Y junctions of Heisenberg spin chains

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Outline

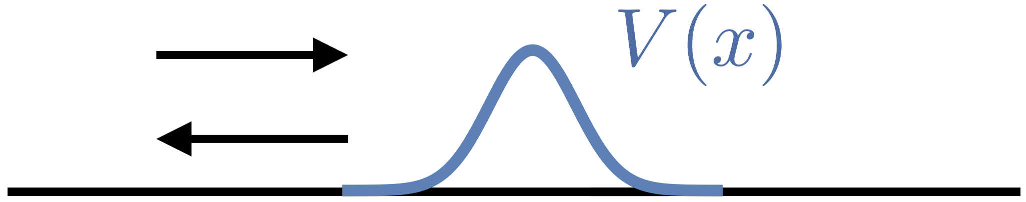
- Junctions of quantum wires and spin chains
- Y junction model and its fixed points
- Numerics (DMRG)
- From Y junction to networks

In collaboration with **Francesco Buccheri** and **Reinhold Egger** (Düsseldorf)
Flavia Ramos (IIP Natal)

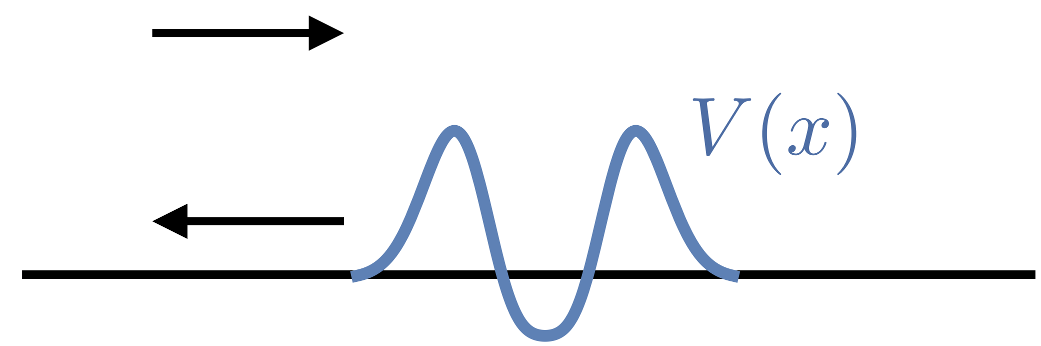
Junction of two quantum wires

- Kane and Fisher: impurities in 1D electron liquid. [1992]

- Renormalization of scattering amplitudes depends on electron-electron interactions in the wires.



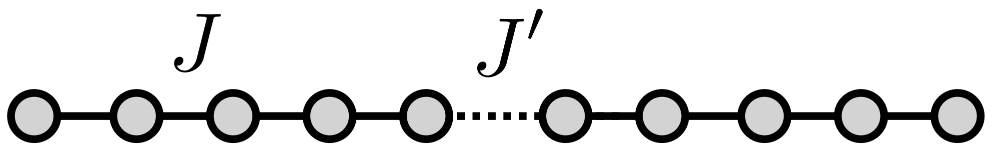
single barrier: total reflection at low energy fixed point for repulsive interactions

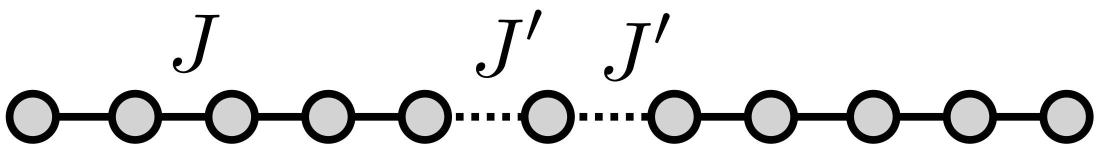


double barrier: resonant tunnelling (ideal conductance) possible by fine tuning

Junction of two spin chains

- Eggert and Affleck: impurities in XXZ spin chains. [1992]





Junction of two spin chains

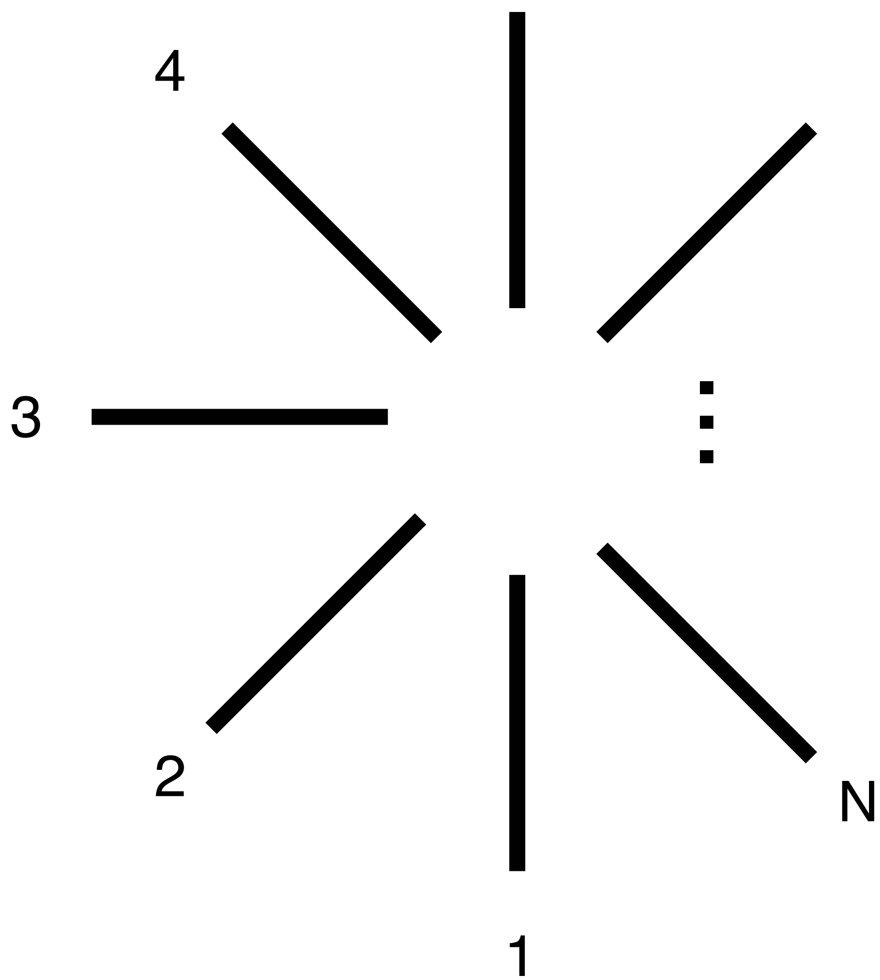
- Eggert and Affleck: impurities in XXZ spin chains. [1992]

(analogy with two-channel Kondo effect)

- Integrable spin chains with impurities.

  [Andrei and Johanesson 1984; Zvyagin and Frahm 1997]

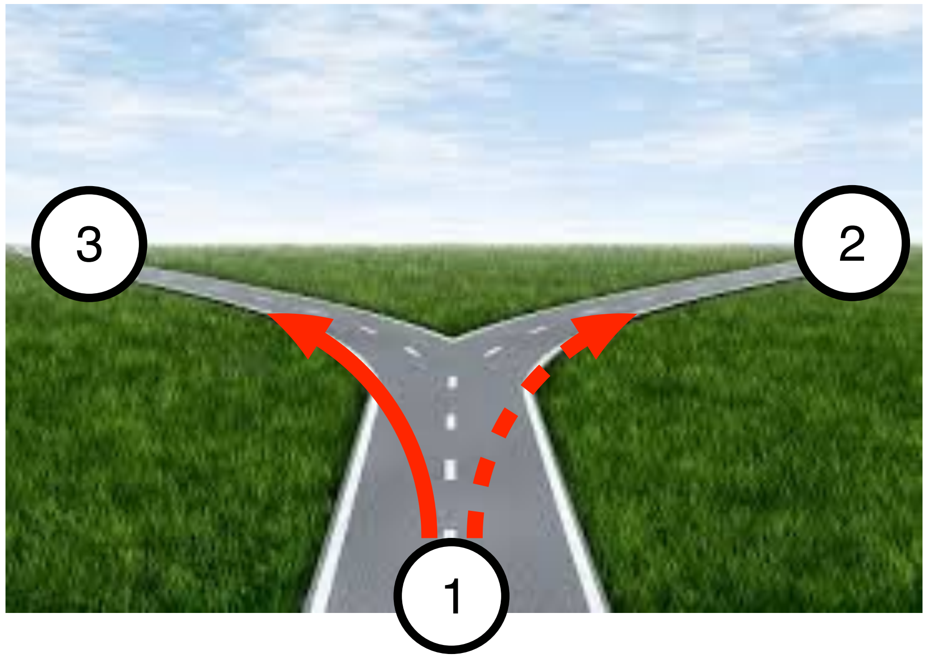
Multiple-wire junctions



Y junctions (N = 3)

- Conductance matrix

$$I_\alpha = \sum_{\alpha'=1}^{3} G_{\alpha\alpha'} V_{\alpha'}$$



$$G_{\alpha+1,\alpha} \neq G_{\alpha-1,\alpha} \Rightarrow \text{ chiral junction}$$

$(\alpha + 3 \equiv \alpha)$

- Breaks reflection symmetry (P) as well as time reversal symmetry (T).

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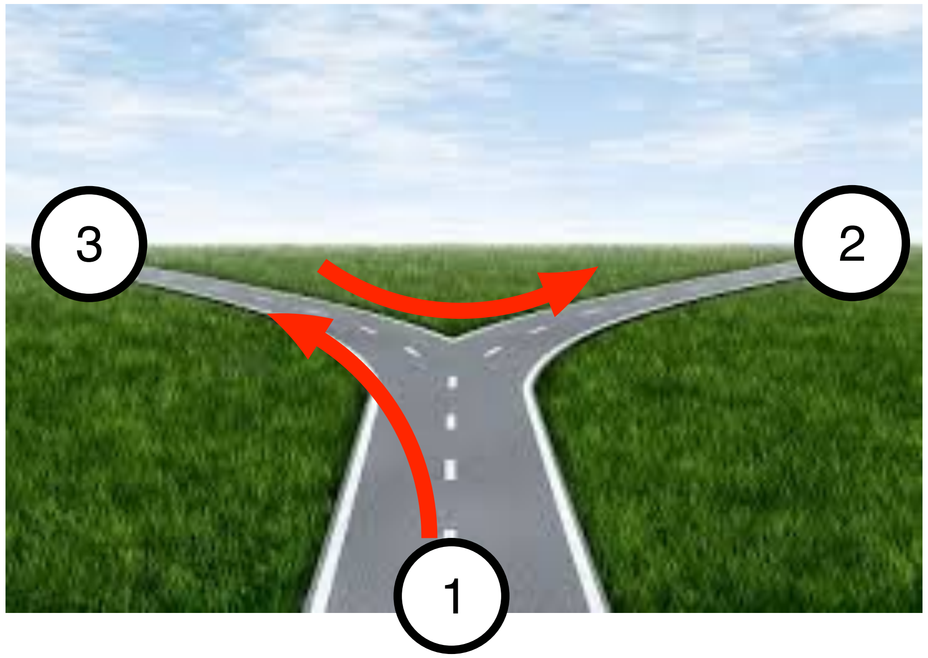

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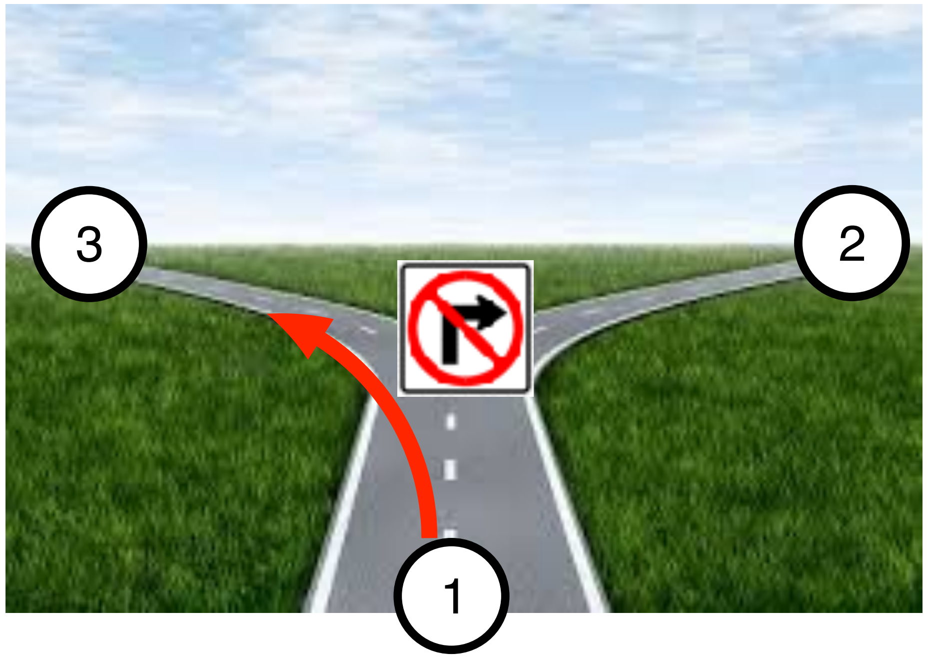


$$G_{\alpha+1,\alpha} \neq G_{\alpha-1,\alpha} \Rightarrow \text{ chiral junction} \qquad (\alpha + 3 \equiv \alpha)$$

- Breaks reflection symmetry (P) as well as time reversal symmetry (T).

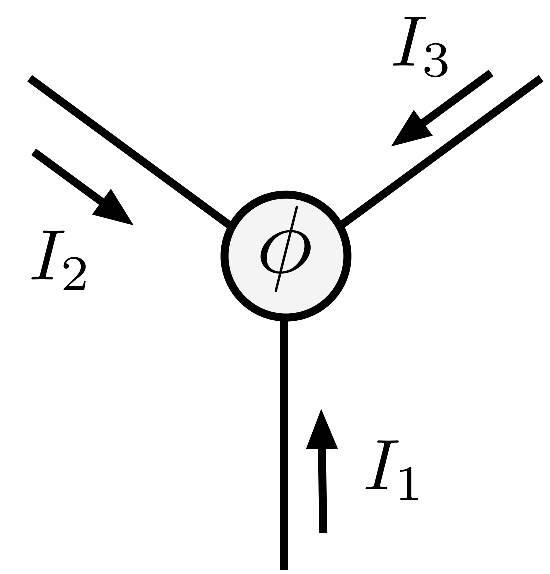
- Ideal circulator: $G_{\alpha,\alpha-1} = 1$ (in proper units)

$$G_{\alpha,\alpha+1} = 0$$

Y junction of quantum wires

- Tunneling in the presence of a magnetic flux:

  [Oshikawa, Chamon & Affleck 2003; 2006]



$$H_b = t' \sum_{\alpha=1}^{3} e^{i\phi/3} \psi^\dagger_{\alpha+1}(0)\psi_\alpha(0) + \text{h.c.}$$

$$G_{\alpha\alpha'} = \frac{e^2}{h}\left[\frac{G_S}{2}(3\delta_{\alpha\alpha'} - 1) + \frac{G_A}{2}\epsilon_{\alpha\alpha'}\right]$$

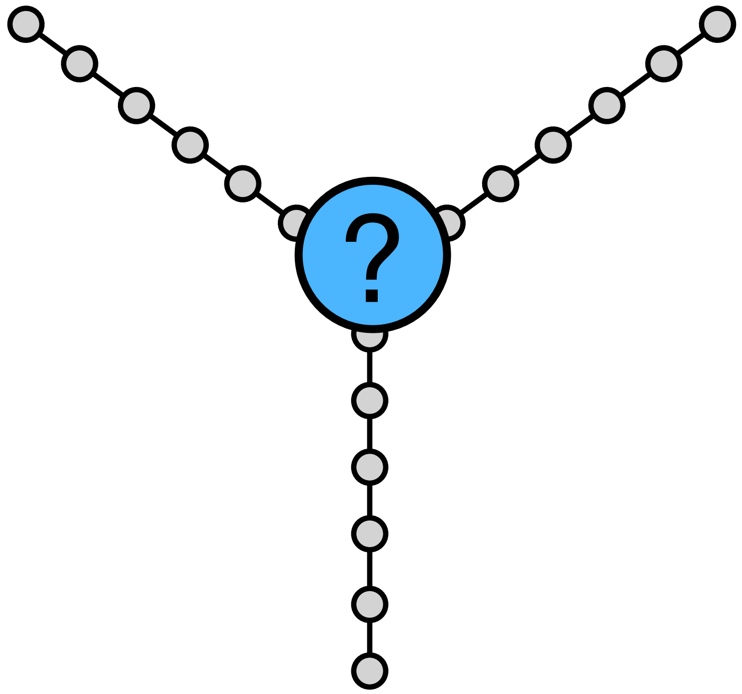
$$(\epsilon_{\alpha\alpha'} = -\epsilon_{\alpha'\alpha})$$

- Stable chiral fixed points with $G_A \neq 0$ for flux $\phi \neq 0, \pi$ and attractive electron-electron interactions in the wires (Luttinger parameter $1 < K < 3$).

Question

- Can we get a chiral fixed point with spin-1/2 XXX chains?

- First guess: no!
  Antiferromagnetic spin chain equivalent to spinless fermions with <u>repulsive</u> interactions.



- But maybe it exists at finite coupling…

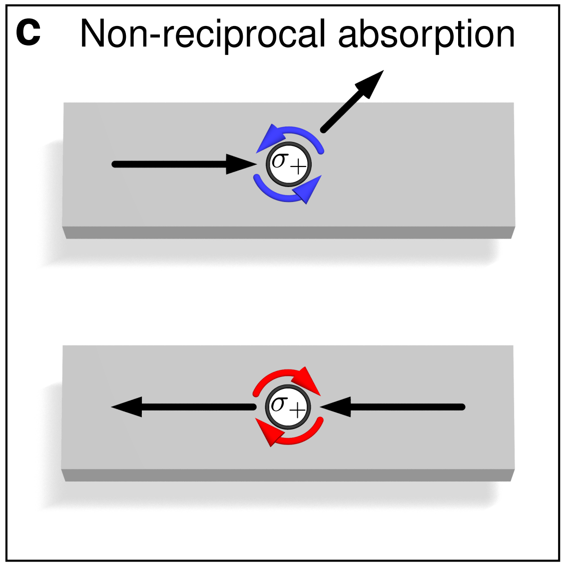
Why we want to try

- Quantum circulators have been realized in nanophotonic devices: chiral quantum optics.

  [Scheucher et al., Science 2016]

  Analogue for antiferromagnetic spintronics?

  [Baltz … Tserkovnyak, RMP 2018]



[Lodahl et al., Nature 2017]

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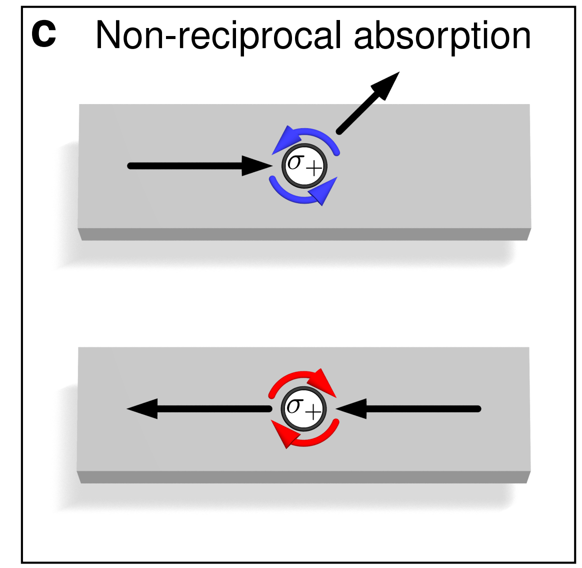
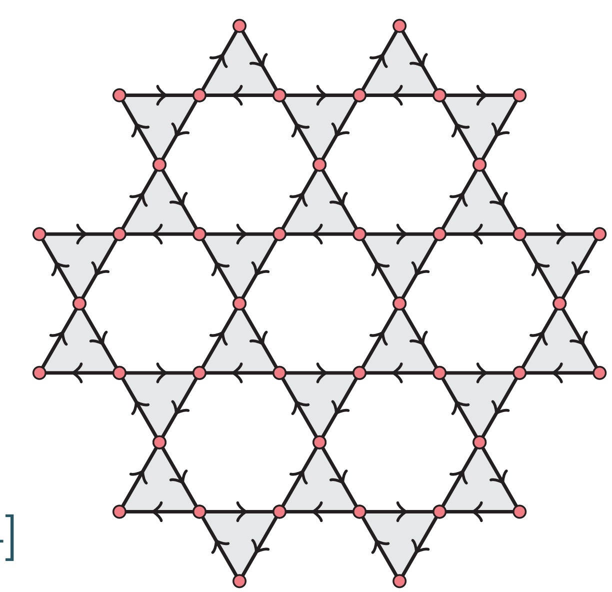


[Lodahl et al., Nature 2017]

- Relation to chiral spin liquids?

  [Kalmeyer & Laughlin 1987; Wen, Wilczek & Zee 1989]

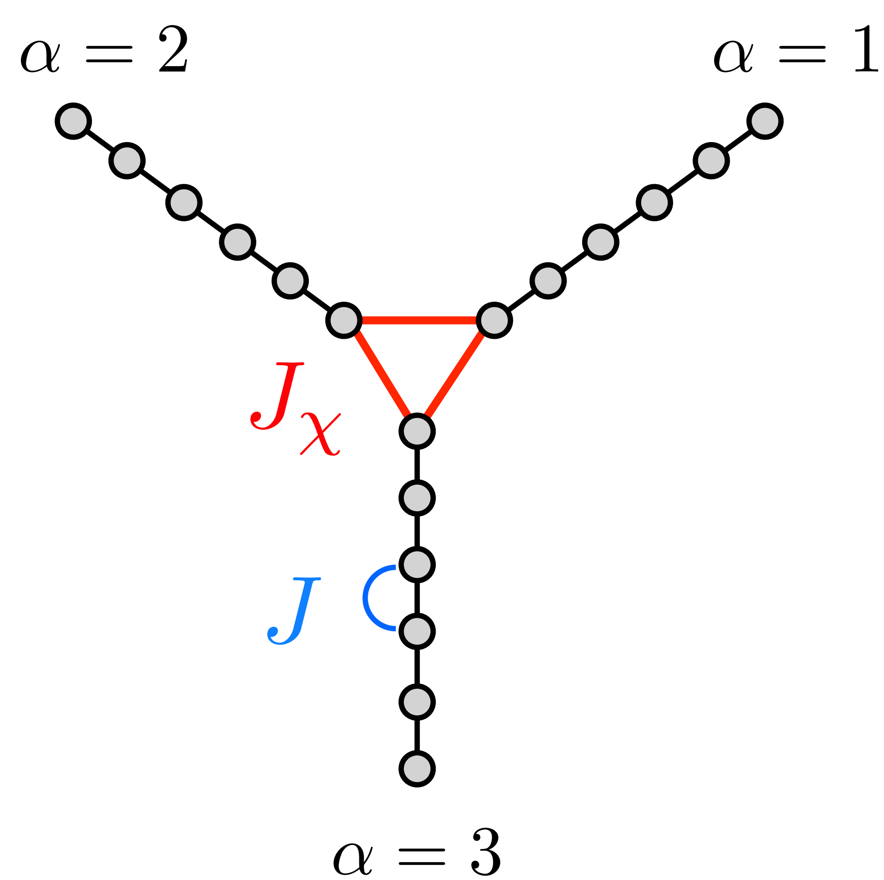
[Bauer et al., Nat. Phys. 2014]



Model: Y junction of spin-1/2 Heisenberg chains

- Three-spin interaction at the boundary preserves SU(2) but breaks reflection (*P*) and time reversal (*T*) symmetries.

[Buccheri, Egger, R.P. & Ramos, PRB 2018]



$$H = J \sum_{\alpha=1}^{3} \sum_{j \geq 1} \mathbf{S}_\alpha(j) \cdot \mathbf{S}_\alpha(j+1) + J_\chi \mathbf{S}_1(1) \cdot [\mathbf{S}_2(1) \times \mathbf{S}_3(1)]$$

$$\mathcal{P} : \alpha \mapsto -\alpha \qquad \mathcal{T} : \mathbf{S} \mapsto -\mathbf{S}$$

- J_χ can be realized as a Floquet spin model.

[Claassen, Jiang, Moritz & Devereaux 2017]

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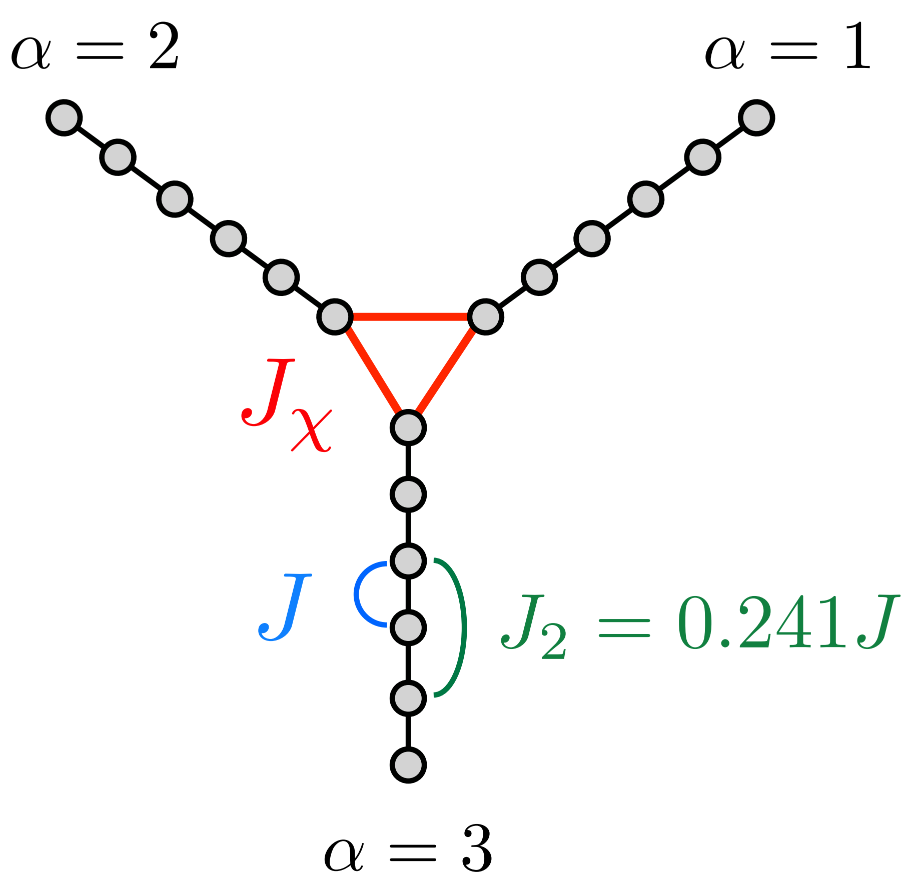

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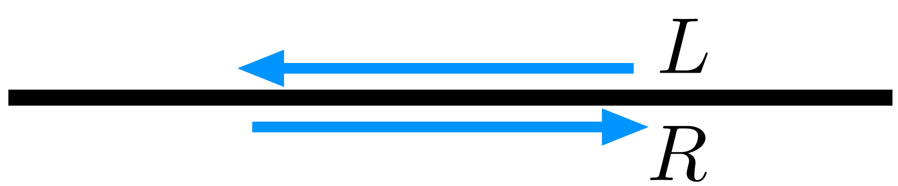
Effective field theory for a single Heisenberg chain

- Spin-1/2 chain described by CFT with central charge c=1: $SU(2)_1$ Wess-Zumino-Witten (WZW) model. [Affleck & Haldane 1987]

- Hamiltonian: $$H = \frac{2\pi v_s}{3} \int dx (\mathbf{J}_R^2 + \mathbf{J}_L^2)$$

$$J_L^z(x) \sim \partial_x \varphi_L(x)$$

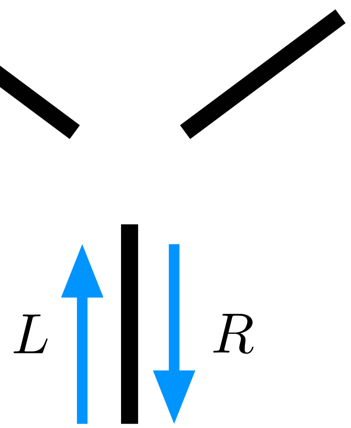
$$J_L^{\pm}(x) \sim e^{\pm i\sqrt{4\pi}\varphi_L(x)}$$



- Spin operator:

$$\mathbf{S}_j \to \mathbf{S}(x) \sim \mathbf{J}_R(x) + \mathbf{J}_L(x) + (-1)^x \mathbf{n}(x)$$

Y junction at weak coupling



- Continuum limit for $J_\chi = 0$:.

$$H_0 = \sum_{\alpha=1}^{3} \frac{2\pi v}{3} \int_0^\infty dx \, \left[\mathbf{J}_{R,\alpha}^2 + \mathbf{J}_{L,\alpha}^2 \right]$$

- Open (O) boundary conditions at x = 0: $\mathbf{J}_{R,\alpha}(x) = \mathbf{J}_{L,\alpha}(-x)$

- Boundary spin operator: $\mathbf{S}_\alpha(j=1) \sim \mathbf{J}_{L,\alpha}(0)$

- Boundary operators are irrelevant: stable fixed point.

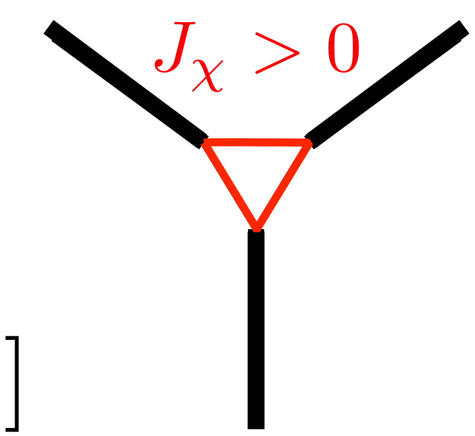
$$\delta H = C_1 \sum_\alpha \mathbf{J}_{L,\alpha}(0) \cdot \mathbf{J}_{L,\alpha+1}(0) + C_2 \mathbf{J}_{L,1}(0) \cdot [\mathbf{J}_{L,2}(0) \times \mathbf{J}_{L,3}(0)] + \dots$$

dimension 2   dimension 3

Y junction at strong coupling

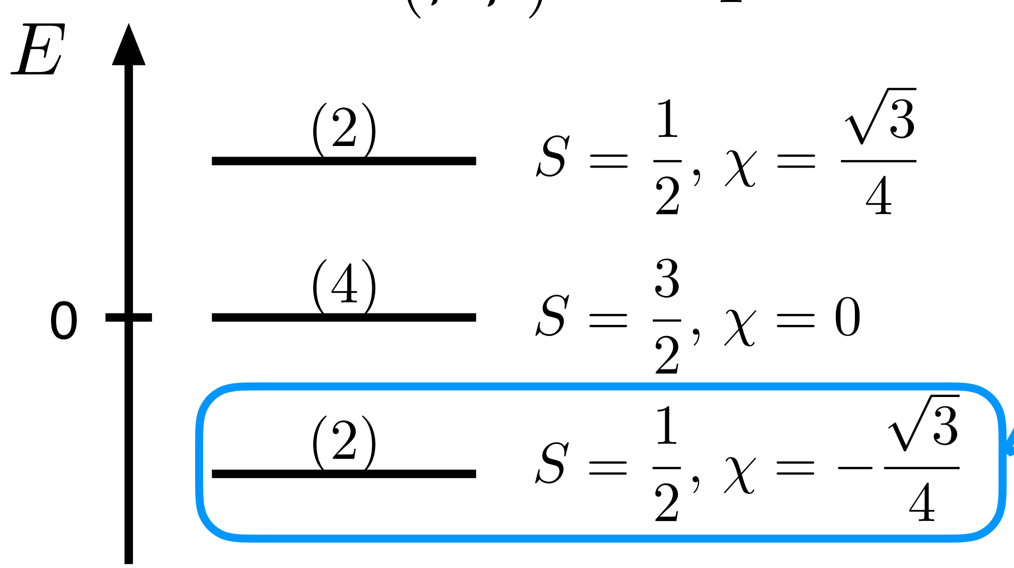
- For $J_\chi \gg J$, start by diagonalizing boundary interaction:

$$H_b = J_\chi \hat{\chi} \qquad \hat{\chi} = \mathbf{S}_1(1) \cdot [\mathbf{S}_2(1) \times \mathbf{S}_3(1)]$$



- Low-energy states form an effective pseudospin 1/2:

$$(\mathcal{PT})^2 = -1$$



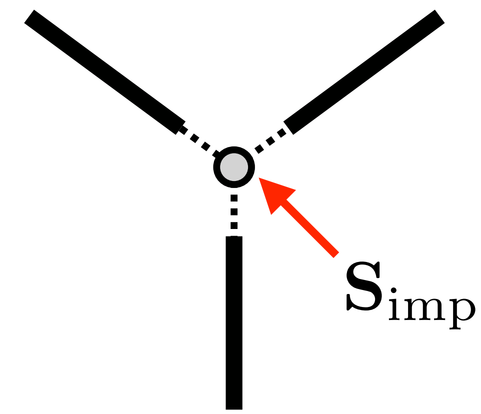
$$|+\rangle = \frac{i}{\sqrt{3}}\left(|\downarrow\uparrow\uparrow\rangle + \omega|\uparrow\uparrow\downarrow\rangle + \omega^2|\uparrow\downarrow\uparrow\rangle\right)$$

$$|-\rangle = -\frac{i}{\sqrt{3}}\left(|\uparrow\downarrow\downarrow\rangle + \omega^2|\downarrow\uparrow\downarrow\rangle + \omega|\downarrow\downarrow\uparrow\rangle\right)$$

$$\omega = e^{i2\pi/3}$$

Effective Hamiltonian at strong coupling

- Projection onto low-energy subspace gives "impurity spin" coupled to three chains.



$$H_K = J_K \mathbf{S}_{\text{imp}} \cdot \sum_\alpha \mathbf{S}_\alpha(2)$$

$$J_K = \frac{J}{3}$$

- Kondo coupling is marginally relevant:

$$\ell = \ln(\Lambda_0/\Lambda)$$

$$\lambda_K = \frac{J_K}{2\pi v} \qquad \frac{d\lambda_K}{d\ell} = \lambda_K^2 - \frac{3}{2}\lambda_K^3 + \ldots$$

- Flow to strong coupling: three-channel Kondo fixed point (K).

- Can be described by boundary CFT. [Cardy; Affleck and Ludwig]

Boundary CFT approach

- Conformally invariant boundary condition for Kondo fixed point obtained by fusion with spin-1/2 primary of WZW model representing the sum of spin currents.

- Conformal embedding for two chains:

$$\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_1 \to \underbrace{\mathrm{SU}(2)_2}_{\text{total spin}} \times \underbrace{\mathrm{Ising}}_{\text{"flavor"}}$$

[Eggert and Affleck]

central charge:

$$c = \frac{3}{2} + \frac{1}{2} = 2$$

- For three chains: [Buccheri, Egger, R.P. & Ramos, PRB 2018]

$$\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_1 \times \mathrm{SU}(2)_1 \to \mathrm{SU}(2)_3 \times \mathbb{Z}_3^{(5)}$$

$$c = \frac{9}{5} + \frac{6}{5} = 3$$

belongs to family of $Z_3^{(p)}$ CFTs with W_3 algebra

[Zamolodchikov & Fateev 1987; Fateev & Lukyanov 1988; Affleck, Oshikawa & Saleur 2001]

Stability of the Kondo fixed point

- Operator content from partition function Z_{AB} on the cylinder (even-length chains).

$$\Delta = \Delta_s + \Delta_f$$

AB	$s = 0\,(\Delta_s = 0)$	$s = 1\,(\Delta_s = 2/5)$
OO	$0, 2(\times 2)$	$3/5(\times 2), 8/5$
KK	$0, 3/5(\times 2), 8/5, 2(\times 2)$	$0, 3/5(\times 2), 8/5, 2(\times 2)$
CC	$0, 1/2(\times 3), 2(\times 2)$	$1/10(\times 3), 3/5(\times 2), 8/5$
KO	$3/5(\times 2), 8/5$	$0, 3/5(\times 2), 8/5, 2(\times 2)$

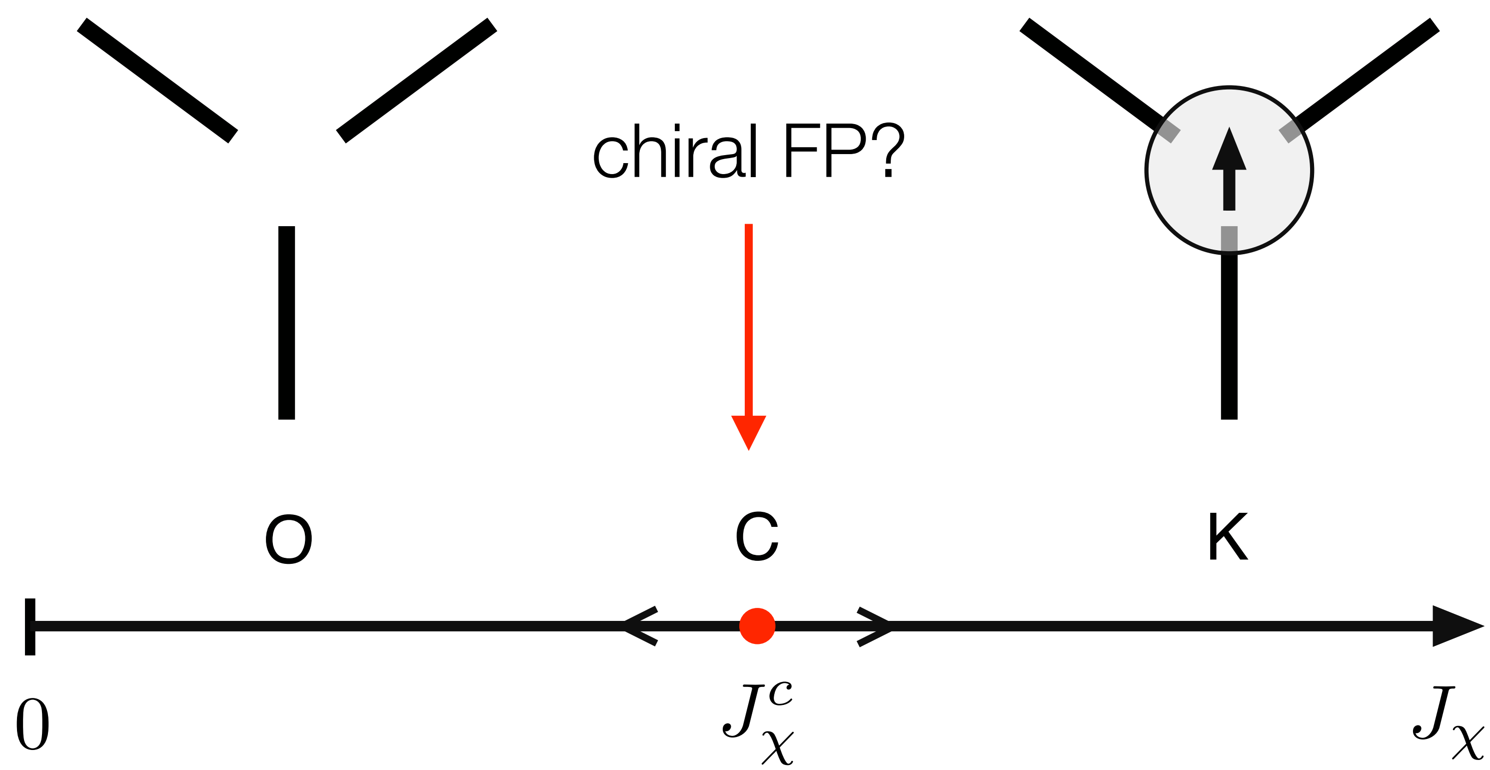
- Z_3 symmetry (cyclic permutation of chains) rules out relevant perturbation and stabilizes three-channel Kondo fixed point.

- Boundary operators: $\delta H = C_1' \mathbf{J}_{-1} \cdot \boldsymbol{\phi}_1 \; + \; C_2' \Omega \; + \; \ldots$

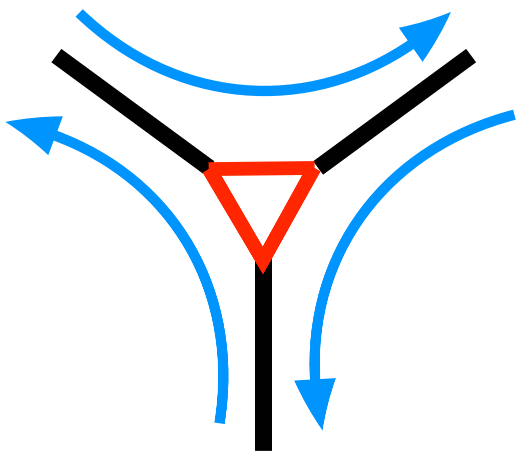
(irrelevant!) dimension 7/5 dimension 8/5

Looking for a critical point

- *P* and *T* symmetries are restored at stable fixed points.



Imposing chiral boundary conditions



- SU(2) invariant chiral fixed points (C):

$$\mathbf{J}_{R,\alpha}(x) = \mathbf{J}_{L,\alpha\pm 1}(-x)$$

- Boundary CFT: fusion with Z_3-charged fields in $Z_3^{(5)}$ sector.

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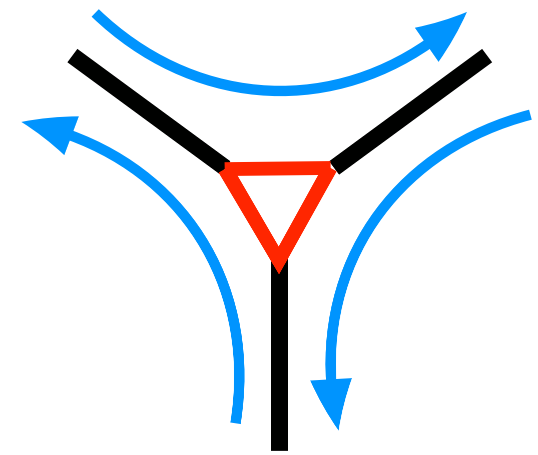

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$$\mathbf{J}_{R,\alpha}(x) = \mathbf{J}_{L,\alpha\pm1}(-x)$$

- Boundary CFT: fusion with Z_3-charged fields in $Z_3^{(5)}$ sector.
- Relevant perturbation: $\delta H = \lambda_1 \sum_\alpha \cos\left\{\sqrt{\pi}\left[\varphi_{L,\alpha}(0) - \varphi_{L,\alpha+1}(0)\right]\right\}$
- C point requires fine tuning λ_1=0; magnetization switches from integer to half-integer values (cf. double barrier).

$$S^z_{\rm tot} \sim \frac{1}{\sqrt{4\pi}} \sum_\alpha \int_0^\infty dx\ (\partial_x\varphi_{L,\alpha} - \partial_x\varphi_{R,\alpha}) = \begin{cases} n \in \mathbb{Z}, & \lambda_1 < 0 \\ n + \frac{1}{2}, & \lambda_1 > 0 \end{cases}$$

A check: boundary entropy

$$\ln Z = \frac{\pi c}{6v}\frac{L}{\beta} + \ln g + \ldots \qquad L \gg v\beta$$

[Affleck, Ludwig 1991]

- Critical point must have the highest “ground state degeneracy” *g* because *g* decreases under the renormalization group flow (*g*-theorem).

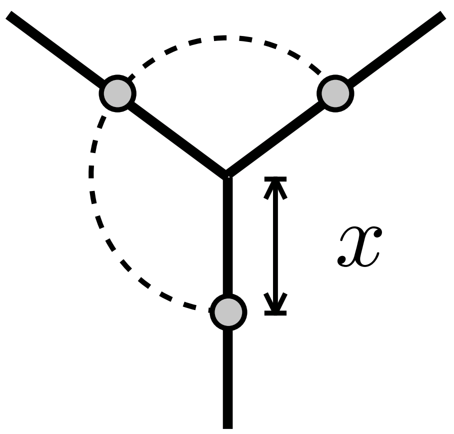
Open: $g_{\mathrm{O}} = 1$

Kondo: $g_{\mathrm{K}} = \frac{1+\sqrt{5}}{2} \approx 1.618$

Chiral: $g_{\mathrm{C}} = 2$ (same as double barrier/resonant level)

Three-spin correlations

$$G_3(x) = \langle \mathbf{S}_1(x) \cdot [\mathbf{S}_2(x) \times \mathbf{S}_3(x)] \rangle$$



- At the chiral fixed point:

$$G_3(x) \sim (-1)^x \langle e^{i\sqrt{\pi}[\varphi_{L1}(x)-\varphi_{L2}(-x)]} e^{i\sqrt{\pi}[\varphi_{L2}(x)+\varphi_{L3}(-x)]} e^{-i\sqrt{\pi}[\varphi_{L3}(x)+\varphi_{L1}(-x)]} \rangle$$

$$G_3(x) \sim \frac{(-1)^x}{x^{3/2}}$$

- At O and K points, apply first-order perturbation theory in the irrelevant chiral operator (scaling dimension Δ):

$$G_3(x) \sim \frac{(-1)^x}{x^{\Delta + \frac{1}{2}}}$$

Open: $\Delta = 3$

Kondo: $\Delta = \frac{8}{5}$

Numerics: pinpointing the critical point

- Density matrix renormalization group (DMRG) for Y junctions with open boundary at $j = L$. [Guo and White 2006]

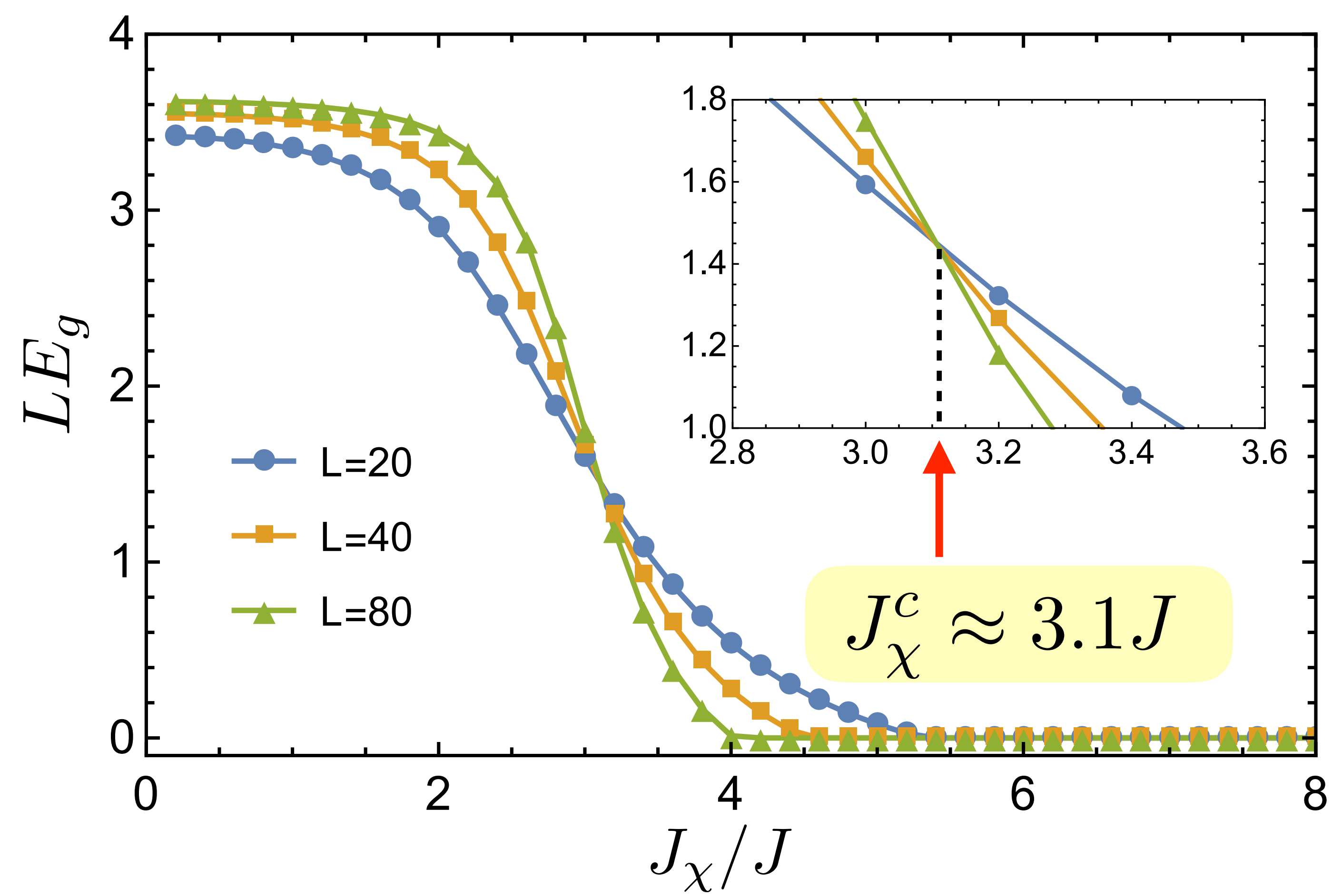
- Finite-size gap: $E_g = E_0(S^z_{\rm tot} = 1) - E_0(S^z_{\rm tot} = 0)$

OO boundary conditions (singlet ground state)

$$E_g = \pi v / L$$

KO boundary conditions (triplet ground state)

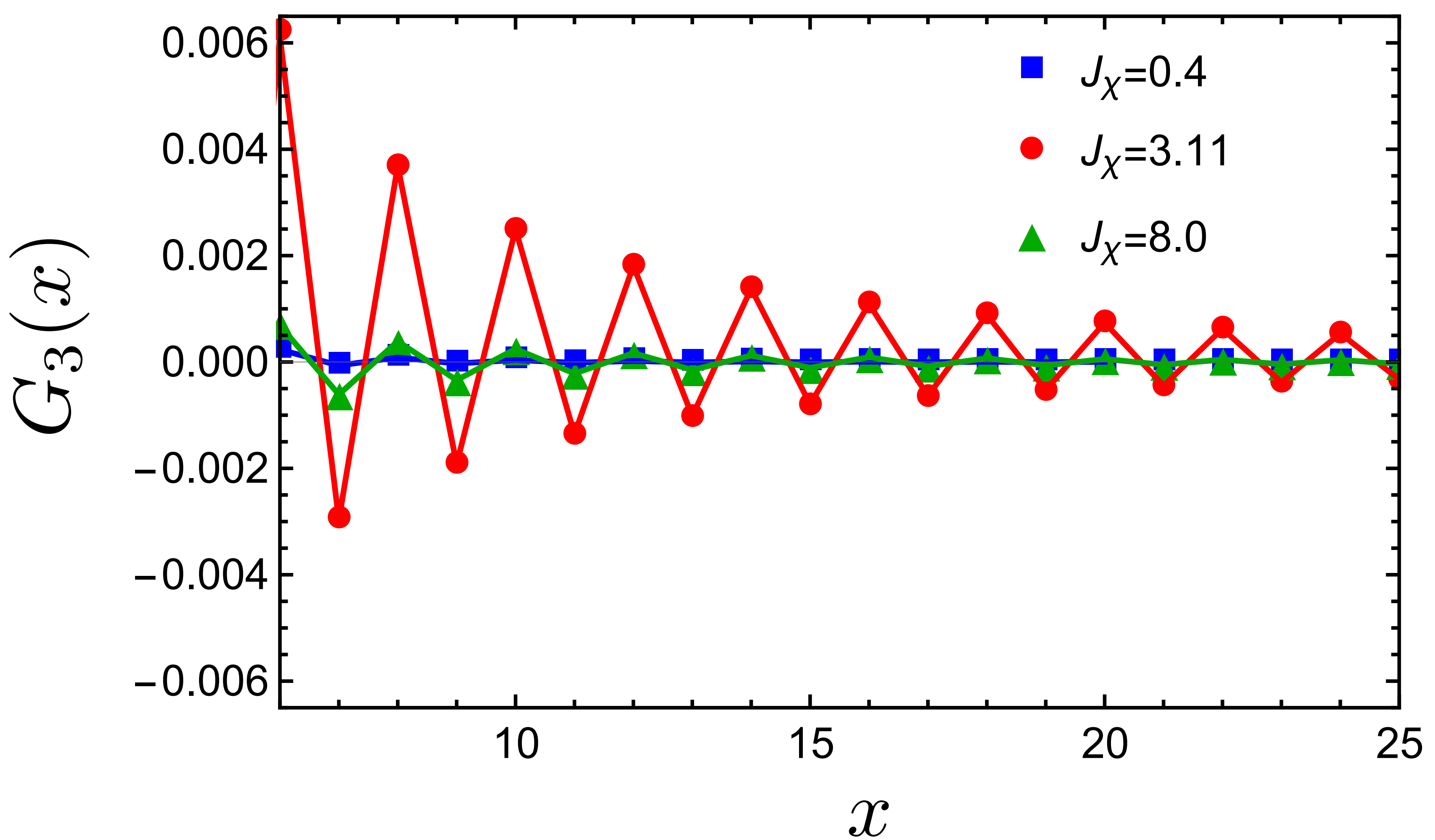
$$E_g = 0$$



Numerics: large-distance decay of correlations

exponents:

J_χ/J	$L=40$	$L=60$	$L=80$	Extrap.	Expected	
0.4	3.56	3.51	3.49	3.45	3.5	**Open**
3.11	1.89	1.79	1.74	1.59	1.5	**Chiral**
8	2.31	2.22	2.18	2.08	2.1	**Kondo**



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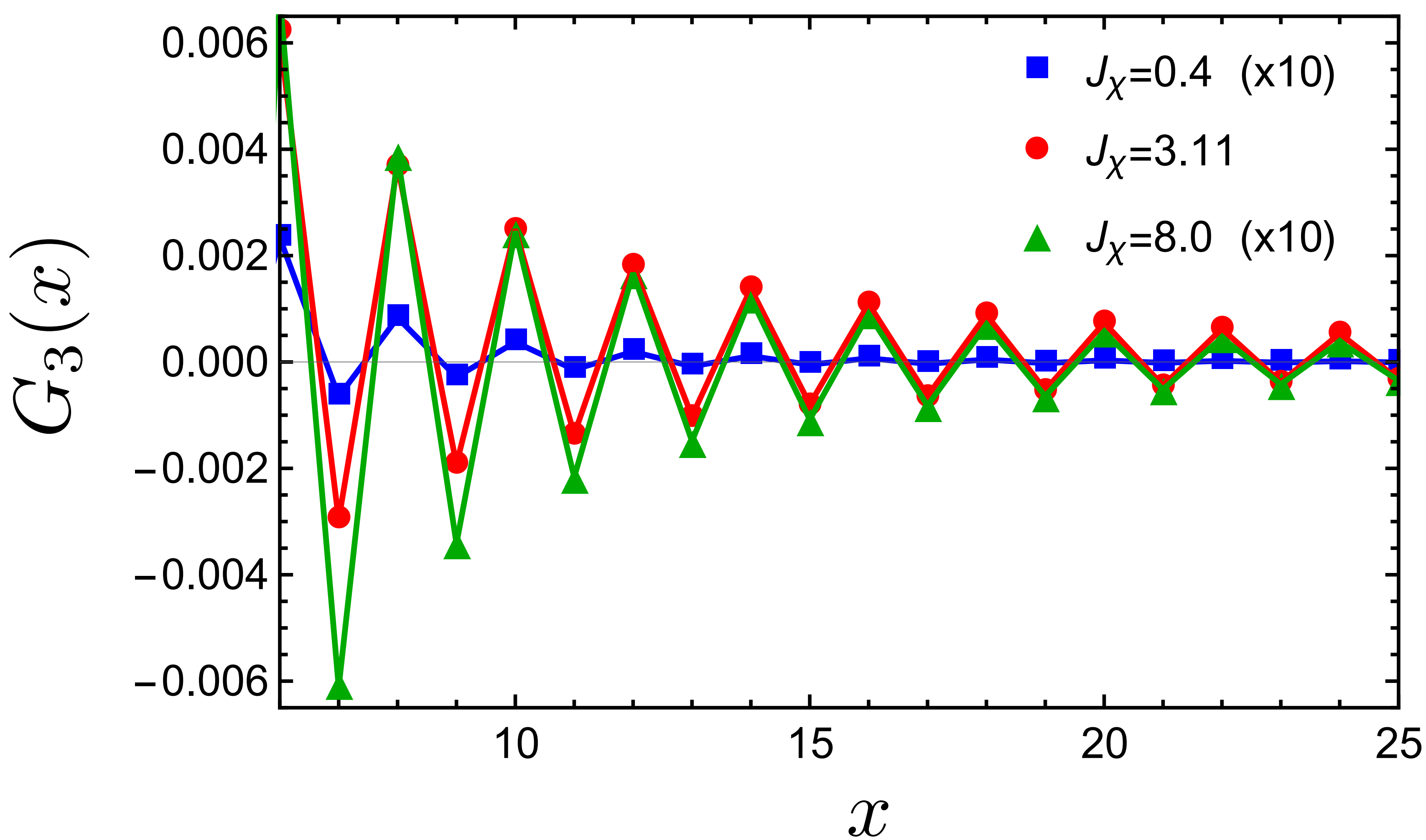


Spin conductance

- Kubo formula:

$$G^{ab}_{\alpha\alpha'} = -\lim_{\omega\to 0+}\frac{1}{\omega L}\int_{-\infty}^{+\infty} d\tau \int_0^L dx\, \langle T_\tau J^a_\alpha(y,\tau) J^b_{\alpha'}(x,0)\rangle$$

- Spin current operator: $\mathbf{J}_\alpha(x,\tau) = \mathbf{J}_{R,\alpha}(\bar{z}) - \mathbf{J}_{L,\alpha}(z)$

- At the SU(2)-symmetric chiral fixed point, conductance is maximally asymmetric: ideal quantum spin circulator.

$$G^{ab}_{\alpha\alpha'} = G_0 \delta^{ab}\left(\delta_{\alpha\alpha'} - \delta_{\alpha,\alpha'\pm 1}\right)$$

$$G_0 = \frac{1}{2}\frac{(g\mu_B)^2}{h}$$

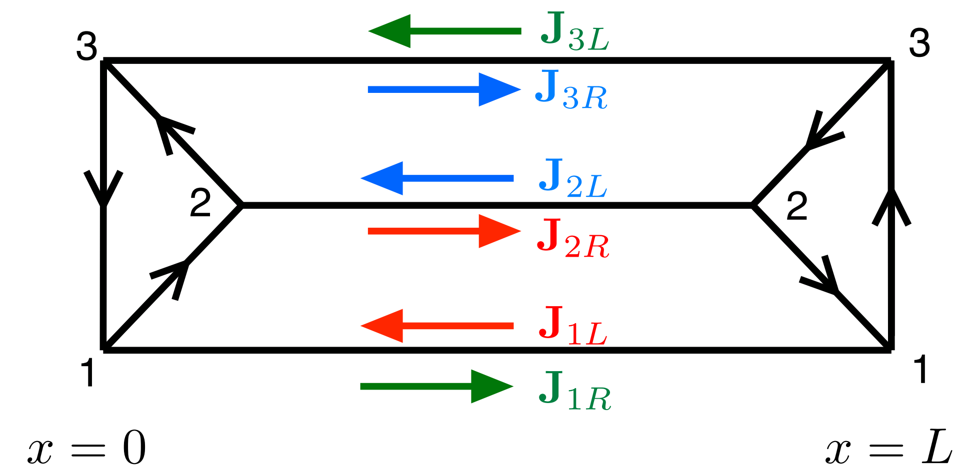
Numerical result for the spin conductance

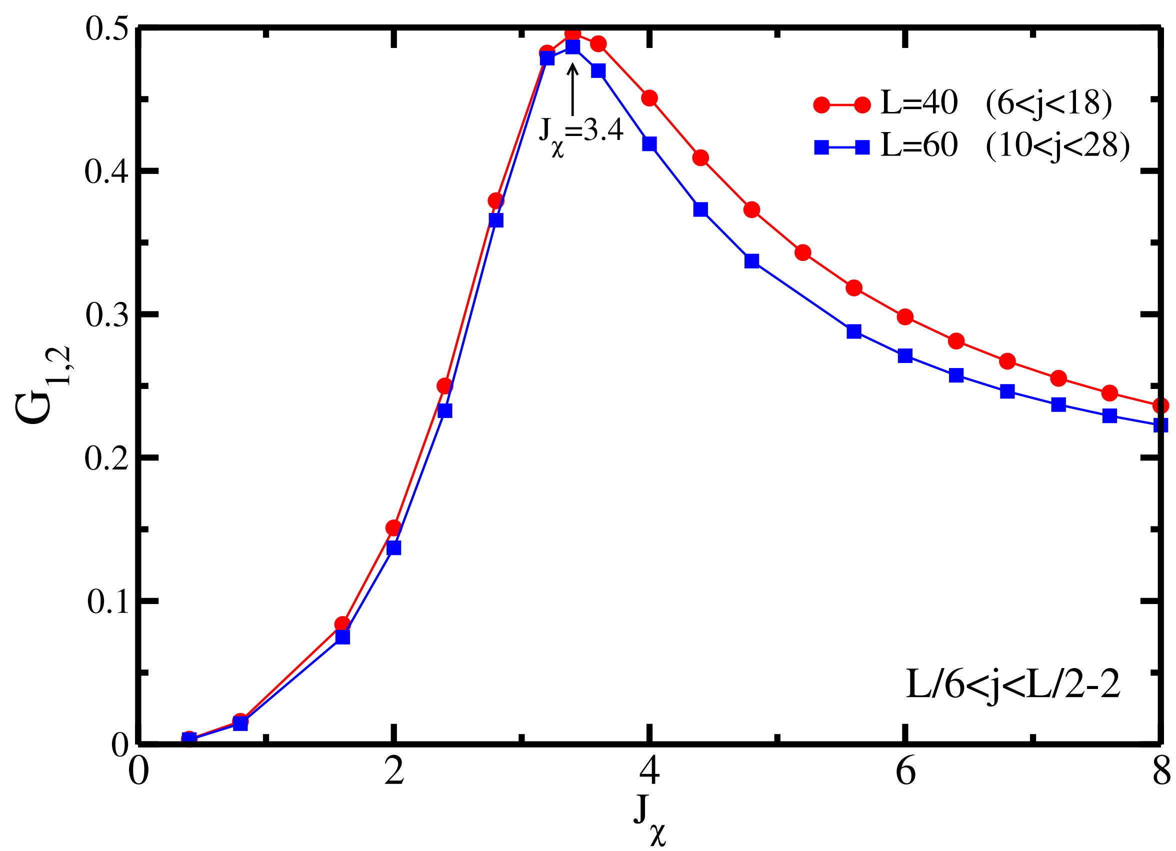
- Using conformal symmetry, conductance can be calculated from static correlations on strip with the same boundary condition at both ends.

  [Rahmani et al. 2010]

- DMRG for finite chains: conductance peak at critical value of $J\chi$.

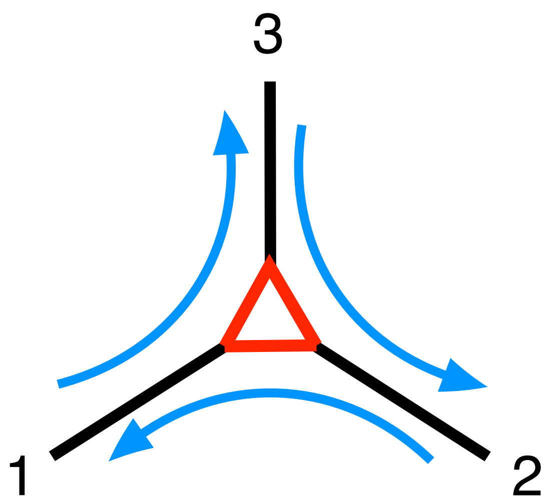
  [Buccheri, Egger, R.P. & Ramos, in preparation]



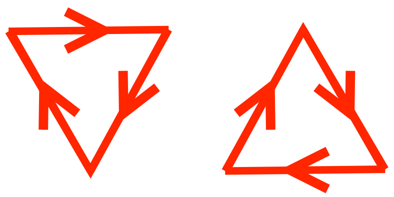


From Y junction to networks: uniform chirality


Gabriel Ferraz



$$\mathbf{J}_{R,\alpha}(0) = \mathbf{J}_{L,\alpha+1}(0)$$

$J_\chi > 0$  $J_\chi > 0$

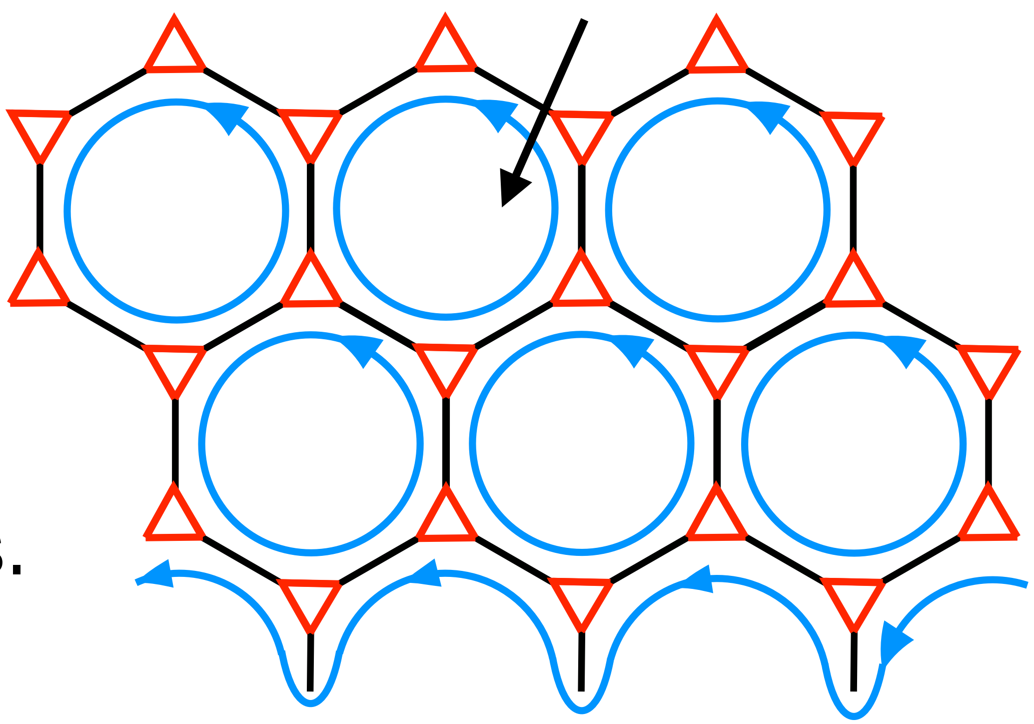
- Finite spin gap in the bulk and gapless edge states (chiral SU(2)$_1$ WZW model): Kalmeyer-Laughlin chiral spin liquid. [Kalmeyer & Laughlin 1987]

- Cf. coupled-chain constructions.

  [Gorohovsky, R.P. & Sela 2015]

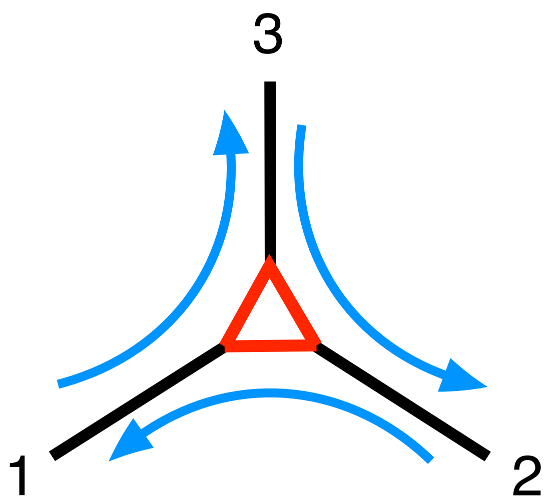
  [Huang … Chamon & Mudry 2016]

$E_{\text{gap}} \sim v/L$

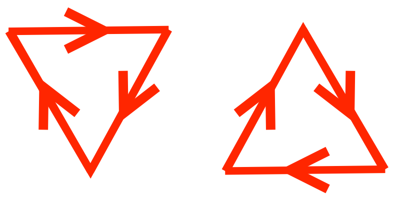


From Y junction to networks: uniform chirality

Gabriel Ferraz



$$\mathbf{J}_{R,\alpha}(0) = \mathbf{J}_{L,\alpha+1}(0)$$

$J_\chi > 0$  $J_\chi > 0$

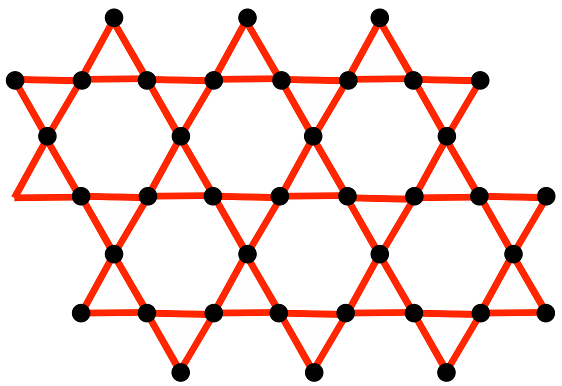
- Finite spin gap in the bulk and gapless edge states (chiral SU(2)$_1$ WZW model): Kalmeyer-Laughlin chiral spin liquid. [Kalmeyer & Laughlin 1987]

- Cf. coupled-chain constructions.

  [Gorohovsky, R.P. & Sela 2015]

  [Huang … Chamon & Mudry 2016]

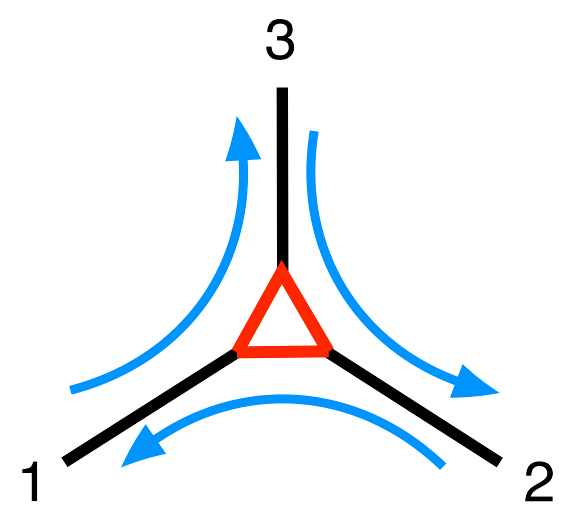
$$L \to 0, \; E_{\text{gap}} \sim J_\chi$$



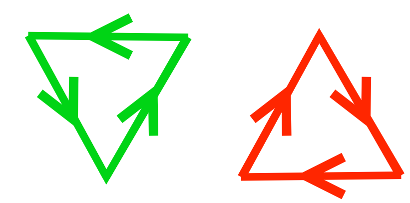
cf. numerical evidence for gapped chiral spin liquid on the kagome lattice

[Bauer et al. 2014]

From Y junction to networks: staggered chirality

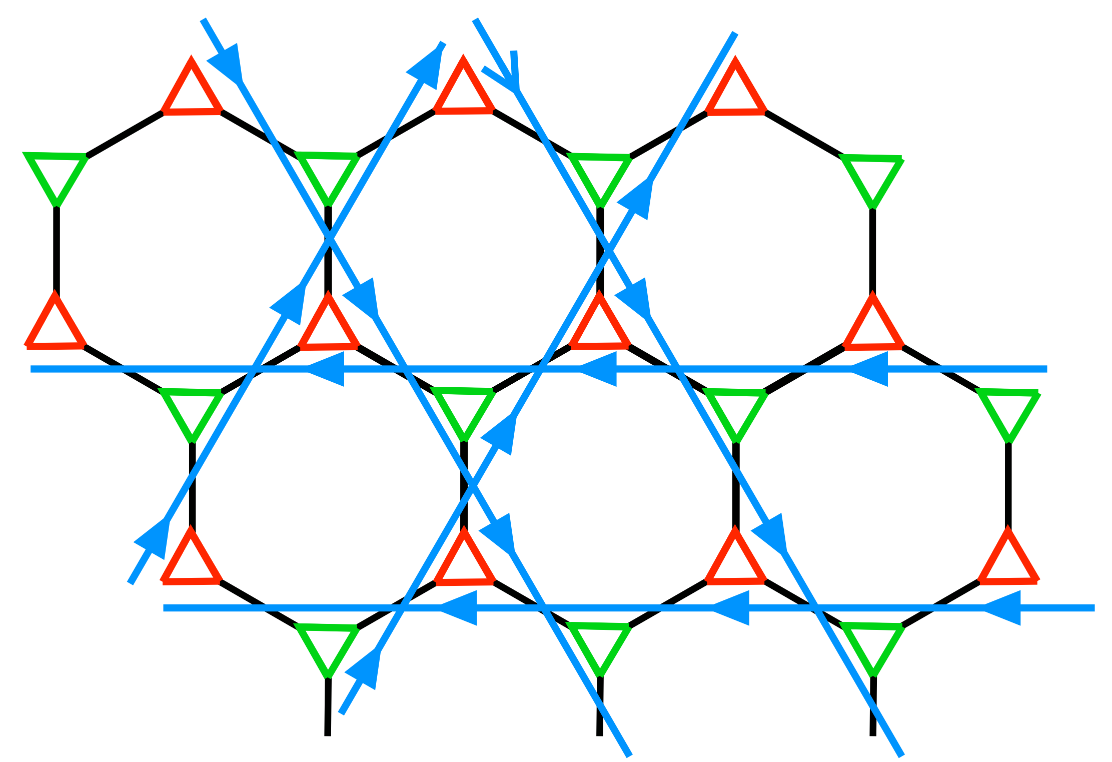


$$\mathbf{J}_{R,\alpha}(0) = \mathbf{J}_{L,\alpha\pm1}(0)$$



- Gapless 1D modes in the bulk: gapless chiral spin liquid?
- Sliding Luttinger liquid equivalent to a spinon Fermi surface state.

[R.P. and Bieri, SciPost 2018]



Conclusions

- A chiral fixed point of Heisenberg spin chains is found at a critical point separating a decoupled-chain fixed point at weak coupling from a three-channel Kondo fixed point at strong coupling.

- At the chiral fixed point we have an ideal spin circulator, which may serve as a building block for network constructions of chiral spin liquids.

[Buccheri, Egger, R.P. & Ramos, PRB 2018]