Workshop: "Exactly Solvable Quantum Chains"

International Institute of Physics, Natal, 25 Juin 2018

Conformal Fishnet Theory in Four Dimensions as Integrable Non-Compact Spin Chain

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Collaborations with

Ö. Gürdogan	arXiv:1512.06704
J. Caetano and Ö. Gürdogan	arXiv:1612.05895
D. Grabner, N. Gromov, G. Korchemsky	arXiv:1711.04786
N. Gromov, G. Korchemsky, S. Negro, G. Sizov	arXiv:1706.04167
D. Chicherin, F. Loebbert, D. Mueller, D. Zhang	arXiv:1704.01967
	arXiv:1708.00007
E. Olivucci	arXiv:1801.09844

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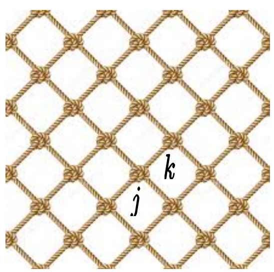
Outline

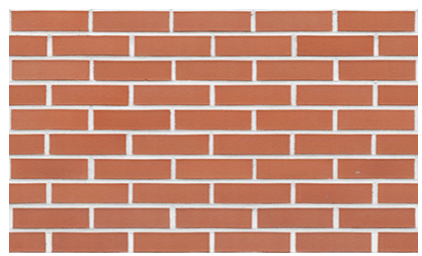
- Exactly solvable conformal field theories (CFT) in D=4 dimensions?
- A unique all-loop integrable 4D CFT: planar N=4 Super-Yang-Mills (SYM)
- Integrable deformations destroying supersymmetry,e.g. γ-deformation of N=4 SYM
- We propose a new family of integrable non-supersymmetric 4D CFTs: Gurdogan, V.K. 2015
  Double scaling limit of N=4 SYM: strong γ-deformation & weak coupling
- Leads to non-unitary, logarithmic “chiral” CFT, d ominated by specific, integrable (computable!) multi-loop 4D Feynman graphs

“fishnet” A. Zamolodchikov 1980
(massless, 4-scalar int.)

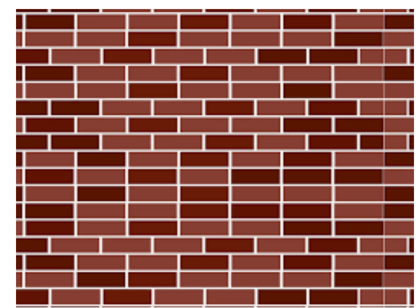
$$\int \prod_i d^4x_i \prod_{<jk>} \frac{1}{(x_j - x_k)^2}$$

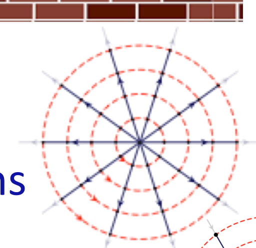


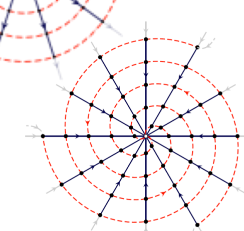
“brick wall” Gurdogan, V.K. 2015
Caetano, Gurdogan, V.K. 2016
(Yukawa ψψϕ interaction)



mix of both







- Integrable 2D lattice models, (1+1)D non-compact SU(2,2) spin chains
- Anomalous dimensions from “wheel” and “spiral” (spiderweb) graphs (computable via Quantum Spectral Curve) Gromov, V.K. ,Leurent, Volin 2013
- Exact 4-point correlation functions and OPE data

γ-twisted N=4 SYM and double scaling limit

- Lagrangian of N=4 SYM has $N_c \times N_c$ matrix gauge, scalar, and fermion fields:

$$\mathcal{L} = N_c \mathrm{tr}[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D^\mu \phi_i^\dagger D_\mu \phi^i + i\bar{\psi}_A^{\dot{\alpha}} D_{\dot{\alpha}}^{\alpha} \psi_\alpha^A] + \mathcal{L}_{\mathrm{int}}$$

$$\mathcal{L}_{\mathrm{int}} = N_c g^2 \,\mathrm{tr}\left(\frac{1}{4}\{\phi_i^\dagger, \phi^i\}\{\phi_j^\dagger, \phi^j\} - e^{-i\epsilon^{ijk}\gamma_k}\phi_i^\dagger \phi_j^\dagger \phi^i \phi^j\right) +$$

$$N_c g\, \mathrm{tr}(-e^{-\frac{i}{2}\gamma_j^-}\bar{\psi}_j \phi^j \bar{\psi}_4 + e^{+\frac{i}{2}\gamma_j^-}\bar{\psi}_4 \phi^j \bar{\psi}_j + i\epsilon_{ijk} e^{\frac{i}{2}\epsilon_{jkm}\gamma_m^+}\bar{\psi}^k \phi^i \bar{\psi}^j + \text{conjugate terms})\,.$$

$$\gamma_1^\pm = -\frac{\gamma_3 \pm \gamma_2}{2}$$

$$\gamma_2^\pm = -\frac{\gamma_1 \pm \gamma_3}{2}$$

$$\gamma_3^\pm = -\frac{\gamma_2 \pm \gamma_1}{2}$$

- γ-twisted N=4 SYM Lagrangian: product of matrix fields → star-product

$$A\,B \to A \star B \equiv q_{A,B}\, A\, B \qquad \text{where} \qquad q_{A,B} = e^{-\frac{i}{2}\epsilon^{mjk}\gamma_m J_j^A J_k^B} = (q_{B,A})^{-1}$$

Leigh, Strassler
Frolov, Tseytlin
Beisert, Roiban
Lunin, Maldacena

$J_1^A,\, J_2^A,\, J_3^A \in SO(6)$ - Cartan charges of R-symmetry $\quad \gamma_1, \gamma_2, \gamma_3$ - twists

- PSU(2,2|4)→SU(2,2) × U(1)^3 - breaks R-symmetry and all supersymmetry

- Double scaling limit: strong twist, weak coupling

Gurdogan, V.K. 2015

$$g \to 0, \qquad e^{-i\gamma_j/2} \to \infty, \qquad \xi_j = g\, e^{-i\gamma_j/2} - \text{fixed}, \qquad (j = 1, 2, 3.)$$

$$\mathcal{L} = N_c \mathrm{tr}[-\frac{1}{2}\partial^\mu \phi_i^\dagger \partial_\mu \phi^i + i\bar{\psi}_A^{\dot{\alpha}} \partial_{\dot{\alpha}}^{\alpha} \psi_\alpha^A] + \mathcal{L}_{\mathrm{int}}$$

$$\mathcal{L}_{\mathrm{int}} = N_c \mathrm{tr}[\xi_1^2\, \phi_2^\dagger \phi_3^\dagger \phi_2 \phi_3 + \xi_2^2\, \phi_3^\dagger \phi_1^\dagger \phi_3 \phi_1 + \xi_3^2\, \phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2 +$$

$$+ i\sqrt{\xi_2 \xi_3}(\psi^3 \phi^1 \psi^2 + \bar{\psi}_3 \phi_1^\dagger \bar{\psi}_2) + i\sqrt{\xi_1 \xi_3}(\psi^1 \phi^2 \psi^3 + \bar{\psi}_1 \phi_2^\dagger \bar{\psi}_3) + i\sqrt{\xi_1 \xi_2}(\psi^2 \phi^3 \psi^1 + \bar{\psi}_2 \phi_3^\dagger \bar{\psi}_1)\,].$$

Gurdogan, V.K. 2015

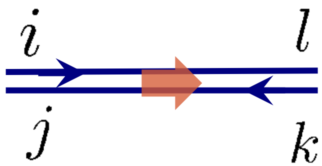
Special case of "fishnet" CFT_4 (Feynman rules)

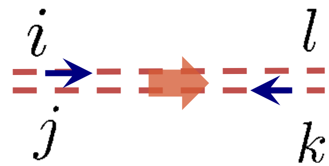
- Single coupling: $\xi := \xi_3 \; - \; \text{fixed}, \qquad \xi_1 = \xi_2 = 0$

$$\mathcal{L}[\phi_1, \phi_2] = \frac{N_c}{2} \text{tr} \; \left(\partial^\mu \phi_1^\dagger \partial_\mu \phi_1 + \partial^\mu \phi_2^\dagger \partial_\mu \phi_2 + 2\xi^2 \, \phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2 \right) \; .$$

Zero dimensional analogue:
Kostov, Staudacher 1995

- Propagators

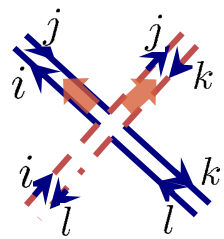




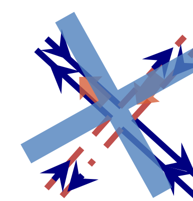
$i, j, k, l = 1 \ldots N_c$

$$\left\langle \phi_1^{*ij}(y) \phi_1^{kl}(x) \right\rangle_0 \quad = \quad \left\langle \phi_2^{*ij}(y) \phi_2^{kl}(x) \right\rangle_0 \quad = \quad \delta^{il} \delta^{jk} \frac{1}{(x-y)^2}$$

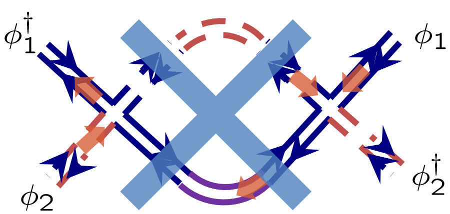
- Vertex: $\xi^2 \text{tr} \, (\phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2)$

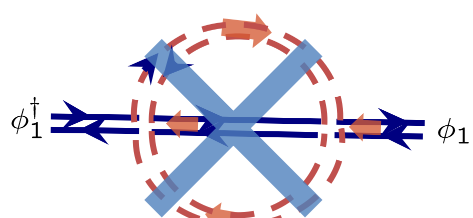


Missing
"anti-chiral" vertex

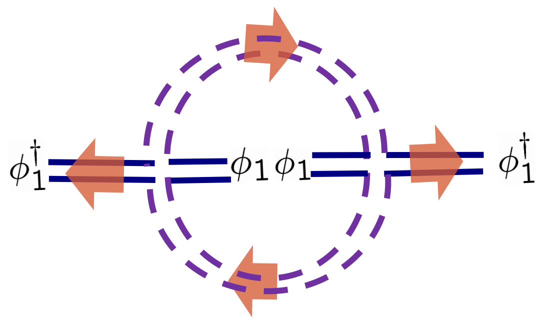


- Very limited number of planar graphs
- No mass or vertex renormalization in planar limit!

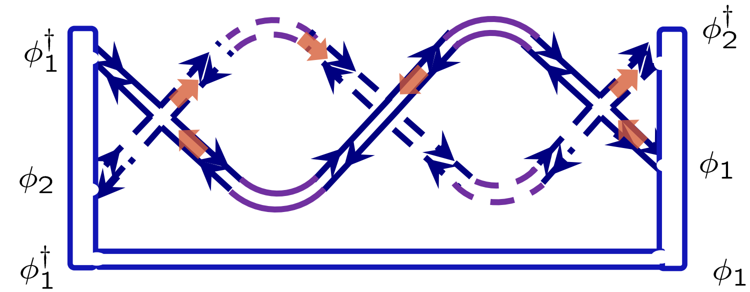




Double trace terms



$\mathrm{tr}(\phi_1\phi_1)\mathrm{tr}(\phi_1^\dagger\phi_1^\dagger)$



$\mathrm{tr}(\phi_1^\dagger\phi_2)\mathrm{tr}(\phi_2^\dagger\phi_1)$

- ξ dosn't run in planar limit, but tr^2 couplings do run. Still CFT at critical point!
- Double-trace counter-terms should be added

$$\mathcal{L}_{\rm dt} = \alpha_1^2 \sum_{i=1}^{2} \mathrm{tr}(\phi_i\phi_i)\,\mathrm{tr}(\phi_i^\dagger\phi_i^\dagger) - \alpha_2^2\,\mathrm{tr}(\phi_1\phi_2)\mathrm{tr}(\phi_2^\dagger\phi_1^\dagger) - \alpha_2^2\mathrm{tr}(\phi_1\phi_2^\dagger)\mathrm{tr}(\phi_2\phi_1^\dagger)\,,$$

- Beta-function quadratic in double-trace coupling (agrees with general arguments)

Pomoni, Rastelli 2009

Grabner, Gromov, V.K., Korchemsky '17

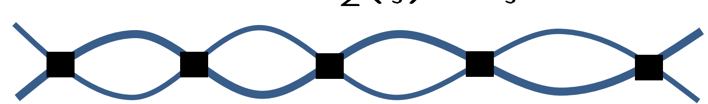
$$\beta_{\alpha_1^2} = a(\xi) + b(\xi)\alpha_1^2 + c(\xi)\,\alpha_1^4 \qquad \text{where} \qquad \frac{1}{2}\sqrt{b^2-4ac} = \gamma_{\mathrm{tr}\phi_1^2} = \sqrt{1-\sqrt{1+4\xi^4}}$$

- As in 4D, it is a non-unitary CFT at any ξ , if we tune the double trace coupling

$$\alpha_{1,\pm}^2 = \pm\frac{i\xi^2}{2} - \frac{\xi^4}{2} \mp \frac{3i\xi^6}{4} + \xi^8 \pm \frac{65i\xi^{10}}{48} - \frac{19\xi^{12}}{10} + O\left(\xi^{14}\right)$$

Sieg, Wilhelm 2016
Grabner, Gromov, V.K., Korchemsky '17

- The other double-trace coupling also tuned to the critical value $\alpha_2^2(\xi) = \xi^2$ to renormalize the only type of divergent graphs:

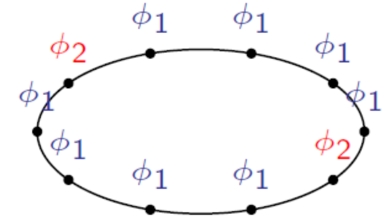


Caetano, Gurdogan, V.K , 2016

Operators, correlators, graphs…

- Operators with M magnons ϕ_2 in the “vacuum” of ϕ_1

$$\mathcal{O}_{L,M}(x) = \mathrm{tr}\left(\underbrace{\phi_2\phi_1\phi_1\phi_1\phi_2\ldots\phi_1}_{L \text{ fields}}\right)(x) + \text{permutations}$$

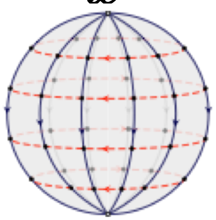


- Correlators $\langle \mathcal{O}(x)\ \mathcal{O}(0)\rangle \sim |x|^{-2L-2\gamma(\xi)}$ anomalous dimension

$\mathrm{tr}[\phi_1(x)]^L$

BMN “vacuum” (non-protected)

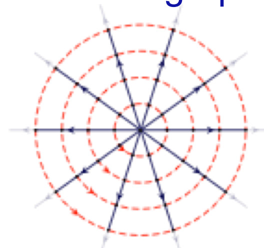
x



0

UV- reduction

“wheel” graphs - divergent! Need $\epsilon = 4 - D$ regularization

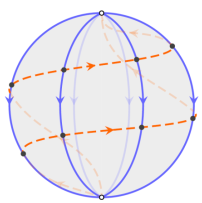


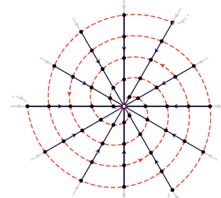
$$= \xi^{2LM}\left(\frac{\#}{\epsilon^M} + \frac{\#}{\epsilon^{M-1}} + \ldots + \frac{C_M}{\epsilon} + \text{finite}\right)$$

$$\gamma(\xi) = \sum_{M=1}^{\infty} C_M\, \xi^{2LM}$$

“spiderweb” graphs

Multi-magnon spiral graphs



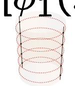


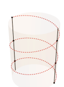
Typical “fishnet” structures in the bulk of graphs. Integrable!

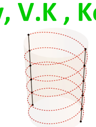
- 4-point functions, computed explicitly

Grabner, Gromov, V.K , Korchemsky, 2017
V.K., Olivucci 2018
Gromov, V.K , Korchemsky (to appear)

$\mathrm{tr}[\phi_1(x_1)\quad \phi_1(x_2)]$







$\mathrm{tr}[\phi_1^\dagger(x_3)\quad \phi_1^\dagger(x_4)]$

Fishnet CFT at any D

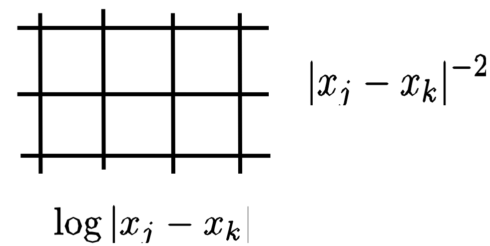
Olivucci, V.K. 2018

- Bi-scalar CFT can be defined at any D

$$\mathcal{L}_\phi = N_c\,\mathrm{tr}[-\phi_1^\dagger(\partial_\mu\partial^\mu)^{\frac{D}{4}+\omega}\phi_1 - \phi_2^\dagger(\partial_\mu\partial^\mu)^{\frac{D}{4}-\omega}\phi_2 + \xi^2\;\phi_1^\dagger\phi_2^\dagger\phi_1\phi_2]$$

- D-dimensional “fishnet” graphs generalize 4D “fishnets”

$$\int\prod_i d^D x_i \prod_{<jk>} \frac{1}{|x_j - x_k|^{D/2}} \xrightarrow{D=4 \qquad \omega=0} \int\prod_i d^4 x_i \prod_{<jk>} \frac{1}{(x_j - x_k)^2}$$

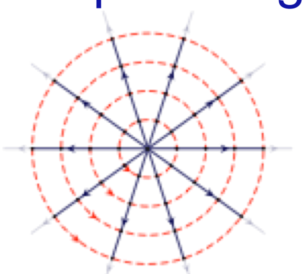


- Anisotropic “fishnet” at D=2, ω=1/2 $\leftrightarrow$ Lipatov’s reggeized gluon model (BFKL)
- Interesting fishnet model at D=2, ω=0 with propagators $\frac{1}{|x_j - x_k|}$
- Integrable conformal SO(1,D) spin chain

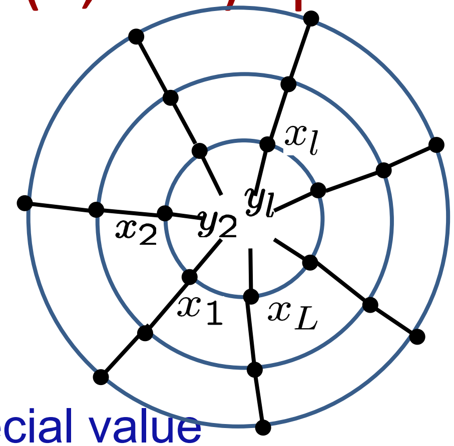
Gurdogan, V.K. 2015 , V.K., Olivucci 2018
Gromov, V.K , Korchemsky, Negro, Sizov 2017

Fishnet wheel graphs and integrable SO(1,D+1) spin chain

- Operator generating a wheel (fishnet) graph



$$\hat{t}_L = \xi^{2L} \prod_{i=1}^{L} \frac{1}{(x_i - x_{i+1})^{D/4}\,(y_i - x_i)^{D/4}}$$

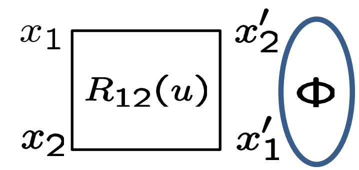


$x_l \in \mathbf{R}^D$

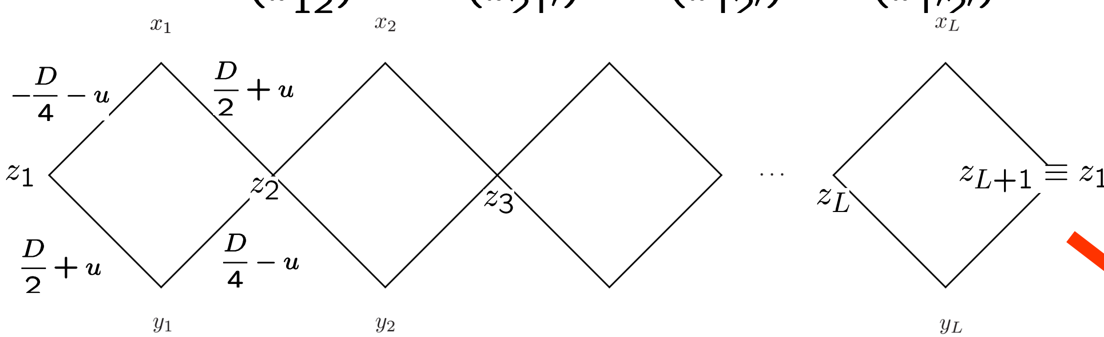
$\sim (\hat{t}_L)^3$

- Integrability: graph-generating operator emerges at special value of spectral parameter of Transfer-matrix in physical irrep on 4D conformal group
- R-operator in principal series representation:

$$[R_{12}\Phi](x_1,x_2)(u) = \int \frac{d^D x_{1'} d^D x_{2'}\, \Phi(x_{1'},x_{2'})}{(x_{12}^2)^{-\frac{D}{4}-u}(x_{21'}^2)^{\frac{D}{2}+u}(x_{12'}^2)^{\frac{D}{2}+u}(x_{1'2'}^2)^{-u+\frac{D}{4}}}$$



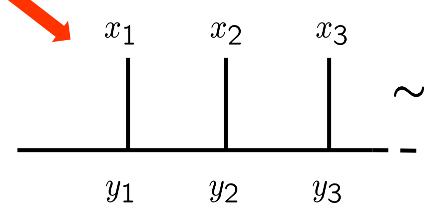
$T_L(u;\mathbf{y}|\mathbf{x}) =$



$$[T_L(u)\,, T_L(u')] = 0$$

Zamolodchikov 1980
Derkachev, Manashov
Derkachev, Manashov , Korchemsky
Chichetin, Derkachev, Isaev

$u = -D/4 + \epsilon$



$\sim \frac{1}{\epsilon^L}\hat{t}_L$

- Hence integrability of bi-scalar model is demonstrated explicitly in all orders!
- Non-compact spin chains not well studied. We use Quantum Spectral Curve!

AdS/CFT and Quantum Spectral Curve (QSC)

AdS5/CFT4 duality: g→1/g

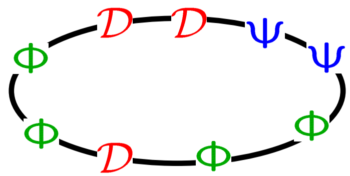
4D CFT (N=4 SYM) → string (2D) sigma model on AdS5xS5

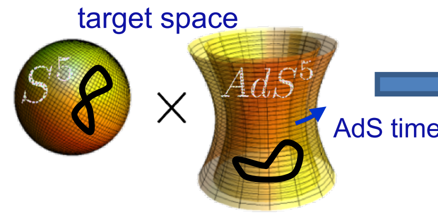
Maldacena 1998
Gubser, Polyakov, Klebanov 2000
Witten 2000

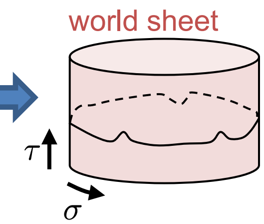
$$\mathcal{L} = N_c \mathrm{tr}[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D^{\mu}\phi_i^{\dagger}D_{\mu}\phi^i + i\bar{\psi}_A^{\dot{\alpha}}D_{\dot{\alpha}}^{\alpha}\psi_{\alpha}^A] + \mathcal{L}_{\mathrm{int}}$$

$$\mathcal{O}(x) = \mathrm{Tr}\,[\mathcal{D}\mathcal{D}\Psi\Psi\Phi\Phi\mathcal{D}\Psi\ldots]\,(x)$$

+permutations





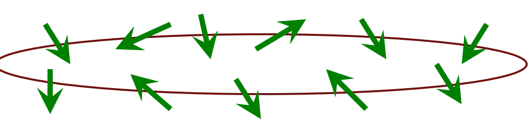


anomalous dimension ɣ(g) → AdS energy

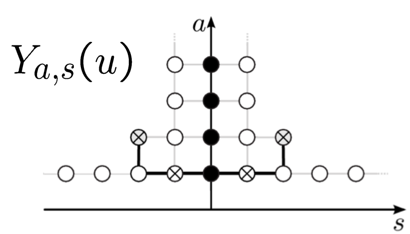
- Integrability in planar, 't Hooft limit (short history):

1-loop integrability
Classical finite gap solution
Asymptotic Bethe ansatz

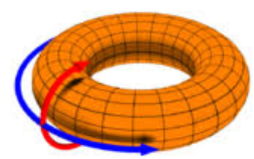
→ Y-system → TBA → QSC



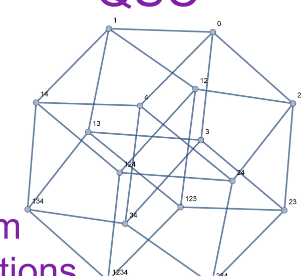
Minahan Zarembo 2002
Kazakov, Marshakov, Minahan, Zarembo 2005
Eden, Beisert, Staudacher 2006



Gromov, V.K., Vieira 2009



Bombardelli,Fioravanti,Tateo
Gromov,V.K.,Kozak,Vieira
Arutyunov,Frolov 2009



Hasse diagram Q-functions

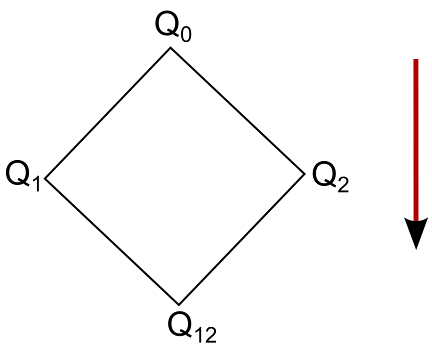
Gromov, V.K., Leurent, Volin 2013

- QSC: full set of 256 Baxter Q-functions for PSU(2,2|4) symmetry, with special analyticity
- The most advanced tool describing exact spectrum of anomalous dimensions of N=4 SYM

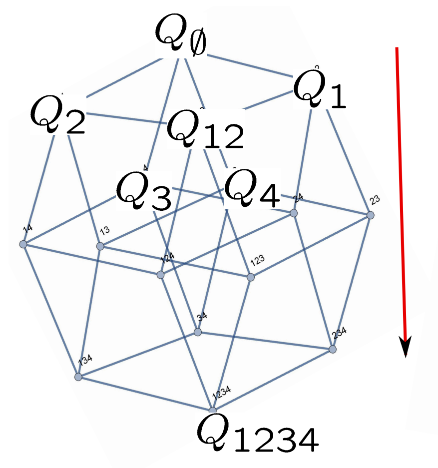
Quantum Spectral Curve of N=4 SYM

Gromov, V.K., Tsuboi 2010
Gromov, V.K., Leurent, Volin 2013
V.K., Leurent, Volin 2015

- QSC eqs. closes on a finite number of Baxter functions of spectral parameter $Q_I(u)$
- The multi-index *I* labels Hasse diagram - N-hypercube (N=8 - rank of PSU(2,2|4))



GL(2)



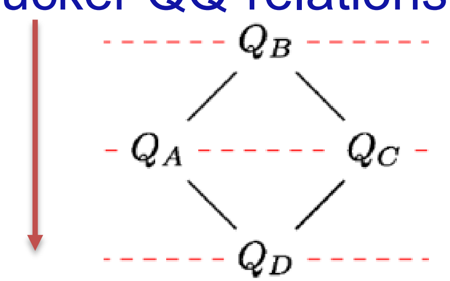
GL(4)

- Parametrize all 2^N Q-functions through one-index Q-functions $Q_1, Q_2, \dots, Q_N$ through determinant formula

$$Q_{j_1,\dots,j_k}(u) = \frac{1}{C(u)} \det_{1\le m,n\le k} Q_{j_m}\left(u + i\frac{2n-k-1}{2}\right)$$

$$C(u) = \prod_{j=0}^{k-1} Q_\emptyset\left(u + i\frac{2j-k+1}{2}\right)$$

- Plücker QQ-relations on each face:



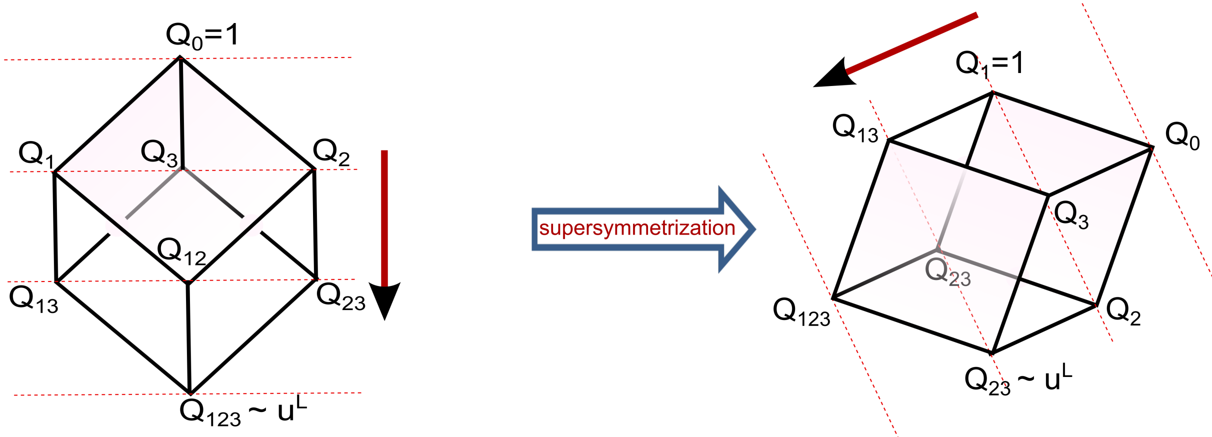
$$\leftrightarrow \quad Q_B(u)Q_D(u) = \begin{vmatrix} Q_A(u+\frac{i}{2}) & Q_C(u+\frac{i}{2}) \\ Q_A(u-\frac{i}{2}) & Q_C(u-\frac{i}{2}) \end{vmatrix}$$

Grassmanian structure!

Krichever, Lupan, Wiegmann, Zabrodin, 1994
V.K., Sorin, Zabrodin 2007
Gromov, V.K., Leurent, Tsuboi 2010, 2011

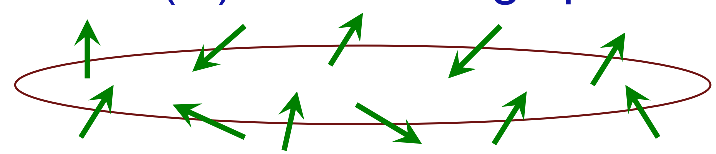
(K|M)-graded Q-system and Heizenberg spin chain

Tsuboi
V.K., Sorin, Zabrodin
V.K., Leurent, Volin



GL(3) GL(1|2)

- Nicest formulation of solution for spectrum of GL(N) Heisenberg spin chain: impose polynomiality of Q-functions



$$Q_k(u) = x_k^{iu} \prod_{j=1}^{J_k} (u - u_j^{(k)})$$

$$Q_\emptyset(u) = 1, \qquad Q_{1,2,\dots,N}(u) \sim u^L \quad \Rightarrow \quad \text{Energy} = L + i\,\partial_u \log \frac{Q_{\bar{k}}(u - i/2)}{Q_{\bar{k}}(u + i/2)}\Big|_{u=0}$$

- "Tilting" the hypercube, i.e. fixing two diametrically opposite Q-functions on Hasse diagram we get Q-system for Heisenberg gl(K|M) super-spin chain!

$$Q_{1,2\dots,K}(u) = 1, \qquad Q_{K+1,K+2,\dots,K+M}(u) \sim u^L$$

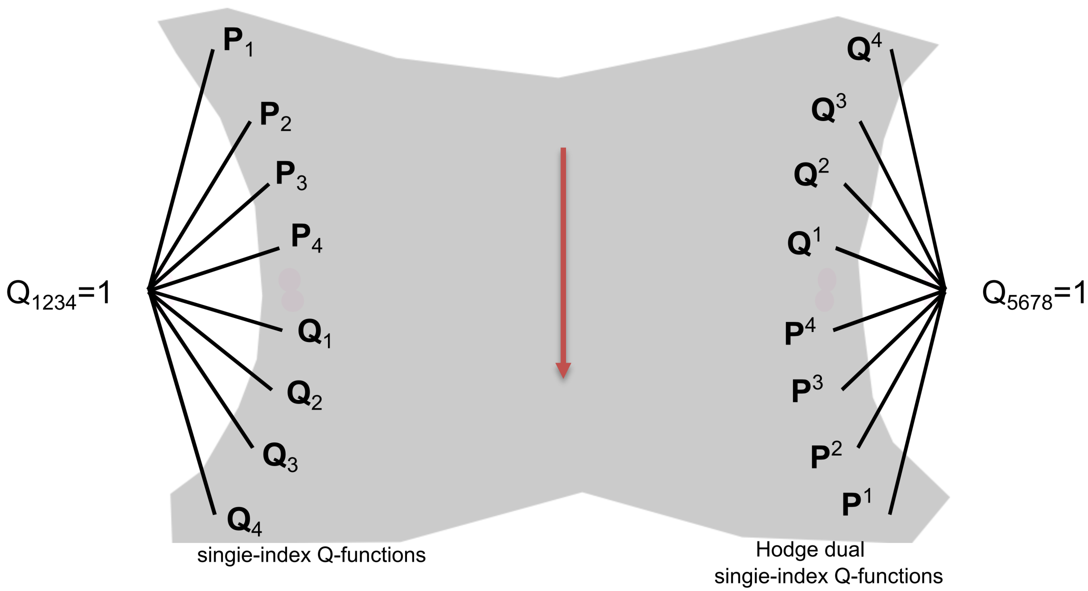
and all other Q-functions are (twisted) polynomials.

- Then the energy is $\quad \text{Energy} = L + i\,\partial_u \log \dfrac{Q_{K,K+1,\dots,K+M}(u + i/2)}{Q_{K,K+1,\dots,K+M}(u - i/2)}\Big|_{u=0}$

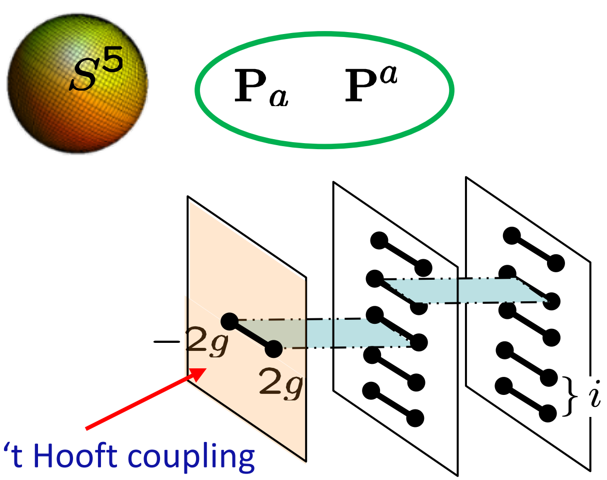
Quantum Spectral Curve of AdS/CFT

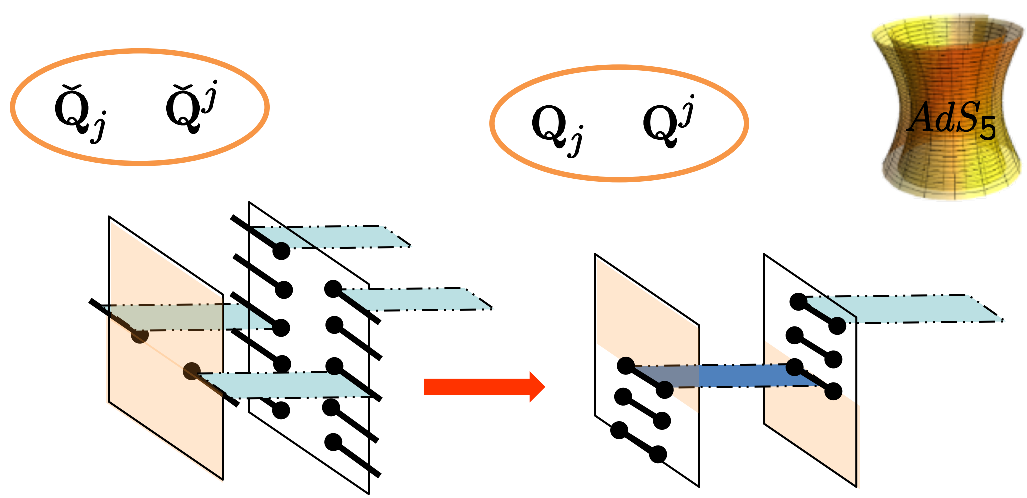
Gromov, V.K., Leurent, Volin 2013
V.K., Leurent, Volin 2014

- 8 + 8 Q-functions on (4|4) Hasse diagram (with nice analyticity on defining sheets)



- Q-functions live on infinitely branching Riemann surface with fixed equidistant cuts





Qantum Spectral Curve: large u, ɣ-twist and RH conditions

Gromov, V.K., Leurent, Volin 2013
V.K., Leurent, Volin 2014

- Large u asymptotics fixed by PSU(2,2|4) Cartan charges $\{\Delta, S_1, S_2 | J_1, J_2, J_3\}$
- ɣ-deformed QSC

$$\mathbf{Q}_j \sim u^{\check{M}_j}, \qquad \mathbf{P}_b \sim x_b^{i\,u}\, u^{-\hat{M}_b} \qquad \{x_1, x_2, x_3, x_4\} \in SU(4)$$

$$\hat{M}_b = \frac{1}{2}\{+J_1+J_2-J_3, \quad +J_1-J_2+J_3, \quad -J_1+J_2+J_3, \quad -J_1-J_2-J_3\}$$

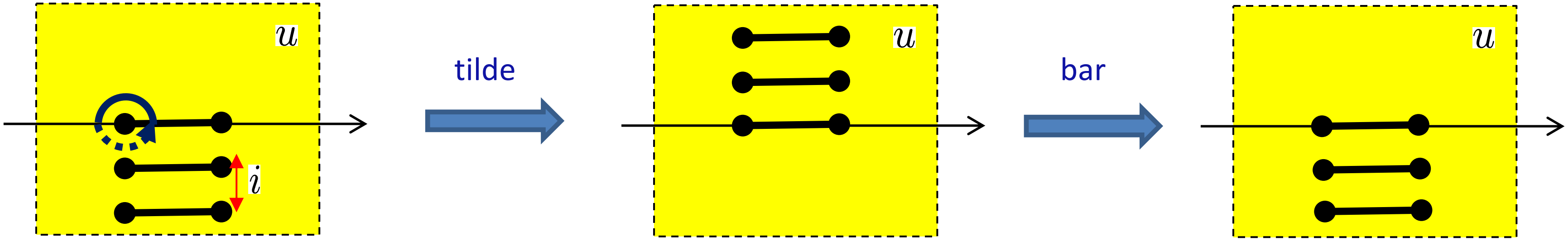
$$\check{M}_j = \frac{1}{2}\{+\Delta-S_1-S_2, \quad +\Delta+S_1+S_2, \quad -\Delta-S_1+S_2, \quad -\Delta+S_1-S_2\}$$

- Monodromy around branchpoints (inspired by classical finite gap solution) given by "gluing" relations at the main cut:

V.K.,Marshakov,Minahan,Zarembo
Beisert,V.K.,Sakai,Zarembo

$$\tilde{\mathbf{Q}}_1 \sim \bar{\mathbf{Q}}^2, \quad \tilde{\mathbf{Q}}_2 \sim \bar{\mathbf{Q}}^1, \quad \tilde{\mathbf{Q}}_2 \sim \bar{\mathbf{Q}}^4, \quad \tilde{\mathbf{Q}}_4 \sim \bar{\mathbf{Q}}^2.$$

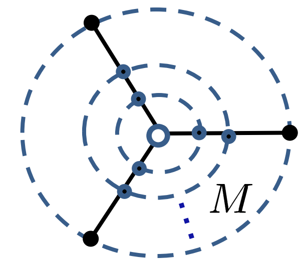
Gromov, V.K. (2010) – in classical limit
Gromov, Sizov, Levkovich-Maslyk (2015)



- These Riemann-Hilbert conditions fix all physical solutions for Q-system and also conformal dimensions $\Delta(g^2)$ (finite set of operators with given charges)

Gromov, V.K., Korchemsky, Negro, Sizov 2017

Double Scaling in QSC



- Consider "vacuum" operator $\mathcal{O}_{\text{vac}}(x) = \text{tr}(\phi_1)^3$ in bi-scalar model. Construct QSC in DS limit:

$$\xi = g\,\kappa, \ \text{where} \ \kappa = e^{-i\gamma_3/2} \to \infty, \ g \to 0$$

- Extra "Left-Right" symmetry for such operators

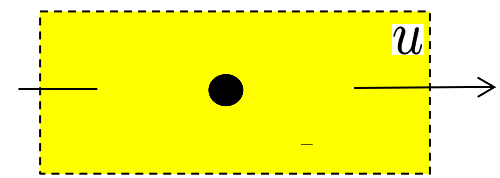
$$\mathbf{Q}^j = -\chi^{jk}\mathbf{Q}_k\,, \qquad \mathbf{P}^a = -\chi^{ab}\mathbf{P}_b$$

$$\chi = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

- Analyticity: asymptotics $u \to \infty$

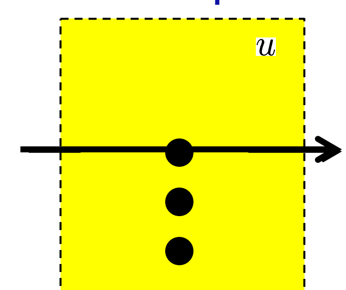
$$\begin{pmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \\ \mathbf{P}_4 \end{pmatrix} \simeq \begin{pmatrix} \kappa^{iu} & u^{-\frac{3}{2}} \\ \kappa^{-iu} & u^{-\frac{3}{2}} \\ \kappa^{iu} & u^{\frac{3}{2}} \\ \kappa^{-iu} & u^{\frac{3}{2}} \end{pmatrix} \times (1 + \mathcal{O}(1/u))$$

one short cut



short cuts in lower half-plane

$$\begin{pmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \\ \mathbf{Q}_3 \\ \mathbf{Q}_4 \end{pmatrix} \simeq \begin{pmatrix} u^{\frac{\Delta}{2}} \\ u^{\frac{\Delta}{2}+1} \\ u^{-\frac{\Delta}{2}+2} \\ u^{-\frac{\Delta}{2}+3} \end{pmatrix} \times (1 + \mathcal{O}(1/u))$$



- Double scaling limit of QSC Baxter functions:
  At week coupling, cuts generate poles – the only source of singularities at finite u:

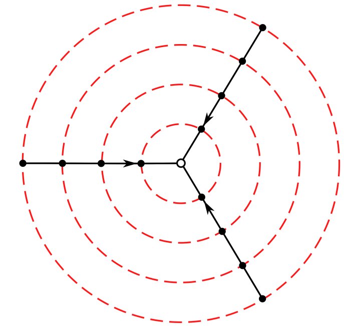
$$\sqrt{u^2 - 4g^2} = u - \frac{2}{u}g^2 - \frac{2}{u^3}g^4 - \frac{4}{u^5}g^6 + \mathcal{O}(g^8)$$

- QSC gluing relations become for this state $\tilde{\mathbf{Q}}_1 = -\beta\,\bar{\mathbf{Q}}_3\,, \qquad \tilde{\mathbf{Q}}_2 = \bar{\beta}\,\bar{\mathbf{Q}}_4$

Gromov, V.K., Korchemsky, Negro, Sizov 2017

Solution for L=3 "vacuum" operator

$\mathrm{tr}(\phi_1)^3$



- In double scaling limit, QSC gives two 2nd order Baxter eqs.

$$\left(\frac{(\Delta-1)(\Delta-3)}{4u^2} \pm \frac{i\,\xi^3}{u^3} - 2\right) q(u) + q(u+i) + q(u-i) = 0$$

notation
$q_j = Q_j\, u^{-3/2}$

for $q_2(u,\xi)$, $q_4(u,\xi)$ and for $q_1(u,\xi) = q_2(u,-\xi)\,,\quad q_3(u,\xi) = q_4(u,-\xi)$

- The gluing conditions, after exclusion of constant β, become

$$\tilde{\mathbf{Q}}_1\, \mathbf{Q}_4 + \overline{\tilde{\mathbf{Q}}_2\, \mathbf{Q}_3} = 0$$

which at u=0 is equivalent to the following quantization condition

$$q_2(0,\xi)\, q_4(0,-\xi) + q_2(0,-\xi)\, q_4(0,\xi) = 0$$

- These q-functions should obey "pure" large u asymptotics

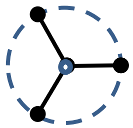
$$q_2(u,\xi) \sim u^{\Delta/2-1/2}\left(1+\frac{\alpha_1}{u}+\frac{\alpha_2}{u^2}+\cdots\right), \qquad q_4(u,\xi) \sim u^{-\Delta/2+3/2}\left(1+\frac{\beta_1}{u}+\frac{\beta_2}{u^2}+\cdots\right)$$

- Solved numerically for anomalous dimensions with virtually arbitrary precision at all relevant couplings

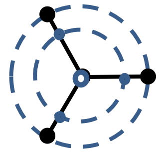
- Perturbative solution in ξ gives the exact periods of "wheel" graphs

L=3 BMN vacuum and all-loop wheel graphs

Gromov, V.K , Korchemsky, Negro, Sizov (2017)

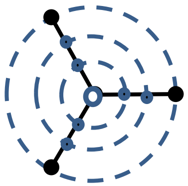


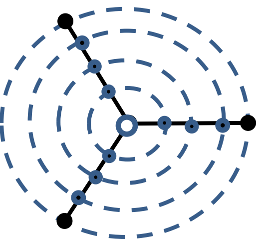
Broadhurst 1980



Ahn, Bajnok, Bombardelli, Nepomechie 2013

E.Panzer, 2015



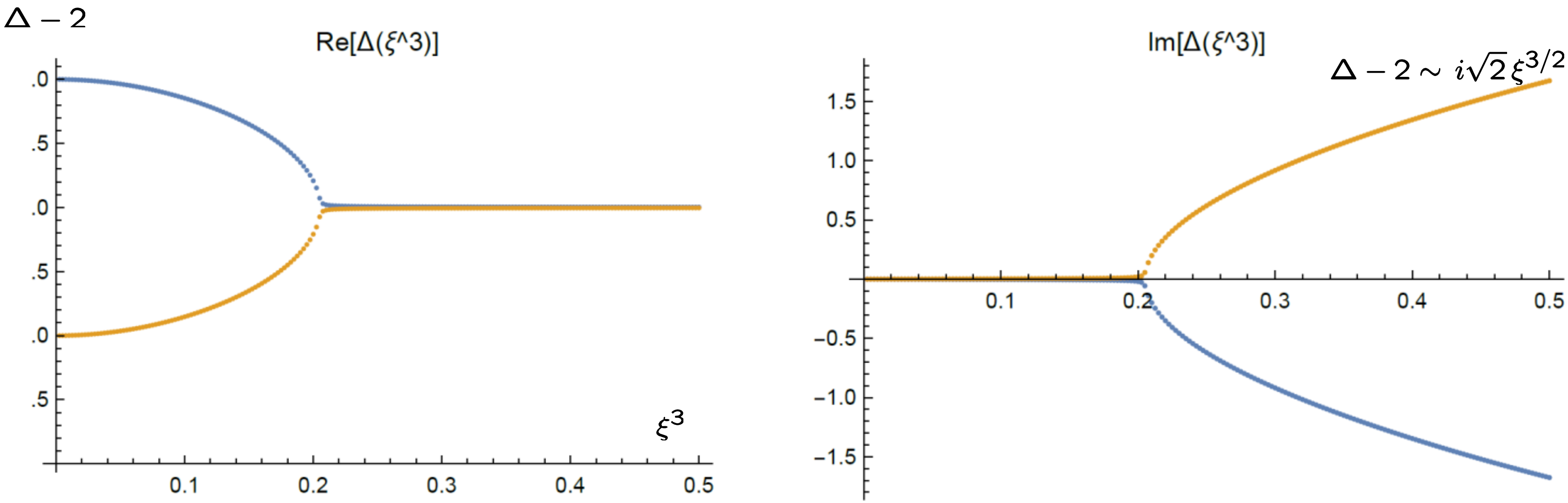


$$
\begin{aligned}
\Delta - 3 = \quad & -12\xi^6\zeta(3) + \xi^{12}\left(189\zeta(7) - 144\zeta(3)^2\right) \\
& +\xi^{18}\Bigg(-1944\zeta(8,2,1) - 3024\zeta(3)^3 - 3024\zeta(5)\zeta(3)^2 + \frac{198\pi^8\zeta(3)}{175} + 6804\zeta(7)\zeta(3) \\
& +\frac{612\pi^6\zeta(5)}{35} + 270\pi^4\zeta(7) + 5994\pi^2\zeta(9) - \frac{925911\zeta(11)}{8}\Bigg) + \\
& \xi^{24}\Bigg(\frac{10368}{5}\pi^4\zeta(8,2,1) + 5184\pi^2\zeta(9,3,1) + 51840\pi^2\zeta(10,2,1) - 148716\zeta(11,3,1) \\
& -1061910\zeta(12,2,1) + 62208\zeta(10,2,1,1,1) - 93312\zeta(3)\zeta(8,2,1) - 288\zeta(3)^5 \\
& +72\gamma\pi^2\zeta(3)^4 - 77760\zeta(3)^4 - \frac{80756\pi^6\zeta(3)^3}{945} - 145152\zeta(5)\zeta(3)^3 - \frac{29}{270}\gamma\pi^8\zeta(3)^2 \\
& +\frac{9504\pi^8\zeta(3)^2}{175} - 879\pi^4\zeta(5)\zeta(3)^2 - 2025\pi^2\zeta(7)\zeta(3)^2 + 244944\zeta(7)\zeta(3)^2 \\
& +186588\zeta(9)\zeta(3)^2 + \frac{2910394\pi^{12}\zeta(3)}{2627625} - 2592\pi^2\zeta(5)^2\zeta(3) + \frac{29376}{35}\pi^6\zeta(5)\zeta(3) \\
& +12960\pi^4\zeta(7)\zeta(3) + 298404\zeta(5)\zeta(7)\zeta(3) + 287712\pi^2\zeta(9)\zeta(3) \\
& -5555466\zeta(11)\zeta(3) + 57672\zeta(5)^3 - 71442\zeta(7)^2 + \frac{13953\pi^{10}\zeta(5)}{1925} + \frac{7293\pi^8\zeta(7)}{175} - \frac{19959\pi^6\zeta(9}{5} \\
& +\frac{119979\pi^4\zeta(11)}{2} + \frac{10738413\pi^2\zeta(13)}{2} - \frac{4607294013\zeta(15)}{80}\Bigg) + O\left(\xi^{25}\right)
\end{aligned}
$$

Riemann (multi)-zeta numbers

- Generalization to any number of spokes L is in work
(Baxter equation is available)

Numerics for L=3 "BMN vacuum" $\mathrm{tr}(\phi_1)^3$



- Around ξ^3=0.2 dimension becomes imaginary: phase transition, finite convergence radius.
- What happens to the string dual?

Perturbative expansion and numerics for operators of length L>3

- More solutions, for operators L=3+4n+2 $\quad \mathcal{O} = \text{tr}\left(\phi_1^3\ (\phi_2^\dagger\phi_2)^n\right)\ +\ \text{permutations}$

$$\Delta_-^{L=5} = 5 - 2i\xi^3 + 3\xi^6 + \frac{31}{4}i\xi^9 + \left(3\zeta_3 - \frac{97}{4}\right)\xi^{12} + i\left(\frac{27\zeta_3}{2} - \frac{5359}{64}\right)\xi^{15} + \left(-\frac{219\zeta_3}{4} - \frac{15\zeta_5}{2} + \frac{4911}{16}\right)\xi^{18} + \dots$$

$$\Delta_+^{L=5} = (\Delta_-^{L=5})^*$$

$$\Delta_-^{L=9} = 9 - \frac{i\xi^3}{3} + \frac{7\xi^6}{216} - \frac{223i\xi^9}{10368} + \xi^{12}\left(\frac{\zeta_3}{432} + \frac{17029}{1119744}\right) + \xi^{15}\left(\frac{1424867i}{214990848} - \frac{31i\zeta_3}{31104}\right) + \dots$$

$$\Delta_+^{L=9} = (\Delta_-^{L=9})^*$$

- At L=3+4n dimensions are real (until tl

$$\Delta_{7,A} = 7 - \frac{\xi^6}{2} - \frac{17\xi^{12}}{64} - \frac{891\xi^{18}}{4096} - \frac{27465\xi^{24}}{131072} + \dots$$

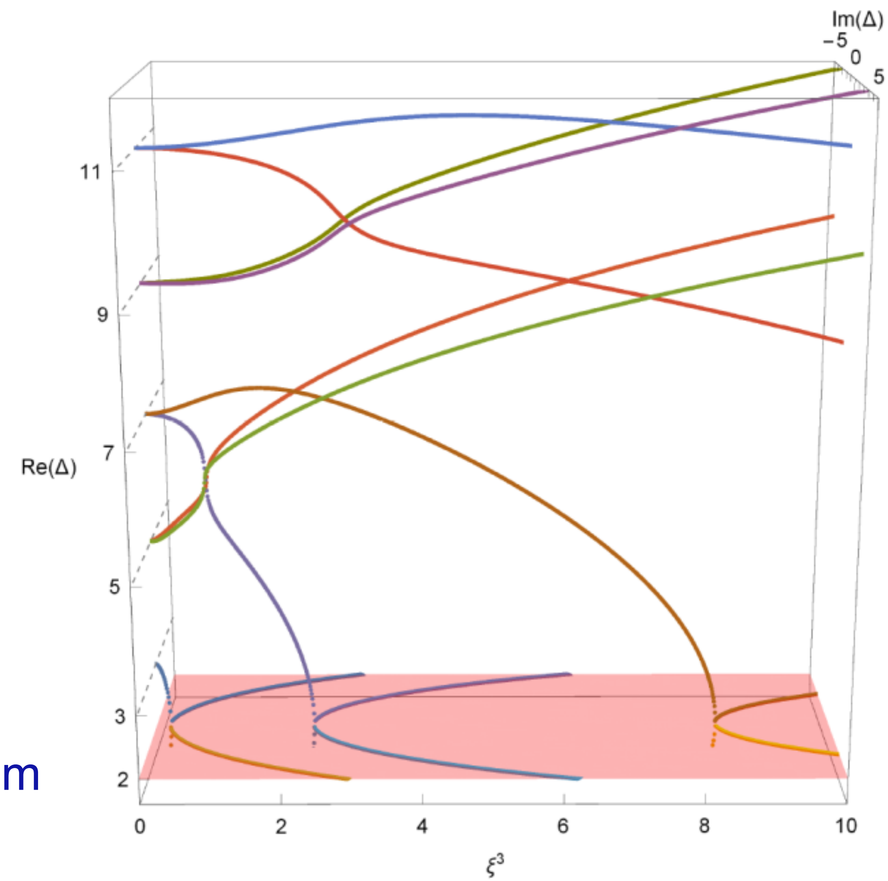
$$\Delta_{7,B} = 7 + \frac{\xi^6}{2} - \frac{23\xi^{12}}{64} + \xi^{18}\left(\frac{15283}{36864} - \frac{\zeta_3}{12}\right) + \xi^{24}\left(\frac{65\zeta}{384}\right.$$

- Numerics:

- Large ξ limit also studied.

$$(\Delta^{L=3} - 2)^2 = -2\xi^3 + \frac{7}{16\xi^3} + O\left(\frac{1}{\xi^6}\right)$$

- Described by a classical integrable system of 3 non-compact spins



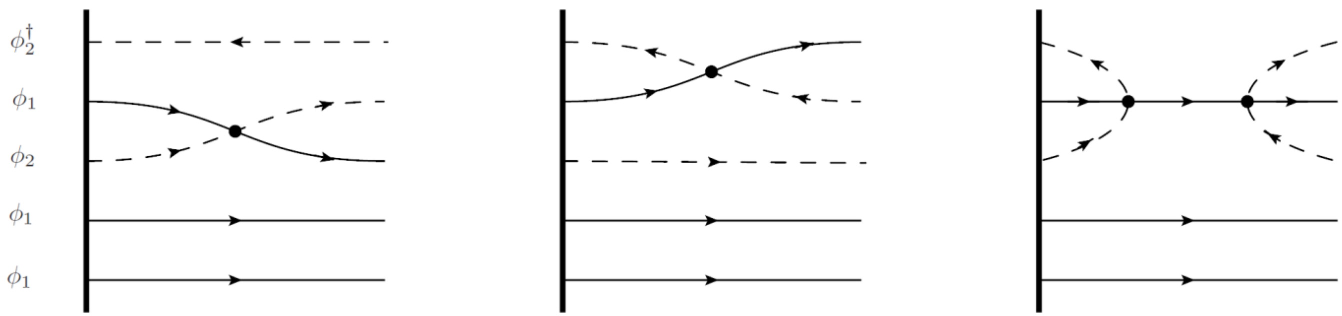
Logarithmic multiplets

Gromov, V.K , Negro, Korchemsky, Sizov
Caetano

- Mixing in multiplet of operators of length L=5:

protected

$$O_1 = \mathrm{tr}\,(\phi_1^3\phi_2\phi_2^\dagger)\,, \qquad O_2 = \mathrm{tr}\,(\phi_1^2\phi_2\phi_1\phi_2^\dagger)\,, \qquad O_3 = \mathrm{tr}\,(\phi_1\phi_2\phi_1^2\phi_2^\dagger)\,, \qquad O_4 = \mathrm{tr}\,(\phi_2\phi_1^3\phi_2^\dagger)\,.$$



- Non-unitary mixing matrix. “Diagonalisation” means bringing to Jordan form

$$\mu\frac{d}{d\mu}O_i(x) = -V_{ij}O_j(x), \qquad V = \frac{1}{4\pi^2}\begin{bmatrix} 0 & \xi^2 & 0 & 0 \\ 0 & 0 & \xi^2 & 0 \\ 0 & -\xi^4 & 0 & \xi^2 \\ 0 & 0 & 0 & 0 \end{bmatrix} = U\cdot\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -i\xi^3 & 0 \\ 0 & 0 & 0 & i\xi^3 \end{pmatrix}\cdot U^{-1}$$

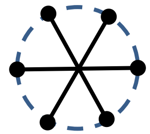
- Lower block contains usual operators with dimensions $\Delta_{\pm}^{L=5} = 5 \pm 2i\left(\frac{\xi}{4\pi}\right)^3 + \dots$

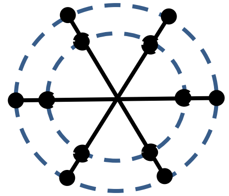
- Upper block is Jordan cell, it gives log-conformal correlators:

Gurarie 1993
Caetano 2016

$$\langle \tilde{O}_\alpha^\dagger(x)\tilde{O}_\beta(0)\rangle_{\mathrm{ren}} = \frac{1}{x^{10}}\begin{bmatrix} 0 & 1 \\ 1 & \log x^2\mu^2 \end{bmatrix}_{\alpha\beta}$$

Wheel graphs at any L and dimension of $\mathrm{tr}[\phi_1(x)]^L$





$$\gamma(L) = \gamma^{(1)}(L)\,\zeta_{2L-3}\,\xi^{2L} + \gamma^{(2)}(L)\,\xi^{4L} + \mathcal{O}(\xi^{6L}),$$

$$\gamma^{(1)}_{\text{vac}}(L) = -2\binom{2L-2}{L-1}\zeta_{2L-3}$$

Broadherst 1980

$$\gamma^{(2)}(L) = \frac{2^3}{\Gamma^2(L)}\Bigg\{-\sum_{j_1=0}^{L-1}\frac{(2L-2)!}{L-j_1}\binom{L+j_1-1}{j_1}\zeta_{2L-3}\zeta_{\substack{L+j_1-2,\\ L-j_1}} + \frac{\Gamma^2(L)}{8}\binom{4L-2}{2L-1}\zeta_{4L-5}$$
$$+\sum_{\substack{j_1,j_2>0\\ j_1+j_2<2L-3}}\frac{\Gamma(2L-j_1-j_2-1)\Gamma(L+j_1)\Gamma(L+j_2)}{\Gamma(j_1+1)\Gamma(j_2+1)\Gamma(L-j_1)\Gamma(L-j_2)}\Bigg[\sum_{k=1}^{L+j_1-2}\Bigg(\binom{L+k+j_2-4}{k-1}\Big(2\zeta_{\substack{L+j_1-k-1,\\ L+j_2+k-3,\\ 2L-j_1-j_2-1}} + \zeta_{\substack{L+j_1-k-1,\\ 3L-j_1+k-4}}\Big)$$
$$-\binom{L+k+j_2-2}{k}\Big(2\zeta_{\substack{L+j_1-k-1,\\ L+j_2+k-2,\\ 2L-j_1-j_2-2}} - 2\zeta_{\substack{L+j_1-k-1,\\ L+j_2+k-1,\\ 2L-j_1-j_2-3}}\Big)\Bigg) + 2\zeta_{\substack{2L+j_1+j_2-3,\\ 2L-j_1-j_2-2}} + 2\zeta_{\substack{2L+j_2-1,\\ L+j_1-1,\\ 2L-j_1-j_2-3}}\ \zeta_{\substack{L+j_1-2,\\ 3L-j_1-3}} - \zeta_{\substack{2L+j_1+j_2-2,\\ 2L-j_1-j_2-3}}\Bigg]$$
$$+\frac{\Gamma^2(2L-1)}{2\Gamma^2(L)}\Bigg[\sum_{k=1}^{2L-4}\Bigg(\binom{2L+k-4}{k}\Big(3\zeta_{\substack{2L-k-3,\\ 2L+k-2}} + 2\zeta_{\substack{2L-k-3,\\ 2L+k-4,\\ 2}}\Big) - 2\Bigg(\binom{2L+k-4}{k-1} + \binom{2L+k-4}{k+1}\Bigg)\zeta_{\substack{2L-k-3,\\ 2L+k-3,\\ 1}}\Bigg)$$
$$+2\zeta_{\substack{2L-3,\\ 2L-4,\\ 2}} - 4(L-3)\zeta_{\substack{2L-3,\\ 2L-3,\\ 1}} + \zeta_{\substack{2L-4,\\ 2L-1}} - 4\zeta_{\substack{2L-2,\\ 2L-3}} + 4\zeta_{\substack{4L-6,\\ 1}} + 2\zeta_{\substack{3L-4,\\ 2L-2,\\ 1}} + \frac{2L+1}{L}\zeta_{2L-3}\zeta_{2L-2} - 5\zeta_{4L-5} - \zeta^2_{2L-3}\Bigg]\Bigg\}$$

Ahn, Bajnok, Bombardelli, Nepomechie 2013

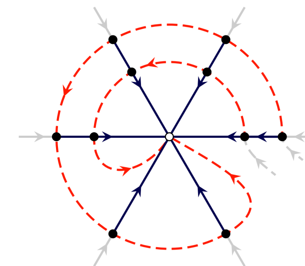
Gurdogan, V.K. 2015

Eden, Beisert, Staudacher, 2005

Dimensions from Asymptotic Bethe Ansatz

Caetano, Gurdogan, V.K 2016

- In bi-scalar model, rapidities live in "mirror" plane and the double-scaled ABA in SU(2) subsector (made of $\phi_1, \ \phi_2$)



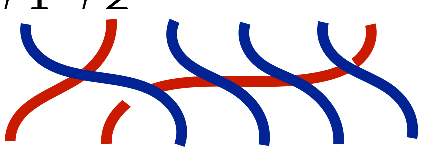
$$(u_j^2 + 1/4)^L = \xi^{2L} \prod_{k \neq j} \frac{u_j - u_k - i}{u_j - u_k + i} \sigma^2(u_j, u_k)\,,$$

Where σ is mirror-mirror ABA dressing phase in g=0 limit

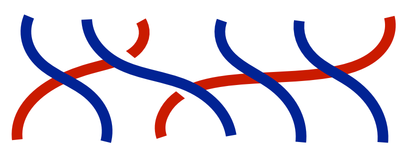
$$\sigma(u,v) = \frac{(1+4u^2)\,\Gamma\left(\frac{1}{2} - iu\right)\,\Gamma\left(\frac{3}{2} - iu\right)\,\Gamma(1+iu-iv)^2\,\Gamma\left(\frac{1}{2}+iv\right)\,\Gamma\left(\frac{3}{2}+iv\right)}{(1+4v^2)\,\Gamma\left(\frac{1}{2} - iv\right)\,\Gamma\left(\frac{3}{2} - iv\right)\,\Gamma(1+iv-iu)^2\,\Gamma\left(\frac{1}{2}+iu\right)\,\Gamma\left(\frac{3}{2}+iu\right)}\,.$$

- The anomalous dimension are given by $\Delta = L - M + 2i \sum_{j=1}^{M} u_j$
- Helps to compute unwrapped magnon graphs entering the mixing matrix:

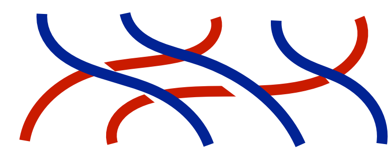
$\phi_1 \ \phi_2$



I_a



I_b



I_c

$$I_b\big|_{1/\epsilon} = -I_a\big|_{1/\epsilon} - \frac{160\zeta(3)}{9} + \frac{53\pi^4}{72} - \frac{187}{5} - \frac{25\pi^2}{12}$$

$$I_c\big|_{1/\epsilon} = I_a\big|_{1/\epsilon} + \frac{418\zeta(3)}{45} + \frac{121\pi^4}{360} + \frac{2\pi^2}{9} - \frac{112}{5}$$

- "Minimally connected" Feynman graph $I_a|_{1/\epsilon}$ is computed directly

Georgoudis, Goncalves, Pereira

Grabner, Gromov, V.K., Korchemsky 2017
Olivucci, V.K. 2018

4-point correlator and exact OPE data

- Exact all-loop calculation of a 4-point correlator (only from conformal symmetry!)

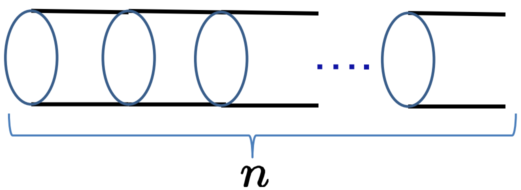
$$G(x_1,x_2|x_3,x_4) \equiv \langle \mathrm{tr}[\phi_1(x_1)\phi_1(x_2)]\ \mathrm{tr}[\phi_1^\dagger(x_3)\phi_1^\dagger(x_4)]\rangle = \frac{\mathcal{G}(u,v)}{(x_{12}^2 x_{34}^2)^{\frac{D}{2}}}$$

cross-ratios

$$u = x_{12}^2 x_{34}^2/(x_{13}^2 x_{24}^2)$$

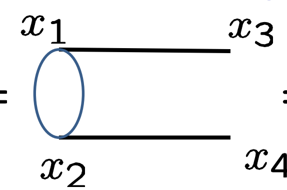
$$v = x_{14}^2 x_{23}^2/(x_{13}^2 x_{24}^2)$$

Dominated by "wheel" graphs, generated by powers of graph-building operator



$$= \mathcal{H}^n$$

$$\mathcal{H} =$$



$$= \frac{1}{x_{12}^4 x_{13}^2 x_{24}^2}$$

- Solving by diagonalization: eigenfunction is easily fixed by conformal symmetry. It is 3-point Polyakov correlation function with spin.

$$\mathcal{H}\ \langle \Delta, S, n\,|\,x_1,x_2\rangle = h^{-1}_{\Delta,S}\ \langle \Delta, S, n\,|\,x_1,x_2\rangle$$

- We obtain integral representation for 4-point correlation function $\Delta = \frac{D}{2} + 2i\nu$

conformal block

$$\mathcal{G}(u,v) = \sum_{S/2\in\mathbb{Z}_+} \int_{-\infty}^{\infty} d\nu\ \mu_{\Delta,S}\ \frac{u^{(\Delta-S)/2} g_{\Delta,S}(u,v)}{1-\xi^4/h_{\Delta,S}} = \sum_{\Delta}\sum_{S/2\in\mathbb{Z}_+} C_{\Delta,S}\, u^{(\Delta-S)/2} g_{\Delta,S}(u,v)$$

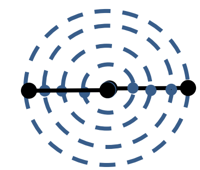
- Integration by residues at poles (physical dimensions): $1-\xi^4/h_{\Delta,S} = 0$ gives the exact OPE over exchange operators $\mathrm{tr}(\phi_1\partial_+^S\phi_1(\phi_2^\dagger\phi_2)^n) + \text{permutations}$

Grabner, Gromov, V.K., Korchemsky 2017
Olivucci, V.K. 2018

4-point correlator: exact results

- Dimensions: $h_{\Delta,S}^{(D=4)} = \frac{16\pi^4}{(-\Delta+S+2)(-\Delta+S+4)(\Delta+S-2)(\Delta+S)} = \xi^4$

- For S=0 it gives dimension of operator $\mathrm{tr}[\phi_1(x)]^2$



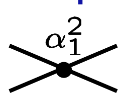
$$\Delta - 2 = \pm i \frac{2\xi^2}{\Gamma\left(\frac{D}{2}\right)} \pm \frac{i}{6} \frac{\xi^6}{\Gamma\left(\frac{D}{2}\right)^3} \left(\pi^2 - 6\psi^{(1)}\left(\frac{D}{2}\right)\right) + O(\xi^{10})$$

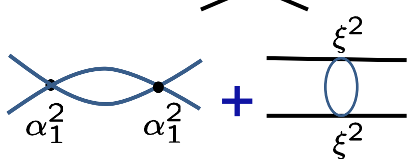
- explicit OPE coefficients (structure constants) for exchange operators, e.g. for D=4

$$C_{\Delta,S} = \frac{(-1)^{-S}}{(2\pi)^3} \frac{\Gamma(S+2)\Gamma\left(\frac{1}{2}(S+\Delta-1)\right)^2 \Gamma(S-\Delta+4)}{\Gamma(S+1)\Gamma\left(\frac{1}{2}(S-\Delta+5)\right)^2 \Gamma(S+\Delta-1)}$$

- Weak-coupling expansion: $\mathcal{G} = \frac{z\bar{z}}{z-\bar{z}} \sum_{n=0}^{\infty} (i\xi^2)^n \, \mathcal{G}_n(z,\bar{z})$

$$\boxed{\begin{aligned} u &= \bar{z}z \\ v &= (1-\bar{z})(1-z) \end{aligned}}$$





$$\begin{aligned} \mathcal{G}_1 &= -(\mathcal{L}_{01} - \mathcal{L}_{10}) \\ \mathcal{G}_2 &= +(\mathcal{L}_{01} - \mathcal{L}_{10}) - \mathcal{L}_{001} + \mathcal{L}_{100} \\ \mathcal{G}_3 &= \frac{3}{2}(\mathcal{L}_{01} - \mathcal{L}_{10}) - \mathcal{L}_{0001} + \mathcal{L}_{0010} - \mathcal{L}_{0100} - \mathcal{L}_{0101} + \mathcal{L}_{1000} + \mathcal{L}_{1010} + 4\zeta_3\mathcal{L}_1 \\ &\dots \end{aligned}$$

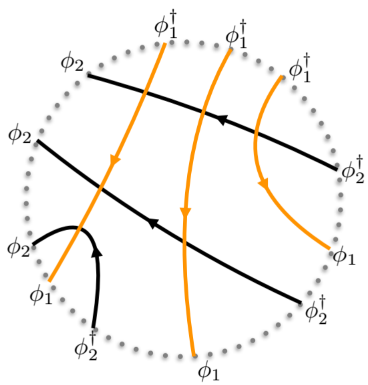
single-valued harmonic polylogarithms

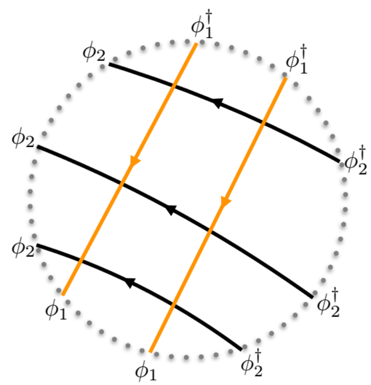
Chicherin, V.K., Loebbert, Mueller, Zhong 2017

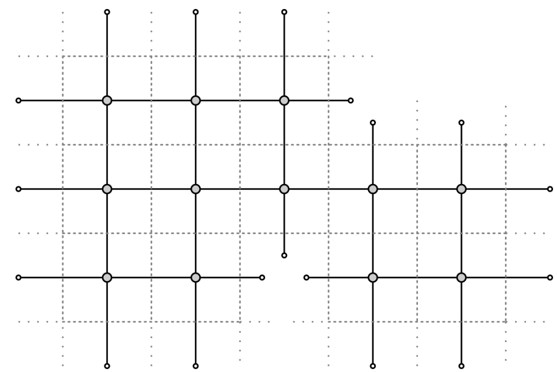
Fishnet amplitudes in bi-scalar model

- Single-trace correlator defined by a single multi-loop graph

$$K(x_1, x_2, \dots x_{2M}) = \langle \mathrm{Tr}\, [\chi_1(x_1)\, \chi_2(x_2), \dots \chi_{2M}(x_{2M})] \rangle \,, \qquad \chi_i \in \{\, \phi_1^\dagger, \phi_2^\dagger, \phi_1, \phi_2 \}$$





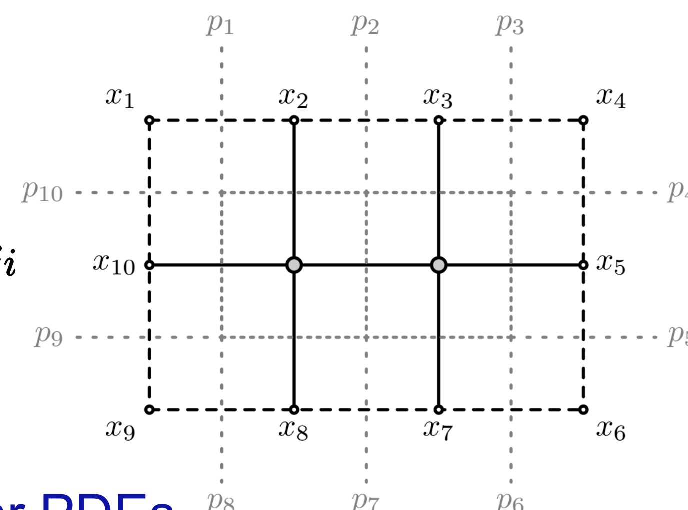


- Typical structure –
  regular square lattice – “fishnet”:
  A disc cut out of square lattice

- Dual graph with light-like momenta
  defines bi-scalar amplitude

$$p_i^2 = x_{i,i+1}^2 = 0$$

$$p_i = x_{i+1} - x_i$$



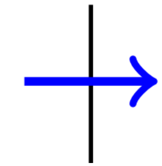
- Manifestly finite, dually conformal invariant
- Integrable, Yangian symmetry! A set of linear PDEs

Chicherin, V.K., Loebbert, Mueller, Zhong (2017)

Yangian invariance of bi-scalar amplitudes

- Conformal Lax operator

$$L_{\alpha\beta}(u_+;u_-) = u\,\delta_{\alpha\beta} + \frac{1}{2} s^{ab}_{\alpha\beta}\,\rho^{\Delta}_{ab} = \begin{pmatrix} 1 & 0 \\ \mathbf{x} & 1 \end{pmatrix} \begin{pmatrix} u_+ \cdot 1 & \mathbf{p} \\ 0 & u_- \cdot 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\mathbf{x} & 1 \end{pmatrix} =$$



Chicherin, Derkachev, Isaev (2012)

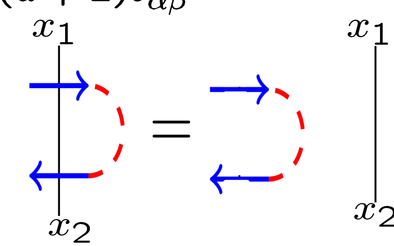
SU(4) generators in fundamental   generators in irrep (0,Δ,0) $\{P_{i,\mu}, L_{i,\mu\nu}, D_i, K_{i,\mu}\} \in su(2,2)$

$$u_+ \equiv u + \frac{\Delta - 4}{2}, \qquad u_- \equiv u - \frac{\Delta}{2} \qquad \mathbf{x} \equiv -i\bar{\boldsymbol{\sigma}}_\mu x_\mu, \quad \mathbf{p} \equiv -\frac{i}{2}\boldsymbol{\sigma}_\mu \partial_{x_\mu}$$

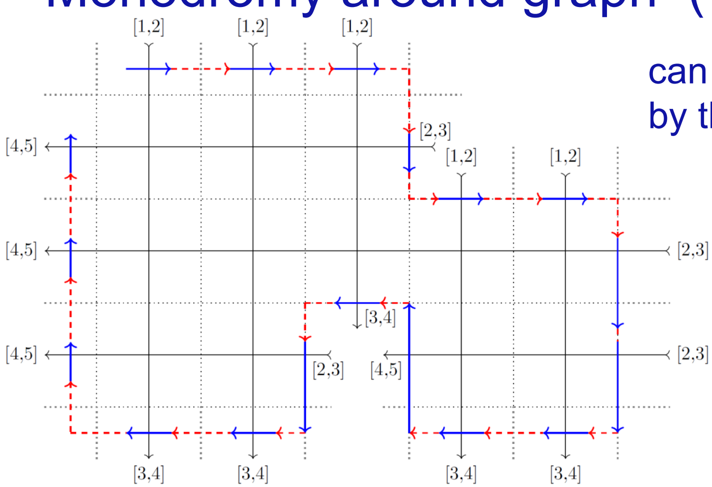
- diagonal action on unity $\quad L_{\alpha\beta}(u, u+2)\cdot 1 = L^T_{\alpha\beta}(u+2, u)\cdot 1 = (u+2)\delta_{\alpha\beta}$

Chicherin, Derkachev, Kirschner (2009)

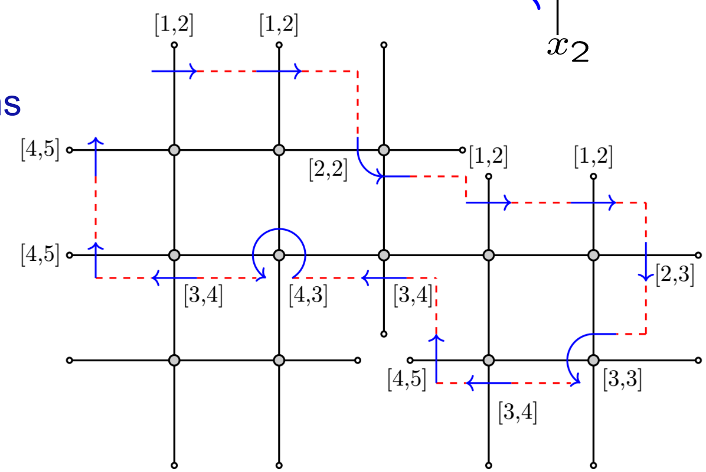
- intertwining $\quad x_{12}^{-2} L_1(u,v) L_2(w, u+1) = L_1(u+1, v) L_2(w,u) x_{12}^{-2}.$



- Monodromy around graph ("lasso")



can be undone
by these operations

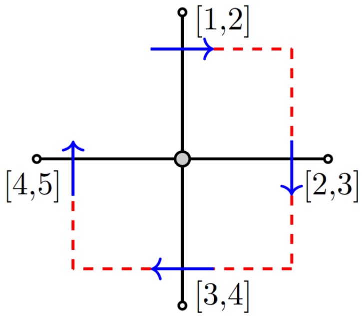


- Feynman graph |G > invariant under action of monodromy around boundary

$$\left(\prod_{i\in\mathcal{C}} L_i[\delta_i^+, \delta_i^-]\right)|G\rangle = \left(\prod_{i\in\mathcal{C}_{\text{out}}} [\delta_i^+][\delta_i^-]\right)|G\rangle\,, \qquad [\delta^\pm] \equiv u_\pm + \delta$$

Examples: Yangian PDE on cross and double-cross integrals

- Expanding in u we get Yangian generators of level 1, etc.
- Differential relation of Yangian symmetry for “cross” integral

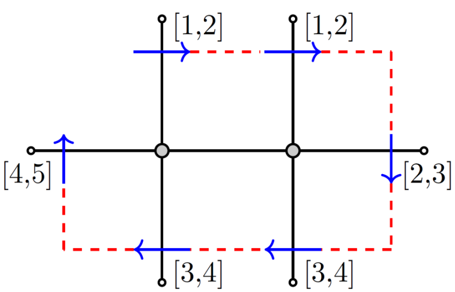


$$\widehat{P}^{\mu}_{\text{cr}}\,|\text{cross}\rangle = 0$$

$$\widehat{P}^{\mu}_{\text{cr}} = -\frac{i}{2}\sum_{j<k=1}^{4}\left[(L_j^{\mu\nu}+\eta^{\mu\nu}D_j)P_{k,\nu}-(j\leftrightarrow k)\right] - P_2^{\mu} - 2P_3^{\mu} - 3P_4^{\mu}$$

$\{P_{i,\mu}, L_{i,\mu\nu}, D_i, K_{i,\mu}\} \in su(2,2)$ - generators of conformal algebra

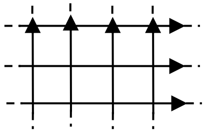
- For “double-cross”:



$$\widehat{P}^{\mu}_{2-\text{cr}}\,|\text{cross}\rangle = 0$$

$$\widehat{P}^{\mu}_{2-\text{cr}} = -\frac{i}{2}\sum_{j<k=1}^{6}\left[(L_j^{\mu\nu}+\eta^{\mu\nu}D_j)P_{k,\nu}-(j\leftrightarrow k)\right] - P_2^{\mu} - 2P_3^{\mu} - 2P_4^{\mu} - 3P_5^{\mu} - 4P_6^{\mu}$$

Conclusions and prospects



- New 4D chiral CFTs from double scaling limit of ɣ-twisted N=4 SYM
- Planar limit dominated by “fishnet” graphs, explicitly integrable at each loop order
- Fishnet limit sheds some light on the origins of integrability of N=4 SYM
- Wheel graphs from QSC or from conformal SU(2,2) spin chain (L=2,3, L>3 in work)
- Some all-loop 4-point correlation function computed. Exact all loop OPE data!

Grabner, Gromov, V.K., Korchemsky ’17 Olivucci, V.K. ‘18

- Bi-scalar amplitudes are finite and exhibit explicit Yangian invariance

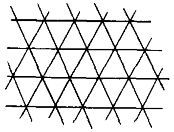
Chicherin, V.K., Loebbert, Mueller, Zhong 2017

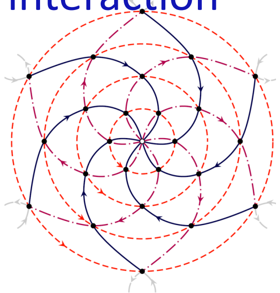
Correa, Maldacena, Sever 2012
Cavaglia, Gromov, Levkovich-Maslyuk 2018

- Structure constants? 1/N corrections? quark-antiquark potential ……. in DS limit
- Similar observation for DS limit of ɣ-deformed 3D ABJM – 6-scalar chiral interaction

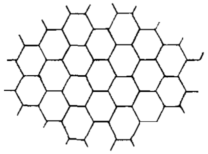
Caetano, Gurdogan, V.K ‘16

$$\mathcal{L}^{(DS)}_{\text{ABJM}} = \text{Tr}\left[-\partial^{\mu}Y_j^{\dagger}\partial_{\mu}Y^j + \quad \xi^3 Y^1 Y_2^{\dagger} Y^3 Y_1^{\dagger} Y^2 Y_3^{\dagger}\right]$$



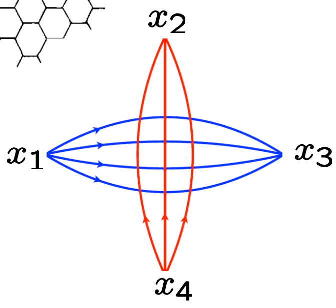


- 6D tri-scalar theory: hexagonal graphs



Torrens, Mamroud ‘17

- Basso-Dixon 4-point correlation function



Basso, Dixon 2017

- String dual of fishnet CFT? Sigma-model on AdS_5 !

Basso, Zhong 2018

Thank you!