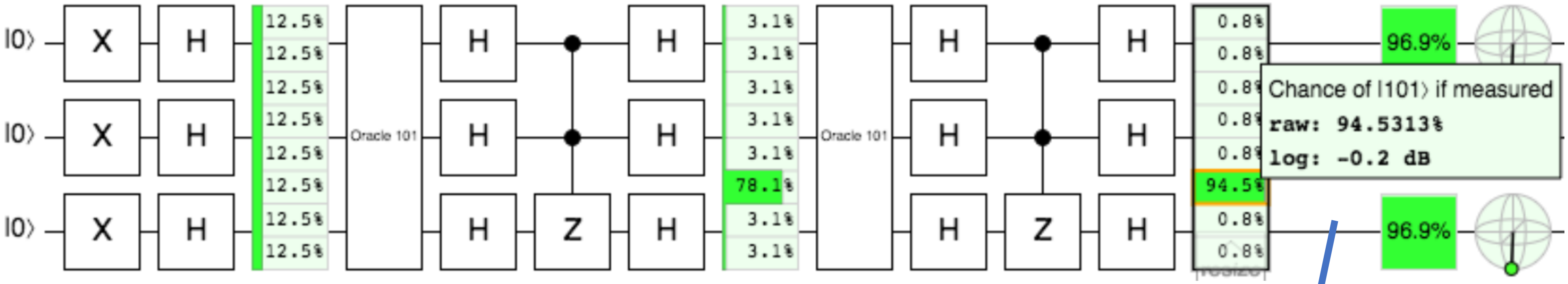
Grover search: finding tagged element in size-N database in $O(\sqrt{N})$ steps



3-qubit Grover search

finds 1 tagged
element out of 8
in two steps

quantum circuit on Quirk simulator:



Oracle tagging the element |101>

Initializing the qubits to |0>

**read-out gives
tagged element |101>
with 94.5% chance**

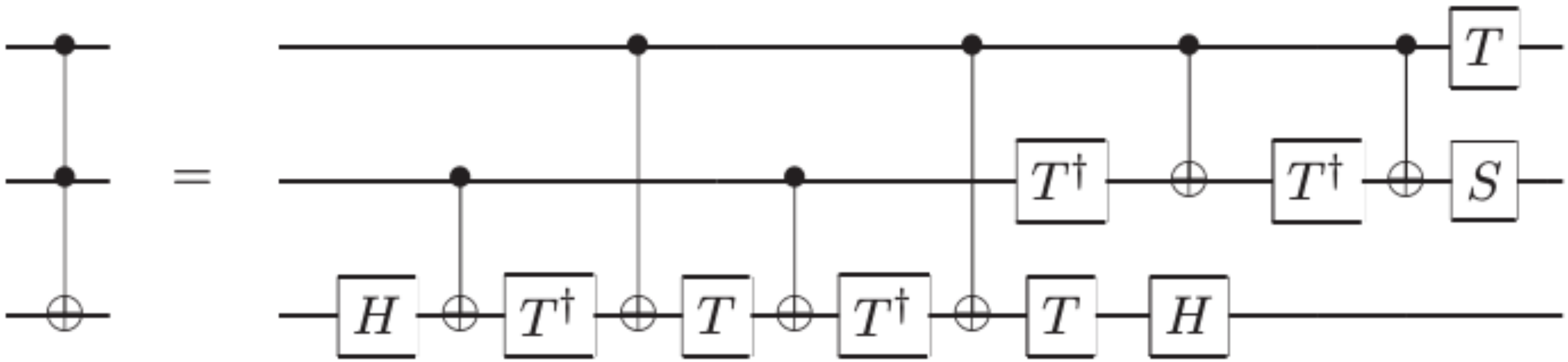
multi-qubit gates

- quantum algorithms such as Grover search use gates like

$$\text{CCNOT (Toffoli), CCZ, } \dots \text{ , } C^{N-1}\text{NOT, } C^{N-1}\text{Z, etc}$$

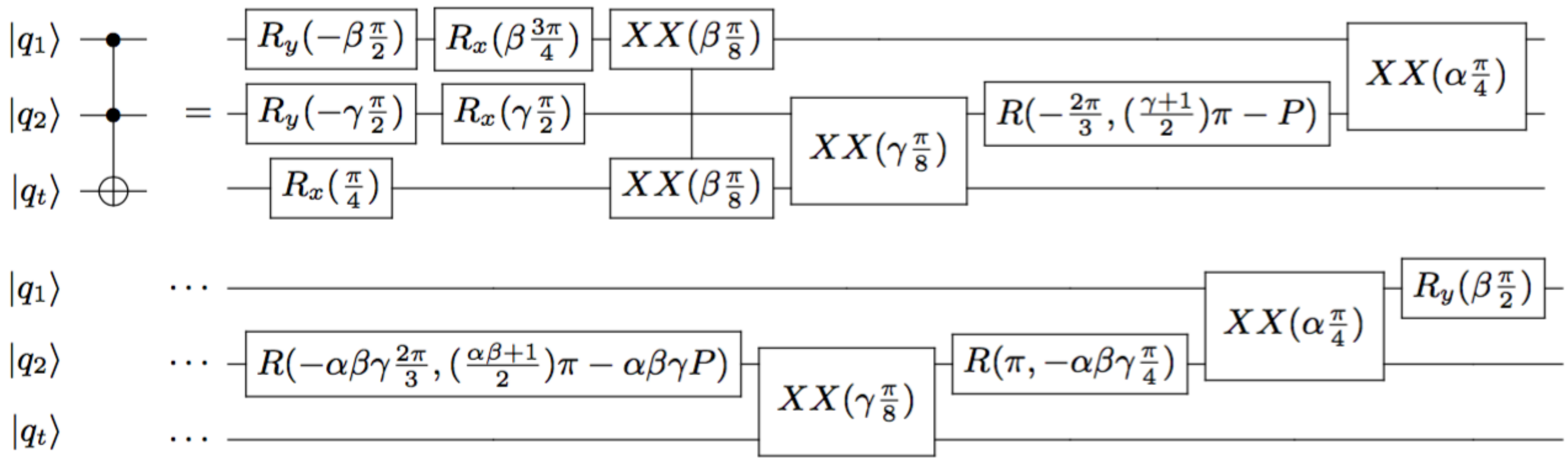
- building these from 1-qubit and 2-qubit gates requires lengthy circuits

multi-qubit gates



Toffoli-3 using standard Clifford + T gate library

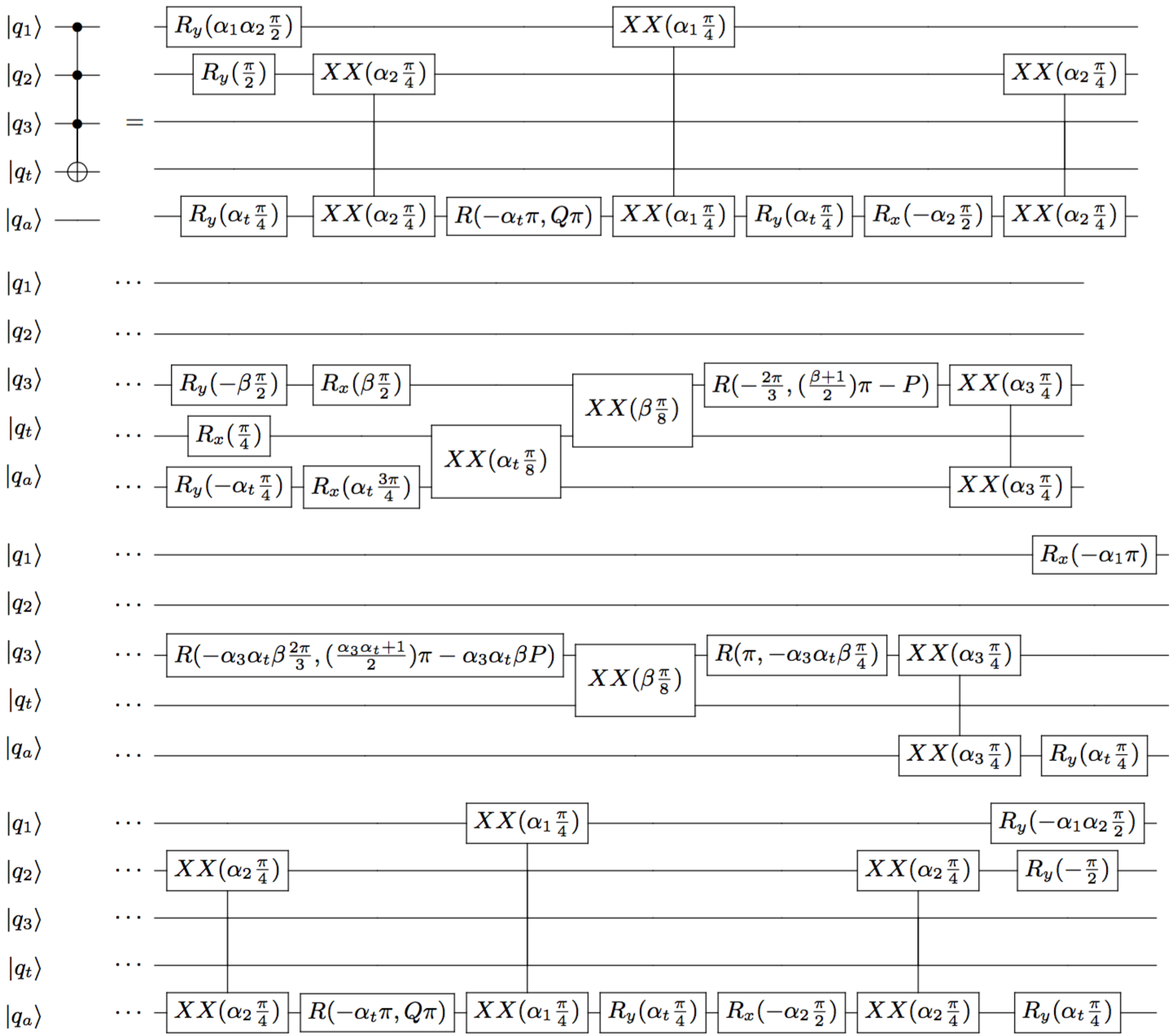
multi-qubit gates

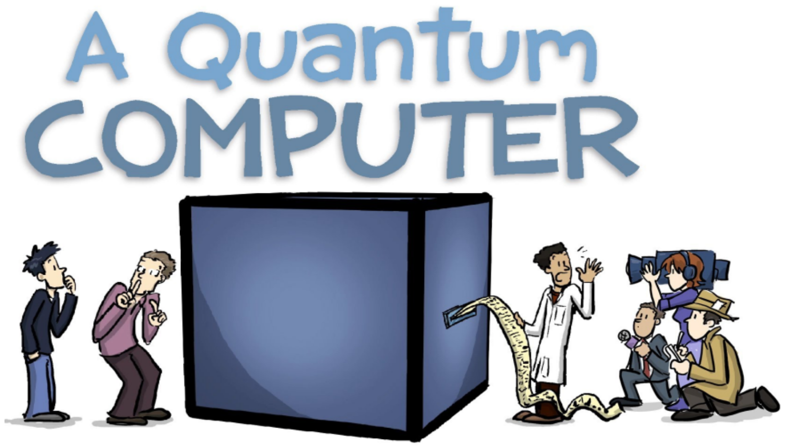


Toffoli-3 using XX/R gate library

multi-qubit gates

Toffoli-4 using XX/R gate library





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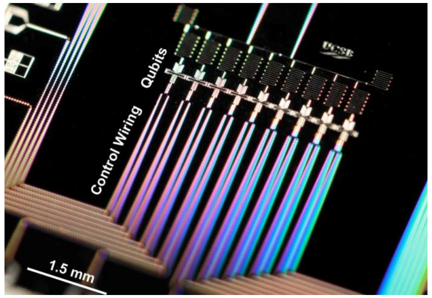
many-body strategies



idea

couple N qubits, leading to a many-body spectrum

proposed protocol

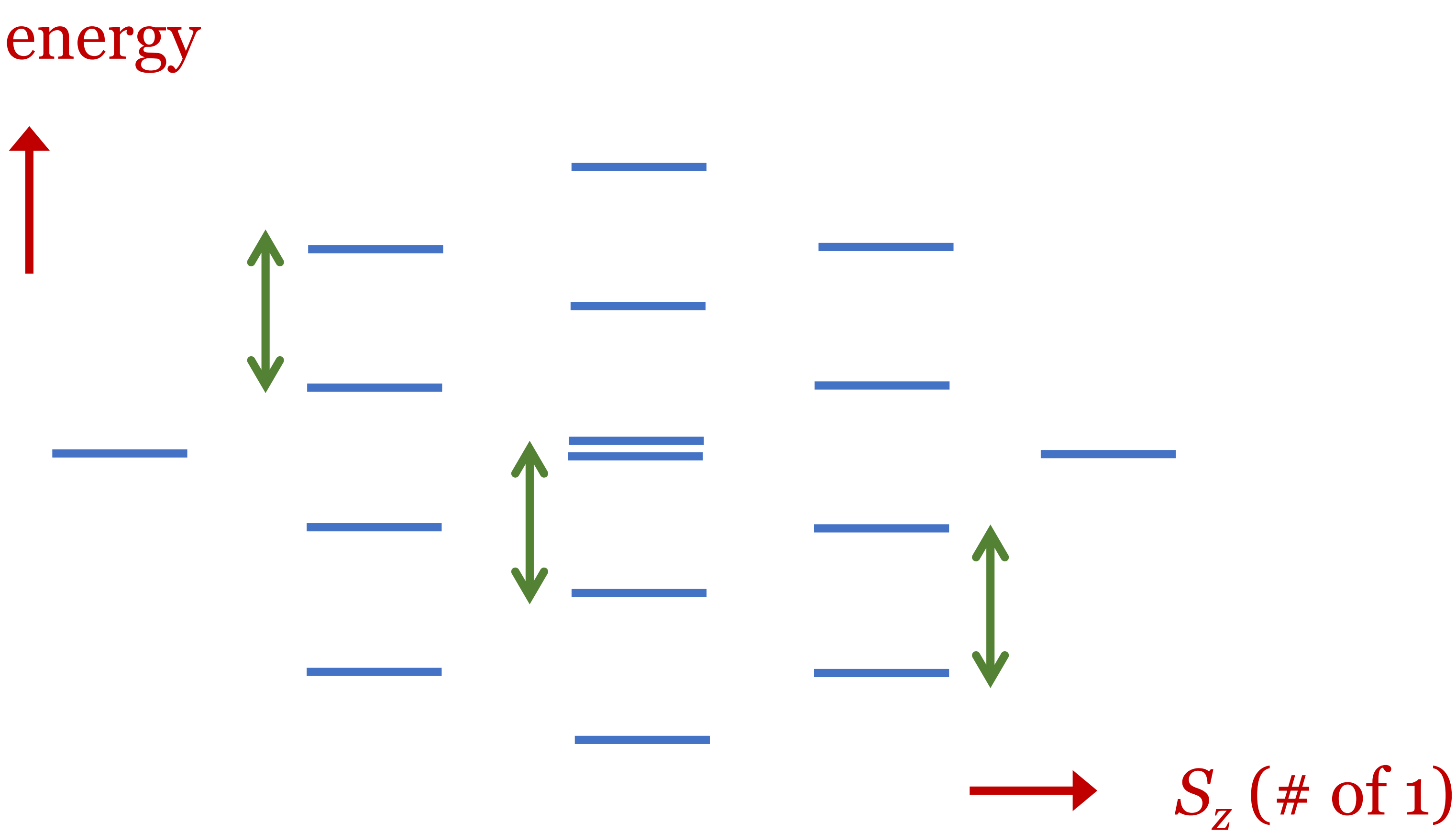
Step 1. Apply quantum circuit for *eigengate* to produce eigenstates from states in computational basis.

Step 2. Use resonant driving to selectively couple and interchange 2 out of 2^N eigenstates.

Step 3. Apply eigengate to return to computational basis.

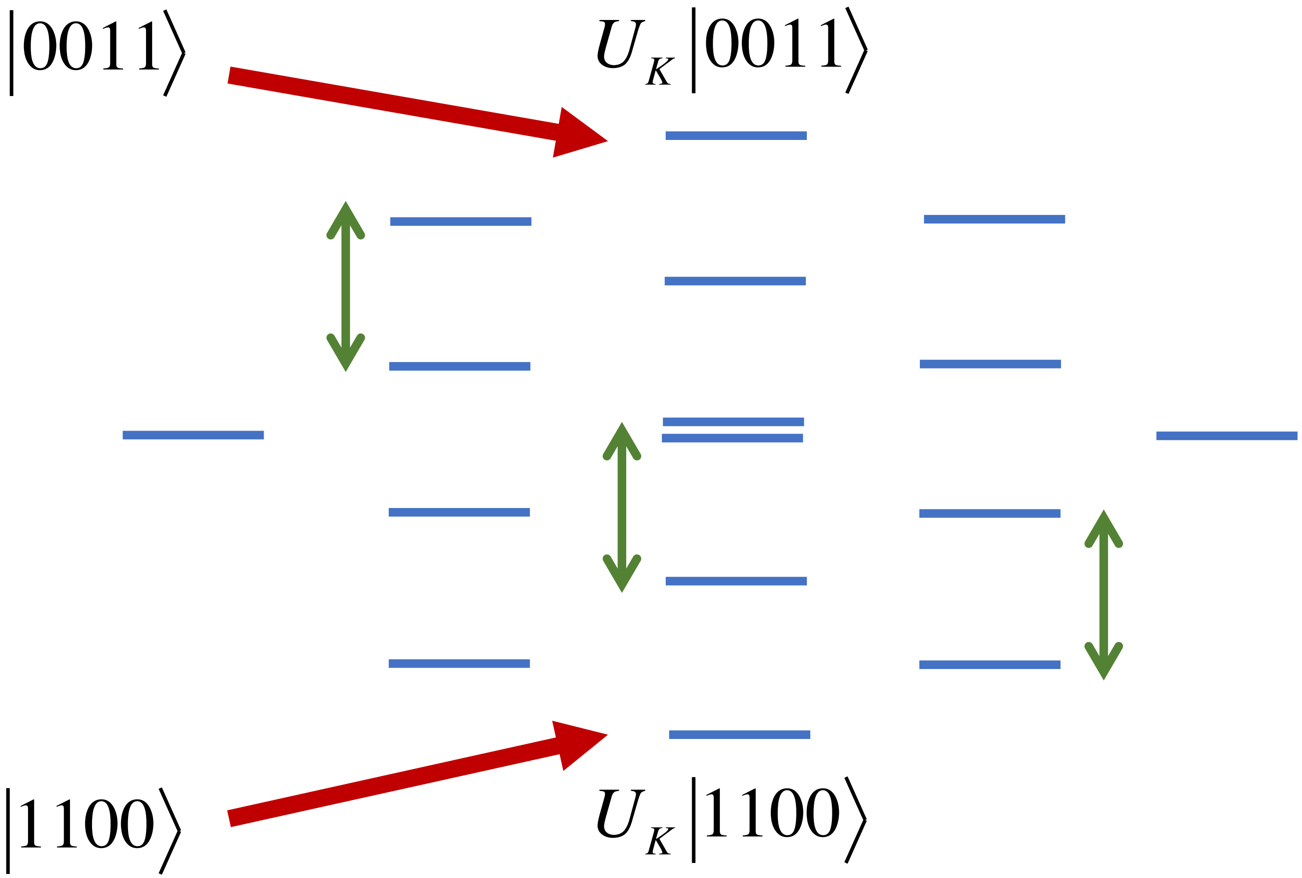
step 0:

many-body energy spectrum (*N*=*4*)



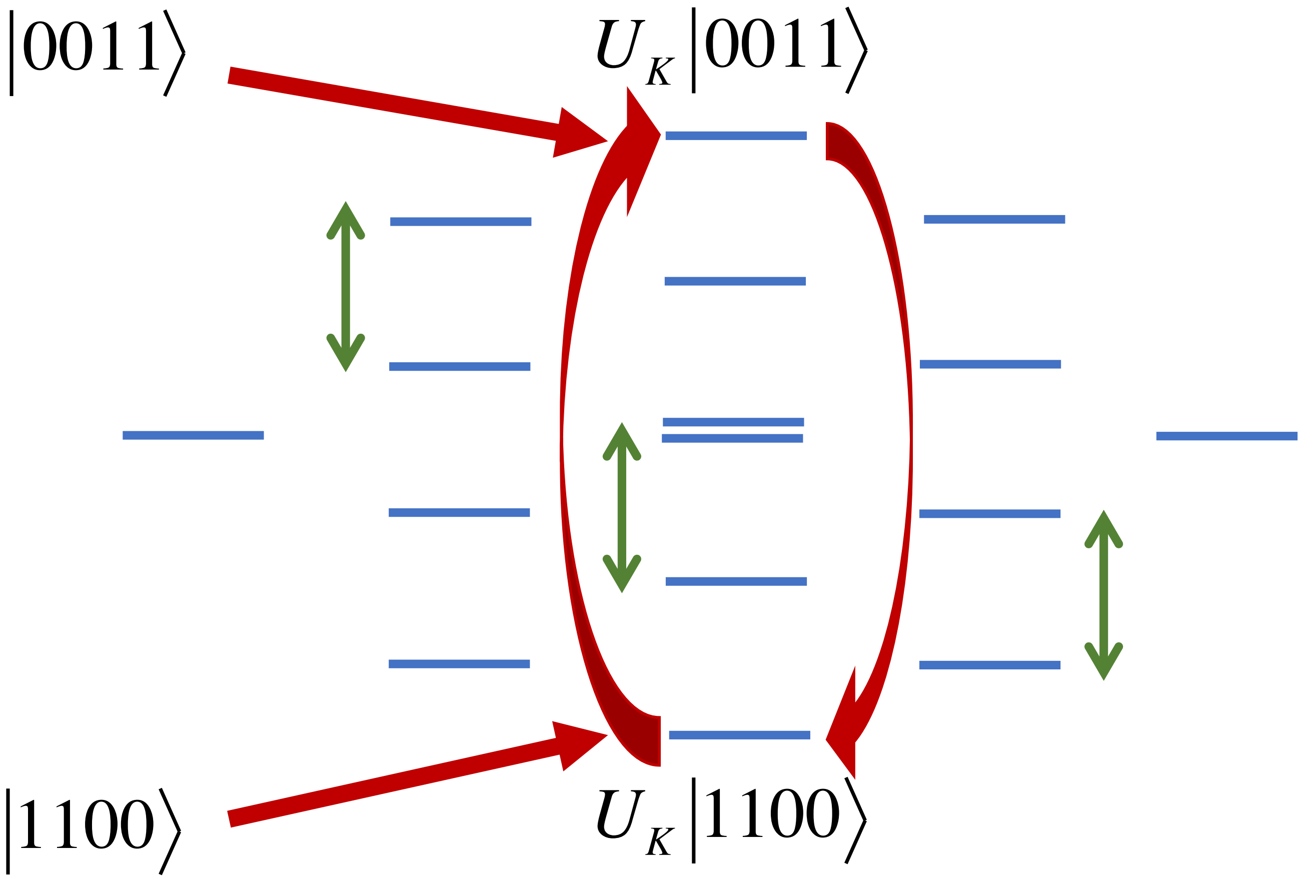
step 1:

eigengate U_K maps computational basis to eigenstates



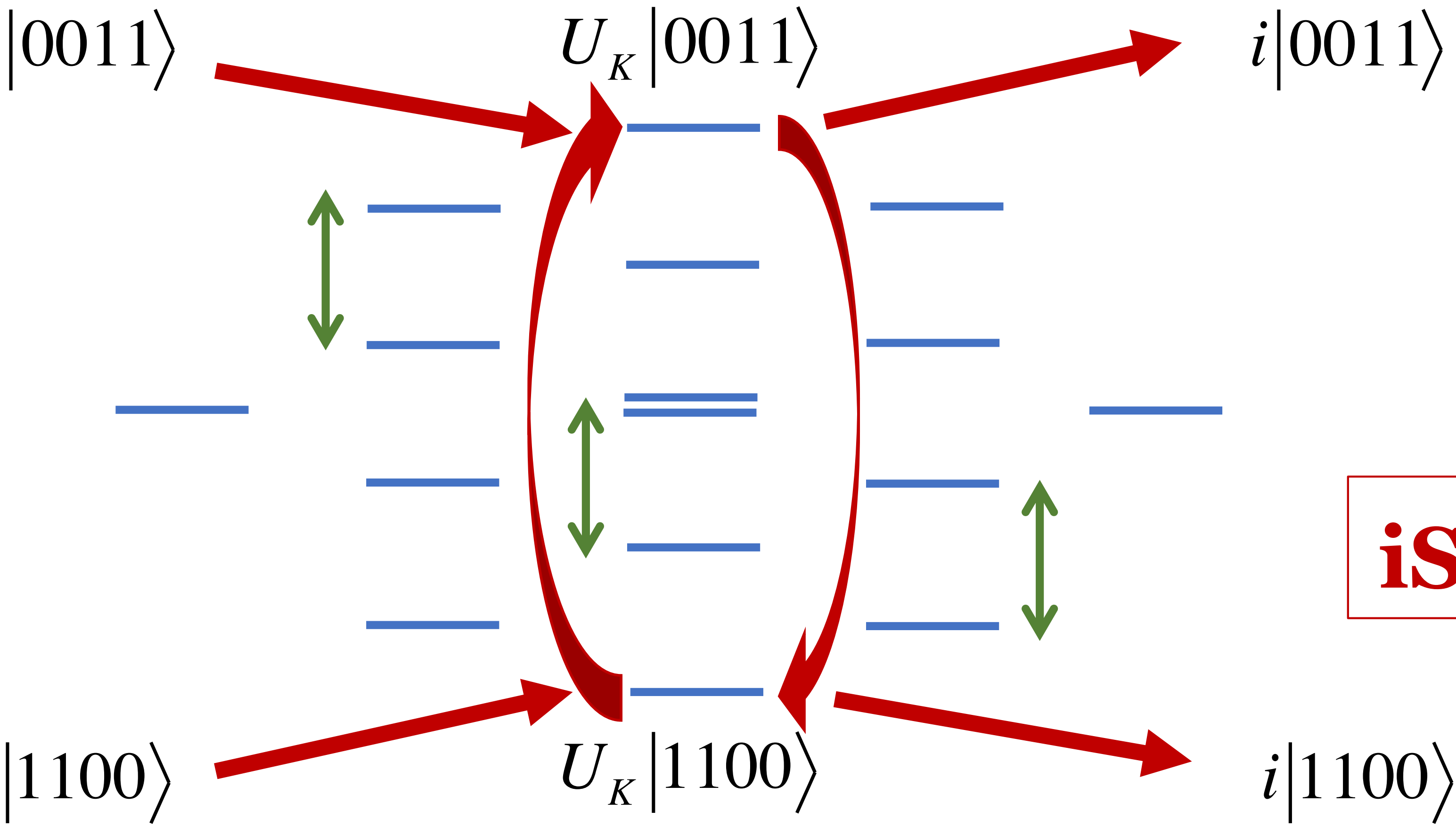
step 2:

resonant driving interchanges a single pair of eigenstates

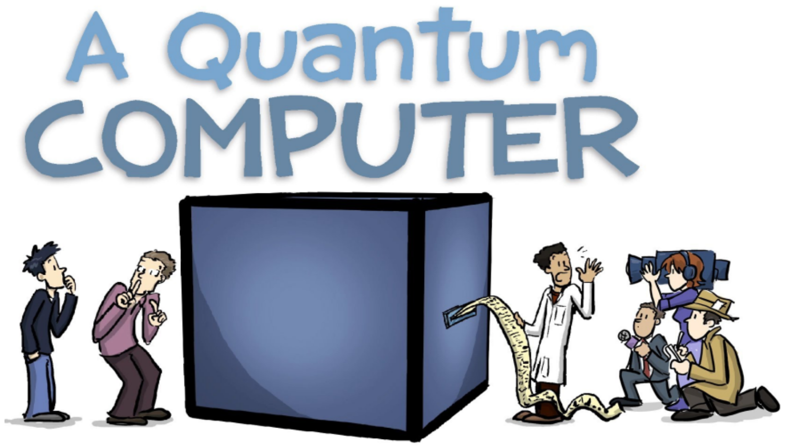


step 3:

inverse eigengate U_K maps back to computational basis



iSWAP$_4$



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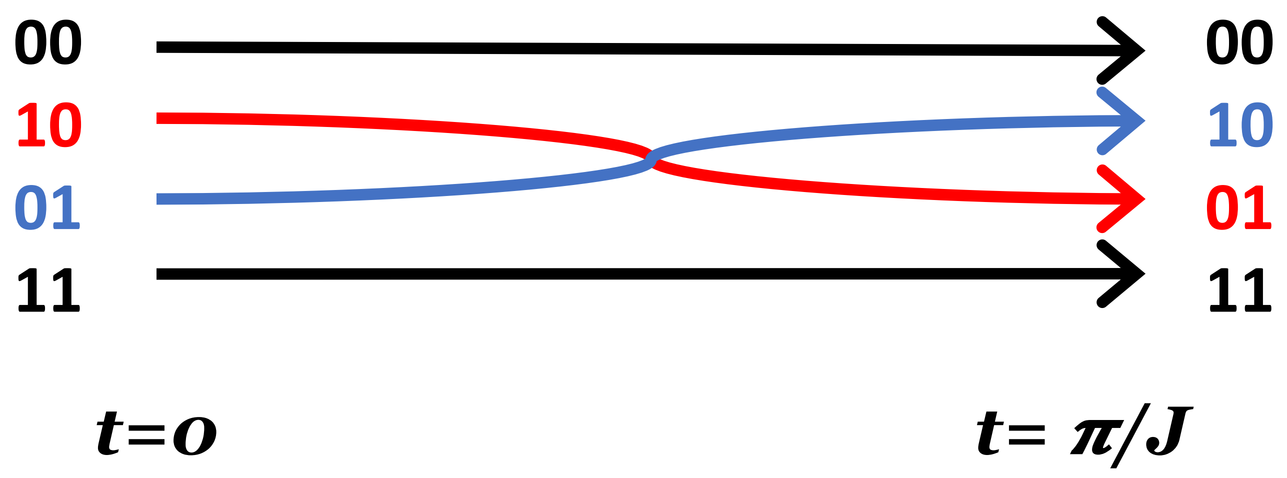
2-qubit $XX+YY$ coupling

$$H^{(2)} = -\frac{J}{2}(X_1 X_2 + Y_1 Y_2)$$

$t=\pi/J$ pulse of $H^{(2)}$ gives gate iSWAP$_2$

$|00\rangle \rightarrow |00\rangle$, $|01\rangle \rightarrow i\,|10\rangle$, $|10\rangle \rightarrow i\,|01\rangle$, $|11\rangle \rightarrow |11\rangle$



Krawtchouk chain ($N=n+1$)

$$H^K = -\frac{J}{2}\sum_{x=0}^{n}\sqrt{(x+1)(n-x)}\,\left[X_x X_{x+1} + Y_x Y_{x+1}\right]$$

- 1-body energies are all commensurate

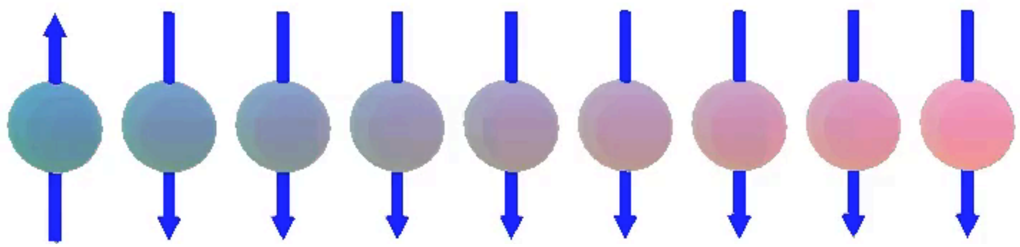
$$\lambda_k = J(k - \frac{N-1}{2}), \qquad k = 0, 1, .., n$$

- many-body energies are (free) sums of 1-body energies thanks to mapping to free fermions

Krawtchouk chain dynamics, I

Time evolution over time $t^{*}=\pi/J$ gives Perfect State Transfer (PST) for state with single `particle' or `spin-flip'

Christandl-Datta-Ekert-Landahl 2004



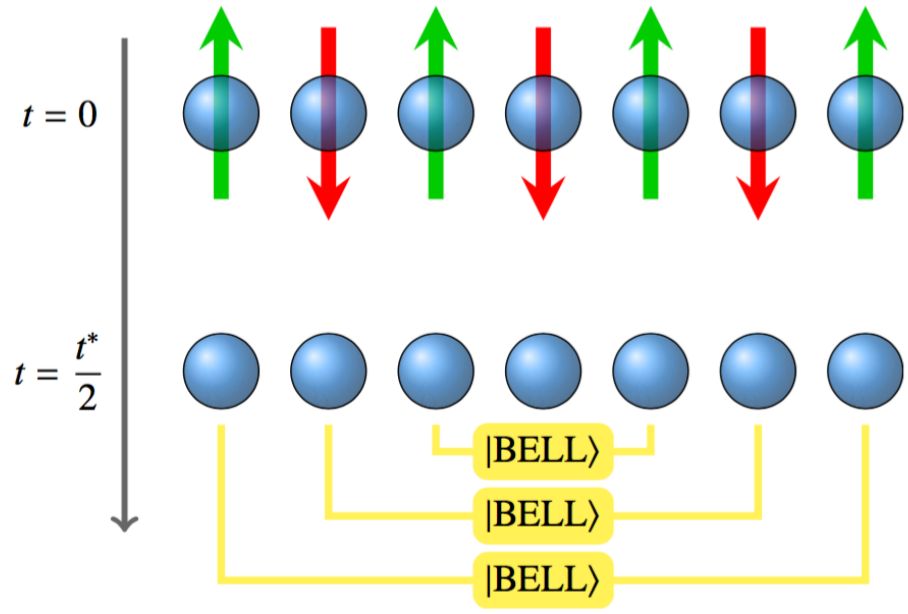
animation:
van der Jeugt

Krawtchouk chain dynamics, II

pulse of time $t^*/2=\pi/(2J)$

on Néel state |1010..>

generates **Rainbow state**:



nested Bell pairs with

maximal block entanglement

entropy $S_{LR}=N/2\ \ln(2)$

Alkurtass-Banchi-Bose 2014

Krawtchouk chain, details (*N*=*n*+1)

$$H^K = -\frac{J}{2}\sum_{x=0}^{n}\sqrt{(x+1)(n-x)}\left[X_x X_{x+1} + Y_x Y_{x+1}\right]$$

- 1-body spectrum

$$\lambda_k = J\left(k - \frac{N-1}{2}\right), \qquad k = 0, 1, .., n$$

- eigenstates

$$\left|\{k\}\right\rangle_{H^K} = \sum_{x=0}^{n}\phi_{k,x}^{(n)}\left|\{x\}\right\rangle \qquad \phi_{k,x}^{(n)} = K_{k,x}^{(n)}\sqrt{\frac{\binom{n}{x}}{\binom{n}{k}2^n}}$$

with $K^{(n)}$ the **Krawtchouk polynomials**

$$K_{k,x}^{(n)} = \sum_{j=0}^{k}(-1)^j\binom{x}{j}\binom{n-x}{k-j}$$

Krawtchouk chain, details (*N*=*n*+*1*)

$$H^K = -\frac{J}{2}\sum_{x=0}^{n}\sqrt{(x+1)(n-x)}\left[X_x X_{x+1} + Y_x Y_{x+1}\right]$$

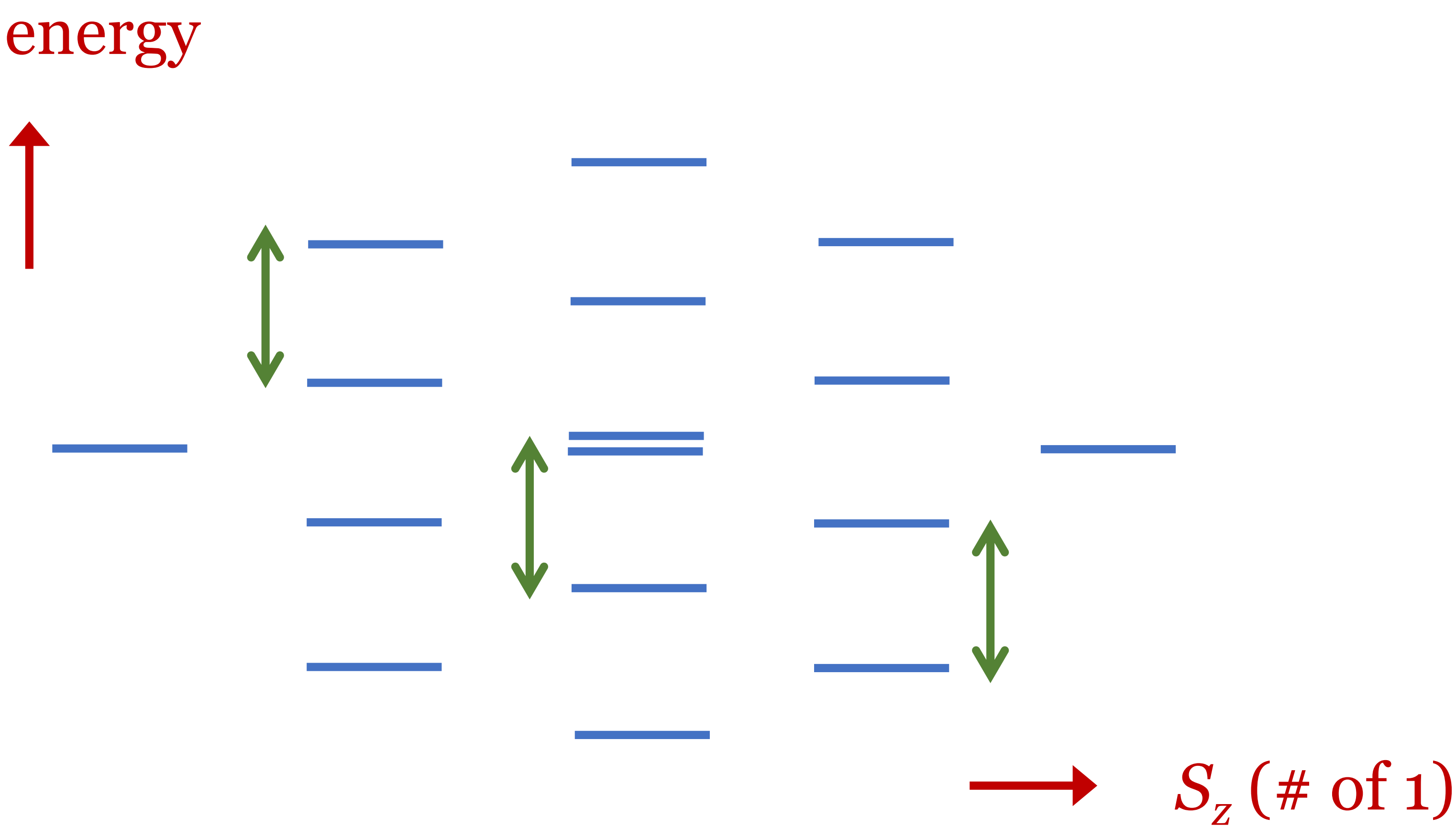
- mapping to free fermions through Jordan-Wigner transformation

$$\frac{1}{2}\left(X_j + iY_j\right) = \prod_{i=0}^{j-1}(1-2n_i)f_j \qquad \frac{1}{2}\left(X_j - iY_j\right) = \prod_{i=0}^{j-1}(1-2n_i)f_j^+$$

- many-body eigenstates built from fermionic eigenmodes

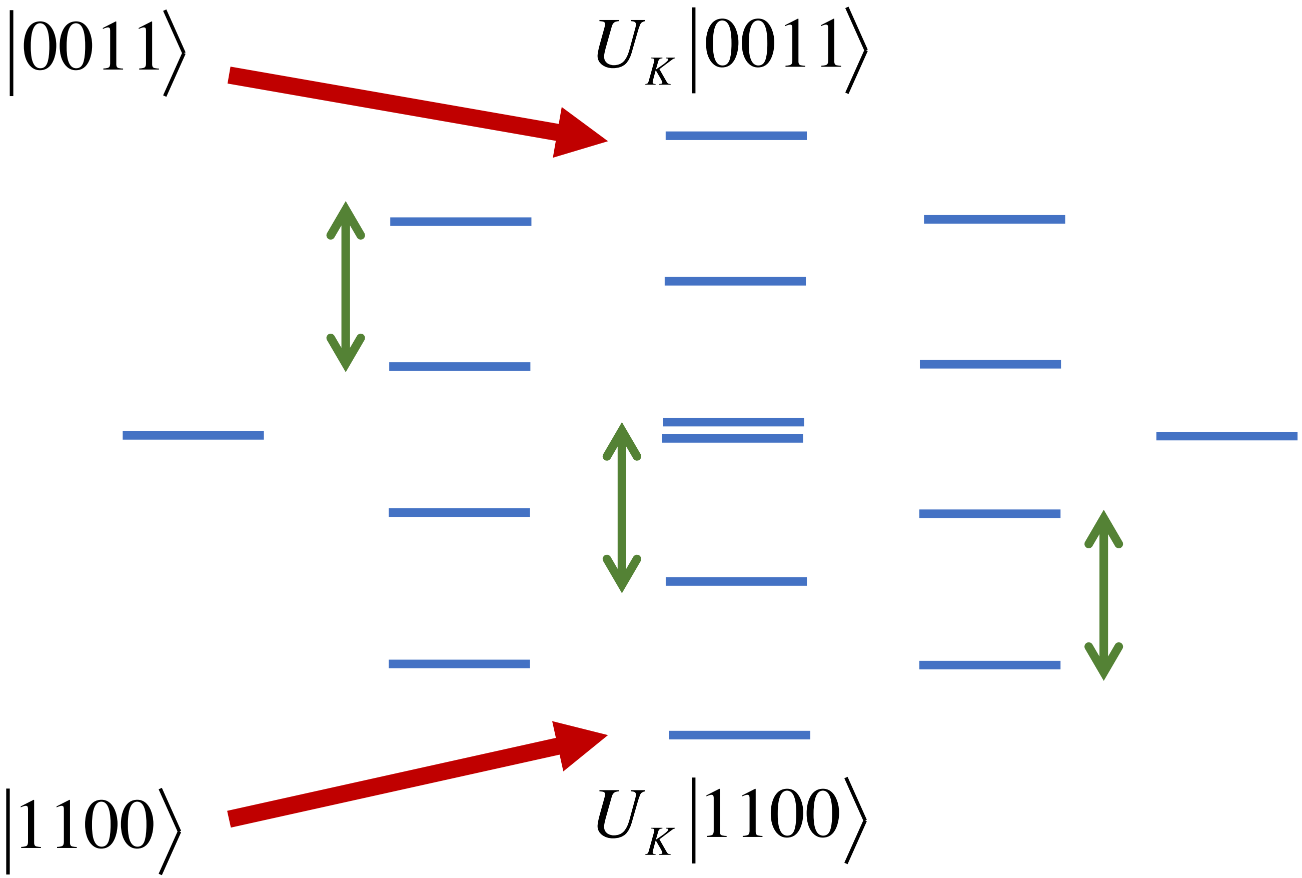
$$c_k^+ = \sum_{j=0}^{n}\phi_{k,j}^{(n)} f_j^+$$

Krawtchouk chain energy spectrum (*N=4*)



step 1:

eigengate U_K maps computational basis to eigenstates



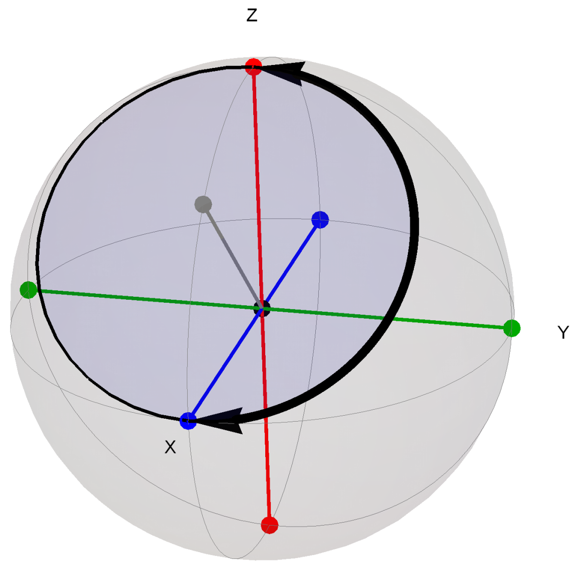
Krawtchouk eigengate

- exact *eigengate* for Krawtchouk chain eigenstates

$$U_K = \exp\left(-i\frac{\pi}{J}\frac{(H^K + H^Z)}{\sqrt{2}}\right)$$

with

$$H^Z = \frac{J}{2}\sum_{x=0}^{n}(x - \frac{n}{2})(I - Z)_x$$

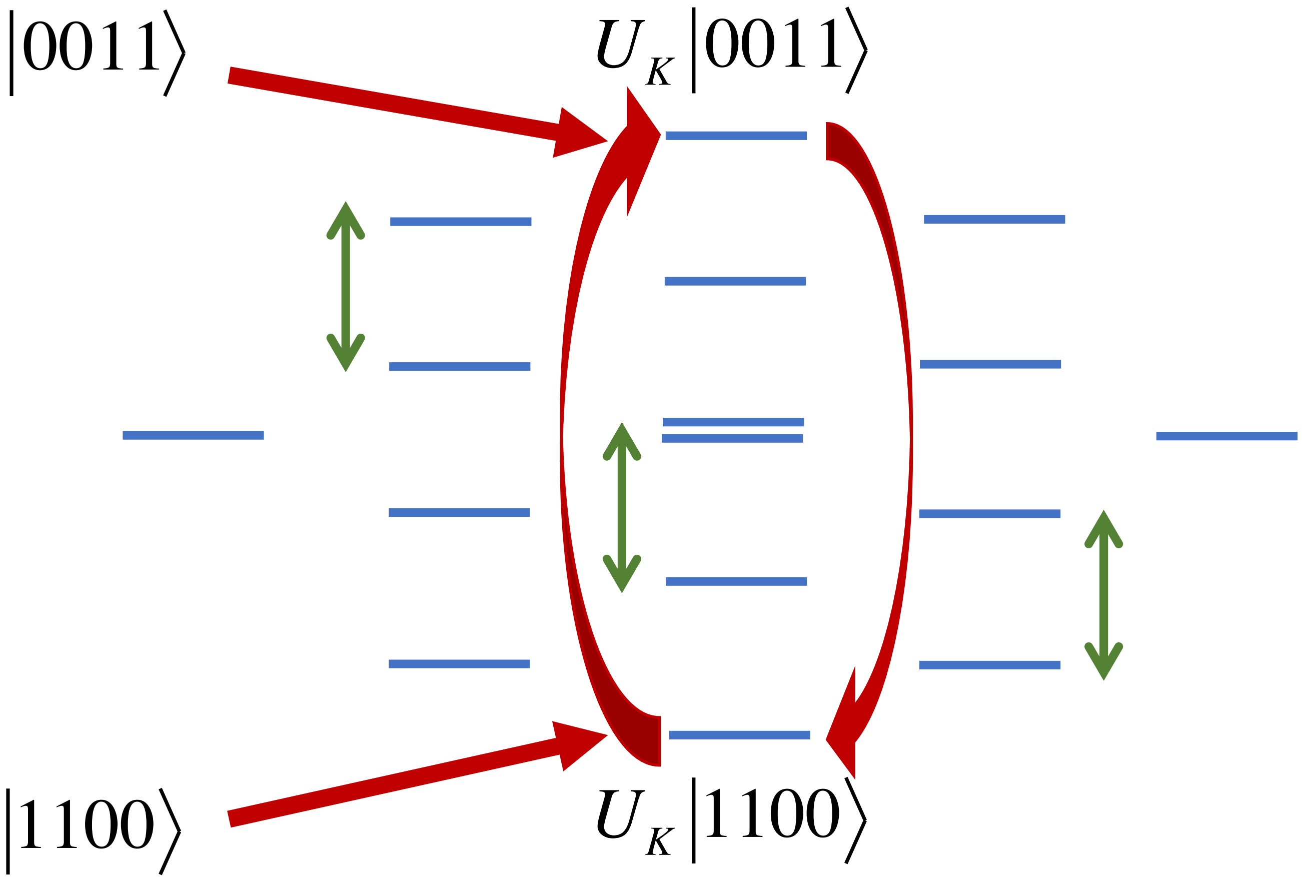


- proof: use angular momentum commutation relations of Krawtchouk operators $L_X = H^K$ and $L_Z = H^Z$ to show that

$$U_K H^Z = H^K U_K$$

step 2:

resonant driving interchanges a single pair of eigenstates



multi-qubit gate: iSWAP$_N$

- need driving term $H_D(t)$ that resonantly couples the

  highest energy state $U_K|00...01...11>$

  to the

  lowest energy state $U_K|11...10...00>$

- need to annihilate the fermionic modes with $\lambda_k>0$ and create the fermionic modes with $\lambda_k<0$ (or $\lambda_k \leq 0$)

- can be done by suitable 1-qubit or 2-qubit operator (!)

multi-qubit gate: iSWAP$_N$

- N odd, $N=n+1$, need to couple

  $U_K\,|0^{n/2+1}\,1^{n/2}>$ with $U_K\,|1^{n/2+1}\,0^{n/2}>$

- need to

  annihilate the $n/2$ fermionic modes with $\lambda_k>0$

  and

  create the $n/2+1$ modes with $\lambda_k \leq 0$

- can be done by the 1-qubit operator

$$\sigma^-_{n/2} = [1-2f_1^+ f_1]\ldots[1-2f^+_{n/2-1} f_{n/2-1}]f^+_{n/2}$$

multi-qubit gate: iSWAP$_N$

- N odd, N=n+1, matrix element for single qubit resonant driving

$$\left\langle 1^{n/2+1}0^{n/2}\right|U_K\,\sigma^-_{n/2}\,U_K\left|0^{n/2+1}1^{n/2}\right\rangle$$

$$= 2^{n/2}\left|\phi^{(n)}_{\{0,\ldots,n/2\},\{0,\ldots,n/2\}}\right|\left|\phi^{(n)}_{\{0,\ldots,n/2-1\},\{n/2+1,\ldots,n\}}\right|$$

$$= \ldots$$

$$= (-2)^{-n^2/4}$$

- exponential decay implies that driving time for resonant transition grows quickly with N

multi-qubit gate: iSWAP$_N$

- N even, need to couple

  $U_K\,|0^{N/2}\,1^{N/2}>$ to $U_K\,|1^{N/2}\,0^{N/2}>$

- need to

  annihilate the $N/2$ fermionic modes with $\lambda_k>0$

  and

  create the $N/2$ fermionic modes with $\lambda_k<0$

- can be done by the 2-qubit operator

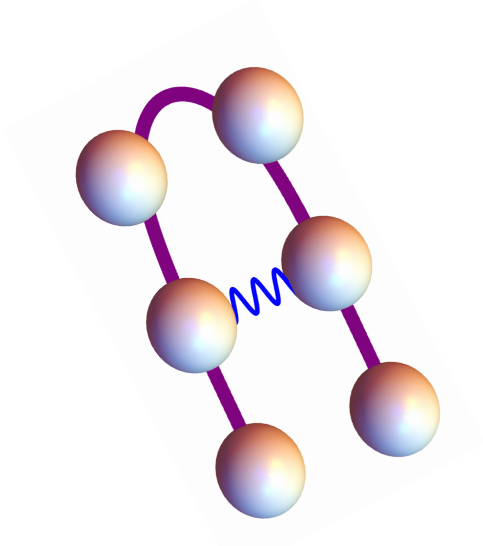
$$\sigma_j^- \sigma_{j+N/2}^+ = f_j^+[1-2f_{j+1}^+ f_{j+1}]\dots[1-2f_{j+N/2-1}^+ f_{j+N/2-1}]f_{j+N/2}$$

multi-qubit gate: iSWAP$_N$

- for N=6: matrix element

$$\langle 111000 | U_K(\sigma_1^+\sigma_4^- - \sigma_4^+\sigma_1^-)U_K | 000111\rangle = \frac{5}{32}$$

- resonant driving term



$$H_D^{(1,-)}(t) = i\,J_D \cos[9Jt]\,[\sigma_1^+\sigma_4^- - \sigma_4^+\sigma_1^-]$$

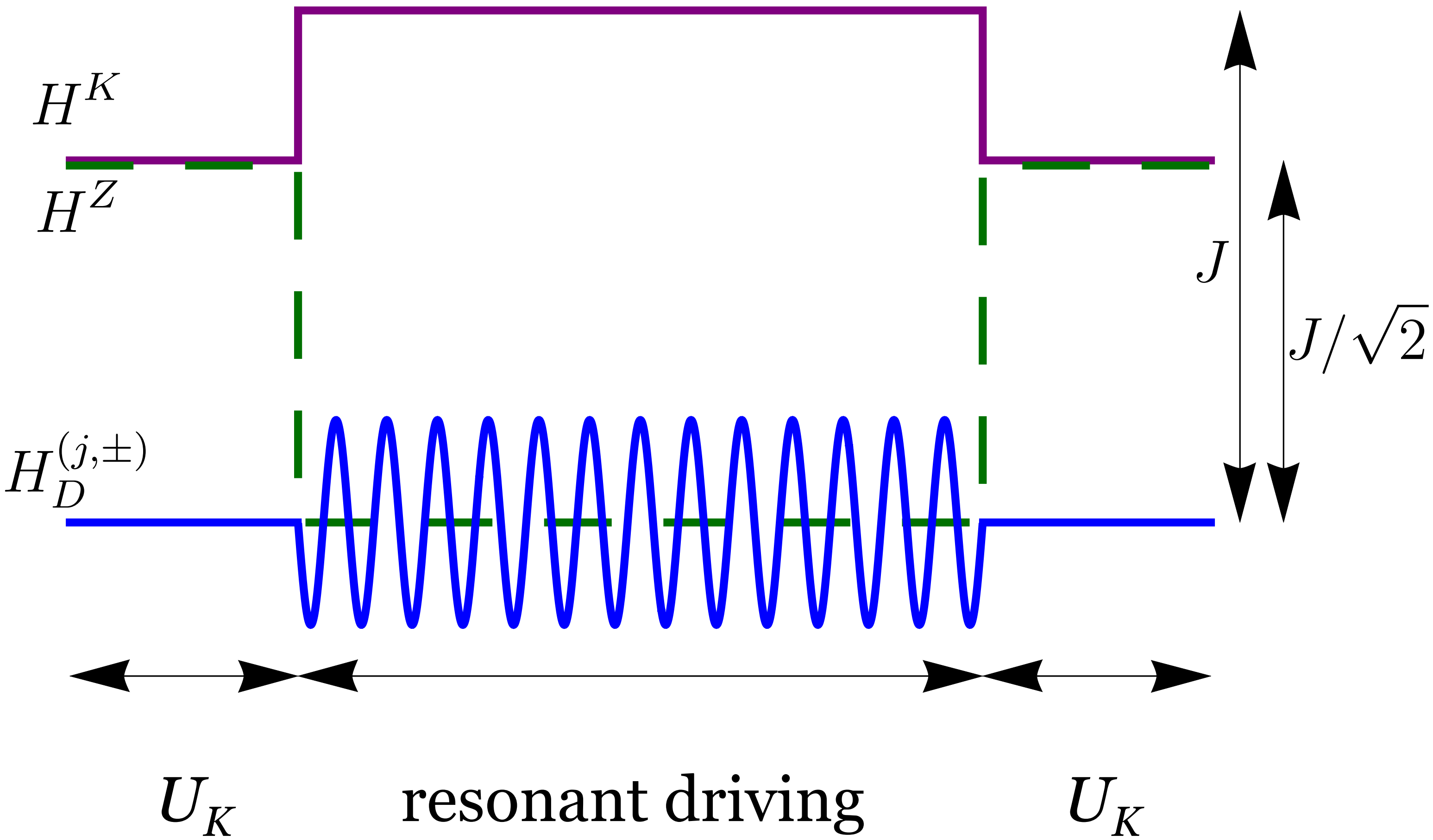
- conditions on driving time τ_D

$$\tau_D(5J_D/64) = \pi/2 \qquad \tau_D = M(2\pi/J)$$

so that (in leading order) |000111> and |111000> are interchanged and all dynamical phases return to 1

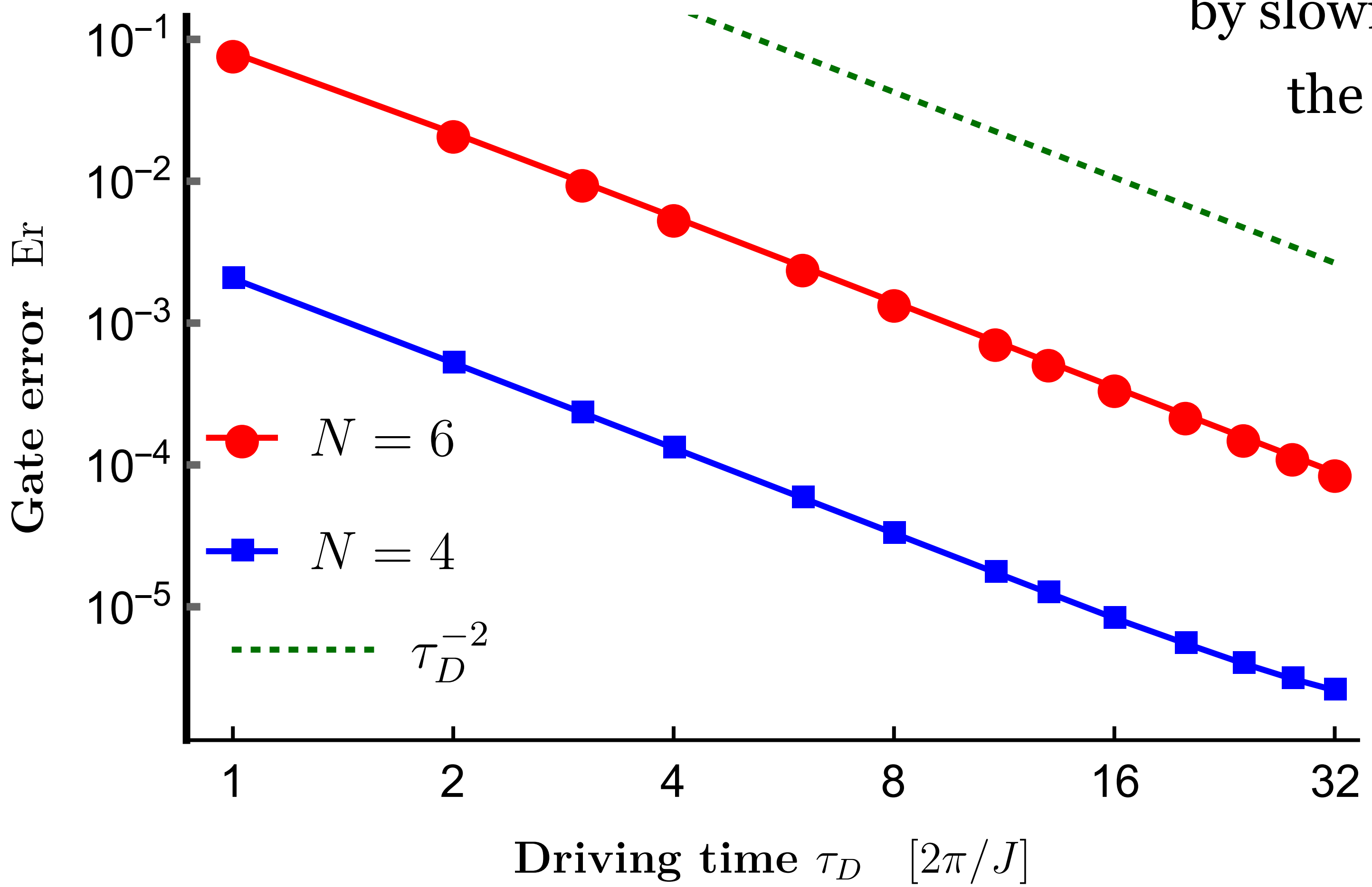
many-body protocol for iSWAP$_6$

$$|000111\rangle \rightarrow i|111000\rangle, \qquad |111000\rangle \rightarrow i|000111\rangle$$



fidelities for $iSWAP_4$ and $iSWAP_6$

gate fidelity enhanced
by slowing down
the resonant
driving



multi-qubit gates ...

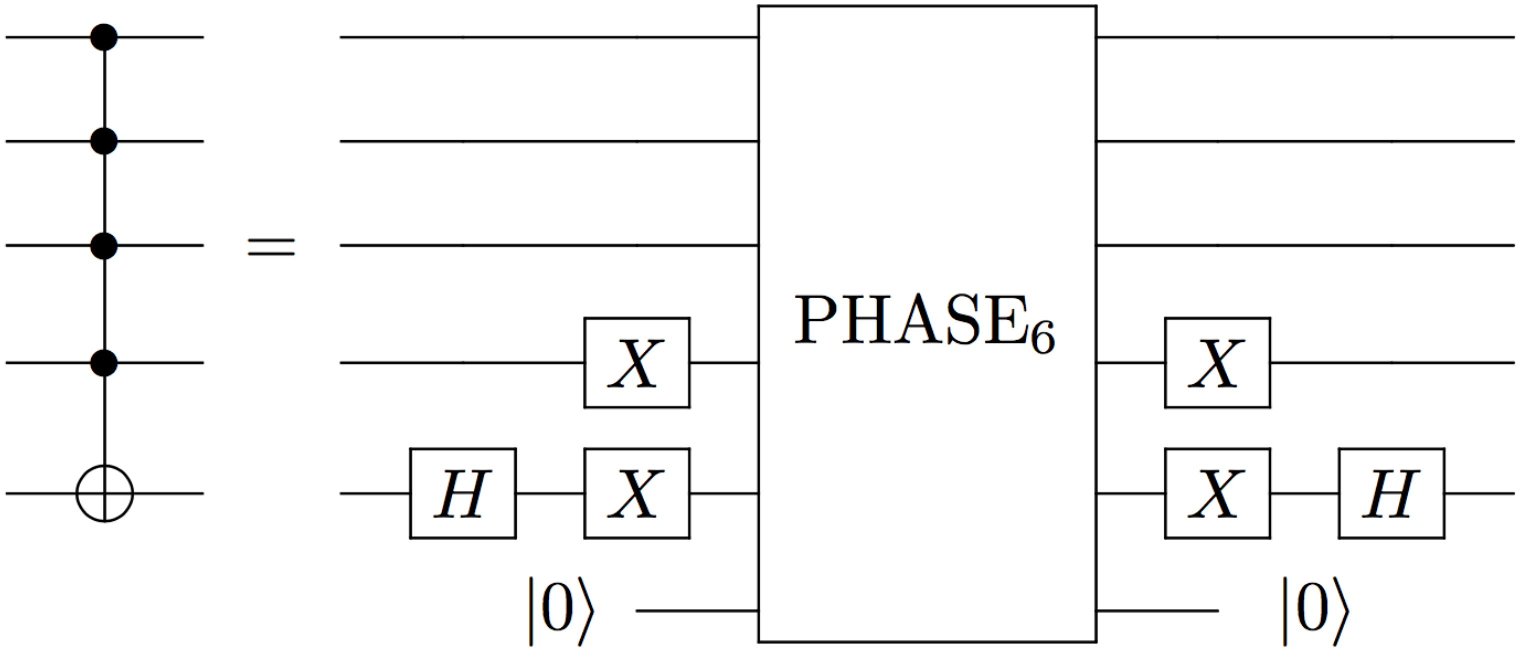
combining Krawtchouk pulse with **resonant driving** produces N-qubit gate **iSWAP**$_N$

$$|000\ldots111\ldots\rangle \rightarrow i\,|111\ldots000\ldots\rangle,$$
$$|111\ldots000\ldots\rangle \rightarrow i\,|000\ldots111\ldots\rangle$$

double–time **iSWAP**$_N$ gives **PHASE**$_N$

$$|000\ldots111\ldots\rangle \rightarrow -\,|000\ldots111\ldots\rangle,$$
$$|111\ldots000\ldots\rangle \rightarrow -\,|111\ldots000\ldots\rangle$$

multi-qubit gates …



Toffoli-5 using double strength iSWAP$_6$ gate called PHASE$_6$

Outlook

further results (with Koen Groenland)

- sensitivity to noise
- various optimizations
- other systems, such as Polychronakos/Frahm spin chains with inverse-square exchange
- …

Outlook

questions, questions …

- for large *N*, our gate times grow rapidly due to suppression of matrix elements – can this be avoided?
- fundamental `speed limits' for quantum gates – given the maximum strength of qubit-qubit interactions, how much time is needed to achieve a gate?