

η -DEFORMATION OF THE $\text{ads}_5 \times \text{s}^5$ PURE SPINOR SUPERSTRING

Hector A. Benitez, Victor O. Rivelles

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Universidade de São Paulo

- We present an integrable deformation of the $\text{AdS}_5 \times \text{S}^5$ pure spinor superstring.
- The deformed model preserves the local symmetries of the theory.
- The resulting model describes a pure spinor superstring in a η -deformed background.

η -DEFORMATION OF THE $\text{ads}_5 \times \text{s}^5$ GREEN-SCHWARZ SUPERSTRING

-Superstring in AdS₅ × S⁵ can be formulated as a supercoset sigma model on

$$\frac{\text{PSU}(2, 2, |4)}{\text{SO}(4, 1) \times \text{SO}(5)} .$$

-The superalgebra $\mathfrak{psu}(2, 2|4)$ possesses a \mathbb{Z}_4 automorphism

$$\mathfrak{g} = \mathfrak{g}_0 + \mathfrak{g}_1 + \mathfrak{g}_2 + \mathfrak{g}_3 .$$

-Then, the Maurer-Cartan form for an element $g \in \text{PSU}(2, 2, |4)$, which takes values on the $\mathfrak{psu}(2, 2|4)$ decomposes as

$$A = -dg g^{-1} = A_0 + A_1 + A_2 + A_3 .$$

-The classical action for the Green-Schwarz superstring in AdS₅ × S⁵ can be written as

$$S_{\text{GS}} = -\frac{1}{4}(g^{\mu\nu} - \epsilon^{\mu\nu}) \int 2\text{Str}(A_{\mu 2} A_{\nu 2}) + \text{Str}(A_{\mu 3} A_{\nu 1} - A_{\mu 1} A_{\nu 3}).$$

-By introducing a linear combination of projectors on $\mathfrak{psu}(2, 2|4)$ given by $d_{\text{GS}} = P_1 + 2P_2 - P_3$ the action can be written as

$$S_{\text{GS}} = -\frac{1}{4}(g^{\mu\nu} - \epsilon^{\mu\nu}) \int \text{Str}(A_{\mu}, d_{\text{GS}} A_{\nu}).$$

-Invariance under local κ -symmetry

$$\delta_{\kappa} g g^{-1} = \{\kappa_{+\mu}^1, A_2^{\mu}\} + \{\kappa_{-\mu}^3, A_2^{\mu}\}.$$

-Classical integrability: The classical equations of motion can be cast in a zero curvature representation.

-The Yang-Baxter deformation of the $\text{AdS}_5 \times S^5$ GS superstring (Delduc, Magro, and Vicedo 2014. [arXiv:hep-th/1309.5850])

$$S_{\text{GS}} = -\frac{1}{4}(\gamma^{\mu\nu} - \epsilon^{\mu\nu}) \int \text{Str}(A_\mu, d_{\text{GS}} \frac{1}{1 - \eta R_g \circ d_{\text{GS}}} A_\nu).$$

Here $R_g = \text{Ad}_g \circ R \circ \text{Ad}_g^{-1}$ where,

$$[R(X), R(Y)] - R([R(X), Y] + [X, R(Y)]) = c[X, Y].$$

- The deformation preserves κ -symmetry

$$\delta_\kappa g g^{-1} = \{\kappa_{+\mu}^1, J_{-2}^\mu\} + \{\kappa_{-\mu}^3, J_{+2}^\mu\},$$

where, we have defined the deformed currents

$$J_- = \frac{1}{1 - \eta R_g \circ d_{\text{GS}}} A, \quad J_+ = \frac{1}{1 + \eta R_g \circ \hat{d}_{\text{GS}}} A.$$

-The background fields do not satisfy the type IIB supergravity equations of motion but rather a generalization (Arutyunov et al. 2016. [arXiv:hep-th/1511.05795]).

-The e.o.m depends on two vector fields instead of a scalar field.

$$\begin{aligned} R_{ab} + 2\nabla_{(a}X_{b)} - \frac{1}{4}H_{acd}H_b{}^{cd} &= 0, \\ \nabla^c H_{abc} - 2X^c H_{abc} - 4\nabla_{[a}X_{b]} &= 0, \\ \nabla^a X_a - 2X_a X^a + \frac{1}{12}H^{abc}H_{abc} &= 0. \end{aligned}$$

-For the particular solution $X_a = \nabla_a \Phi$ type IIB supergravity is recovered.

-In particular for the η -model, this supergravity condition translates into an algebraic relation on the R-matrices, the so-called unimodular condition (Borsato and Wulff 2016. [arXiv:hep-th/1608.03570])

$$R^{AB}f_{AB}^C = 0 \implies \text{Supergravity.}$$

THE $ads_5 \times s^5$ PURE SPINOR SUPERSTRING

-The classical PS superstring action in an arbitrary background is given by (Berkovits and Howe 2002. [arXiv:hep-th/0112160])

$$S_{\text{BH}} = \frac{1}{2\pi\alpha'} \int dz^2 \left(\frac{1}{2} E^a \bar{E}^b \eta_{ab} + \frac{1}{2} E^A \bar{E}^B B_{AB} + d_\alpha \bar{E}^\alpha + d_{\hat{\alpha}} E^{\hat{\alpha}} + d_\alpha d_{\hat{\alpha}} P^{\alpha\hat{\alpha}} + \Omega_\alpha^\beta \lambda^\alpha \omega_\beta + \hat{\Omega}_{\hat{\alpha}}^{\hat{\beta}} \hat{\lambda}^{\hat{\alpha}} \hat{\omega}_{\hat{\beta}} + \lambda^\alpha \omega_\beta \hat{d}_\gamma C_\alpha^{\beta\gamma} + \hat{\lambda}^{\hat{\alpha}} \hat{\omega}_{\hat{\beta}} d_\gamma \tilde{C}_{\hat{\alpha}}^{\beta\gamma} + \lambda^\alpha \omega_\beta \hat{\lambda}^{\hat{\alpha}} \hat{\omega}_{\hat{\beta}} S_{\alpha\hat{\alpha}}^{\beta\hat{\beta}} + S_{\text{gh}} \right).$$

-To present the correct number of degrees of freedom, it is required to present:

1. A Nilpotent BRST charge

$$Q_{\text{BRST}} = \int j_-^{\text{B}} + j_+^{\text{B}}, \quad j_-^{\text{B}} = \lambda^\alpha d_\alpha, \quad j_+^{\text{B}} = \hat{\lambda}^{\hat{\alpha}} d_{\hat{\alpha}}.$$

2. The holomorphicity of the BRST currents

$$\partial_+ j_-^{\text{B}} = 0, \quad \partial_- j_+^{\text{B}} = 0.$$

-Similarly to the GS model, these requirements imply some constraints on the target superspace.

-It was proven that on-shell supergravity backgrounds satisfy these constraints.

$$S_0 = \int \text{Str} \left(\frac{1}{4} A_+ d_{\text{PS}} A_- + \omega_{1+} \partial_- \lambda_3 + \omega_{3-} \partial_+ \lambda_1 + N_{0+} A_{0-} + N_{0-} A_{0+} - N_{0-} N_{0+} \right)$$

$$d_{\text{PS}} = P_1 + 2P_2 + 3P_3.$$

-The bosonic ghosts λ^α and $\hat{\lambda}^{\hat{\alpha}}$ are constrained to satisfy the pure spinor condition

$$\lambda \gamma^a \lambda = \hat{\lambda} \gamma^a \hat{\lambda} = 0.$$

-The ghost Lorentz generators are given by

$$N_{0-} = -\{\omega_{1+}, \lambda_3\}, \quad N_{0+} = -\{\omega_{3-}, \lambda_1\}.$$

-The action is BRST invariant under:

$$\begin{aligned} \epsilon Q(g) &= (\epsilon \lambda_1 + \epsilon \lambda_3)g, \\ \epsilon Q(w_{3-}) &= -A_{3-}, \\ \epsilon Q(w_{1+}) &= -A_{1+}. \end{aligned}$$

$$\begin{aligned}
D_- A_{1+} + [A_{1-}, N_{0+}] - [N_{0-}, A_{1+}] &= 0, \\
D_- A_{2+} + [A_{1-}, A_{1+}] + [A_{2-}, N_{0+}] - [N_{0-}, A_{2+}] &= 0, \\
D_- A_{3+} + [A_{1-}, A_{2+}] + [A_{2-}, A_{1+}] - [A_{3-}, N_{0-}] - [N_{0-}, A_{3+}] &= 0, \\
D_+ A_{1-} + [A_{2+}, A_{3-}] + [A_{3+}, A_{2-}] + [A_{1-}, N_{0+}] - [N_{0-}, A_{1+}] &= 0, \\
D_+ A_{2-} + [A_{3+}, A_{3-}] + [A_{2-}, N_{0+}] - [N_{0-}, A_{2+}] &= 0, \\
D_- A_{3+} + [A_{3-}, N_{0-}] - [N_{0-}, A_{3+}] &= 0, \\
D_- N_{0+} - [N_{0-}, N_{0+}] = 0, \quad D_+ N_{0-} - [N_{0+}, N_{0-}] &= 0. \\
D_- = \partial_- + [A_{0-}, \cdot], \quad D_+ = \partial_+ + [A_{0+}, \cdot], &
\end{aligned}$$

-The Lax pair is given by (Vallilo 2004. [arXiv:hep-th/1203.0677])

$$\begin{aligned}
L_+(z) &= A_{0+} + z^{-3}A_{1+} + z^{-2}A_{2+} + z^{-1}A_{3+} + (z^{-4} - 1)N_{0+}, \\
L_-(z) &= A_{0-} + zA_{1-} + z^2A_{2-} + z^3A_{3-} + (z^4 - 1)N_{0-}, \\
\partial_- L_+ - \partial_+ L_- + [L_-, L_+] &= 0.
\end{aligned}$$

BRST DEFORMATION OF THE $\text{ads}_5 \times \text{s}^5$ PURE SPINOR SUPERSTRING

-On the pure spinor side there are few examples of consistent deformations.

-A remarkable example is given by (Bedoya et al. 2011. [arXiv:hep-th/1005.0049]). In this work the $\text{AdS}_5 \times S^5$ pure spinor superstring was deformed in a BRST invariant way (based on the homological perturbation theory).

$$\begin{aligned} S_{\text{def}} &= S_0 + \eta \int V_1^2 + \eta^2 \int V_2^2 + \dots, \\ Q_{\text{def}} &= Q_0 + \eta Q_1 + \eta^2 Q_2 + \dots \end{aligned}$$

-In this approach, the first step is to consider a massless vertex operator $V[B]$ (which represents a physical state of the model).

$$V[B](\epsilon, \epsilon') = B^{AB} (g^{-1}(\epsilon\lambda_1 - \epsilon\lambda_3)g)_A (g^{-1}(\epsilon'\lambda_1 - \epsilon'\lambda_3)g)_B .$$

-At the linearized level, this states yields a deformation proportional to the integrated vertex operator V_1 .

$$V_1^2 = \frac{\eta}{4} \int \text{Str}(Bj_+, j_-) .$$

-Once V_1 is known, the full deformation can be constructed as a series expansion in η for the action and the BRST charge

$$\begin{aligned} S_{\text{def}} &= S_0 + \eta \int V_1^2 + \eta^2 \int V_2^2 + \dots , \\ Q_{\text{def}} &= Q_0 + \eta Q_1 + \eta^2 Q_2 + \dots . \end{aligned}$$

-The coefficients are determined by imposing BRST invariance

$$Q_{\text{def}} S_{\text{def}} = 0 .$$

order by order in η .

-Solving order by order in η we find up to order η^3

$$Q_0 S_0 = 0, \quad (\text{by definition}) \quad (1)$$

$$Q_1 S_0 + Q_0 \int V_1 = 0, \quad (2)$$

$$Q_2 S_0 + Q_1 \int V_1 + Q_0 \int V_2 = 0, \quad (3)$$

$$Q_3 S_0 + Q_2 \int V_1 + Q_1 \int V_2 + Q_0 \int V_3 = 0. \quad (4)$$

-For example, the first step is to solve (2) for

$$V_1^2 = \frac{\eta}{4} \int \text{Str}(B_{j_+, j_-}).$$

-The full deformation can be written as

$$\begin{aligned}
 S_{\text{def}} &= \int \left[\frac{1}{4} \text{Str}(\bar{A}, d_{\text{PS}} J_-) + \text{Str}(N_{0+} J_{0-} + N_{0-} \bar{J}_{0+}) \right. \\
 &\quad \left. - \text{Str}(N_{0-} (1 - 4\eta \mathcal{O}_{\text{PS}-}^{-1} B_g) N_{0+}) + \text{Str}(\omega_{1+} \partial_- \lambda_3 + \omega_{3-} \partial_+ \lambda_1) \right].
 \end{aligned}$$

-This action must be invariant under the BRST transformations

$$\begin{aligned}
 Q(g) &= g((1 - \eta B_g) \lambda_1 + (1 + \eta B_g) \lambda_3), \\
 Q(w_{3-}) &= -J_{3-} - 4\eta P_3 \circ \mathcal{O}_{\text{PS}-}^{-1} B_g N_{0-}, \\
 Q(w_{1+}) &= -\bar{J}_{1+} + 4\eta P_1 \circ \mathcal{O}_{\text{PS}+}^{-1} B_g N_{0+}.
 \end{aligned}$$

-We have defined the operators

$$\begin{aligned}
 \mathcal{O}_{\text{PS}-} &= 1 - \eta B_g d_{\text{PS}}, & \mathcal{O}_{\text{PS}+} &= 1 + \eta B_g \hat{d}_{\text{PS}}, \\
 J_- &= \mathcal{O}_{\text{PS}-}^{-1} A, & J_+ &= \mathcal{O}_{\text{PS}+}^{-1} A.
 \end{aligned}$$

- B^{AB} is a R-matrix: $Q^2 = 0$ requires that the matrix B^{AB} must satisfy the mCYBE.

-Not all the vertex of the type (1) belongs to the cohomology:

It includes states transforming in the adjoint representation when B^{AB} takes the form $B^{AB} = f_C^{AB} A^C$.

This family of states corresponds to exact (non-physical) elements,

$$A^C [g^{-1}(\epsilon\lambda_1 - \epsilon\lambda_3)g, g^{-1}(\epsilon'\lambda_1 - \epsilon\lambda_3)g]_C = A^C \epsilon Q(g^{-1}(\epsilon\lambda_1 - \epsilon\lambda_3)g)_C. \quad (5)$$

-However, the trivial states (5) produce a well defined deformation.

Hence, one expect that the complete deformation presents these type of fields in its target background.

-This can be understood when we take into account conformal invariance; the vertex $V[B]$ is a primary field only when the unitary condition holds

$$B^{AB} f_{AB}^C = 0, \implies \text{conformal invariance.}$$

INTEGRABILITY

-We can write them in a more suggestive manner by defining the currents \mathcal{J}_- and \mathcal{J}_+ as

$$\mathcal{J}_- := \mathbf{J}_- + 4\eta \mathcal{O}_{\text{PS}^-}^{-1} \mathbf{R}_g \mathbf{N}_{0-}, \quad \mathcal{J}_+ = \bar{\mathbf{J}}_+ - 4\eta \mathcal{O}_{\text{PS}^+}^{-1} \mathbf{R}_g \mathbf{N}_{0+},$$

then, the equations of motion are take the form

$$\begin{aligned} \mathcal{D}_- \mathcal{J}_{1+} &+ [\mathcal{J}_{1-}, \mathbf{N}_{0+}] - [\mathbf{N}_{0-}, \mathcal{J}_{1+}] = 0, \\ \mathcal{D}_- \mathcal{J}_{2+} &+ [\mathcal{J}_{1-}, \mathcal{J}_{1+}] + [\mathcal{J}_{2-}, \mathbf{N}_{0+}] - [\mathbf{N}_{0-}, \mathcal{J}_{2+}] = 0, \\ \mathcal{D}_- \mathcal{J}_{3+} &+ [\mathcal{J}_{1-}, \mathcal{J}_{2+}] + [\mathcal{J}_{2-}, \mathcal{J}_{1+}] - [\mathcal{J}_{3-}, \mathbf{N}_{0-}] - [\mathbf{N}_{0-}, \mathcal{J}_{3+}] = 0, \\ \mathcal{D}_+ \mathcal{J}_{1-} &+ [\mathcal{J}_{2+}, \mathcal{J}_{3-}] + [\mathcal{J}_{3+}, \mathcal{J}_{2-}] + [\mathcal{J}_{1-}, \mathbf{N}_{0+}] - [\mathbf{N}_{0-}, \mathcal{J}_{1+}] = 0, \\ \mathcal{D}_+ \mathcal{J}_{2-} &+ [\mathcal{J}_{3+}, \mathcal{J}_{3-}] + [\mathcal{J}_{2-}, \mathbf{N}_{0+}] - [\mathbf{N}_{0-}, \mathcal{J}_{2+}] = 0, \\ \mathcal{D}_- \mathcal{J}_{3+} &+ [\mathcal{J}_{3-}, \mathbf{N}_{0-}] - [\mathbf{N}_{0-}, \mathcal{J}_{3+}] = 0, \\ \mathcal{D}_- \mathbf{N}_{0+} &- [\mathbf{N}_{0-}, \mathbf{N}_{0+}] = 0, \quad \mathcal{D}_+ \mathbf{N}_{0-} - [\mathbf{N}_{0+}, \mathbf{N}_{0-}] = 0. \end{aligned}$$

-At this stage it should be clear that the ansatz for the Lax pair should be found by exchanging \mathcal{J}_- and \mathcal{J}_+ for A_- and \bar{A}_+ respectively, in the undeformed Lax pair.

-Moreover, the BRST density charges can be written as

$$j_-^B = \text{Str}(\lambda_1, \mathcal{J}_{3-}), \quad j_+^B = \text{Str}(\lambda_3, \mathcal{J}_{1+}). \quad (6)$$

-The (anti) holomorphicity of $(j_+^B) j_-^B$ can be easily proven by using the above equations of motion.

READING THE TARGET SPACE FIELDS

-This is achieved by comparing the deformed model with the standard Berkovits-Howe action (Berkovits and Howe 2002.

[arXiv:hep-th/0112160])

$$S_{\text{BH}} = \frac{1}{2\pi\alpha'} \int dz^2 \left(\frac{1}{2} E^a \bar{E}^b \eta_{ab} + \frac{1}{2} E^A \bar{E}^B B_{AB} + d_\alpha \bar{E}^\alpha + d_{\hat{\alpha}} E^{\hat{\alpha}} + d_\alpha d_{\hat{\alpha}} P^{\alpha\hat{\alpha}} + \right. \\ \left. + \Omega_\alpha^\beta \lambda^\alpha \omega_\beta + \hat{\Omega}_{\hat{\alpha}}^{\hat{\beta}} \hat{\lambda}^{\hat{\alpha}} \hat{\omega}_{\hat{\beta}} + \lambda^\alpha \omega_\beta \hat{d}_\gamma C_\alpha^{\beta\hat{\gamma}} + \hat{\lambda}^{\hat{\alpha}} \hat{\omega}_{\hat{\beta}} d_\gamma \tilde{C}_{\hat{\alpha}}^{\hat{\beta}\gamma} + \lambda^\alpha \omega_\beta \hat{\lambda}^{\hat{\alpha}} \hat{\omega}_{\hat{\beta}} S_{\alpha\hat{\alpha}}^{\beta\hat{\beta}} + S_{\text{gh}} \right).$$

-The supervielbeins E^A of the deformed geometry are given by

$$E_2^a = J_{\text{GS}2+}^a, \quad E_1^\alpha = \text{Ad}_h J_{\text{GS}1+}^\alpha, \quad E_3^{\hat{\alpha}} = J_{\text{GS}3-}^{\hat{\alpha}},$$

where h is an element of the isotropy group.

-The metric and the B-field can be read from the GS sector

$$G_{MN} \partial Z^M \bar{\partial} Z^N = \text{Str}(\bar{J}_{\text{GS}-}, J_{\text{GS}-}), \quad B = \frac{1}{2} (P_1 - P_3 + \eta \hat{d}_{\text{GS}} \circ R_g \circ d_{\text{GS}}).$$

-The Ramond-Ramond bispinor

$$P_{\alpha\hat{\alpha}} = \frac{1}{2} (P_1 \circ \vartheta_+ P_1 \circ \text{Ad}_h^{-1})_\alpha^{\hat{\beta}} \mathcal{K}_{\hat{\alpha}\beta}.$$

CONCLUSION

-In this work we have found an integrable deformation of the PS model in $\text{AdS}_5 \times S^5$.

-The GS η -model and the pure spinor deformation of $\text{AdS}_5 \times S^5$, develop the same geometry and target space content, which implies an extended supergravity background.

-Moreover, we expect that the central charge of the theory should be proportional to the unimodular condition on the R-matrices.

-Having found that BRST symmetry allows an extended supergravity background, it would be interesting to understand how the BRST constraints imply the correct equations of motion for extended supergravity as they were obtained from kappa-symmetry in the GS formulation Borsato and Wulff 2016. [arXiv:hep-th/1608.03570].