A mathematical model for the spatiotemporal epidemic spreading of COVID-19

Wesley Cota

with

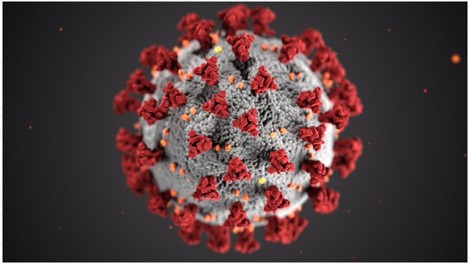
Alex Arenas (Universitat Rovira i Virgili, Tarragona, Spain)
Jesús Gómez-Gardeñes (Universidad de Zaragoza, Zaragoza, Spain)
Sergio Gómez (Universitat Rovira i Virgili, Tarragona, Spain)
Clara Granell (Universidad de Zaragoza, Zaragoza, Spain)
Joan T. Matamalas (Harvard Medical School, Boston, USA)
David Soriano-Paños (Universidad de Zaragoza, Zaragoza, Spain)
Benjamin Steinegger (Universitat Rovira i Virgili, Tarragona, Spain)

UFV
Universidade Federal de Viçosa

wcota.me | wesley.cota@ufv.br | @wlcota

Spread of COVID-19: the new challenge

The importance of quantifying the emergence of new cases



- One of the major difficulties is the **early detection of cases**.
- Crucial feature: an *asymptomatic phase*, where the individual is *infectious* but presents mild or no symptoms.
  - This phase can last up to 14 days!

How to predict local contagion?

- Motivation:

  Critical regimes driven by recurrent mobility patterns of reaction–diffusion processes in networks

  J. Gómez-Gardeñes, D. Soriano-Paños & A. Arenas

  Nature Physics **14**, 391–395(2018)
- Markovian framework that includes **real data**:
  - spatial distribution of populations
  - commuting mobility patterns
- Both data can be compiled from census data.

Motivation

Nature Phys. **14**, 391–395 (2018)

Metapopulation substrate

- $i = 1, \ldots, N$ patches populated by n_i individuals.
- Each individual i is associated to its residence i.
- With probability p_d, it moves from i to some patch j.
- The patch j is chosen proportionally to

$$R_{ij} = \frac{\overbrace{W_{ij}}^{\text{weight}}}{\sum_l W_{il}}.$$

- And then they go back to their homes.

SIR/S dynamics with recurrent mobility

- ρ_i is the fraction of infected individuals that live in i.
- The time evolution is given by Markovian equations:

$$\rho_i(t+1) = \overbrace{(1-\mu)}^{\text{does not recover}} \rho_i(t) + \underbrace{[1-\rho_i(t)]}_{\text{susceptibles}} \overbrace{\Pi_i(t)}^{\text{prob. of being infected}},$$

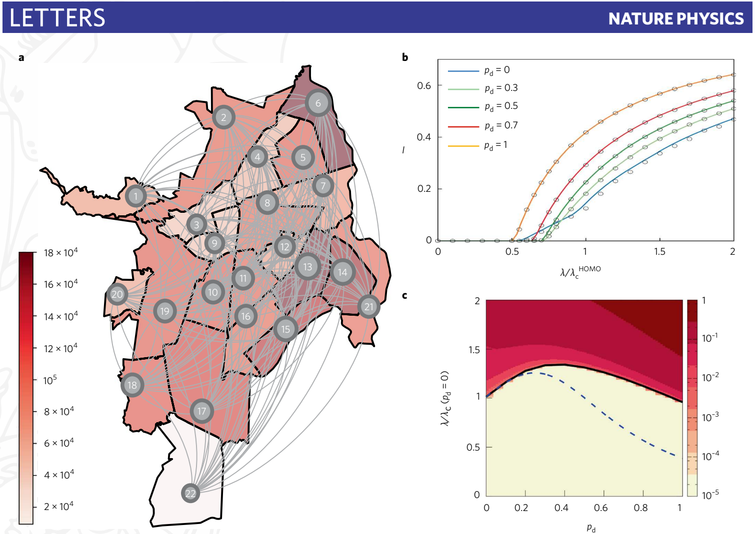
where

$$\Pi_i(t) = (1-p_d)P_i(t) + p_d \sum_j R_{ij} P_j(t)$$

and

$$P_i(t) = 1 - \prod_{j=1}^{N} \left[1 - \lambda \rho_j(t)\right]^{n_{j \to i}}$$

Motivation



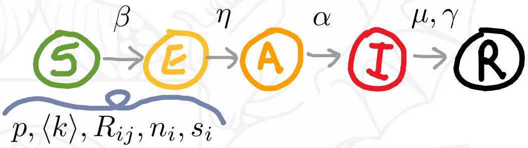
J. Gómez-Gardeñes, D. Soriano-Paños and A. Arenas. Nature Phys. **14**, 391–395 (2018)

Map of propagation risk of COVID-19 by local contact in Spain

https://covid-19-risk.github.io/map

The SEAIR model with human mobility

Susceptible-Exposed-Asymptomatic-Infected-Recovered



SARS-COV-2 data: *

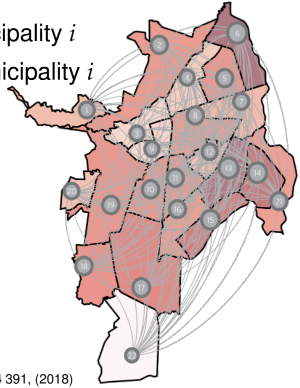
- β: infection probability of A and I agents
- $\eta + \alpha$: (Latent period)$^{-1}$ + (Asymptomatic period)$^{-1}$
- α: symptoms onset probability
- γ: hospitalization probability
- μ: recovery probability

Demographic and human behavior data:

- p: degree of mobility
- $\langle k \rangle$: average number of contacts

PLoS Medicine **5**, 0381 (2008)

- R_{ij}: recurrent mobility rate between municipalities i and j
- n_i: population of municipality i
- s_i: surface area of municipality i



Nat. Phys. **14** 391, (2018)

– * M. Chinazzi, et al., medRxiv p. 2020.02.09.20021261 (2020).
– V. E. Pitzer, G. M. Leung, and M. Lipsitch, American Journal of Epidemiology 166, 355 (2007), ISSN 0002-9262
– Q. Li, et al., New England Journal of Medicine (2020), ISSN 0028-4793
– L. Danon, E. Brooks-Pollock, M. Bailey, and M. J. Keeling, medRxiv p. 2020.02.12.20022566 (2020).
– J. M. Read, J. R. Bridgen, D. A. Cummings, A. Ho, and C. P. Jewell, medRxiv p. 2020.01.23.20018549 (2020).
– J. Mossong, et al., PLoS Medicine 5, 0381 (2008), ISSN 15491277.

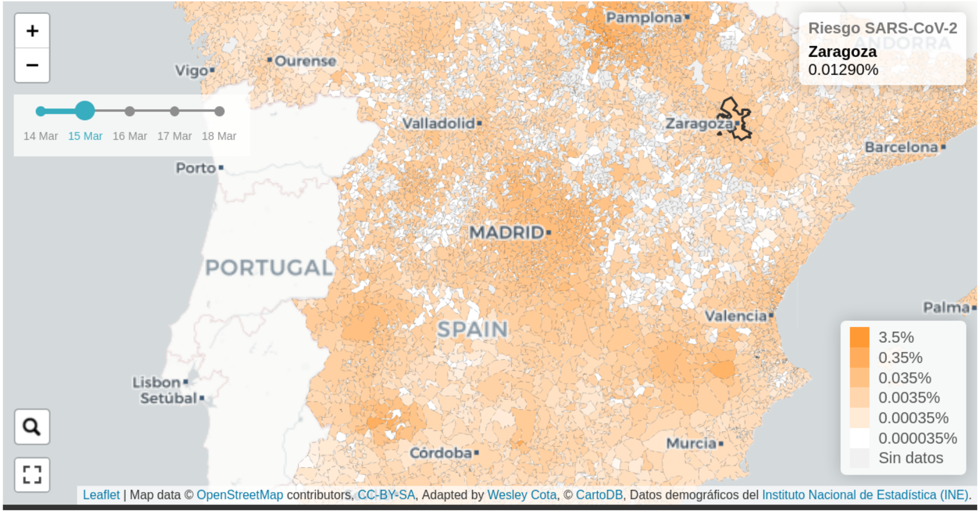
The SEAIR model with human mobility

Markovian formalism for the model

- By having the reported cases for each **municipality** as seeds, the model is evolved.
- For each municipality: fraction of the population estimated to have been infected by local contact.
- The number of contacts is tuned to take into account the density of individuals living in a given municipality.*

`https://covid-19-risk.github.io/map`

* H. Hu, K. Nigmatulina, and P. Eckhoff, Mathematical Biosciences 244, 125 (2013), ISSN 0025-5564



https://covid-19-risk.github.io/map

CATALUNYA MIGDIA

Un expert en epidèmies alerta que "cap sistema sanitari de la UE està preparat per rebre aquest cop"

... han elaborat, ... iabilitat a quatre dies ... ia anticipar les

elCorreo DE ANDALUCÍA — jueves, 12 marzo 2020 02:00, última actualización

SEVILLA PROVINCIA ANDALUCÍA OPINIÓN CULTURA MÁS PASIÓN SEVILLA FC BETIS +SECCIONES

El mapa que predice el riesgo de nuevos contagios

Crean un mapa de predicción de riesgo de coronavirus por muni

HERALDO

CORONAVIRUS

La Universidad de Zaragoza diseña un mapa que predice los nuevos contagios por coronavirus

EL MUNDO España Opinión Economía Internacional

‹ Ciencia y Salud | Salud

SALUD

El mapa de riesgo de contagio por coronavirus en España

Científicos españoles elaboran un mapa que permite ubicar las zonas con mayor riesgo de contagio por coronavirus Covid-19 a nivel municipal

Mapa coronavirus a Espanya: tots els casos d'afectats en directe

El Nacional
Barcelona. Dimecres, 11 de març de 2020
2 minuts

Estudante da UFV estuda proliferação do Coronavírus na Espanha

INSTITUCIONAL
Campus Viçosa

Doutorando da UFV participa da criação de modelo computacional que prevê risco de incidência do coronavírus

el Periódico de Aragón

ARAGÓN

UNIVERSIDAD DE ZARAGOZA

El campus diseña un modelo matemático que predice el riesgo de nuevos contagios de coronavirus

LAVANGUARDIA

Vida

COVID-19

Un algoritmo predice el riesgo de contagio por coronavirus en España

URVACTIV@
DIARI DIGITAL DE LA UNIVERSITAT ROVIRA I VIRGILI

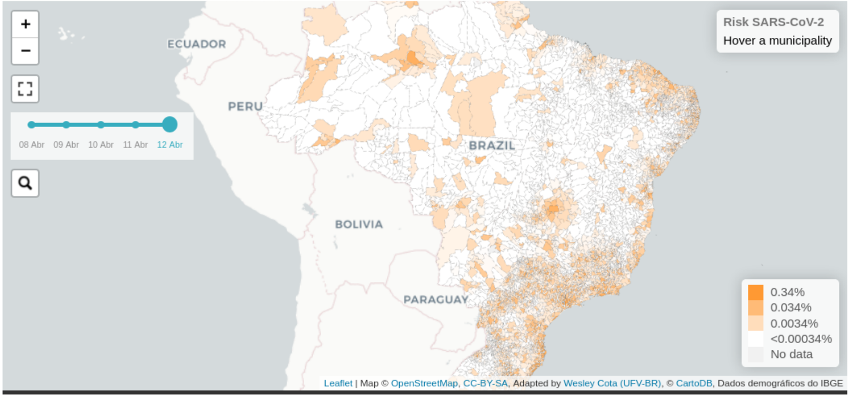
Un modelo matemático predice el riesgo de nuevos contagios por coronavirus en España

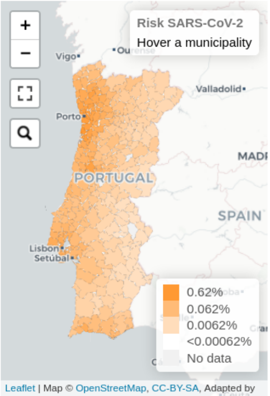
eldiario.es

Coronavirus Juan Carlos I 8M Desahucios Alberto Núñez Feijóo Memoria histórica

MAPA | El riesgo de contagiarse de coronavirus en España que calcula un modelo matemático, municipio a municipio

Also for Brazil and Portugal!





`https://covid-19-risk.github.io/map`

with Silvio C. Ferreira (UFV), Nuno Araújo (U Lisboa), Hygor P. Melo (U Lisboa)

Testing different epidemic containment scenarios

medRxiv
THE PREPRINT SERVER FOR HEALTH SCIENCES

A mathematical model for the spatiotemporal epidemic spreading of COVID19

Alex Arenas, Wesley Cota, Jesus Gomez-Gardenes, Sergio Gómez, Clara Granell, Joan T. Matamalas, David Soriano-Panos, Benjamin Steinegger

doi: https://doi.org/10.1101/2020.03.21.20040022

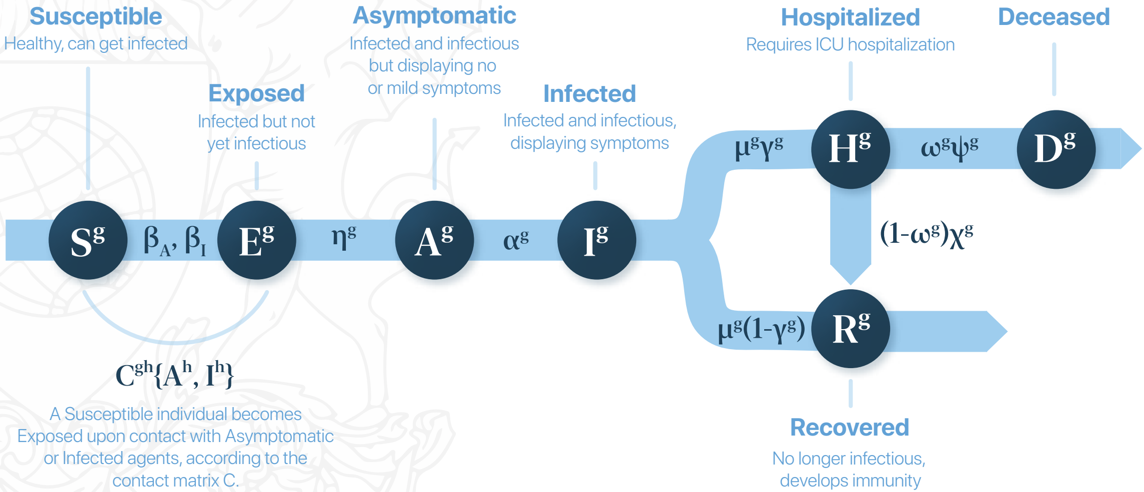
medRxiv
THE PREPRINT SERVER FOR HEALTH SCIENCES

Derivation of the effective reproduction number R for COVID-19 in relation to mobility restrictions and confinement

Alex Arenas, Wesley Cota, Jesus Gomez-Gardenes, Sergio Gomez, Clara Granell, Joan T. Matamalas, David Soriano-Panos, Benjamin Steinegger

doi: https://doi.org/10.1101/2020.04.06.20054320

Urban demography + age strata + age-structured contact patterns + daily recurrent mobility flows



The model

medRxiv 2020.04.06.20054320 | medRxiv 2020.03.21.20040022

$$
\begin{aligned}
\rho_i^{S,g}(t+1) &= \rho_i^{S,g}(t)\,\left[1-\Pi_i^g(t)\right]\,,\\
\rho_i^{E,g}(t+1) &= \rho_i^{S,g}(t)\,\Pi_i^g(t) + (1-\eta^g)\,\rho_i^{E,g}(t)\,,\\
\rho_i^{A,g}(t+1) &= \eta^g\,\rho_i^{E,g}(t) + (1-\alpha^g)\,\rho_i^{A,g}(t)\,,\\
\rho_i^{I,g}(t+1) &= \alpha^g\,\rho_i^{A,g}(t) + (1-\mu^g)\,\rho_i^{I,g}(t)\,,\\
\rho_i^{H,g}(t+1) &= \mu^g\,\gamma^g\,\rho_i^{I,g}(t) + \omega^g\,(1-\psi^g)\,\rho_i^{H,g}(t) + (1-\omega^g)\,(1-\chi^g)\,\rho_i^{H,g}(t)\,,\\
\rho_i^{D,g}(t+1) &= \omega^g\,\psi^g\,\rho_i^{H,g}(t) + \rho_i^{D,g}(t)\,,\\
\rho_i^{R,g}(t+1) &= \mu^g\,(1-\gamma^g)\,\rho_i^{I,g}(t) + (1-\omega^g)\,\chi^g\,\rho_i^{H,g}(t) + \rho_i^{R,g}(t)\,.
\end{aligned}
$$

β_A	Infectivity of asymptomatic	0.06
β_I	Infectivity of infected	0.06
η^g	Latent rate	$\frac{1}{2.34}$
α^g	Asymptomatic infectious rate	$\left(\frac{1}{8.86},\frac{1}{2.86},\frac{1}{2.86}\right)$
μ^g	Escape rate	$\left(\frac{1}{1.0},\frac{1}{7.0},\frac{1}{7.0}\right)$

γ^g	Fraction of cases requiring ICU	$(0.002, 0.009, 0.05)$
ψ^g	Death rate	$\frac{1}{7.0}$
ω^g	Fatality rate of ICU patients	0.42
χ^g	ICU discharge rate	$\frac{1}{20.0}$

The model

medRxiv 2020.04.06.20054320 | medRxiv 2020.03.21.20040022

$$\Pi_i^g(t) = (1 - p^g)\, P_i^g(t) + p^g \sum_{j=1}^{N_P} R_{ij}^g \, P_j^g(t)\,,$$

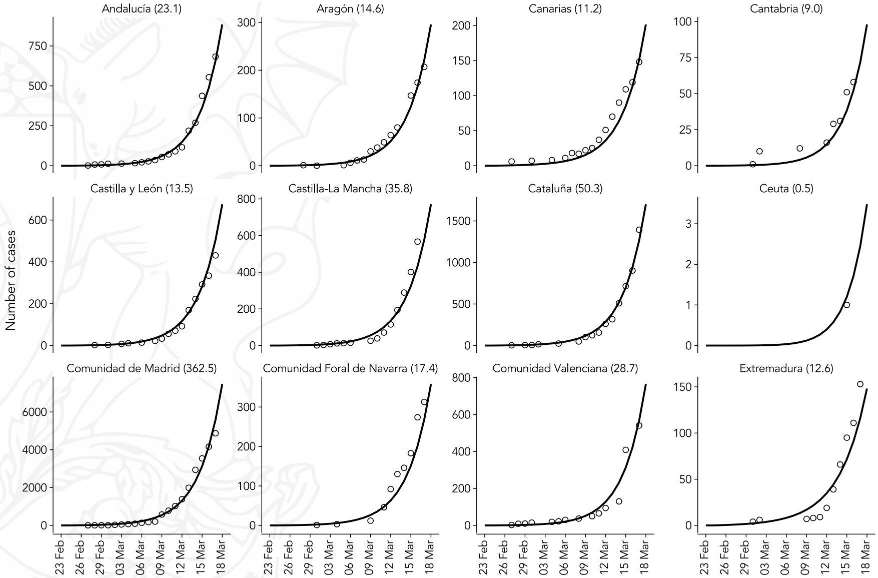
$$P_i^g(t) = 1 - \prod_{h=1}^{N_G} \prod_{j=1}^{N_P} (1-\beta_A)^{z^g \langle k^g \rangle f\left(\frac{\tilde{n}_i}{s_i}\right) C^{gh} \frac{n_{j\to i}^{A,h}(t)}{\tilde{n}_i^h}} (1-\beta_I)^{z^g \langle k^g \rangle f\left(\frac{\tilde{n}_i}{s_i}\right) C^{gh} \frac{n_{j\to i}^{I,h}(t)}{\tilde{n}_i^h}}\,.$$

$$f(x) = 1 + (1 - e^{-\xi x})$$

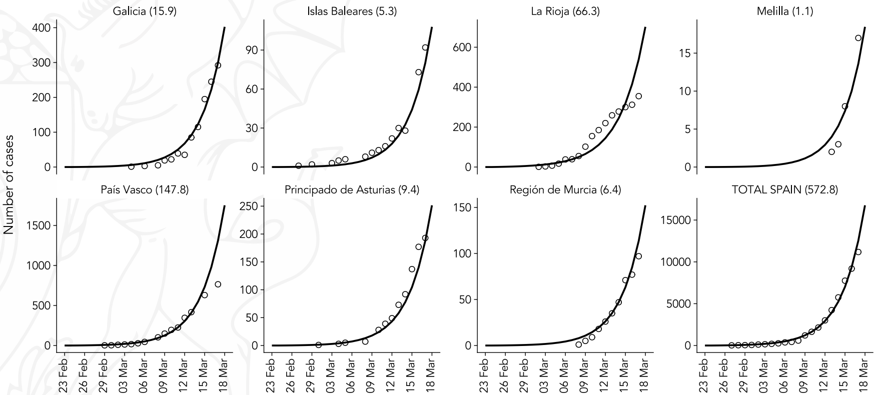
$\langle k^g \rangle$	Average number of contacts	$(11.8, 13.3, 6.6)$
C^{gh}	Contacts-by-age matrix	$\begin{pmatrix} 0.5980 & 0.3849 & 0.0171 \\ 0.2440 & 0.7210 & 0.0350 \\ 0.1919 & 0.5705 & 0.2376 \end{pmatrix}$
p^g	Mobility factor	$(0.0, 1.0, 0.0)$

ξ	Density factor	0.01 km^{-2}
n_i^g	Regional population	Data provided by INE
R_{ij}^g	Mobility matrix	Data provided by INE

The results


Andalucía (23.1)
Aragón (14.6)
Canarias (11.2)
Cantabria (9.0)
Castilla y León (13.5)
Castilla-La Mancha (35.8)
Cataluña (50.3)
Ceuta (0.5)
Comunidad de Madrid (362.5)
Comunidad Foral de Navarra (17.4)
Comunidad Valenciana (28.7)
Extremadura (12.6)
Number of cases
23 Feb
26 Feb
29 Feb
03 Mar
06 Mar
09 Mar
12 Mar
15 Mar
18 Mar

The results



Overall, the total number of infectious seeds is 47 individuals which represents 0.2% of the number of cases reported by March 20, 2020.

Imposing mobility restrictions

- We isolate a fraction κ_0 of the adult population.
- $\kappa_0 = 1$ reflects a total lockdown.
- That way, the average numbers of contacts are:

$$\langle k_c^Y \rangle = \langle k_c^O \rangle = \sigma - 1 \,,$$
$$\langle k_c^M \rangle = \kappa_0 \, (\sigma - 1) + (1 - \kappa_0) \langle k^M \rangle \,,$$

  where σ is the average household size.
- Applying the containment at time t_c, we have

$$\langle k^g \rangle(t) = [1 - \Theta(t - t_c)] \; \langle k^g \rangle + \Theta(t - t_c) \, \langle k_c^g \rangle,$$

  and

$$p^g(t) = [1 - \kappa_0 \, \Theta(t - t_c)] \; p^g$$

- Young and elderly people stay at home

- We also introduce a permeability factor ϕ.
- Some of the original equations are modified:

$$\begin{aligned}
\rho_i^{S,g}(t+1) &= \rho_i^{S,g}(t) \, [1 - \delta_{t,t_c} \, (1-\phi) \, \kappa_0 \, CH_i(t_c)] \, \left[1 - \Pi_i^g(t)\right] \,, \\
\rho_i^{E,g}(t+1) &= \rho_i^{S,g}(t) \, [1 - \delta_{t,t_c} \, (1-\phi) \, \kappa_0 \, CH_i(t_c)] \, \Pi_i^g(t) + (1 - \eta^g) \, \rho_i^{E,g}(t) \,, \\
\rho_i^{CH,g}(t) &= \rho_i^{S,g}(t_c) \, (1-\phi) \, \kappa_0 \, CH_i(t_c) \, \Theta(t - t_c)
\end{aligned}$$

- A new compartment CH quantifies the probability of one individual living in a household without any infected one:

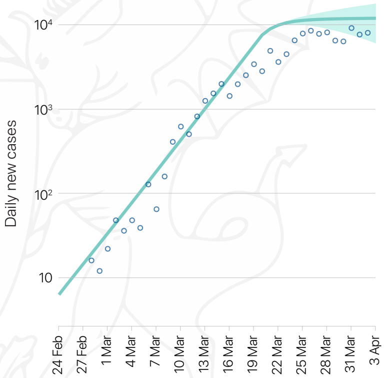
$$CH_i(t_c) = \left\{ \frac{1}{n_i} \sum_{g=1}^{N_G} \left[\rho_i^{S,g}(t_c) + \rho_i^{R,g}(t_c)\right] n_i^g \right\}^{\sigma} \,.$$

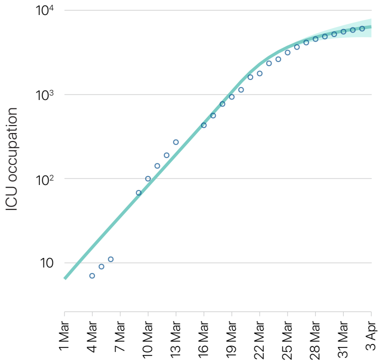
The contacts will also be time-dependent.

$$P_i^g(t) = 1 - \prod_{h=1}^{N_G} \prod_{j=1}^{N_P} (1 - \beta_A)^{z^g(t) \, \langle k^g \rangle(t) \, f\left(\frac{\tilde{n}_i(t)}{s_i}\right) \, C^{gh} \, \frac{n_{j \to i}^{A,h}(t)}{\tilde{n}_i^h(t)}} \, (1 - \beta_I)^{z^g(t) \, \langle k^g \rangle(t) \, f\left(\frac{\tilde{n}_i(t)}{s_i}\right) \, C^{gh} \, \frac{\nu \, n_{j \to i}^{I,h}(t)}{\tilde{n}_i^h(t)}} \,,$$

where ν is a isolation factor that reduces the infectiousness of symptomatic agents

Number of cases and ICU occupation in Spain



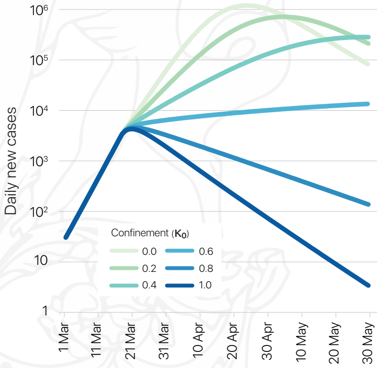


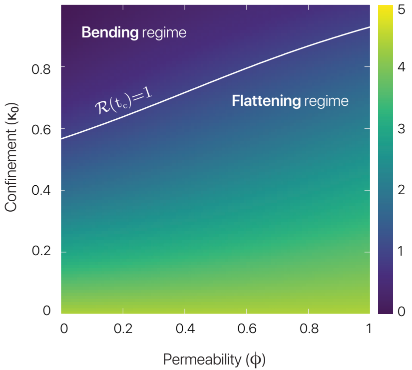
Left: H+R+D versus real cases reported. Right: number of patients predicted in compartment H and real ICU occupation.

$\kappa_0 \in [0.6, 0.8]$, $\phi \in [0.2, 0.4]$.

Effective reproduction number $\mathcal{R}$: from bending the curve to flattening it

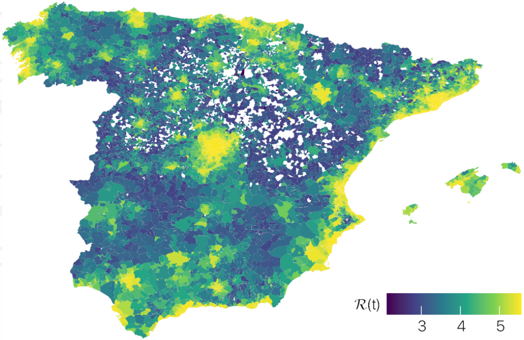
- By using this formalism, we can analytically relate the reduction of mobility and confinement measures with the reproductive ratio $\mathcal{R}$.
- Definition: $\mathcal{R}(t)$ is the number of secondary cases that an individual, becoming infectious at time t, will produce over time.
- The result takes into account the epidemiological characteristics of the disease along with the social, demographic and mobility patterns.



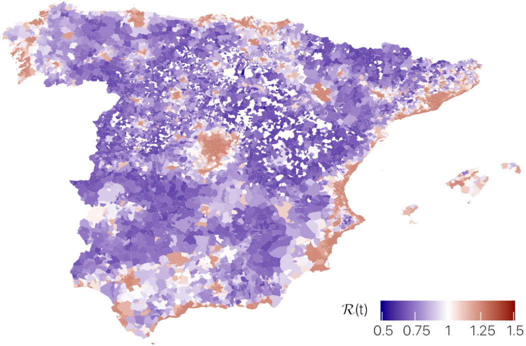


Spatial distribution of the effective reproduction number of adult population

Scenario A
No confinement measures



Scenario B
3 weeks into confinement



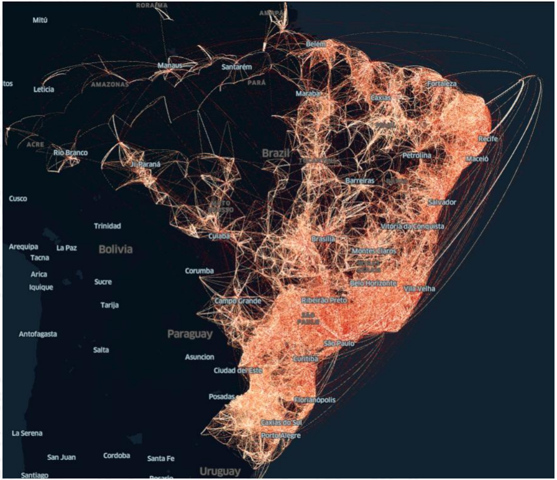
$$(\kappa_0, \phi) = (0.70, 0.20)$$

Stochastic version of the model: Brazil

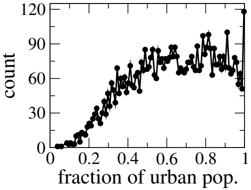
with Silvio C. Ferreira (UFV) and Guilherme H. Costa (UFV)

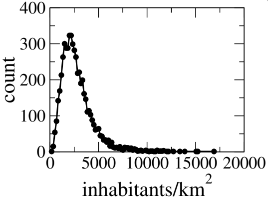
Brazil: a huge country

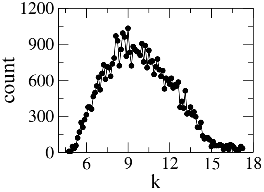
Brazil is not well represented by a single patch.



IBGE + ANAC data

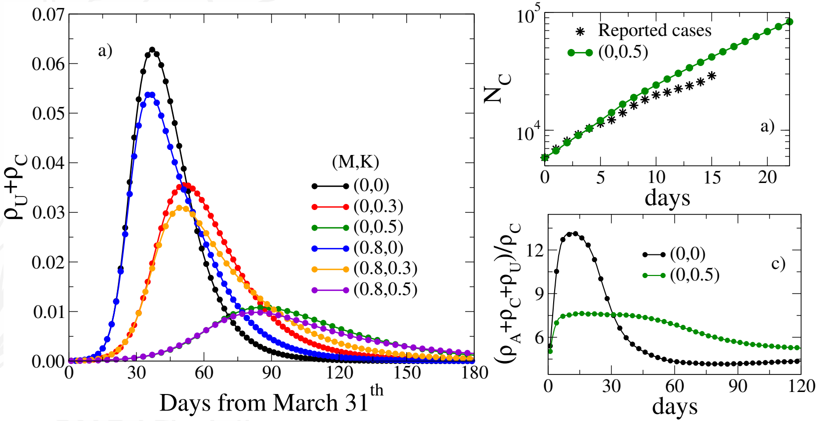






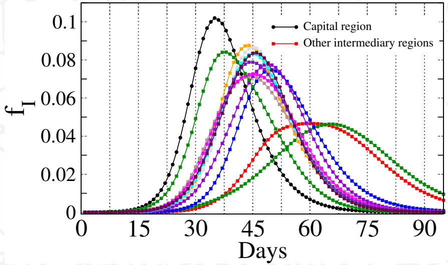
Applying the model

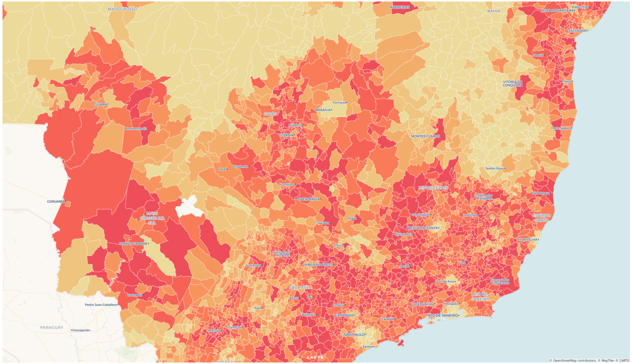
- We can study the effect of changing the mobility by M, and the contacts by K (social distance).
- As expected, the social distancing is the most effective measure against the spreading.
- Of course, we have a problem with under reporting of the cases.



The spreading from big cities to the smaller ones

- The spreading behaves differently across different cities!
- While the “peak” happens first at the capital cities, we can have dramatically different behaviors in other cities.





Resources:

Brazil data and COVID-19 APS Collection

Collecting and sharing the data at level of municipalities - Brazil

Open access data with time and municipality resolution: https://github.com/wcota/covid19br

- It is difficult to have data at municipality level in Brazil.
- The official platform only gives reports by states.
- Solution: read the states reports!

`https://covid19br.wcota.me`

Coronavirus (COVID-19) Collection - American Physical Society (APS)

PHYSICAL REVIEW JOURNALS

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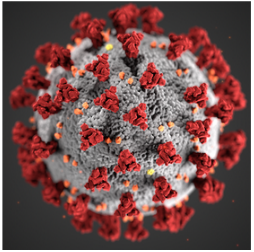
Coronavirus (COVID-19) Collection



In support of global efforts to address the COVID-19 pandemic, the American Physical Society (APS) has committed to making potentially relevant, peer-reviewed articles from our *Physical Review* journals more discoverable, accessible, and usable.

We have identified a collection of articles potentially relevant to researchers, health professionals, and others working on the COVID-19 pandemic, and are making this collection free to read for the duration of the crisis. The scope of the collection includes any articles that mention coronavirus, as well as those classified as relevant to epidemiology and epidemic spreading models. We will continue to add to the collection as additional potentially relevant articles are identified, and as new articles with potential relevance are published.

Read the up-to-date informational page for more details about the *Physical Review* journals' response to the COVID-19 pandemic.

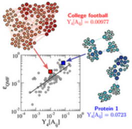
`https://journals.aps.org/collections/covid19`

Coronavirus (COVID-19) Collection - American Physical Society (APS)

Spectral properties and the accuracy of mean-field approaches for epidemics on correlated power-law networks

Diogo H. Silva, Silvio C. Ferreira, Wesley Cota, Romualdo Pastor-Satorras, and Claudio Castellano

Phys. Rev. Research **1**, 033024 (2019) – Published 15 October 2019



The authors show how the accuracy of mean-field estimates of the epidemic threshold in real and synthetic complex networks are related to their spectral properties. The results allow to gauge the predictive effectiveness of the different theories, enabling the selection of the minimal representative approach in order to obtain the desired accuracy in predictions for real-world topologies.

Complex Systems | Interdisciplinary Physics | Statistical Physics

Robustness and fragility of the susceptible-infected-susceptible epidemic models on complex networks

Wesley Cota, Angélica S. Mata, and Silvio C. Ferreira

Phys. Rev. E **98**, 012310 (2018) – Published 18 July 2018

Sampling methods for the quasistationary regime of epidemic processes on regular and complex networks

Renan S. Sander, Guilherme S. Costa, and Silvio C. Ferreira

Phys. Rev. E **94**, 042308 (2016) – Published 14 October 2016

Collective versus hub activation of epidemic phases on networks

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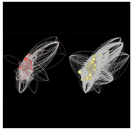

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UFV Universidade Federal de Viçosa

Universidad Zaragoza 1542

UNIVERSITAT ROVIRA i VIRGILI

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wcota.me | wesley.cota@ufv.br | @wlcota