Factorization and Criticality in Spin Systems

Marco Cerezo[1]

In collaboration with R. Rossignoli[12], N. Canosa[1]

[1]IFLP - UNLP - CONICET, [2]CIC

Exactly Solvable Quantum Chains
June 22, 2018 @ Natal

UNIVERSIDAD NACIONAL DE LA PLATA

cerezo@fisica.unlp.edu.ar

CONICET

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- Conclusions and perspectives

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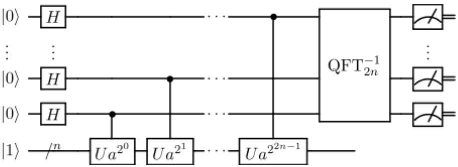

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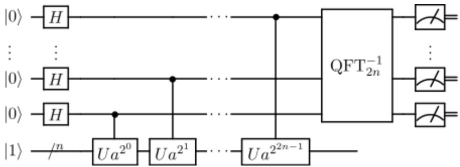
- New forms of **information transmission** : quantum teleportation, quantum cryptography.



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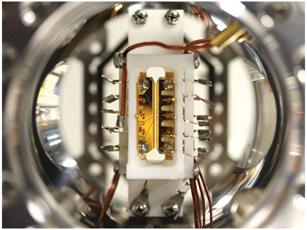

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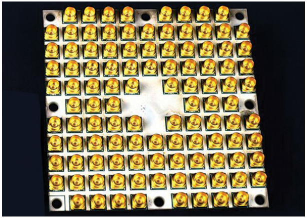

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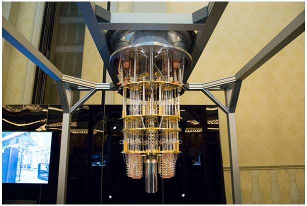

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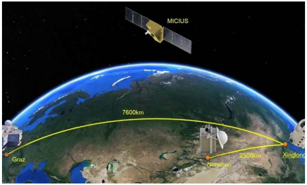

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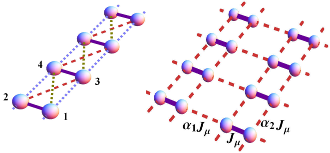
What about **spin systems** ?

$$H = -\sum_i \boldsymbol{h}^i \cdot \boldsymbol{S}_i - \frac{1}{2}\sum_{i,j} \boldsymbol{S}_i \cdot \mathcal{J}^{ij} \boldsymbol{S}_j$$

$$H = -\sum_{i,\mu} h^i_\mu S^\mu_i - \frac{1}{2}\sum_{i,j,\mu,\nu} J^{ij}_{\mu\nu} S^\mu_i S^\nu_j \,,$$

Some (basic) considerations



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In the absence of magnetic fields:

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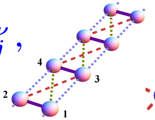

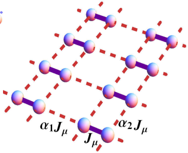

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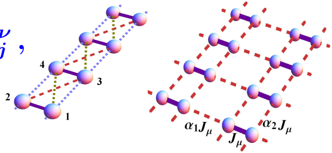

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With pairwise entanglement's range similar to that of the **interactions** .

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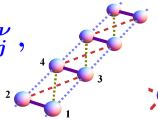

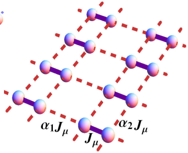

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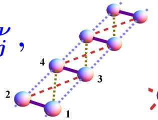

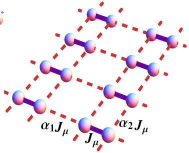

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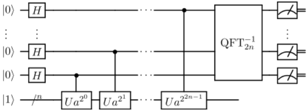
When spin systems are immersed in **finite magnetic fields** , the **GS** still remains an **entangled state** .

If we want a **separable GS** (**initialization**): "turn off" the interaction or apply strong magnetic fields ($h^i_\mu >> J^{ij}_{\mu\nu}$):

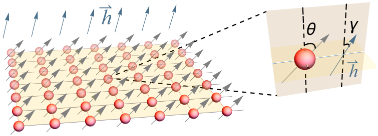
$$H = -\sum_{i,\mu} h^i_\mu S^\mu_i \cancel{- \frac{1}{2}\sum_{i,j,\mu,\nu} J^{ij}_{\mu\nu} S^\mu_i S^\nu_j} \,,$$

the spins align with the magnetic fields.



Factorizing Field

Is it possible to have a separable GS in the presence of spin interactions and finite magnetic fields?



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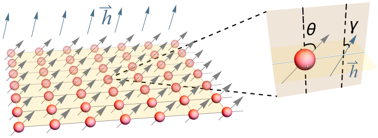

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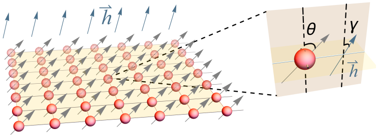

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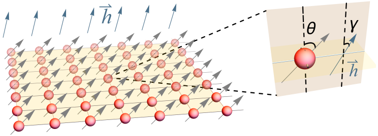

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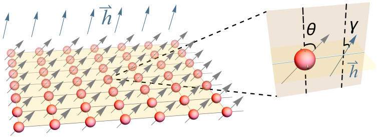

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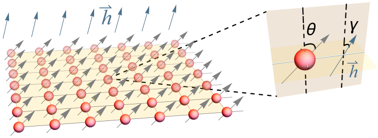

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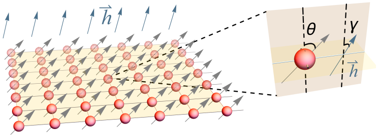

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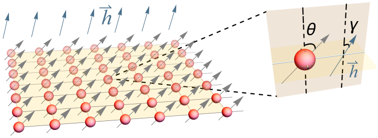

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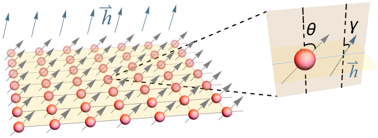

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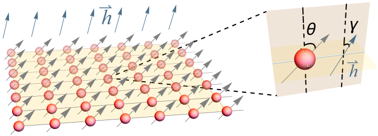

Factorized states can be used as **initial states for quantum information protocols** .

A brief history of factorization

Physica **112A** (1982) 235–255 *North-Holland Publishing Co.* Received 12 October 1981

ANTIFERROMAGNETIC LONG-RANGE ORDER IN THE ANISOTROPIC QUANTUM SPIN CHAIN

Josef KURMANN and Harry THOMAS

Institut Für Physik, Universität Basel, CH-4056 Basel, Switzerland

and

Gerhard MÜLLER

Department of Physics, University of Rhode Island, Kingston, R.I. 02881, USA

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VOLUME 93, NUMBER 16 PHYSICAL REVIEW LETTERS week ending 15 OCTOBER 2004

Studying Quantum Spin Systems through Entanglement Estimators

Tommaso Roscilde,[1] Paola Verrucchi,[2] Andrea Fubini,[2,3] Stephan Haas,[1] and Valerio Tognetti[2,3,4]

[1]*Department of Physics and Astronomy, University of Southern California, Los Angeles, California 90089-0484, USA*

[2]*Istituto Nazionale per la Fisica della Materia, UdR Firenze, Via G. Sansone 1, I-50019 Sesto F.no (FI), Italy*

[3]*Dipartimento di Fisica dell'Università di Firenze, Via G. Sansone 1, I-50019 Sesto F.no (FI), Italy*

[4]*Istituto Nazionale di Fisica Nucleare, Sezione di Firenze, Via G. Sansone 1, I-50019 Sesto F.no (FI), Italy*

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PRL **94,** 147208 (2005) PHYSICAL REVIEW LETTERS week ending 15 APRIL 2005

Entanglement and Factorized Ground States in Two-Dimensional Quantum Antiferromagnets

Tommaso Roscilde,[1] Paola Verrucchi,[2,3] Andrea Fubini,[4,6] Stephan Haas,[1] and Valerio Tognetti[2,4,5]

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[3]*Istituto Sistemi Complessi - C.N.R., Sez. di Firenze, via Madonna del Piano, I-50019 Sesto F.no (FI), Italy*

[4]*Dipartimento di Fisica dell'Università di Firenze, Via G. Sansone 1, I-50019 Sesto F.no (FI), Italy*

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[6]*MATIS-INFM & DMFCI, Università di Catania, V.le A. Doria 6, I-95125 Catania, Italy*

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PRL **100,** 197201 (2008) PHYSICAL REVIEW LETTERS week ending 16 MAY 2008

Theory of Ground State Factorization in Quantum Cooperative Systems

Salvatore M. Giampaolo,[1,2] Gerardo Adesso,[1,2] and Fabrizio Illuminati[1,2,3,*]

[1]Dipartimento di Matematica e Informatica, Università degli Studi di Salerno, Via Ponte don Melillo, I-84084 Fisciano (SA), Italy
[2]CNR-INFM Coherentia, Napoli, Italy; CNISM, Unità di Salerno, Italy; and INFN, Sezione di Napoli—Gruppo Collegato di Salerno, Italy
[3]ISI Foundation for Scientific Interchange, Viale Settimio Severo 65, I-10133 Turin, Italy
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Probing Quantum Frustrated Systems via Factorization of the Ground State

Salvatore M. Giampaolo,[1] Gerardo Adesso,[2] and Fabrizio Illuminati[1,*]
[1]*Dipartimento di Matematica e Informatica, Università degli Studi di Salerno, CNR-SPIN, CNISM, Unità di Salerno, and INFN, Sezione di Napoli-Gruppo Collegato di Salerno, Via Ponte don Melillo, I-84084 Fisciano (SA), Italy*
[2]*School of Mathematical Sciences, University of Nottingham, University Park, Nottingham NG7 2RD, United Kingdom*
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A brief history of factorization

Physica **112A** (1982) 235–255 *North-Holland Publishing Co.* Received 12 October 1981

ANTIFERROMAGNETIC LONG-RANGE ORDER IN THE ANISOTROPIC QUANTUM SPIN CHAIN

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...asel, Switzerland

VOLUME 93, NUMBER 16 PHYSICAL REVIEW LETTERS week ending 15 OCTOBER 2004

Studying Quantum Spin Systems through Entanglement Estimators

Tommaso Roscilde,[1] Paola Verrucchi,[2] Andrea Fubini,[2,3] St...
[1]Department of Physics and Astronomy, University of Southern Califor...
[2]Istituto Nazionale per la Fisica della Materia, UdR Firenze, Via ...
[3]Dipartimento di Fisica dell'Università di Firenze, Via G. San...
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PRL **94**, 147208 (2005) PHYSICAL REVIEW LETTERS week ending 15 APRIL 2005

Entanglement and Factorized Ground States in Two-Dimensional Quantum Antiferromagnets

...[2,3] Andrea Fubini,[4,6] Stephan Haas,[1] and Valerio Tognetti[2,4,5]
...iversity of Southern California, Los Angeles, CA 90089-0484, USA
...eria, UdR Firenze, Via G. Sansone 1, I-50019 Sesto F.no (FI), Italy
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PRL **100**, 197201 (2008) PHYSICAL REVIEW LETTERS week ending 16 MAY 2008

Theory of Ground State Factorization in Quantum Cooperative Systems

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PRL **104**, 207202 (2010) PHYSICAL REVIEW LETTERS week ending 21 MAY 2010

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PHYSICAL REVIEW A **77**, 052322 (2008)

Entanglement of finite cyclic chains at factorizing fields

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Transverse field!

Factorization general equations

System of N spins s_i interacting through XYZ Heisenberg couplings of arbitrary range in the presence of a general magnetic fields h^i

$$\boldsymbol{H = -\sum_i h^i \cdot S_i - \frac{1}{2}\sum_{i,j} S_i \cdot \mathcal{J}^{ij} S_j}$$

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The completely separable state

$$|\Theta\rangle = \otimes_{i=1}^{n} e^{-\imath\phi_i S_i^z} e^{-\imath\theta_i S_i^y} |\uparrow_i\rangle = |\nearrow\swarrow\nwarrow\ldots\rangle,$$

is an **exact eigenstate** iff it satisfies[1] :

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1. Field independent equations: **Which state?**

$$\boldsymbol{n}_i^{x'} \cdot \mathcal{J}^{ij} \boldsymbol{n}_j^{x'} = \boldsymbol{n}_i^{y'} \cdot \mathcal{J}^{ij} \boldsymbol{n}_j^{y'}, \quad \boldsymbol{n}_i^{x'} \cdot \mathcal{J}^{ij} \boldsymbol{n}_j^{y'} = -\boldsymbol{n}_i^{y'} \cdot \mathcal{J}^{ij} \boldsymbol{n}_j^{x'}$$

with $\boldsymbol{n}_i^{x',y'} \perp \boldsymbol{n}_i \equiv \langle \boldsymbol{S}_i \rangle / s_i$ spin alignment direction

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2. The field-dependent conditions: **What fields?**

$$\boldsymbol{n}_i \times (\boldsymbol{h}_i + \sum_j \mathcal{J}^{ij} \langle \boldsymbol{S}_j \rangle) = \boldsymbol{0}\,,$$

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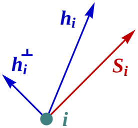
which implies $\boldsymbol{h}_i = \boldsymbol{h}_i^{\perp} + \boldsymbol{h}_i^{\parallel}$. $\quad (\boldsymbol{h}_i^{\parallel} = h_i \boldsymbol{n}_i\,, \quad \boldsymbol{n}_i \cdot \boldsymbol{h}_i^{\perp} = 0)$



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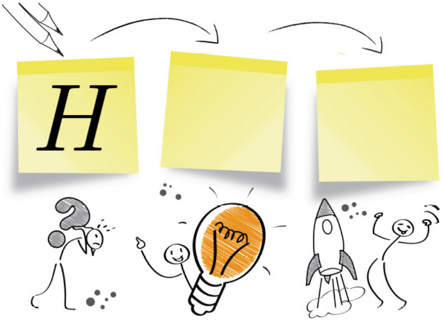

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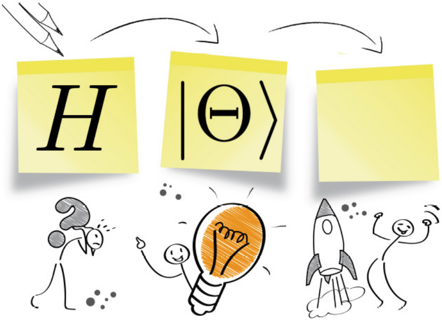

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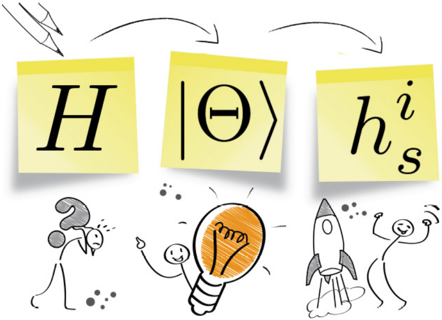

The XYZ case

Spin array with anisotropic XYZ couplings in a general fields.

$$H = -\sum_i \boldsymbol{h}^i \cdot \boldsymbol{S}_i - \sum_{i \neq j} J_x^{ij} S_i^x S_j^x + J_y^{ij} S_i^y S_j^y + J_z^{ij} S_i^z S_j^z$$

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Recap results in **FM** and **AFM** systems

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Recap results in **FM** and **AFM** systems $\Rightarrow$ **Revise** them with our general equations.

Antiferromagnetic spin chain, Recap

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possess a **Néel-type** separable GS $|\nwarrow\nearrow\nwarrow\nearrow\ldots\rangle$ if the factorizing fields point to the surface of an **ellipsoid** .

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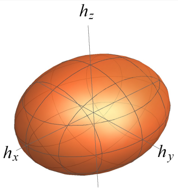
$$\frac{h_x^2}{(J_x+J_z)(J_x+J_y)} + \frac{h_y^2}{(J_y+J_z)(J_y+J_x)} + \frac{h_z^2}{(J_z+J_x)(J_z+J_y)} = 1$$



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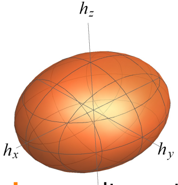
$$\frac{h_x^2}{(J_x+J_z)(J_x+J_y)} + \frac{h_y^2}{(J_y+J_z)(J_y+J_x)} + \frac{h_z^2}{(J_z+J_x)(J_z+J_y)} = 1$$



The Néel separable GS breaks **translational invariance** , it must arise at a GS level crossing and be **two-fold degenerate** .

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Ferromagnetic spin systems, Recap

Anisotropic XYZ systems with first neighbor couplings immersed in transverse fields [1]

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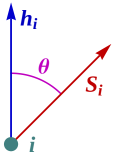

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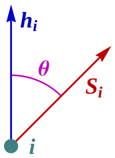

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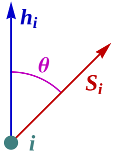
- $|\Theta\rangle$ is a **GS** in FM-type systems ($0 \leq |J_{ij}^y| \leq J_x^{ij}$ etc.)
- $|\Theta\rangle$ breaks **parity symmetry** ($[H, P_z] = 0, \quad P_z = e^{i\pi S_z}$). The GS is **two-fold degenerate** : linear combinations of the symmetry preserving entangled crossing states.



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Anisotropic FM and AFM XYZ systems, Revised

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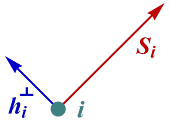

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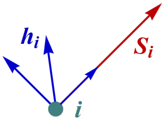

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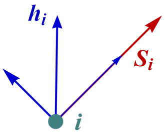

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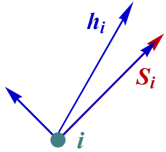

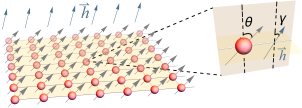

Feasible with a **uniform field** !

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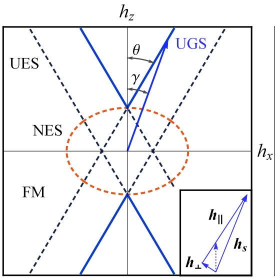

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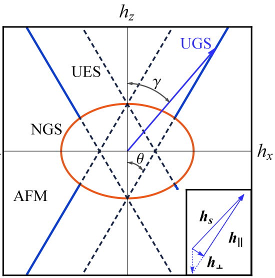

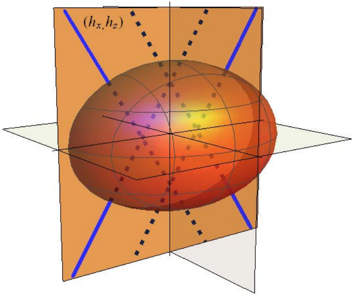

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Factorization and entanglement

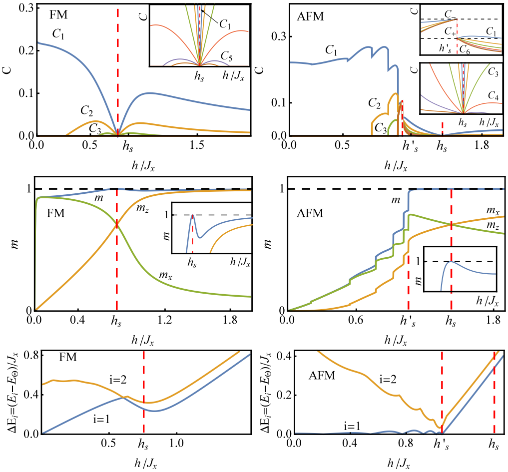
Spin-$1/2$ chain of $N = 12$ spins.

FM **AFM**

Concurrence

Magnetization

Gap



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The XYZ case

General arrays of spins s_i with XXZ couplings immersed in **nonuniform transverse fields**

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Since $[H, S^z] = 0$, its eigenstates can be characterized by their **total magnetization** M along z.

Factorization General Equations

The completely separable state

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1. Field independent equations: **Which state** ?

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2. The field-dependent conditions: **What fields?**

$$h_s^i = \sum_j s_j \nu_{ij} J^{ij} \sqrt{\Delta_{ij}^2 - 1}$$

with $\nu_{ij} = \pm 1$ the sign in (1). This Eq. is independent of the angles θ_i and must fulfill the **zero sum condition** $\sum_i s_i h_s^i = 0$.

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- The components of $|\Theta\rangle$ with definite M are also eigenstates of H with the same energy E_Θ .

$$P_M|\Theta\rangle \propto \sum_{\substack{m_1,\ldots,m_N \\ \sum_i m_i = M}} [\prod_{i=1}^{N} \sqrt{\binom{2s_i}{s_i - m_i}} \eta_{i,i+1}^{\sum_{j=1}^{i} m_j}]|m_1 \ldots m_N\rangle\,, \quad M = -S \ldots S\,,$$

where $P_M = \frac{1}{2\pi}\int_0^{2\pi} e^{i\varphi(S^z - M)} d\varphi$ is the **projector** onto total magnetization M . These states are **entangled** $\forall |M| \leq S-1$.

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Fundamental Properties

$$|\Theta\rangle = \otimes_{i=1}^{n} e^{-\imath\phi_i S_i^z} e^{-\imath\theta_i S_i^y} |\uparrow_i\rangle = |\nearrow\swarrow\nwarrow\ldots\rangle$$

- The energy of the separable state is $E_\Theta = -\sum_{i<j} s_i s_j J_z^{ij}$, which **coincides with the energy of the maximally aligned state** $|M| = S \equiv \sum_i s_i$.
- If $J_z^{ij} \geq 0\ \forall i,j$, $|\Theta\rangle$ is the **GS** of the system.
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- In the vicinity of factorization, entanglement reaches **full range** .

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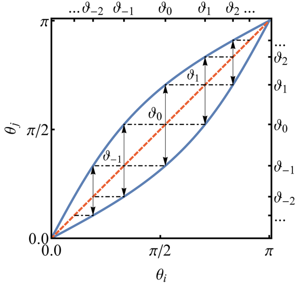

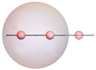

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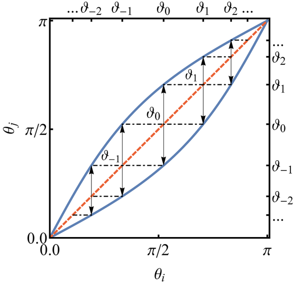

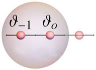

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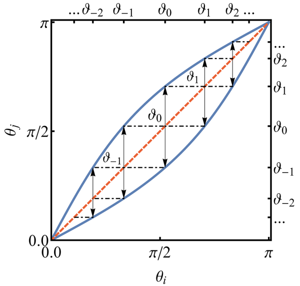

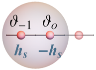

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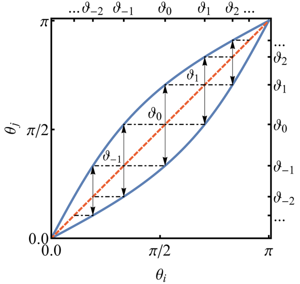

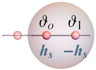

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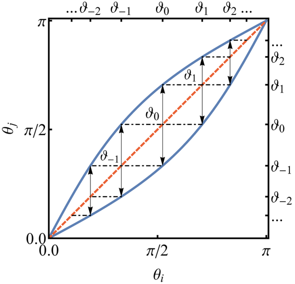

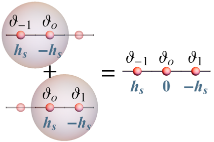

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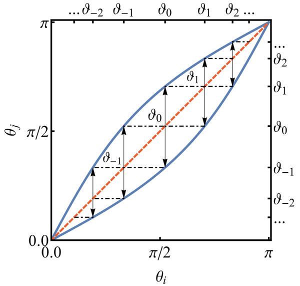

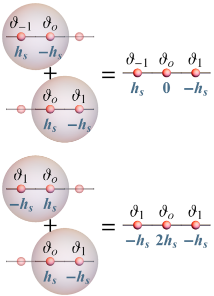

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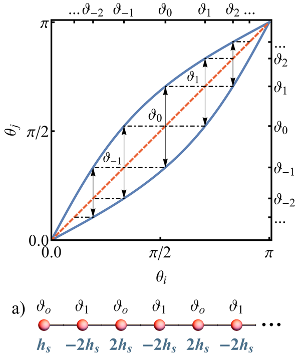

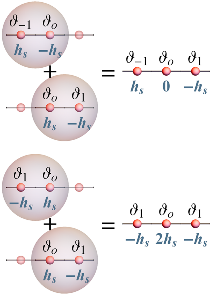

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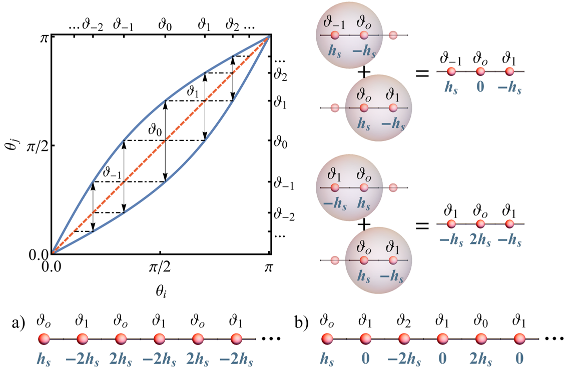

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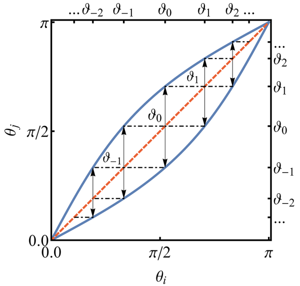

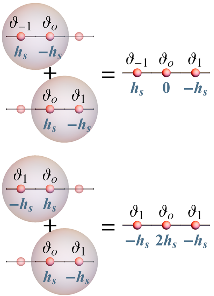

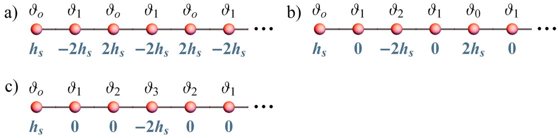

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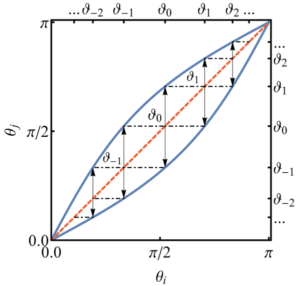

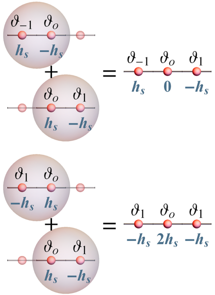

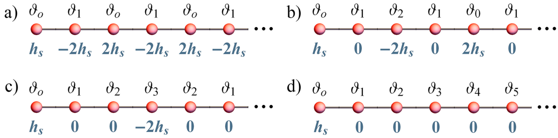

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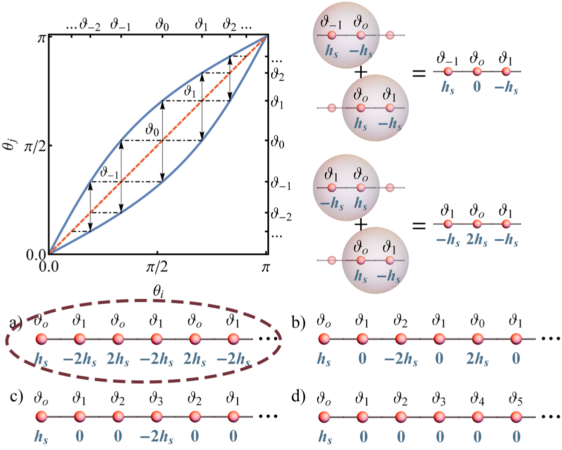

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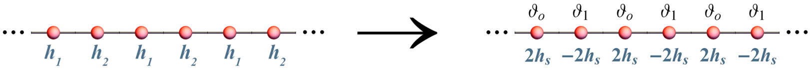

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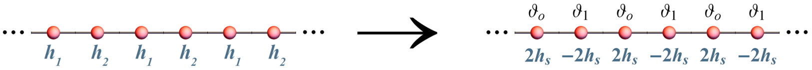

By projecting onto magnetization M we can determine **analytical expressions for the reduce state of any spin pair** . For a d -dimensional spin-s system with uniform anisotropy Δ and **alternating fields** , there are just 3 distinct reduced pair states ρ_{oe}^{M} (odd-even), ρ_{oo}^{M} y ρ_{ee}^{M}

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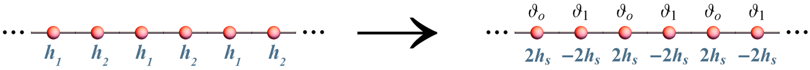

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$$(\rho_{ij}^M)^m_{m_j,m_j'} = \eta^{f_{ij}} \frac{\sqrt{C^{s,m}_{m_j} C^{s,m}_{m_j'}}\, Q^{M-m,(\delta+2l_{ij})s}_{Ns-2s-M+m}(\eta)}{Q^{M,\delta s}_{Ns-M}(\eta)}$$

with $m = m_i + m_j = m_i' + m_j'$ the pair magnetization ($[\rho^M_{i,j}, S_i^z + S_j^z] = 0$), $Q_n^{m,k}(\eta) = (\eta^2-1)^n P_n^{m-k,m+k}(\frac{\eta^2+1}{\eta^2-1})$ with $P_n^{\alpha,\beta}(x)$ the **Jacobi polynomials** . $C_k^{s,m} = \binom{2s}{s-k}\binom{2s}{s-m+k}$ and $f_{ij} = 2s - m_j - m_j', 0, 4s-2m$, $l_{ij} = 0, -1, 1$ for the oe, oo, ee pairs, and $\delta = 0(1)$ if N is even (odd).

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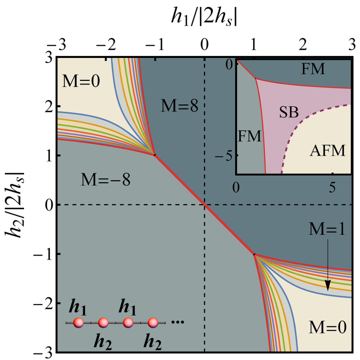
Magnetization and Entanglement



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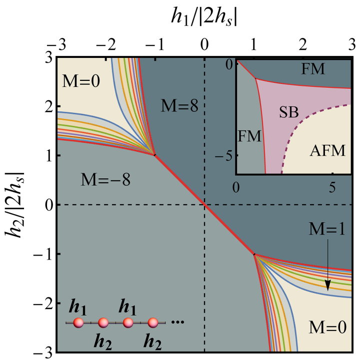

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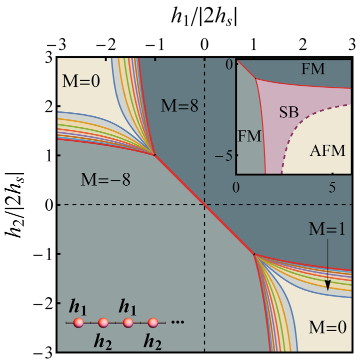

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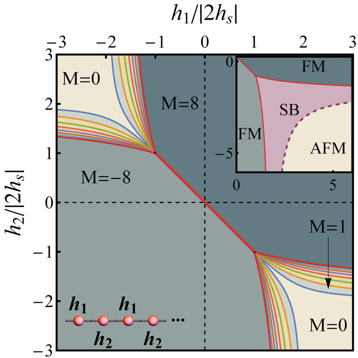

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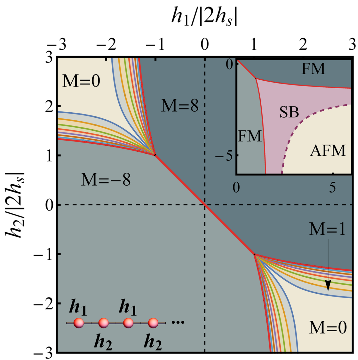

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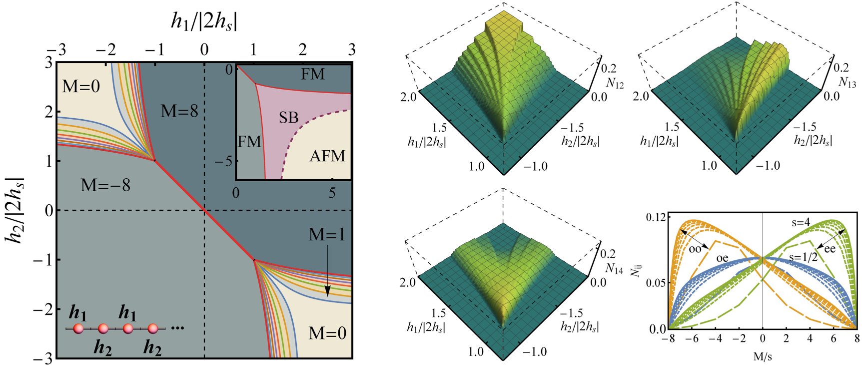

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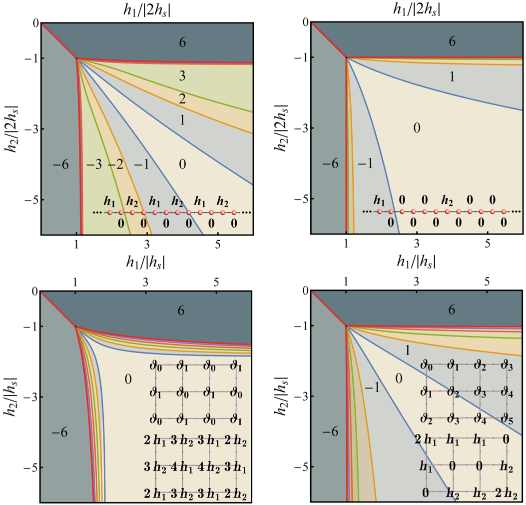
Other field configuration



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Bonus

How many state / field configurations are there?

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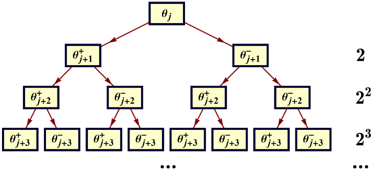

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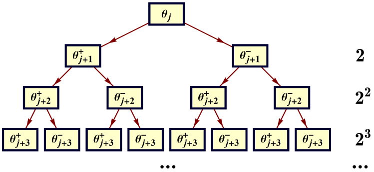

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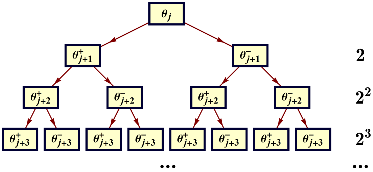

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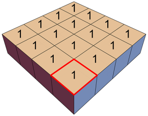
ϑ_o	ϑ_1	ϑ_2	ϑ_3
ϑ_1	ϑ_2	ϑ_3	ϑ_4
ϑ_2	ϑ_3	ϑ_4	ϑ_5
ϑ_3	ϑ_4	ϑ_5	ϑ_6

$2h_s$	h_s	h_s	0
h_s	0	0	$-h_s$
h_s	0	0	$-h_s$
0	$-h_s$	$-h_s$	$-2h_s$



$(i+j-k)$ with i , j the site
k the number of steps from θ_{11}

nondecreasing terrace forms

ϑ_o	ϑ_1	ϑ_o	ϑ_1
ϑ_1	ϑ_o	ϑ_1	ϑ_o
ϑ_o	ϑ_1	ϑ_o	ϑ_1
ϑ_1	ϑ_o	ϑ_1	ϑ_o

$2h_s$	$-3h_s$	$3h_s$	$-2h_s$
$-3h_s$	$4h_s$	$-4h_s$	$3h_s$
$3h_s$	$-4h_s$	$4h_s$	$-3h_s$
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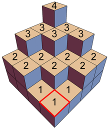
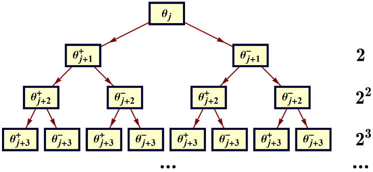


[1] MC, R. Rossignoli, N. Canosa, and E. Rios, Phys. Rev. Lett. **119**, 220605 (2017).

Bonus

How many state / field configurations are there?



For a chain there are 2^{N-1} configurations.

ϑ_o	ϑ_1	ϑ_2	ϑ_3
ϑ_1	ϑ_2	ϑ_3	ϑ_4
ϑ_2	ϑ_3	ϑ_4	ϑ_5
ϑ_3	ϑ_4	ϑ_5	ϑ_6

$2h_s$	h_s	h_s	0
h_s	0	0	$-h_s$
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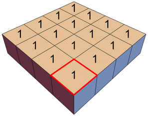

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ϑ_1	ϑ_o	ϑ_1	ϑ_o
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ϑ_1	ϑ_o	ϑ_1	ϑ_o

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$-3h_s$	$4h_s$	$-4h_s$	$3h_s$
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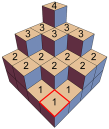

$L(2,N) = 2\times 3^{N-1}$

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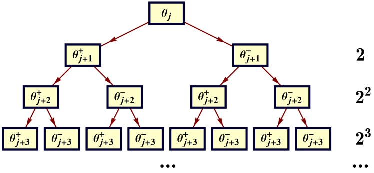

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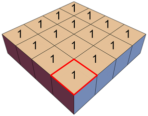

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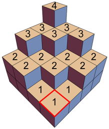
$3h_s$	$-4h_s$	$4h_s$	$-3h_s$
$-2h_s$	$3h_s$	$-3h_s$	$2h_s$



$$L(2,N) = 2\times 3^{N-1}$$
$$L(3,N) = \alpha_+\lambda_+^N + \alpha_-\lambda_-^N$$
$$\lambda_\pm = \frac{5\pm\sqrt{17}}{2}\,,\quad \alpha_\pm = \frac{1\pm 3/\sqrt{17}}{2}$$

[1] MC, R. Rossignoli, N. Canosa, and E. Rios, Phys. Rev. Lett. **119**, 220605 (2017).

Bonus

For $1 \times N$	1,	2,	4,	8,	16
For $2 \times N$	2,	6,	18,	54,	162
For $3 \times N$	4,	18,	82,	374,	1706

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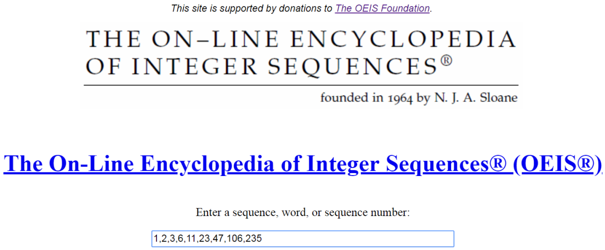
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[1] MC, R. Rossignoli, N. Canosa, and E. Rios, Phys. Rev. Lett. **119**, 220605 (2017).
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Journal of Integer Sequences, Vol. 17 (2014),
Article 14.10.8

Counting Miura-ori Foldings

Jessica Ginepro[1]
Department of Mathematics
University of Connecticut
196 Auditorium Road, Unit 3009
Storrs, CT 06269-3009
USA

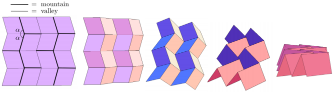

Figure 1: A 4×4 Miura-ori with the standard MV assignment. Bold creases are mountains and non-bold creases are valleys.

1 Introduction

In the mathematics of origami (paper folding), enumerating the number of ways in which a crease pattern can fold up is often difficult. Even the seemingly simple postage-stamp

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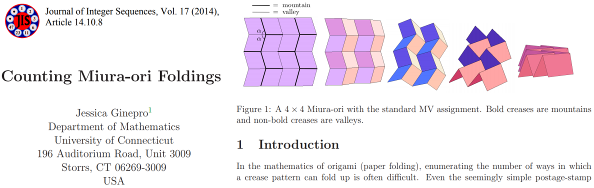

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By defining $A(1) = (1)$ and

$$A(M+1) = \begin{pmatrix} A(M) & A(M)^T \\ 0 & A(M) \end{pmatrix}$$

with $B(M) = A(M) + A(M)^T$, the **total number of configurations** is

$$L(M,N) = \sum_{i,j} (B^{N-1}(M))_{ij}$$

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Separable state engineering

Given a spin system (i.e., given the $\mathcal{J}^{ij}$)

What separable eigenstates can the system posses?

$$\boldsymbol{n}_i^{x'} \cdot \mathcal{J}^{ij} \boldsymbol{n}_j^{x'} = \boldsymbol{n}_i^{y'} \cdot \mathcal{J}^{ij} \boldsymbol{n}_j^{y'}, \quad \boldsymbol{n}_i^{x'} \cdot \mathcal{J}^{ij} \boldsymbol{n}_j^{y'} = -\boldsymbol{n}_i^{y'} \cdot \mathcal{J}^{ij} \boldsymbol{n}_j^{x'} \tag{1}$$

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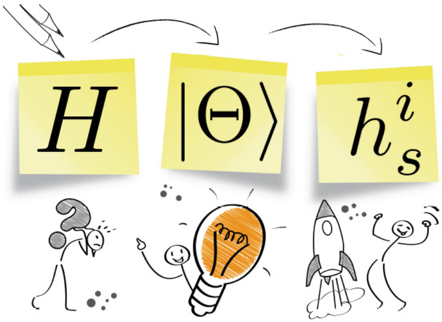

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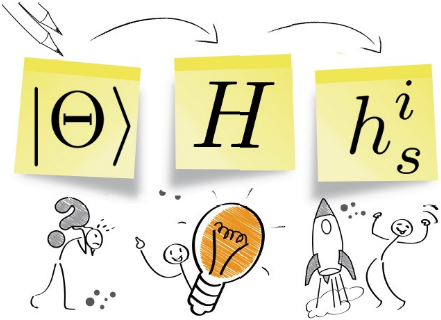
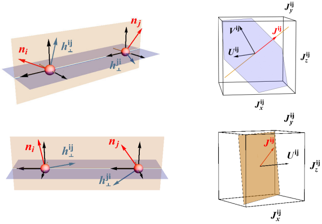


If some control over the **couplings** and the **fields** is feasible, then **YES!**

[1] MC, R. Rossignoli, and N. Canosa, Phys. Rev. A **94**, 042335 (2016).

Properties (I)

- **Lemma 1** : Given $\boldsymbol{n}_i$ and $\boldsymbol{n}_j$ **arbitraries** , there **always exists** a nonzero XYZ -type coupling: $J^{ij}_{\mu\nu} = J^{ij}_{\mu}\delta_{\mu\nu}$ satisfying **(1)** .



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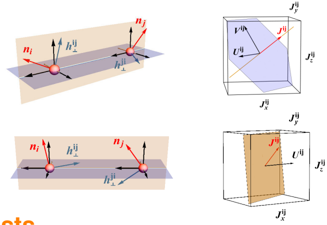

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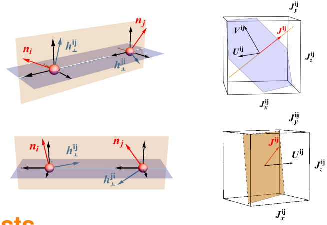

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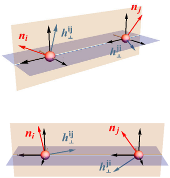

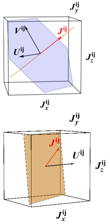

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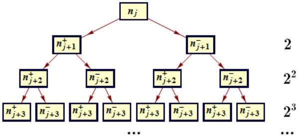

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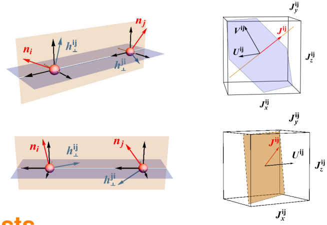

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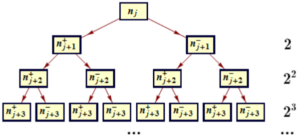
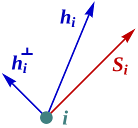


- **Lemma 3** : $|\boldsymbol{\Psi}_s\rangle$ can always become a **nondegenerate GS** of H with a **controllable gap** .



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Properties (II)

- **Lemma 4** : If $s_i = s_j$ and the coupling is of the XYZ -type. Given $\boldsymbol{n}_i$ and $\boldsymbol{n}_j$, there **always exists** a **uniform** factorizing field:

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- **Lemma 5** : Pairwise entanglement reaches **full range** in the vicinity of factorization. $\rightarrow$ **Quantum critical point.**

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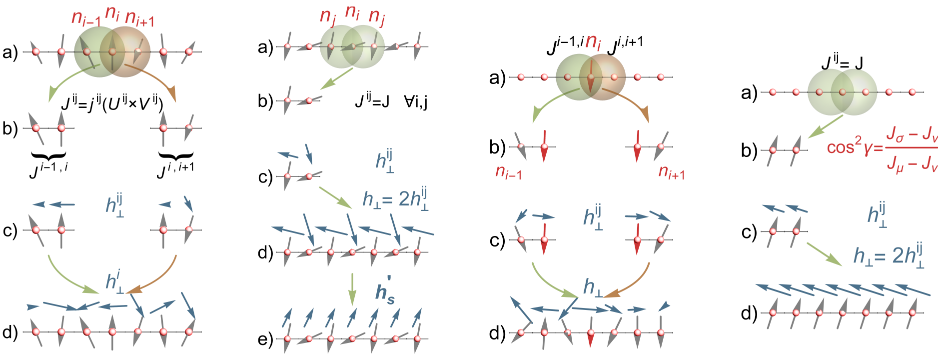
Fixed and tunable couplings

Tunable couplings

Fixed couplings



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Conclusions and perspectives

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Thanks for your attention!

Our works:

M. Cerezo, R. Rossignoli, N. Canosa, and E. Ríos, Phys. Rev. Lett. 119, 220605 (2017).
M. Cerezo, R. Rossignoli, and N. Canosa, Phys. Rev. A 94, 042335 (2016).
M. Cerezo, R. Rossignoli, and N. Canosa, Phys. Rev. B 92, 224422 (2015).