Exactly Solvable Quantum Chains

IIP Natal, 18-29 June 2018

Quantum quenches near criticality

Gesualdo Delfino

SISSA – Trieste

Based on:

G. Delfino, J. Phys. A 47 (2014) 402001

G. Delfino and J. Viti, J. Phys. A 50 (2017) 084004

G. Delfino, Phys. Rev. E 97 (2018) 062138

Quantum quench

for $t < 0$ isolated extended system with Hamiltonian H_0 is in ground state

for $t > 0$ system evolves with Hamiltonian H obtained changing a coupling at $t = 0$

which is time evolution of oservables after the quench (i.e. for $t > 0$)?

Quantum quench

for $t < 0$ isolated extended system with Hamiltonian H_0 is in ground state

for $t > 0$ system evolves with Hamiltonian H obtained changing a coupling at $t = 0$

which is time evolution of oservables after the quench (i.e. for $t > 0$)?

- problem extensively studied as prototype of non-equilibrium quantum dynamics

- special interest in 1D following experiments proposing that integrability allows for lack of relaxation

- the problem proved hard to address theoretically

- post-quench state exactly calculable for transverse field Ising chain (free fermions), but extremely difficult to find interacting integrable cases

- how to gain some general insight about the role of integrability and interaction?

Quenches near a quantum critical point [GD, '14]

• both before and after the quench the system is close to criticality, so that it is described by a massive quantum field theory

• this allows for a general formulation valid for the different universality classes of quantum critical behaviour

• there is a sharp notion of integrability (factorization of scattering amplitudes)

• conclusions on integrability hold for lattice cases admitting a continuum limit (non-integrability in the continuum implies non-integrability on the lattice)

field theory formulation

consider translation invariant near-critical system in 1D

H_0 = pre-quench Hamiltonian

$H = H_0 + \lambda \int dx\, \Psi$ = post-quench Hamiltonian

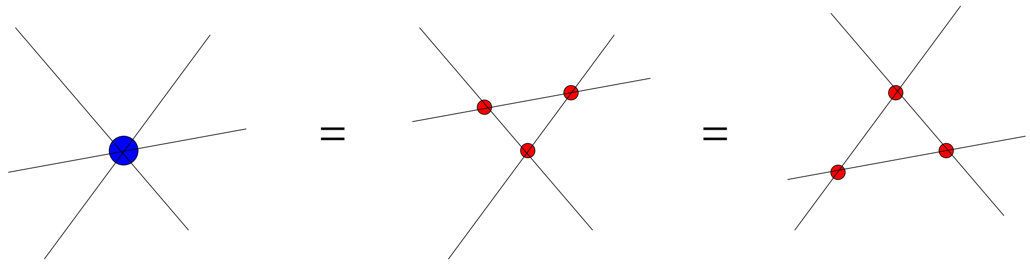
$\mathcal{A}_{CFT}$ = action of quantum critical point

$\mathcal{A}_0 = \mathcal{A}_{CFT} - g \int dt\, dx\, \varphi(x,t)$ = action in absence of quench (equilibrium)

$\mathcal{A} = \mathcal{A}_0 - \lambda \int_0^\infty dt \int_{-\infty}^\infty dx\, \Psi(x,t)$ = action in presence of the quench

integrability at equilibrium

QFT provides sharp notion of integrability at equilibrium, within the particle* description



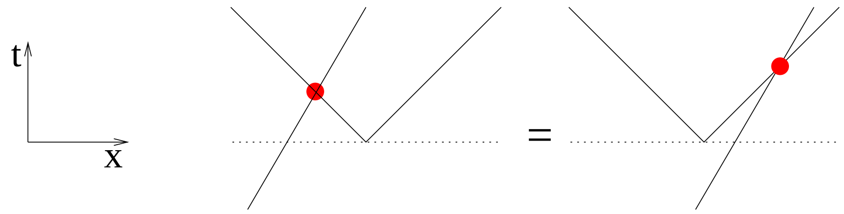
non-trivial integrals of motion generate momentum-dependent space-time translations ⇒ factorization of any scattering amplitude into the product of two-particle ones

*particle (or quasi-particle) = elementary excitation associated to collective near-critical modes

integrable quenches ?

connecting pre- and post-quench theories requires transmission of energy-momentum through $t = 0$; non-conservation of energy also creates particles

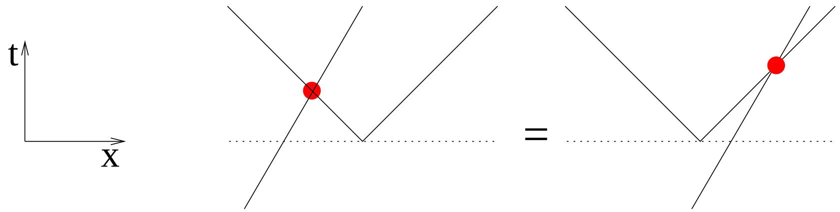
factorization for a typical process



integrable quenches ?

connecting pre- and post-quench theories requires transmission of energy-momentum through $t = 0$; non-conservation of energy also creates particles

factorization for a typical process



$S(s) = S(s') \Rightarrow S = constant \Rightarrow S = \pm 1$ by unitarity

$\Rightarrow$ factorization only for free bosons/fermions

- same follows for spin chains whenever continuum limit can be taken (not the case for Néel state in XXZ)

summarizing: near-critical quenches are exactly solvable (integrable) only if the particles do not interact, both before and after the quench (mass quench for free particles, Bogoliubov solvable)

in presence of interaction near criticality the theory can only proceed perturbatively

perturbation theory in λ

$$\mathcal{A} = \mathcal{A}_0 - \lambda \int_0^\infty dt \int_{-\infty}^\infty dx \, \Psi(x,t)$$

$|0\rangle$, $|p_1, \ldots, p_n\rangle$ = vacuum and particle states of $\mathcal{A}_0$

system is in ground state $|0\rangle$ up to $t = 0$, then quench drives it into the state

$|\psi_0\rangle = S_\lambda |0\rangle$ $\qquad$ $S_\lambda = T \exp\left(-i\lambda \int_0^\infty dt \int_{-\infty}^\infty dx \, \Psi(x,t)\right)$

$$\simeq |0\rangle + \lambda \sum_{n=1}^\infty \frac{2\pi}{n!} \int_{-\infty}^\infty \prod_{i=1}^n \frac{dp_i}{2\pi E_{p_i}} \delta\left(\sum_{i=1}^n p_i\right) \frac{[F_n^\Psi(p_1,\ldots,p_n)]^*}{\sum_{i=1}^n E_{p_i}} |p_1, \ldots, p_n\rangle$$

$$E_p = \sqrt{p^2 + M^2}$$

$F_n^\Psi(p_1, \ldots, p_n) = \langle 0|\Psi(0,0)|p_1, \ldots, p_n\rangle$ form factors of $\mathcal{A}_0$

$$|\psi_0\rangle \simeq |0\rangle + \lambda \sum_{n=1}^{\infty} \frac{2\pi}{n!} \int_{-\infty}^{\infty} \prod_{i=1}^{n} \frac{dp_i}{2\pi E_{p_i}} \delta(\sum_{i=1}^{n} p_i) \frac{[F_n^{\Psi}(p_1,\ldots,p_n)]^*}{\sum_{i=1}^{n} E_{p_i}} |p_1,\ldots,p_n\rangle$$

- **only existing general formula** for post-quench state

- post-quench state made of pairs $|p,-p\rangle$ only if

$$F_n^{\Psi}(p_1,\ldots,p_n) = 0 \quad \text{for } n \neq 2$$

i.e. $\mathcal{A}_0$ free and Ψ quadratic, i.e. mass quench; **"pair structure" absent** in presence of interaction

- present theory provides **only analytic framework** for quenches in presence of interaction; form factors exactly known if pre-quench theory is integrable; first order already non-trivial, exact for small quenches

one-point functions

$$\delta\langle\Phi(t)\rangle = \langle\psi_0|\Phi(x,t)|\psi_0\rangle - \langle 0|\Phi(0,0)|0\rangle + C_\Phi$$

$$\simeq \lambda \sum_{n=1}^{\infty} \frac{2\pi}{n!} \int_{-\infty}^{\infty} \prod_{j=1}^{n} \frac{dp_j}{2\pi E_{p_j}} \frac{\delta(\sum_{j=1}^{n} p_j)}{\sum_{j=1}^{n} E_{p_j}}$$

$$\times\, 2\,\mathrm{Re}\{[F_n^\Psi(p_1,\ldots,p_n)]^* F_n^\Phi(p_1,\ldots,p_n)\, e^{-i\sum_{j=1}^{n} E_{p_j} t}\} + C_\Phi$$

C_Φ determined by $\delta\langle\Phi(0)\rangle = 0$ [GD, Viti, '17]

$$C_\Phi \simeq -\lambda \sum_{n=1}^{\infty} \frac{2\pi}{n!} \int_{-\infty}^{\infty} \prod_{j=1}^{n} \frac{dp_j}{2\pi E_{p_j}} \frac{\delta(\sum_{j=1}^{n} p_j)}{\sum_{j=1}^{n} E_{p_j}}$$

$$\times\, 2\,\mathrm{Re}\{[F_n^\Psi(p_1,\ldots,p_n)]^* F_n^\Phi(p_1,\ldots,p_n)\}$$

$$= -\lambda \int d^2x \langle\Psi(x,-it)\Phi(0,0)\rangle_c \simeq \langle\Phi\rangle_\lambda^{eq} - \langle\Phi\rangle_{\lambda=0}^{eq}$$

$$\delta\langle\Phi(t)\rangle \simeq \lambda \sum_{n=1}^{\infty} \frac{2\pi}{n!} \int_{-\infty}^{\infty} \prod_{j=1}^{n} \frac{dp_j}{2\pi E_{p_j}} \frac{\delta(\sum_{j=1}^{n} p_j)}{\sum_{j=1}^{n} E_{p_j}}$$
$$\times\, 2\,\mathrm{Re}\{[F_n^{\Psi}(p_1,\ldots,p_n)]^* F_n^{\Phi}(p_1,\ldots,p_n)\, e^{-i\sum_{j=1}^{n} E_{p_j} t}\} + C_{\Phi}$$

long time behavior

two time scales: $\frac{1}{M}$ and $t_\lambda \sim \frac{1}{\lambda^{1/(2-X_\Psi)}}$; $t_\lambda \to \infty$ as $\lambda \to 0$

perturbative long time regime: $\frac{1}{M} \ll t \ll t_\lambda$

$t > t_\lambda$ unaccessible in principle in presence of interaction

$$\delta\langle\Phi(t)\rangle \simeq \lambda \sum_{n=1}^{\infty} \frac{2\pi}{n!} \int_{-\infty}^{\infty} \prod_{j=1}^{n} \frac{dp_j}{2\pi E_{p_j}} \frac{\delta(\sum_{j=1}^{n} p_j)}{\sum_{j=1}^{n} E_{p_j}}$$
$$\times\, 2\,\mathrm{Re}\{[F_n^{\Psi}(p_1,\ldots,p_n)]^* F_n^{\Phi}(p_1,\ldots,p_n)\, e^{-i\sum_{j=1}^{n} E_{p_j} t}\} + C_\Phi$$

long time behavior

two time scales: $\frac{1}{M}$ and $t_\lambda \sim \frac{1}{\lambda^{1/(2-X_\Psi)}}$; $t_\lambda \to \infty$ as $\lambda \to 0$

perturbative long time regime: $\frac{1}{M} \ll t \ll t_\lambda$

$t > t_\lambda$ unaccessible in principle in presence of interaction

small momenta dominate for $t \gg \frac{1}{M}$

generically, $[F_n^{\Psi}]^* F_n^{\Phi} \propto \prod_{1\leq i<k\leq n}(p_i - p_k)^2$, $p_j \to 0$

$$\Rightarrow \;\; \delta\langle\Phi(t)\rangle \sim \frac{\lambda}{t^{(n_0^2-1)/2}} A_{\Psi,\Phi} \cos(n_0 M t) + C_\Phi\,, \qquad \frac{1}{M} \ll t \ll t_\lambda$$

n_0 = smallest n for which $F_n^{\Psi}, F_n^{\Phi} \neq 0$, dictated by symmetry

$$\delta\langle\Phi(t)\rangle \sim \frac{\lambda}{t^{(n_0^2-1)/2}} A_{\Psi,\Phi} \cos(n_0 M t) + C_\Phi \,, \qquad \frac{1}{M} \ll t \ll t_\lambda$$

- $n_0 = 1$: for several particle species $a = 1, 2, \ldots, k$

$$\delta\langle\Phi(t)\rangle \sim \lambda \sum_a \frac{2}{M_a^2} [F_{1,a}^{\Psi}]^* F_{1,a}^{\Phi} \cos M_a t + C_\Phi$$

undamped oscillations with frequencies equal to masses

- symmetries can give $n_0 > 1$, normally $n_0 = 2$ and $t^{-3/2}$ damping

- $n_0 > 1$ without interaction ($F_n^{\Psi}|_{\text{free}} \propto \delta_{n,2}$)

- undamped oscillations can only arise in presence of interaction; no role of integrability

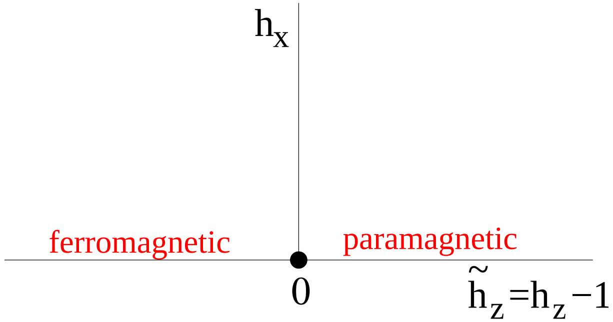
Ising chain [GD, Viti, '17]

at equilibrium :

$$H_{\text{Ising}} = -J \sum_{j=-\infty}^{\infty} [\sigma_j^x \sigma_{j+1}^x + h_z \sigma_j^z + h_x \sigma_j^x]$$

$$\downarrow$$

$$\mathcal{A}_{\text{Ising}} = \mathcal{A}_{CFT} - \tilde{h}_z \int d^2x\, \sigma^z(x) - h_x \int d^2x\, \sigma^x(x)$$

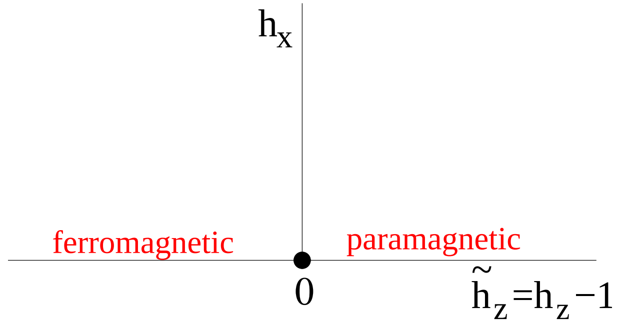


- integrability near criticality: either $\tilde{h}_z$ or h_x equal zero

Ising chain [GD, Viti, '17]

at equilibrium :

$$H_{\text{Ising}} = -J \sum_{j=-\infty}^{\infty} [\sigma_j^x \sigma_{j+1}^x + h_z \sigma_j^z + h_x \sigma_j^x]$$



$$\downarrow$$

$$\mathcal{A}_{\text{Ising}} = \mathcal{A}_{CFT} - \tilde{h}_z \int d^2x\, \sigma^z(x) - h_x \int d^2x\, \sigma^x(x)$$

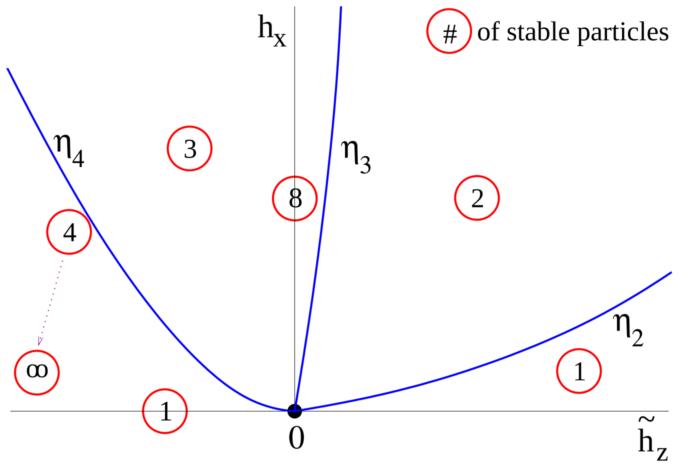
- integrability near criticality: either $\tilde{h}_z$ or h_x equal zero

- particle spectrum:

– depends on $\eta = \tilde{h}_z/|h_x|^{8/15}$
[McCoy, Wu, '78]

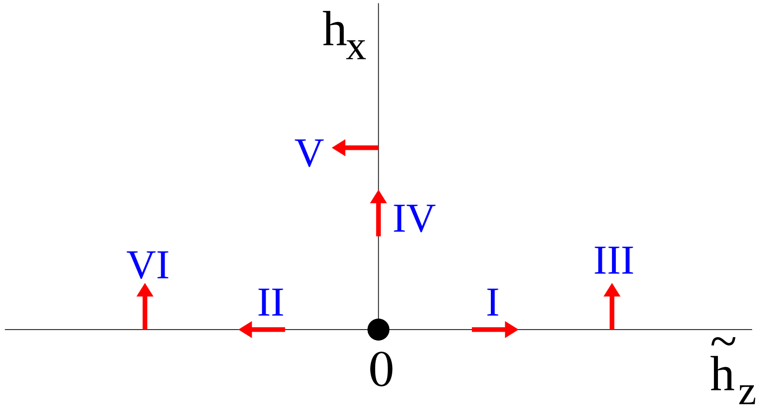
– free fermions at $h_x = 0$

– 8 stable particles at $\tilde{h}_z = 0$
[Zamolodchikov, '88]; heavier 5 decay for $\tilde{h}_z$ however small [GD et al, '06]



quenches :

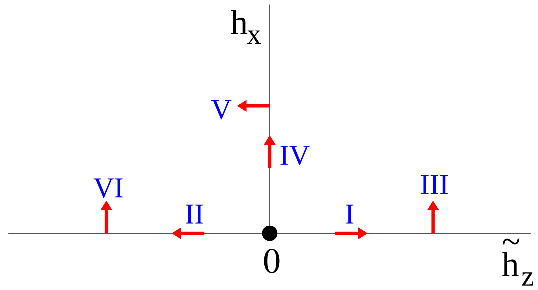
starting from integrable directions we know the form factors and everything is analytic; six different cases



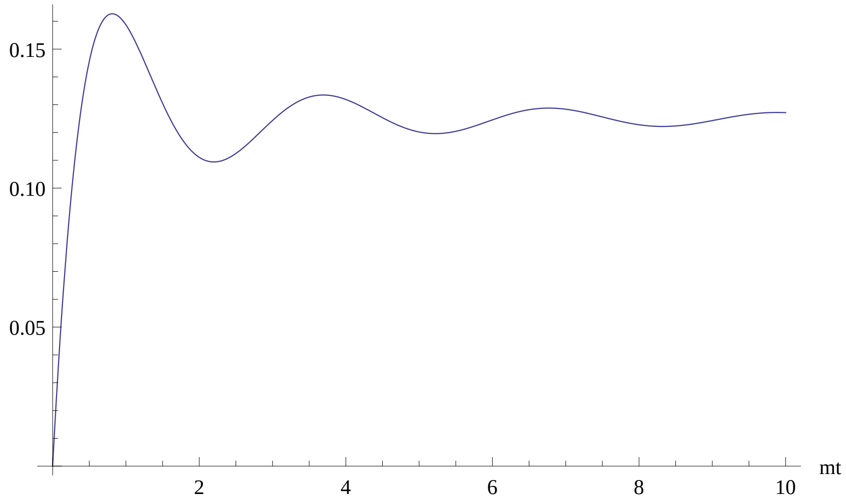
quench	φ	Ψ	$n_0(\sigma^x)$	# of frequencies
I	σ^z (para)	σ^z	$\times$	1
II	σ^z (ferro)	σ^z	2	1
III	σ^z (para)	σ^x	1	1
IV	σ^x	σ^x	1	8
V	σ^x	σ^z	1	8
VI	σ^z (ferro)	σ^x	2	1

quenches I & II: free cases, comparable with other analytic approaches

quench	φ	Ψ	$n_0(\sigma^x)$	# of frequencies
I	σ^z (para)	σ^z	$\times$	1
II	σ^z (ferro)	σ^z	2	1



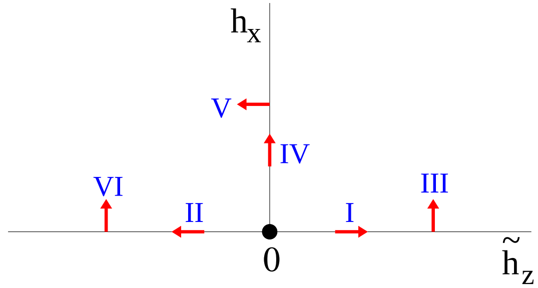
$\lim_{\delta M \to 0} \delta\langle\sigma^x(t)\rangle M/(\delta M \langle\sigma^x(0)\rangle)$



$\langle\sigma^x\rangle_{II}$ exhibits $t^{-3/2}$ damping and $X_{\sigma^x} = 1/8$ asymptotic; reproduced by large t expansion of [Essler, Schuricht, '12] with $\delta M \to 0$

quench III:

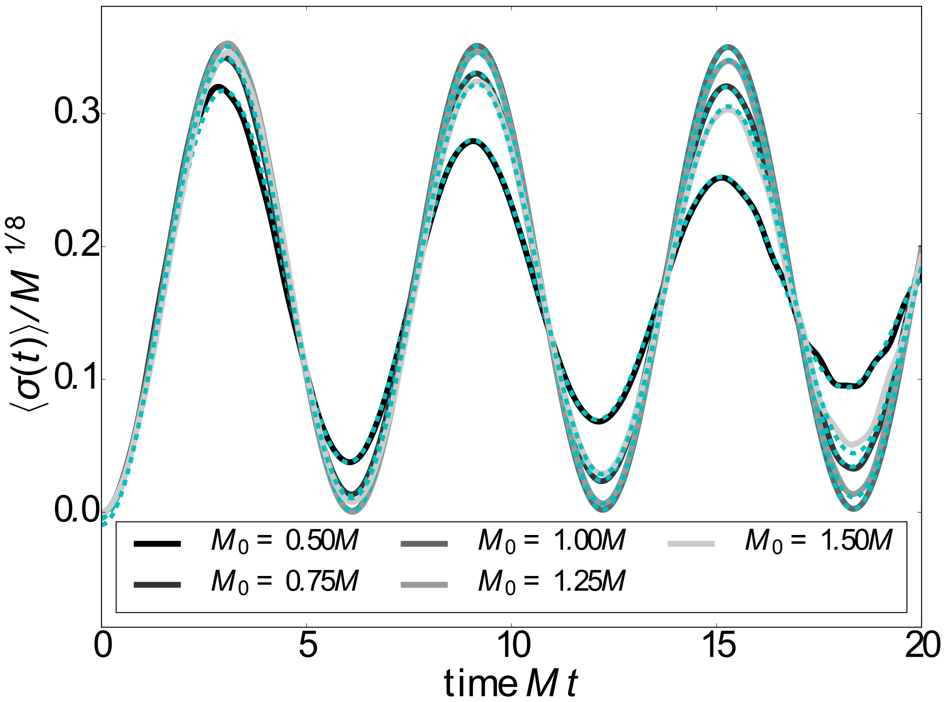
quench	φ	Ψ	$n_0(\sigma^x)$	# of frequencies
III	σ^z (para)	σ^x	1	1



interaction allows for undamped oscillations:

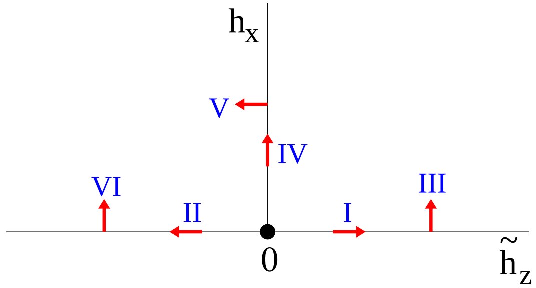
$$\langle \sigma^x(t) \rangle \sim h_x \frac{2}{M^2} |F_1^{\sigma^x}|^2 (\cos Mt - 1)$$

amplitude, frequency and offset of oscillations confirmed by numerical data of [Rakovszki, Mestyan, Collura, Kormos, Takacs, '16]:



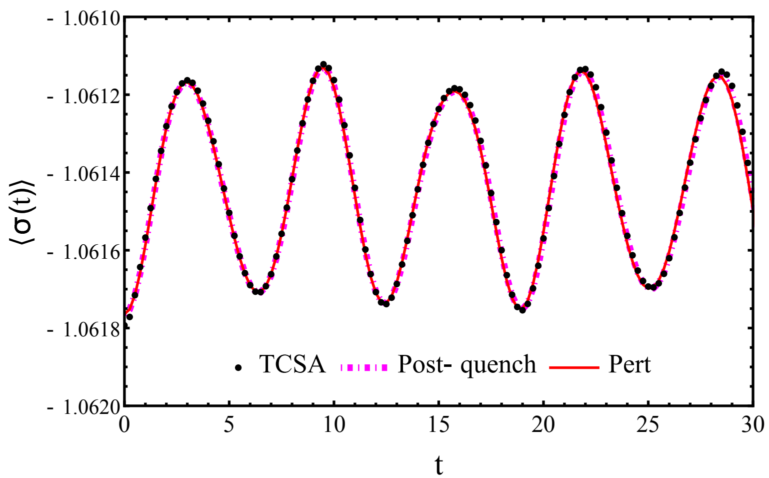
quenches IV & V:

quench	φ	Ψ	$n_0(\sigma^x)$	# of frequencies
IV	σ^x	σ^x	1	8
V	σ^x	σ^z	1	8

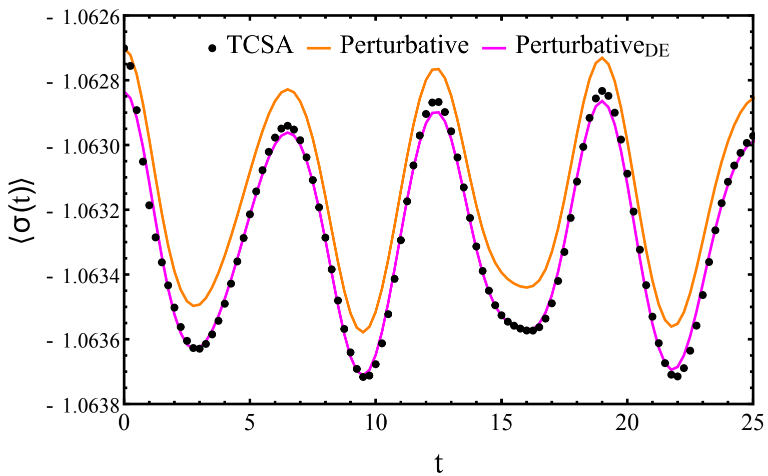


$$\delta\langle\sigma^x(t)\rangle \sim \lambda \sum_{a=1}^{8} \frac{2}{M_a^2}[F_{1,a}^{\Psi}]^* F_{1,a}^{\sigma^x} \cos M_a t + C_{\sigma^x}$$

analytic formulae for undamped oscillations accurately confirmed by numerical data of [Hodsagi, Kormos, Takacs, '18]:



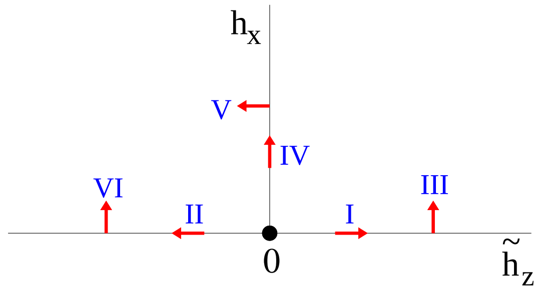
quench IV



quench V

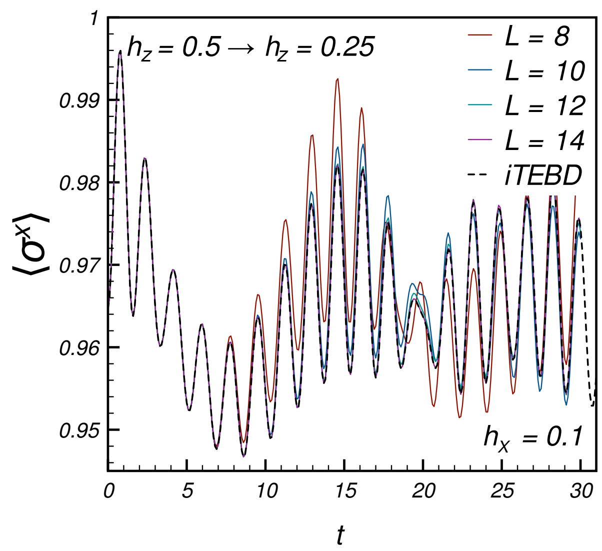
quench VI :

quench	φ	Ψ	$n_0(\sigma^x)$	# of frequencies
VI	σ^z (ferro)	σ^x	2	1



first order result expected to hold for h_x small enough, but no data available for comparison in this regime

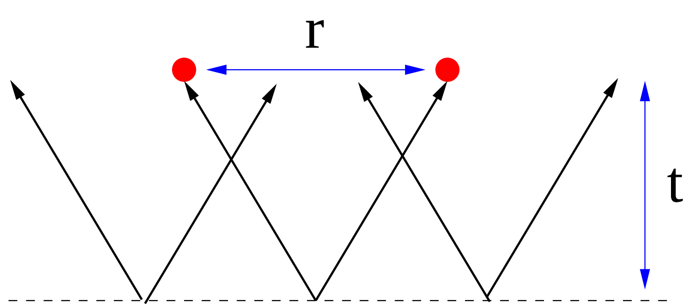
for larger h_x confinement prevails, as visible in data of [Kormos, Collura, Takacs, Calabrese, '16]



Spatial correlations and the light cone

- some analytic calculations exhibited light cone post-quench dynamics: correlations between two points separated by a distance r develop only after a time $t_r \propto r$

- heuristic explanation [Calabrese, Cardy, '06]: quench produces pairs of particles with opposite momenta travelling classically and without scattering with maximal velocity v_{max} $\Rightarrow$ $t_r = r/2v_{max}$



- followed a belief that light cone is related to "pair structure" and non-interacting particles (or integrability)

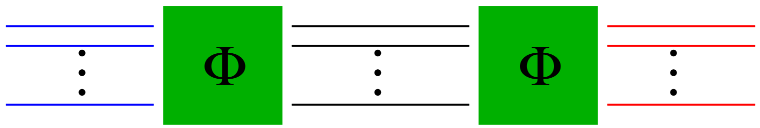
- even experiments were performed in this perspective [Cheneau et al, '12]

first principle derivation [GD, '18]

we showed that the post-quench state has the pair structure only in absence of interaction; more generally

$$|\psi_0\rangle = \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \prod_{i=1}^{n} d\mathbf{p}_i \, \delta\Big(\sum_{i=1}^{n} \mathbf{p}_i\Big) \, f_n(\mathbf{p}_1, \ldots, \mathbf{p}_n) \, |\mathbf{p}_1, \ldots, \mathbf{p}_n\rangle$$

$$\begin{aligned} \langle\psi_0|\Phi(\mathbf{x},t)\Phi(0,t)|\psi_0\rangle \;=\; & \sum_{n_1,n_2,m=0}^{\infty} \int \prod_{i=1}^{n_1} d\mathbf{p}_i \prod_{j=1}^{n_2} d\mathbf{p}'_j \prod_{k=1}^{m} d\mathbf{q}_k \, f^*_{n_2}(\mathbf{p}'_1, \ldots, \mathbf{p}'_{n_2}) \, f_{n_1}(\mathbf{p}_1, \ldots, \mathbf{p}_{n_1}) \\ \times \;\; & F^{\Phi}_{n_2,m}(\mathbf{p}'_1, \ldots, \mathbf{p}'_{n_2}|\mathbf{q}_1, \ldots, \mathbf{q}_m) \, F^{\Phi}_{m,n_1}(\mathbf{q}_1, \ldots, \mathbf{q}_m|\mathbf{p}_1, \ldots, \mathbf{p}_{n_1}) \\ \times \;\; & \delta\Big(\sum_{i=1}^{n_1} \mathbf{p}_i\Big) \, \delta\Big(\sum_{j=1}^{n_2} \mathbf{p}'_j\Big) \, e^{-i\,\varphi(\mathbf{x},t)} \end{aligned}$$



$$F^{\Phi}_{m,n}(\mathbf{q}_1, \ldots, \mathbf{q}_m|\mathbf{p}_1, \ldots, \mathbf{p}_n) = \langle\mathbf{q}_1, \ldots, \mathbf{q}_m|\Phi(0,0)|\mathbf{p}_1, \ldots, \mathbf{p}_n\rangle$$

$$\varphi(\mathbf{x},t) = \mathbf{x} \cdot \textstyle\sum_{k=1}^{m} \mathbf{q}_k + t \left(\sum_{i=1}^{n_1} E_{\mathbf{p}_i} - \sum_{j=1}^{n_2} E_{\mathbf{p}'_j} \right)$$

for $r = |\mathbf{x}|$ large, $e^{-i\,\varphi(\mathbf{x},t)}$ rapidly oscillates unless $\nabla_{\mathbf{q}_k}\varphi = 0$

due to disconnected pieces

$$F^{\Phi}_{m,n}(\mathbf{q}_1,\ldots,\mathbf{q}_m|\mathbf{p}_1,\ldots,\mathbf{p}_n) = \langle \mathbf{q}_1,\ldots,\mathbf{q}_m|\Phi(0,0)|\mathbf{p}_1,\ldots,\mathbf{p}_n\rangle_{\text{connected}}$$

$$+\delta(\mathbf{q}_1-\mathbf{p}_1)\langle \mathbf{q}_2,\ldots,\mathbf{q}_m|\Phi(0,0)|\mathbf{p}_2,\ldots,\mathbf{p}_n\rangle_{\text{connected}} + \cdots$$

a term like

produces the phase $\mathbf{x}\cdot\mathbf{q}+t\left(E_{\mathbf{q}}+\sum_{i=2}^{n_1-1}E_{\mathbf{p}_i}+E_{-\mathbf{q}-(\mathbf{p}_2+\cdots+\mathbf{p}_{n_1-1})}-\sum_{j=1}^{n_2}E_{\mathbf{p}'_j}\right)$

stationary if $\mathbf{x} = -\mathbf{V}t$

$$\mathbf{V} = \mathbf{v}_{\mathbf{q}} + \mathbf{v}_{\mathbf{q}+\mathbf{p}_2+\cdots+\mathbf{p}_{n-1}} \qquad \mathbf{v}_{\mathbf{p}} \equiv \nabla_{\mathbf{p}}E_{\mathbf{p}} = \frac{\mathbf{p}}{\sqrt{\mathbf{p}^2+M^2}}$$

$$|\mathbf{V}| \in (0,2) \quad \Longrightarrow \quad t > \frac{|\mathbf{x}|}{2}$$

- same condition for all terms allowing stationarity

$\Rightarrow$ light cone requires no assumption on quantum state or interaction

Summary

- we formulated the theory of quantum quenches from ground state in presence of interaction near criticality

- it is perturbative, since interaction is incompatible with known notion of solvability in QFT (factorization)

- time scale t_λ emerges beyond which time evolution cannot be determined ($t_\lambda \to \infty$ as $\lambda \to 0$)

- long time behavior (up to t_λ) of one-point functions determined in general

- undamped oscillations exist, require interaction and are not related to integrability; perfect agreement with numerics

- light cone spreading of correlations derived from first principles, requires no assumption on quantum state or interaction