Superconducting quantum criticality on the surface of 3D topological insulators

Joseph Maciejko
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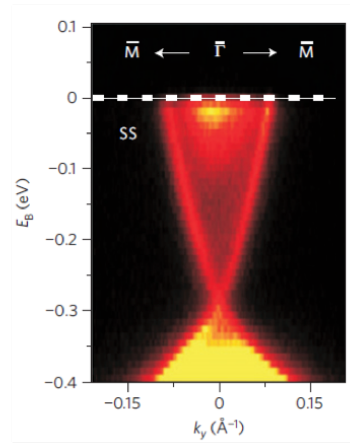

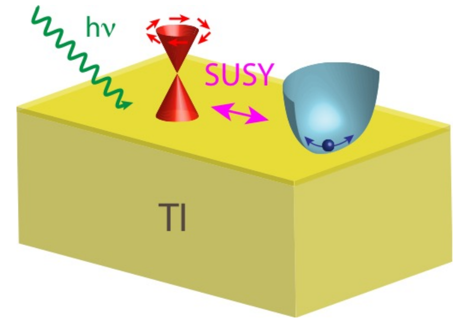

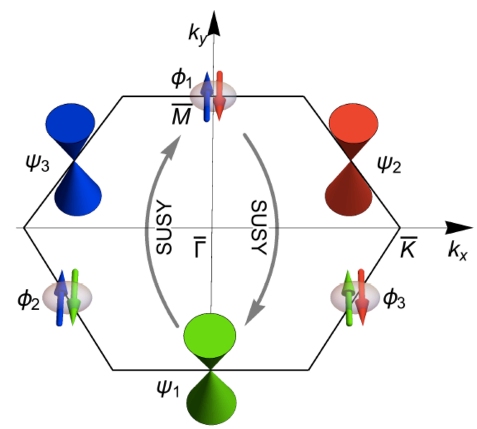

Topological States of Matter @ IIP
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Collaborators

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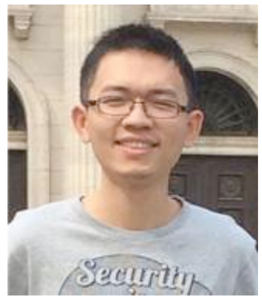

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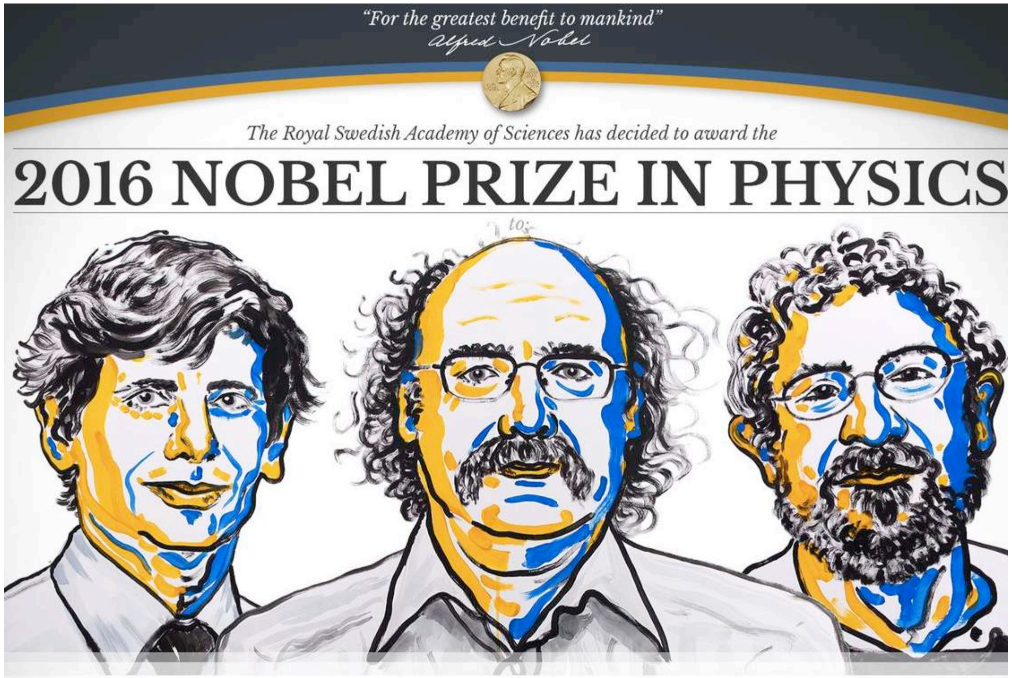
W. Witczak-Krempa and JM, Phys. Rev. Lett. **116**, 100402 (2016)
N. Zerf, C.-H. Lin, and JM, Phys. Rev. B **94**, 205106 (2016)
S.-K. Jian, C.-H. Lin, JM, and H. Yao, arXiv:1609.02146, to appear in PRL



"for theoretical discoveries of topological phase transitions and topological phases of matter"

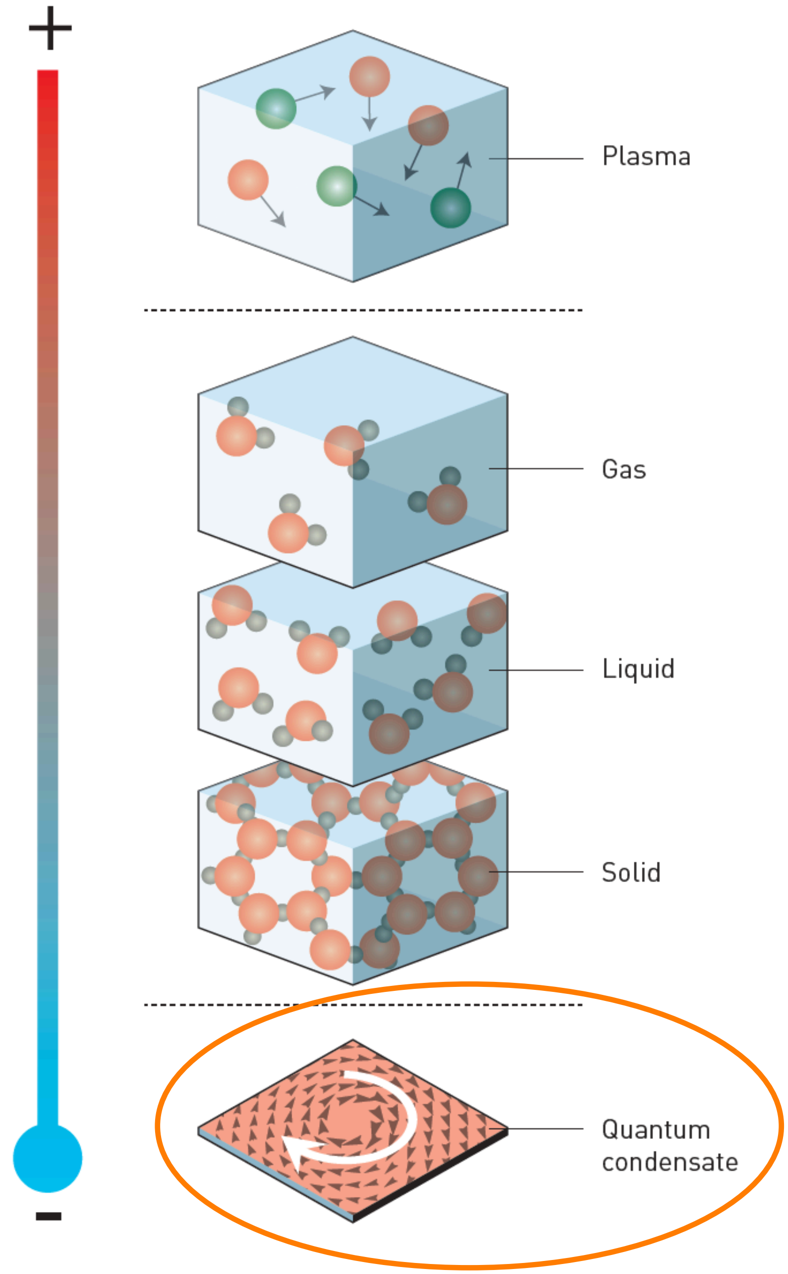
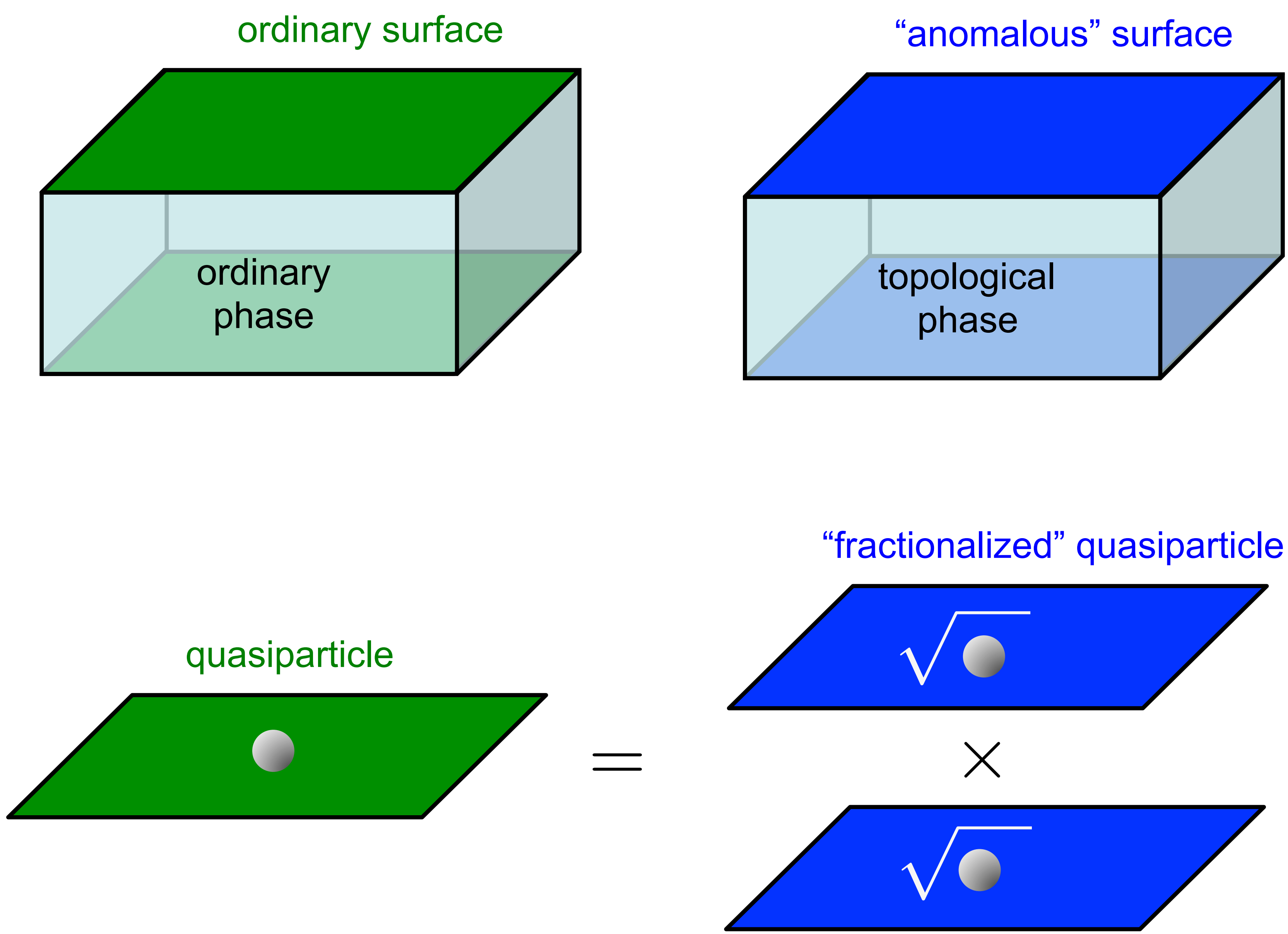
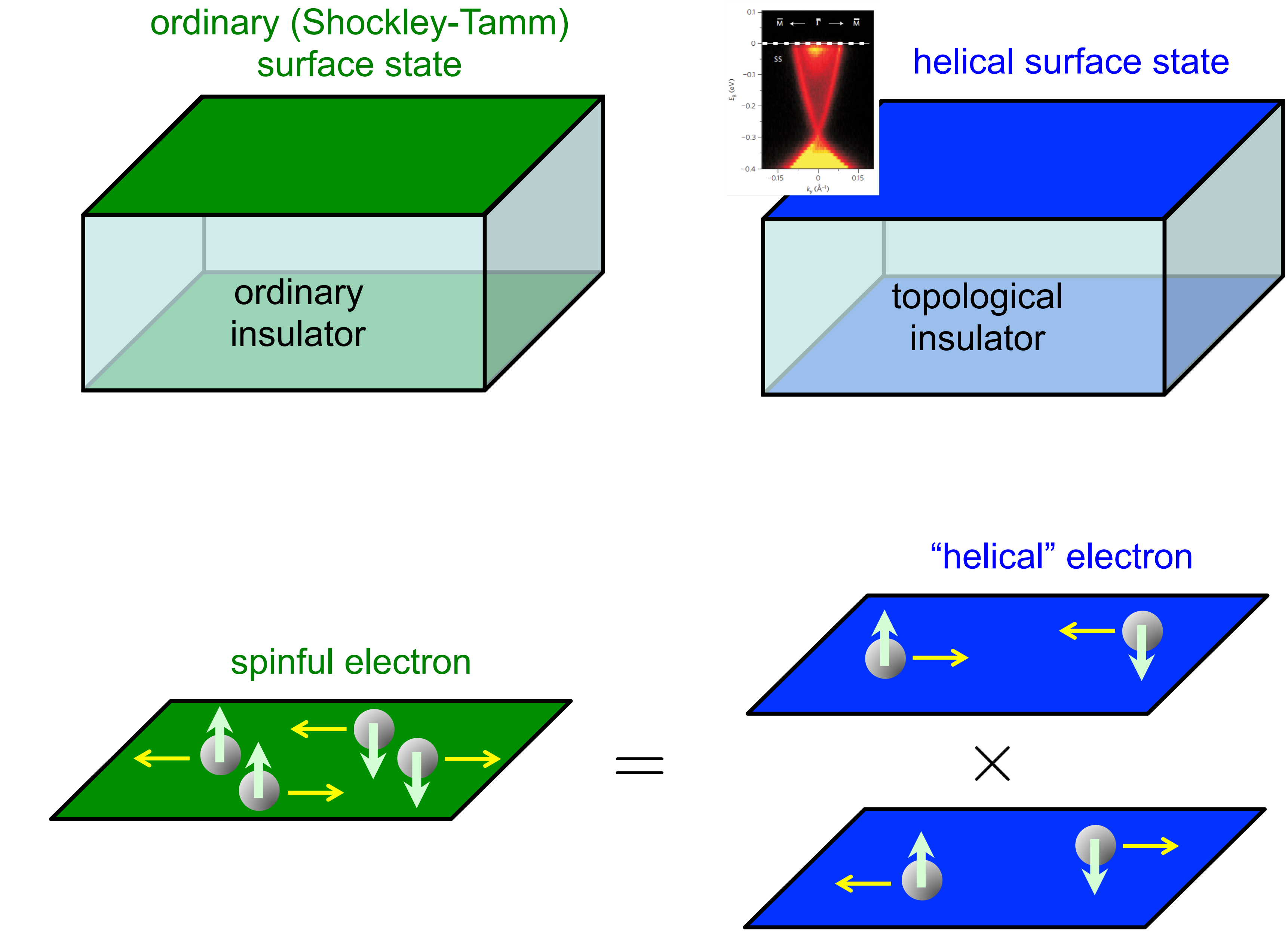
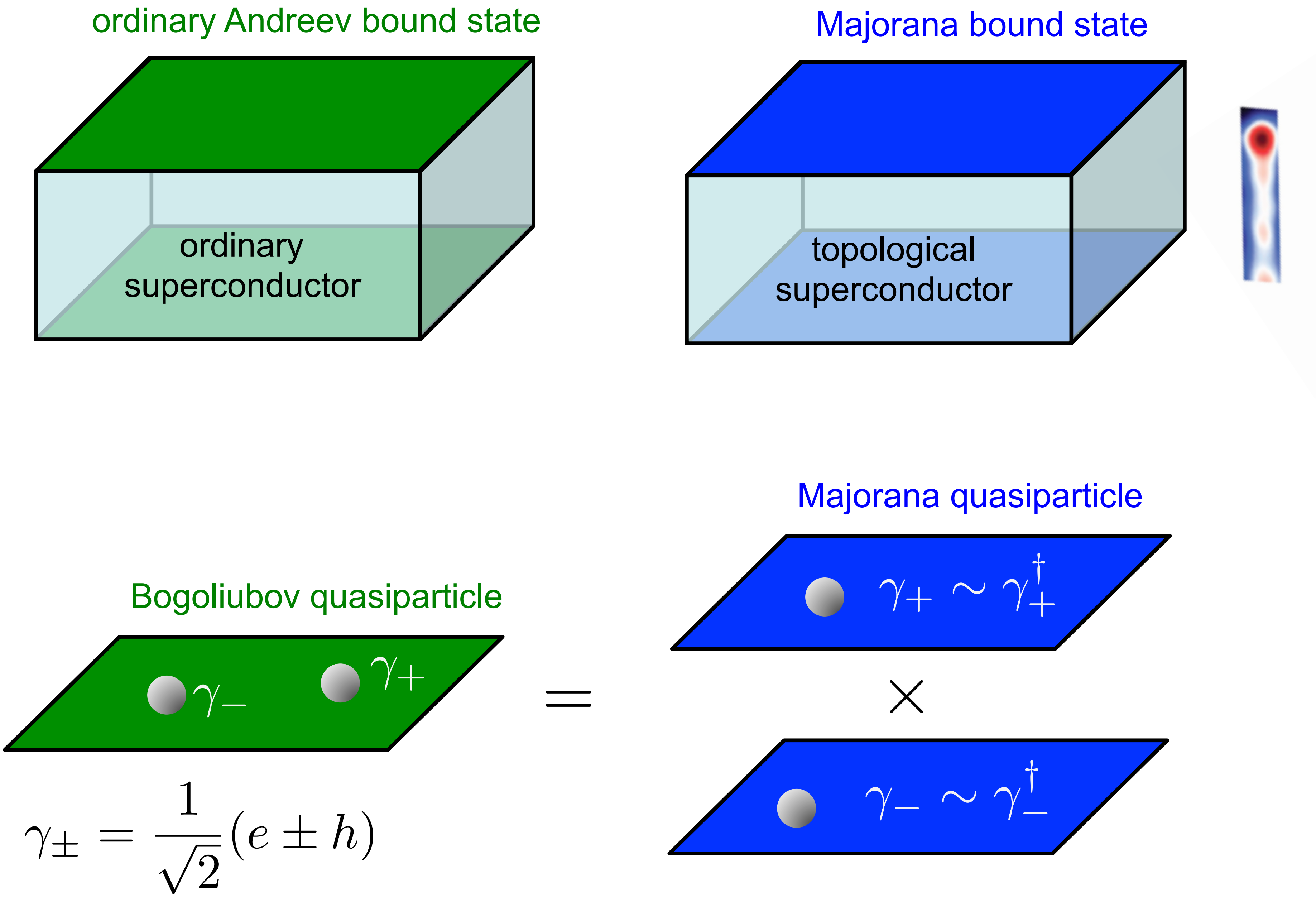


Fig. 1 Phases of matter. The most common phases are gas, liquid and solid matter. However, in extremely high or low temperatures matter assumes other, more exotic states.

Topological phase of matter	Theory	Experiment
IQHE	1981 (Laughlin), 1982 (Thouless et al.)	1980 (von Klitzing et al.)
FQHE	1983 (Laughlin)	1982 (Tsui et al.)
Haldane phase	1983 (Haldane)	1986 (Buyers et al.)
Chern insulator	1988 (Haldane)	2013 (Chang et al.)
QSHE	2005 (Kane, Mele)	2007 (König et al.)
Topological insulator	2007 (Fu, Kane, Mele)	2008 (Hsieh et al.)
Topological superconductor	2001 (Kitaev)	2012 (Mourik et al.)
Weyl semimetal	2011 (Wan et al.)	2015 (Xu et al.)


ordinary surface
ordinary phase
"anomalous" surface
topological phase
quasiparticle
"fractionalized" quasiparticle


ordinary (Shockley-Tamm)
surface state
ordinary
insulator
helical surface state
topological
insulator
spinful electron
"helical" electron


ordinary Andreev bound state
ordinary superconductor
Majorana bound state
topological superconductor
Bogoliubov quasiparticle
γ_-
γ_+
$\gamma_\pm = \frac{1}{\sqrt{2}}(e \pm h)$
=
Majorana quasiparticle
$\gamma_+ \sim \gamma_+^\dagger$
×
$\gamma_- \sim \gamma_-^\dagger$

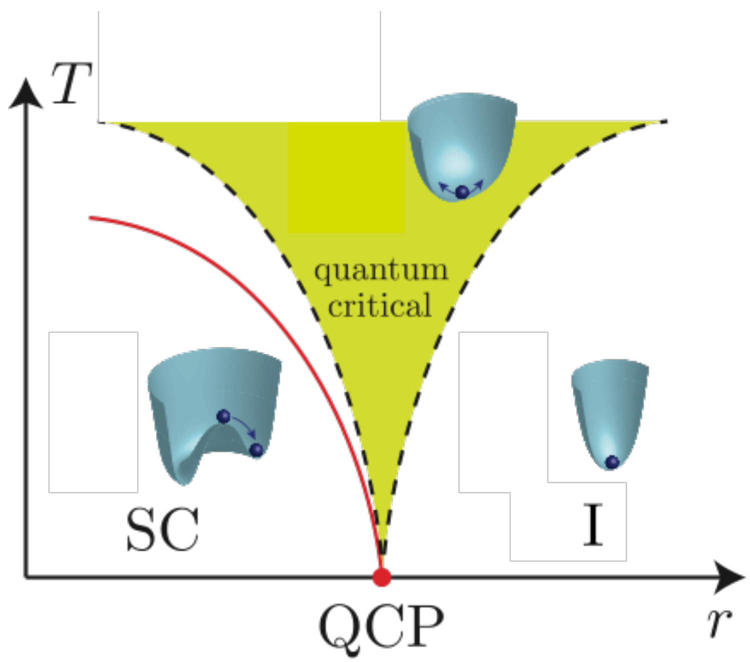
Platforms for novel quantum criticality?

- Topological surface states = novel gapless fermionic vacua with “anomalous” character (can only exist on boundary)
- Possibility of novel “boundary” quantum critical phenomena impossible (or hard) to realize in “bulk” systems?
- Focus on **semimetal-superconductor transition** on surface of 3D TI: odd number of 2D Dirac fermions with U(1) and T symmetries

Outline

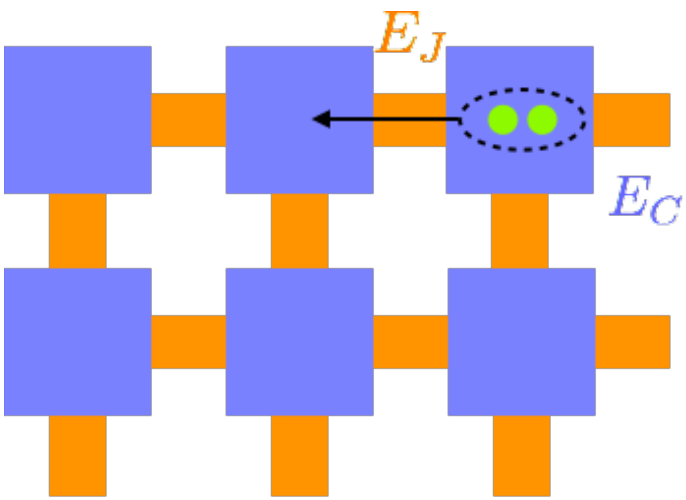
- Warm-up: boson superconductor-insulator transition (SC-I) vs Dirac fermion superconductor-semimetal transition (SC-SM)
- Superconductivity with one Dirac cone (Sb_2Te_3?)
- Superconductivity with three Dirac cones (SmB_6?)

SC-I transition of bosons



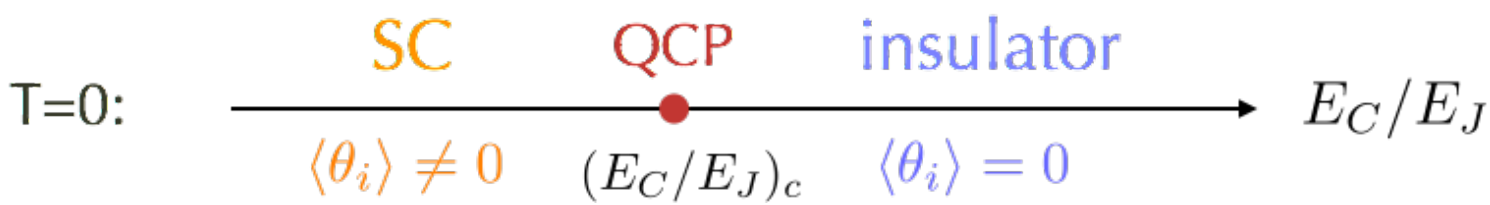
Josephson junction array

- SC islands coupled via Cooper pair tunneling
- Assume $E_C, E_J \ll \Delta$: no low-energy fermions



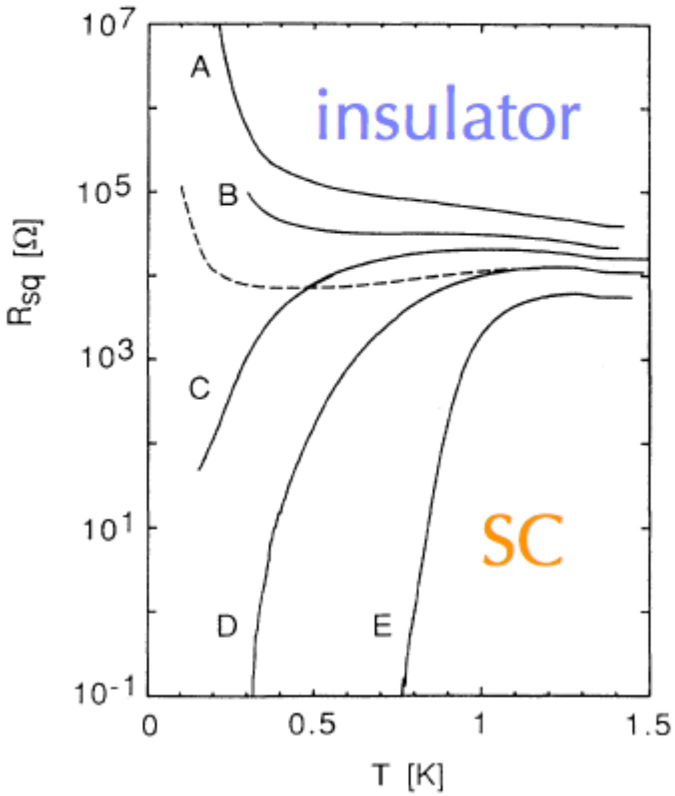
$$H = E_C \sum_i n_i^2 - E_J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

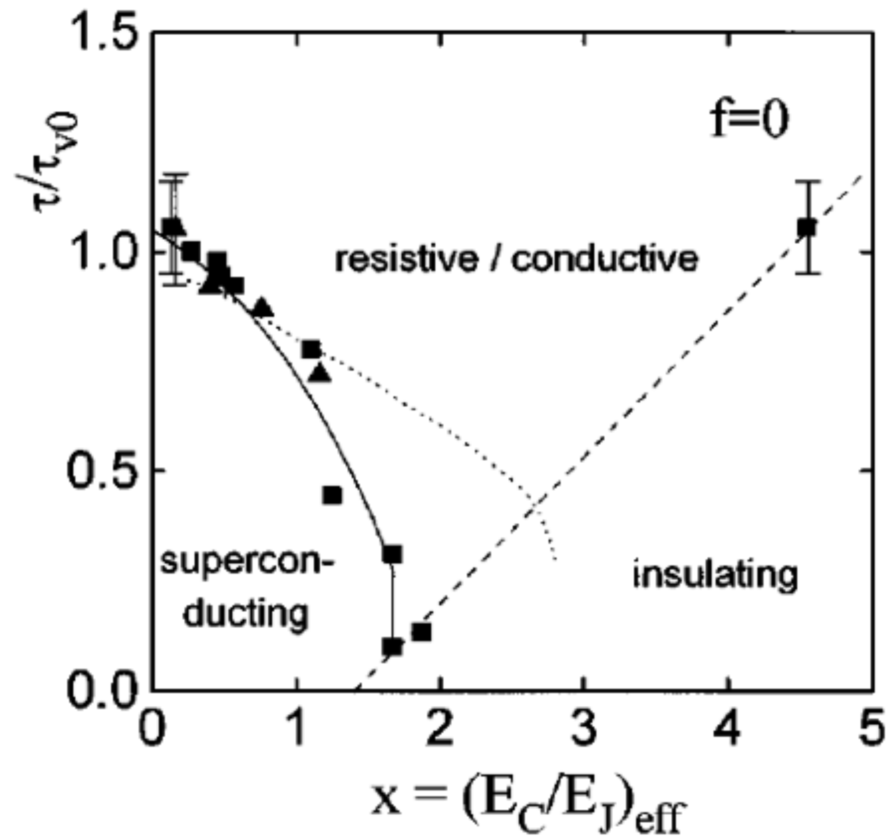
$$[n_i, \theta_j] = i\delta_{ij}$$



(Al junctions)

van der Zant, PRB '96



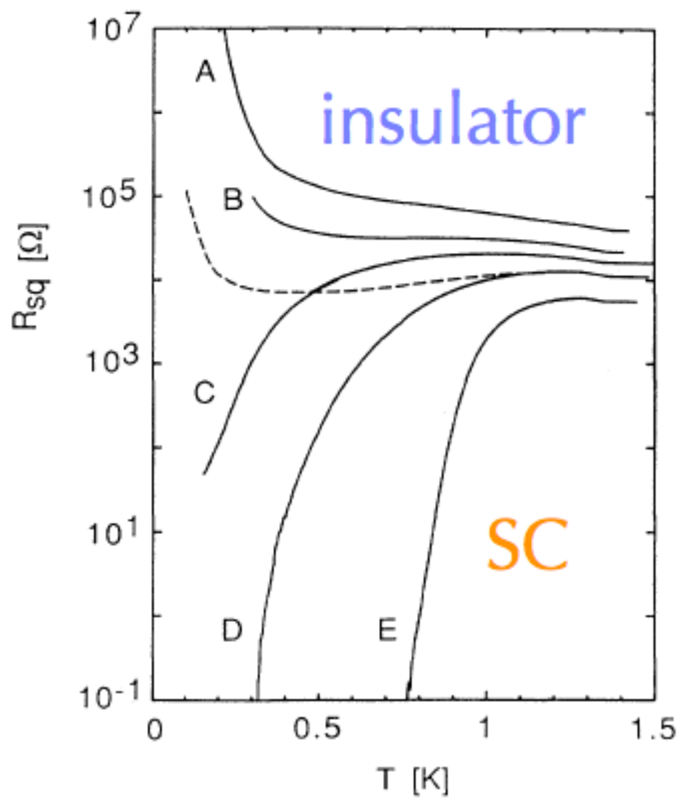


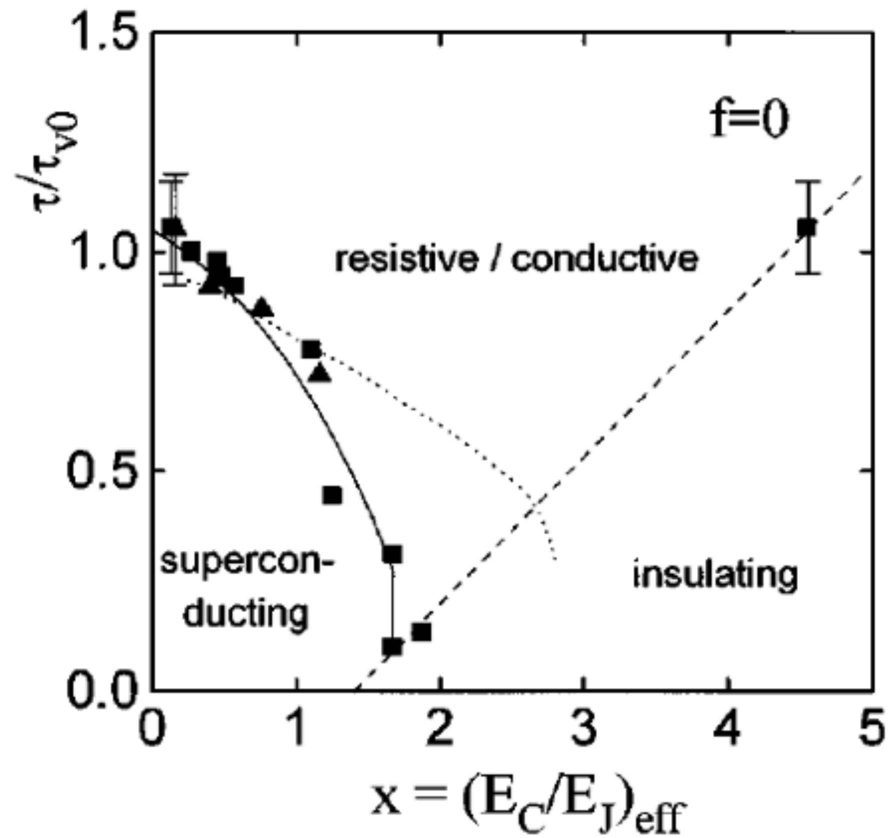
Geerligs et al., PRL '89

$$(E_C/E_J)_c \approx 1.7$$

(Al junctions)

van der Zant, PRB ‘96





Geerligs et al., PRL ‘89

$$(E_C/E_J)_c \approx 1.7$$

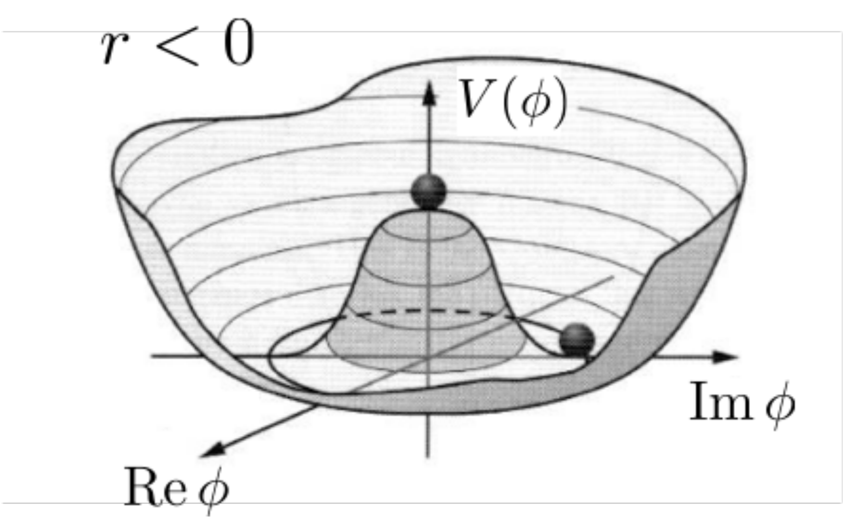
- also Bose-Hubbard model with ^{87}Rb atoms

(Spielman et al., PRL ‘07; Endres et al., Nature ‘12)

Landau-Ginzburg theory

- Coarse-grained description: order parameter = bosonic Cooper pair field $\phi(\boldsymbol{r}, \tau)$

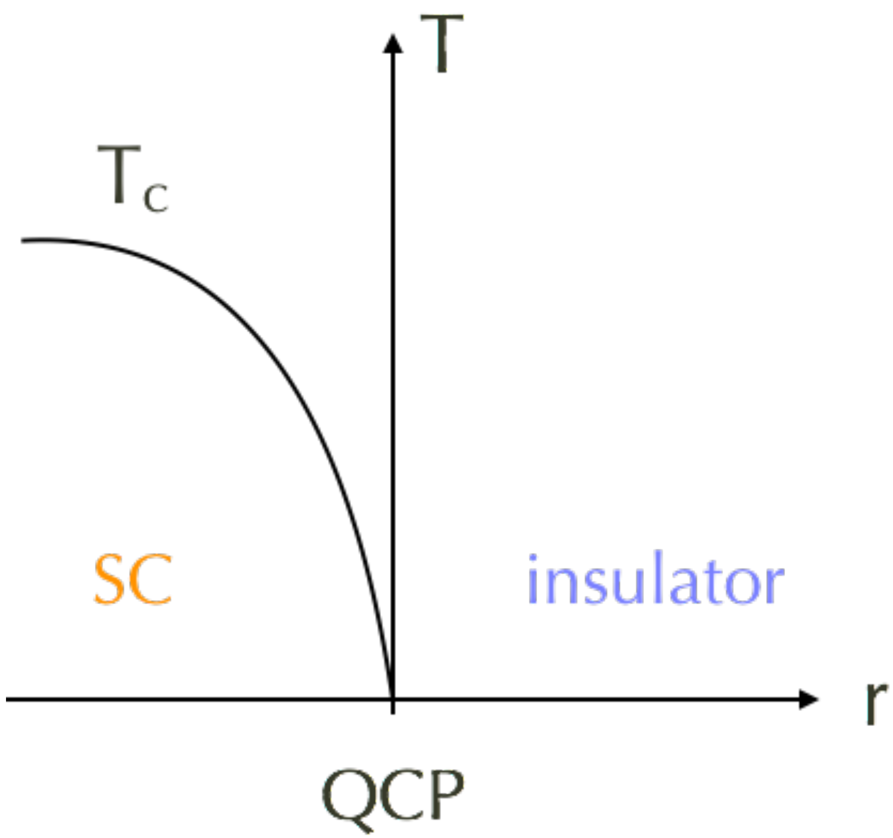
$$\mathcal{L} = |\partial_\tau \phi|^2 + c_b^2 |\nabla \phi|^2 + V(\phi)$$



$$V(\phi) = r|\phi|^2 + u|\phi|^4$$

$$r \sim \frac{E_C}{E_J} - \left(\frac{E_C}{E_J}\right)_c$$

Quantum critical point



- QCP is strongly coupled: O(2) Wilson-Fisher fixed point (3DXY)
- Emergent Lorentz invariance

QCP: optical conductivity

- Universal quantum critical conductivity in d=2 (Damle, Sachdev, PRB ‘97):

$$\sigma(\omega, T) = \frac{e^2}{\hbar}\Sigma\left(\frac{\hbar\omega}{k_B T}\right)$$

- T=0 optical conductivity is frequency-independent: **universal constant**

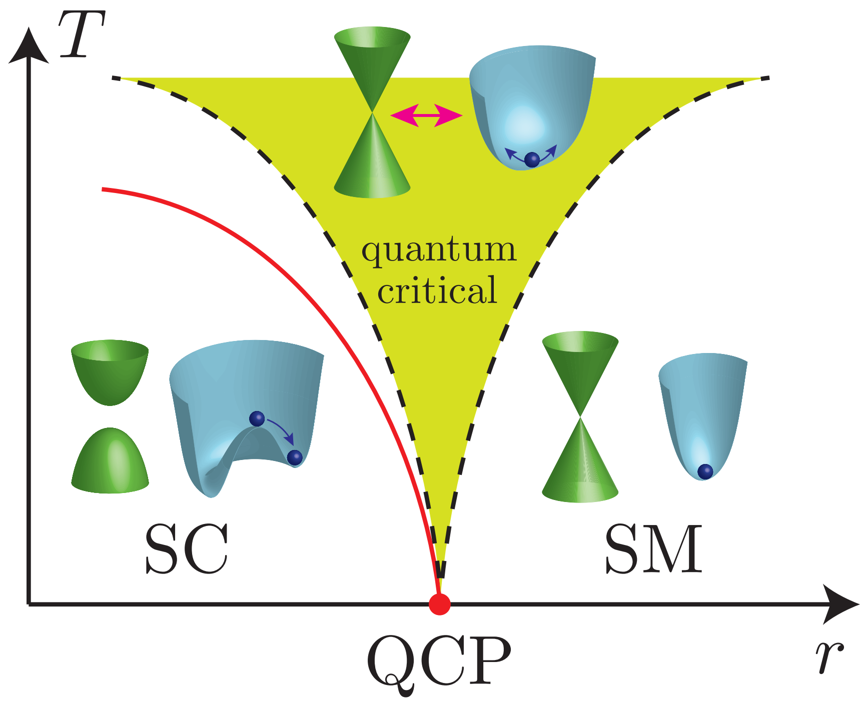
$$\sigma(\omega, 0) = \frac{e^2}{\hbar}\Sigma(\infty) = \frac{e^2}{\hbar}\sigma_\infty$$

- For boson SC-I transition: no exact result, long history – response function of a strongly correlated system with no quasiparticles! (Fisher, Grinstein, Girvin, PRL ‘90; Fazio, Zappalà, PRB ‘96; Šmakov, Sørensen, PRL ‘05…)

- QMC + holography + conformal bootstrap (Katz et al., PRB ‘14; Gazit et al., PRL ‘14; Witczak-Krempa et al., Nat. Phys. ‘14; Kos et al., JHEP ‘15):

$$\sigma_\infty \simeq 0.226$$

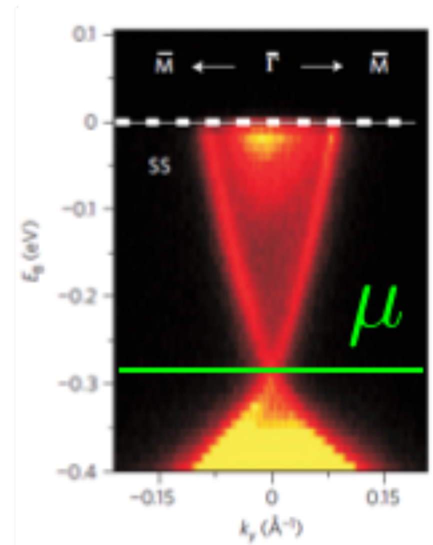
SC-SM transition of Dirac fermions



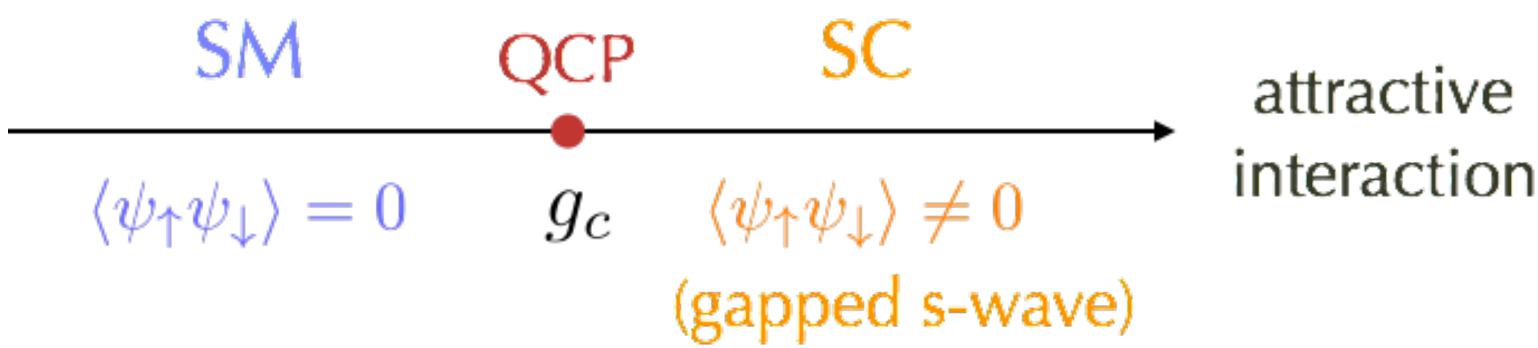
SM-SC transition of 2D Dirac fermions

- Pairing instability of **single** 2D Dirac fermion: 3D TI surface

$$\begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix}$$

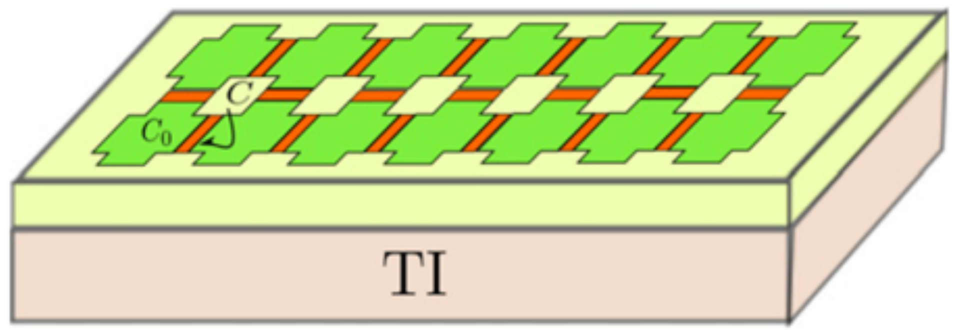


- Consider chemical potential at Dirac point: vanishing DOS implies QCP (Roy, Juričić, Herbut, PRB ‘13; Nandkishore, JM, Huse, Sondhi, PRB ’13)



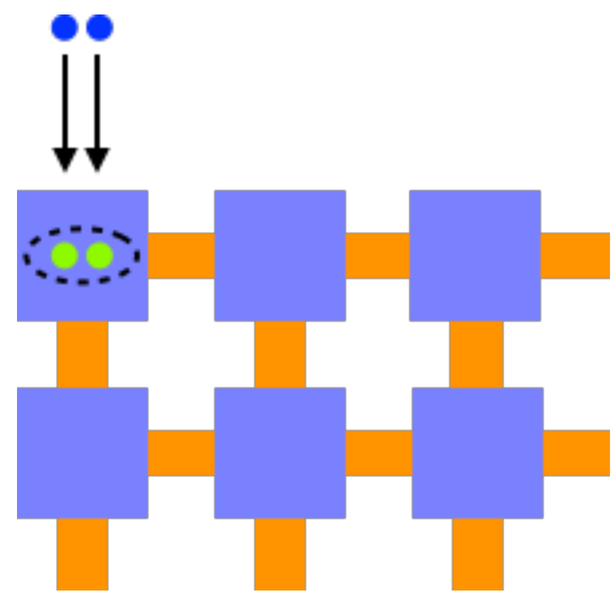
Route #1: Josephson engineering

- JJA on surface of TI



Ponte and Lee, NJP ‘14

- Pairs of Dirac electrons tunnel to SC island and vice-versa



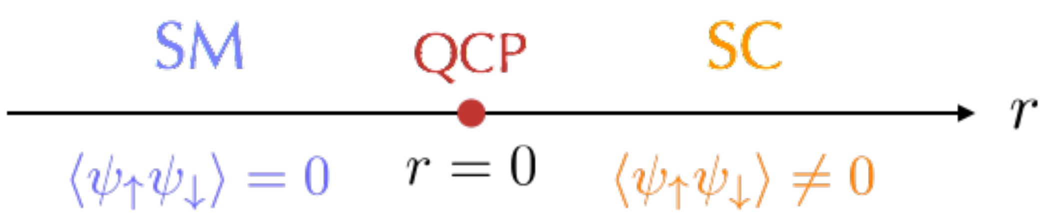
Landau-Ginzburg theory

- Low-energy theory has bosons **and** fermions (pair-breaking effects)

$$\mathcal{L} = i\bar{\psi}(\gamma_0\psi_0 + c_f\gamma_i\partial_i)\psi \qquad \text{Dirac}$$

$$+|\partial_0\phi|^2 + c_b^2|\partial_i\phi|^2 + r|\phi|^2 + \lambda^2|\phi|^4 \qquad \text{JJA}$$

$$+2h(\phi^*\psi_\uparrow\psi_\downarrow + \text{c.c.}) \qquad \text{Dirac-JJA tunneling}$$



Route #2: Intrinsic SC?

- Anisotropic (2D) diamagnetic screening: surface SC

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ARTICLE

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Emergent surface superconductivity in the topological insulator Sb_2Te_3

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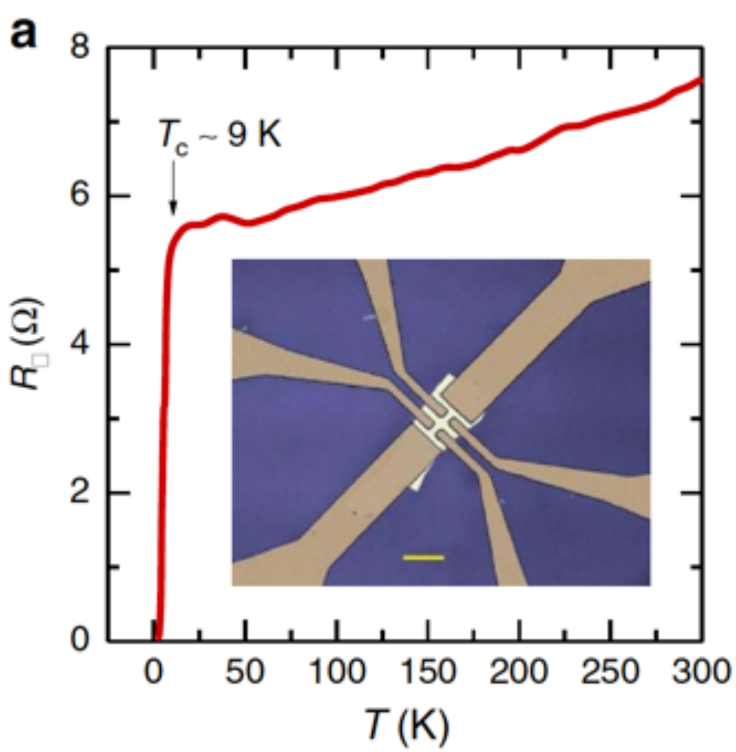
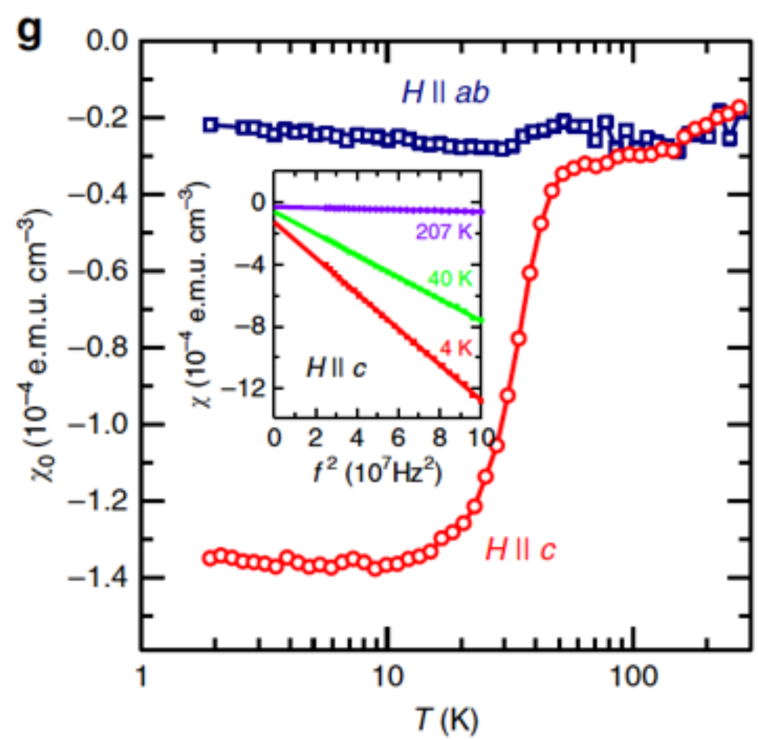
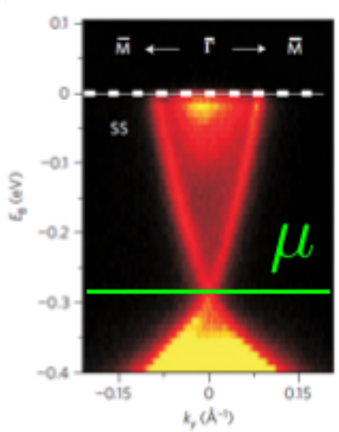
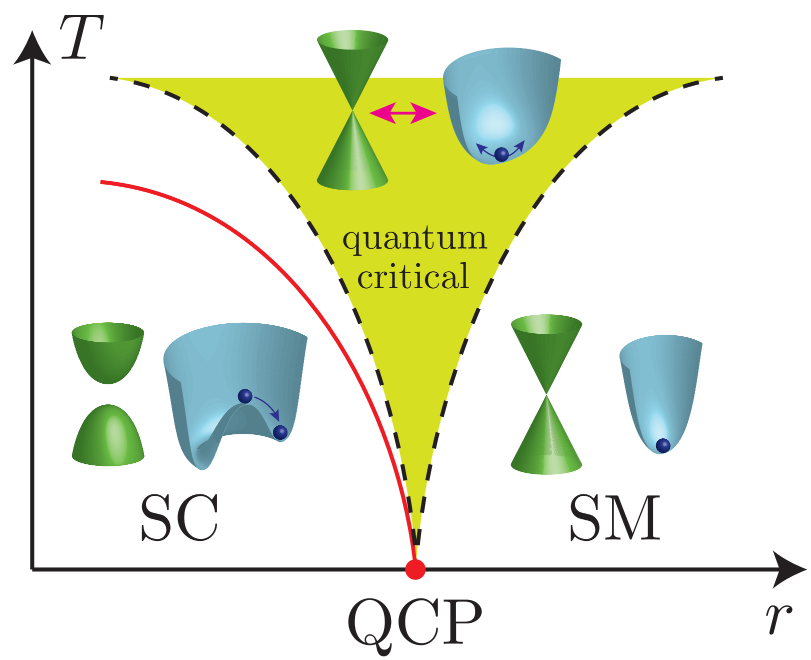




Semimetal-superconductor QCP

- QCP has an emergent (2+1)D **supersymmetry**: N=2 Wess-Zumino model (Grover, Sheng, Vishwanath, Science '14; Ponte, Lee, NJP '14)





$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial_\mu\psi + |\partial_\mu\phi|^2 + r|\phi|^2 + h^2|\phi|^4 + h(\phi^*\psi^T i\sigma^y\psi + \text{h.c.})$$

SUSY QCP: critical exponents

- Strongly coupled QCP: anomalous dimensions exactly known from SUSY (Aharony et al., NPB ‘97)

$$\eta_\phi = \eta_\psi = \frac{1}{3}$$

- Correlation length exponent: $\xi \sim (g - g_c)^{-\nu}$

$$\nu = \frac{1}{2} + \frac{\epsilon}{4} + \mathcal{O}(\epsilon^2) \approx 0.75$$ 1-loop RG (Thomas, ‘05; Lee, PRB ’07)

$$\nu = \frac{1}{2} + \frac{\epsilon}{4} + \frac{\epsilon^2}{24} + \left(\frac{\zeta(3)}{6} - \frac{1}{144}\right)\epsilon^3 + \mathcal{O}(\epsilon^4) \approx 0.985$$

3-loop RG (Zerf, Lin, JM, PRB ’16)

$$\nu \approx 0.9174$$ Padé extrapolation of 3-loop result (Fei et al., PTEP ‘16)

$$\nu \approx 0.9173$$ conformal bootstrap (Bobev et al., PRL ‘15)

QCP: optical conductivity $\sigma(\omega, 0) = \frac{e^2}{\hbar}\Sigma(\infty) = \frac{e^2}{\hbar}\sigma_\infty$

PRL **101,** 196405 (2008) PHYSICAL REVIEW LETTERS week ending 7 NOVEMBER 2008

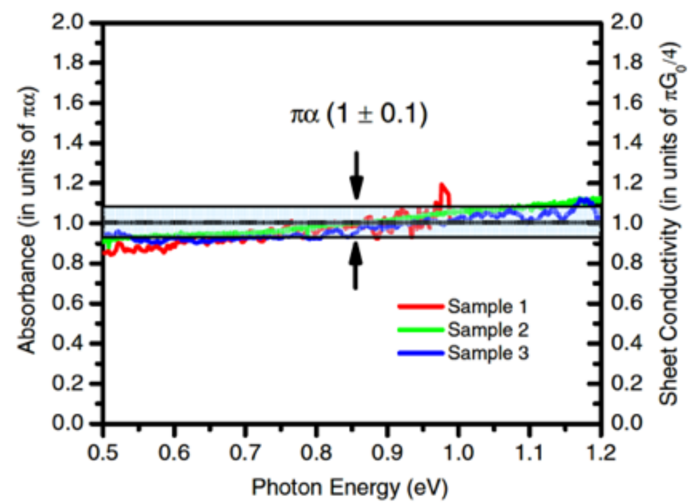
Measurement of the Optical Conductivity of Graphene

Kin Fai Mak,[1] Matthew Y. Sfeir,[2] Yang Wu,[1] Chun Hung Lui,[1] James A. Misewich,[2] and Tony F. Heinz[1,*]
[1]*Departments of Physics and Electrical Engineering, Columbia University, 538 West 120th Street, New York, New York 10027, USA*
[2]*Brookhaven National Laboratory, Upton, New York 11973, USA*
(Received 28 June 2008; published 7 November 2008)

Optical reflectivity and transmission measurements over photon energies between 0.2 and 1.2 eV were performed on single-crystal graphene samples on a SiO_2 substrate. For photon energies above 0.5 eV, graphene yielded a spectrally flat optical absorbance of $(2.3 \pm 0.2)\%$. This result is in agreement with a constant absorbance of $\pi\alpha$, or a sheet conductivity of $\pi e^2/2h$, predicted within a model of noninteracting massless Dirac fermions. This simple result breaks down at lower photon energies, where both spectral

- e.g., graphene = free Dirac CFT:

$$\frac{\hbar\omega}{k_B T} \sim \frac{1\ \text{eV}}{300\ \text{K}} \sim 39 = \infty$$



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$$\sigma_\infty = 1/4 = 4 \times 1/16$$

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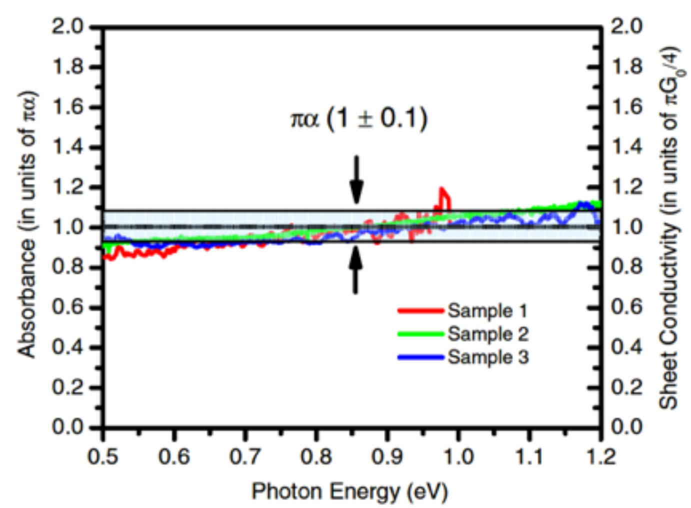

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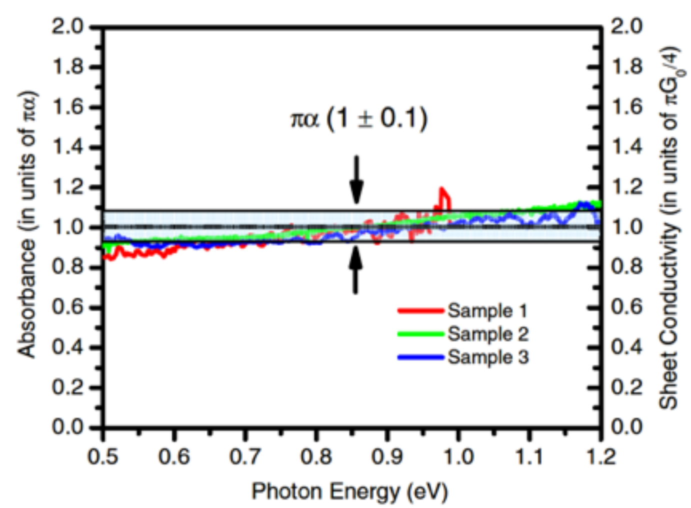
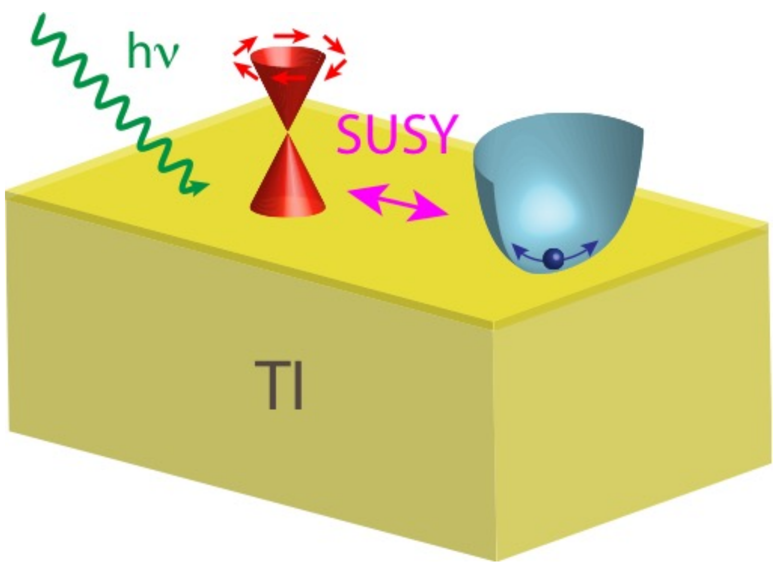
QCP = strongly interacting
Dirac fermions + Cooper pairs!



SUSY QCP: optical conductivity

- Optical conductivity at the strongly correlated Dirac SM-SC QCP can be calculated exactly using SUSY:

$$\sigma(\omega, 0) = \frac{5(16\pi - 9\sqrt{3})}{243\pi} \frac{e^2}{\hbar} \approx 0.2271 \frac{e^2}{\hbar}$$



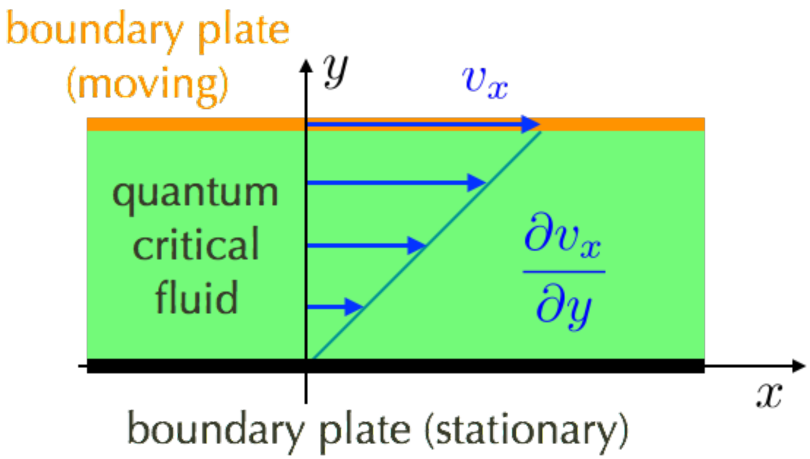
- Reason: 2-point function of stress tensor can be computed from partition function of N=2 WZ model on "squashed" S^3 (Closset et al., JHEP '13; Nishioka, Yonekura, JHEP '13)

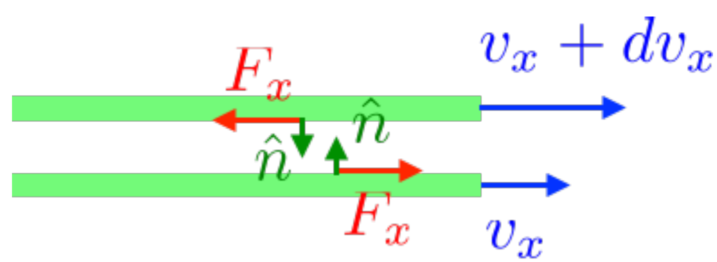
- U(1) current and stress tensor belong to the same SUSY multiplet

Witczak-Krempa and JM, PRL '16

SUSY QCP: shear viscosity

- Optical conductivity and dynamical shear viscosity are related by SUSY:





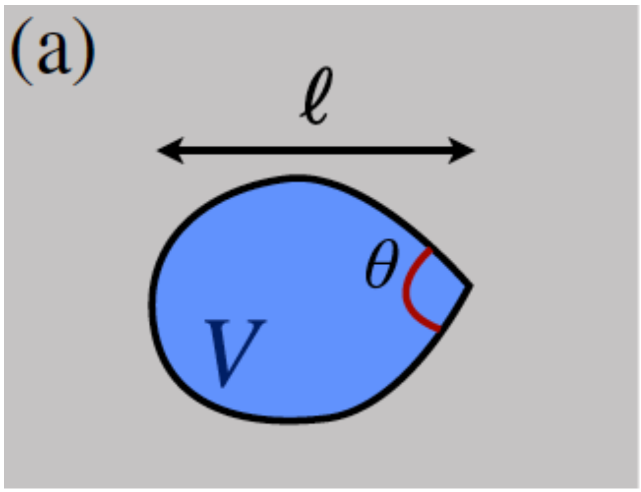
shear stress: $T_{xy} = \frac{F_x}{L} = \eta \frac{\partial v_x}{\partial y}$

$$\eta(\omega, 0) = \frac{\hbar\omega^2}{1944}(16\pi - 9\sqrt{3}) \approx 0.0178\hbar\omega^2$$

Witczak-Krempa and JM, PRL ’16

SUSY QCP: entanglement entropy

- 2-point function of stress tensor also determines corner entanglement entropy



$$S = B\ell/\delta - a(\theta)\ln(\ell/\delta) + \ldots$$

$$a(\theta) \simeq \lambda(\pi - \theta)^2$$

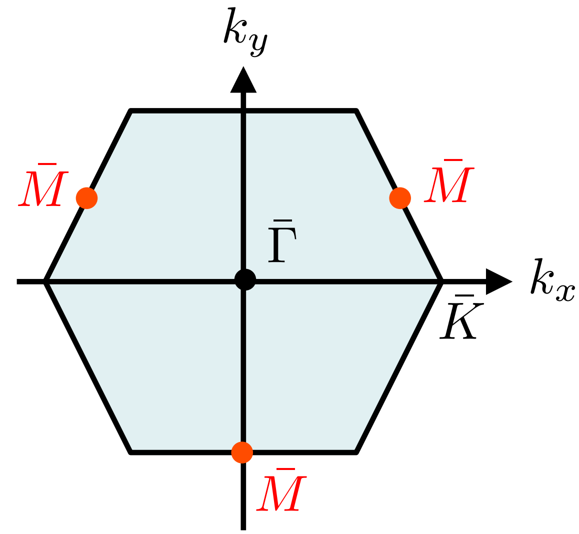
Casini, Huerta, Leitao, NPB ‘09

$$\lambda = \frac{16\pi - 9\sqrt{3}}{972\pi} \simeq 0.011356$$

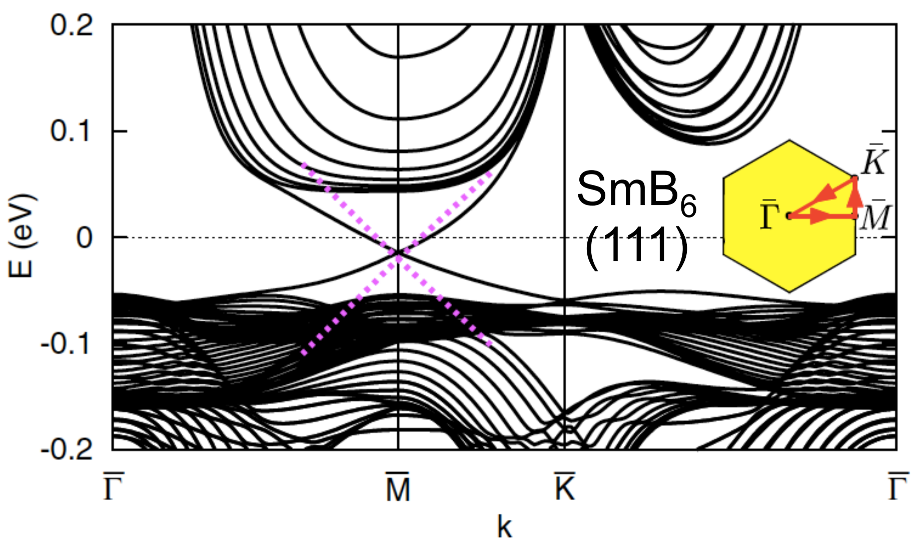
Witczak-Krempa and JM, PRL ’16

From one to three

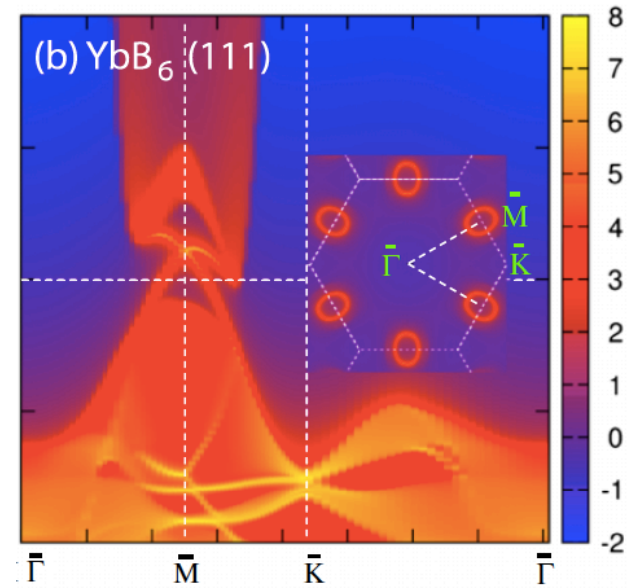
- 3D TI surface has odd # of Dirac cones: consider system with 3 cones
- (111) surface of cubic crystal has C_{3v} symmetry
- Four TRI points in surface BZ: $\bar{\Gamma}$, and three $\bar{M}$ points related by C_3 rotations



- (111) surface of SmB_6 (Ye, Allen, Sun, arXiv ’13; Baruselli, Vojta, PRB ‘14) and YbB_6 (Weng et al., PRL ’14) should host 3 **degenerate** Dirac cones at $\bar{M}$ points

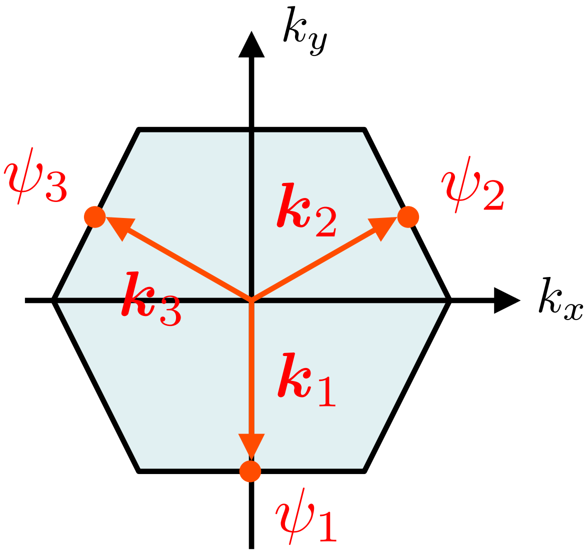


Baruselli and Vojta, PRB ‘16



Weng et al., PRL ‘14

Pairing: intra- vs intervalley



- Consider pairing instabilities for chemical potential at the Dirac point
- 2 possibilities: intravalley or intervalley pairing

- Intravalley pairing ($\langle \psi_1^T i\sigma^y \psi_1 \rangle \neq 0$, etc.) has **Q** = 0 crystal momentum: uniform SC

S.-K. Jian, C.-H. Lin, JM, H. Yao, arXiv '16, to appear in PRL

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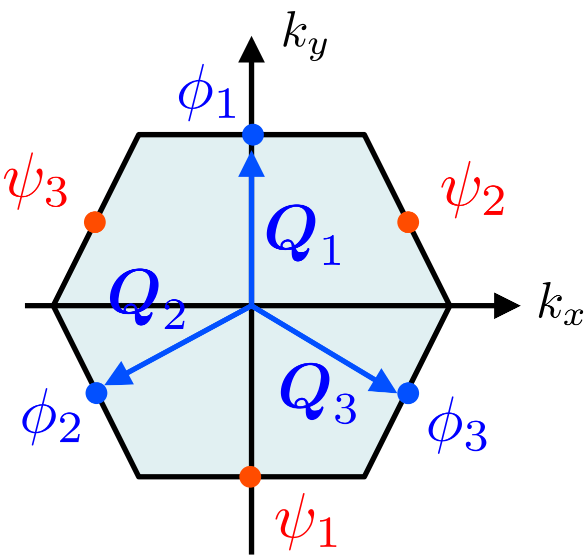

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- **Intervalley** pairing ($\langle\psi_1^T i\sigma^y\psi_2\rangle \neq 0$, etc.) has $\mathbf{Q}_1$, $\mathbf{Q}_2$, $\mathbf{Q}_3 \neq 0$ momentum: **pair-density-wave (PDW)**

$$\boldsymbol{Q}_1 = \boldsymbol{k}_2 + \boldsymbol{k}_3 \qquad \text{(\& cyclic permutations)}$$

$$\phi_i \sim \langle\psi_j^T i\sigma^y\psi_k\rangle,\ ijk = 123, 231, 312$$

S.-K. Jian, C.-H. Lin, JM, H. Yao, arXiv '16, to appear in PRL

Landau theory for PDW instability

- By symmetry (C_{3v} x U(1) x TRS x translation), Landau theory for PDW instability must have the form

$$V = r\sum_i |\phi_i|^2 + u_1 \sum_{i<j} |\phi_i|^2|\phi_j|^2 + u_1' \sum_{i<j}(\phi_i^{*2}\phi_j^2 + \text{h.c.}) + u_2\left(\sum_i |\phi_i|^2\right)^2$$

- In ordered phases, relative phase modes (Leggett modes) are gapped: can ignore at the mean-field level ($u_1 + u_1' \to u_1$)

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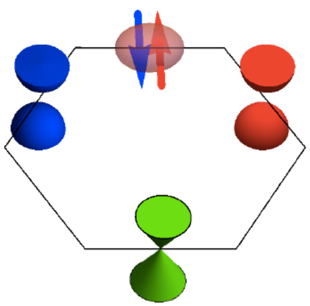
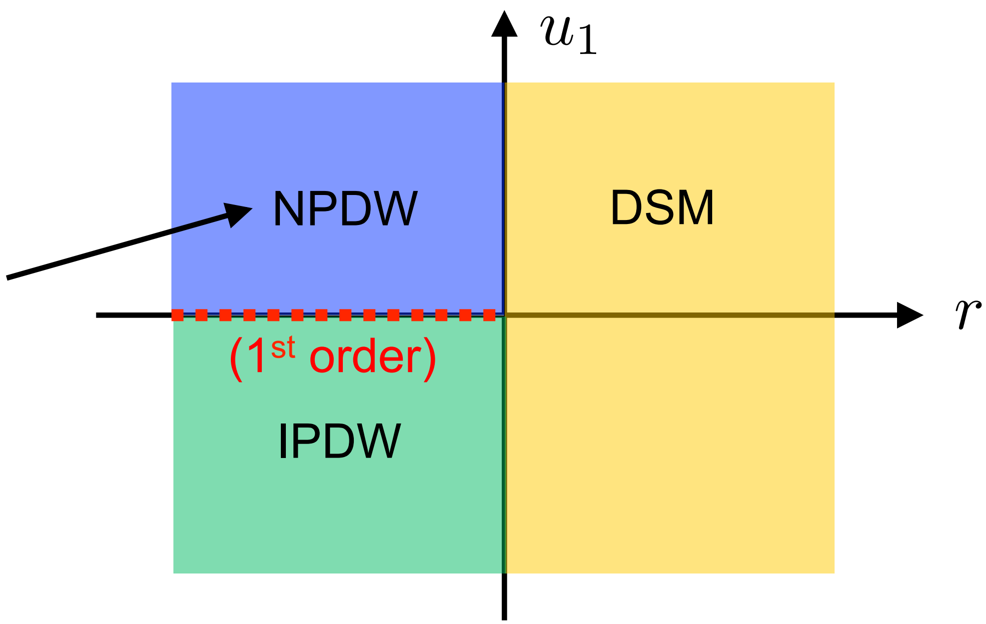
Mean-field phase diagram (I)

- For $u_2 > 0$:





Dirac semimetal (DSM): $\langle \phi_i \rangle = 0$, 3 gapless Dirac cones

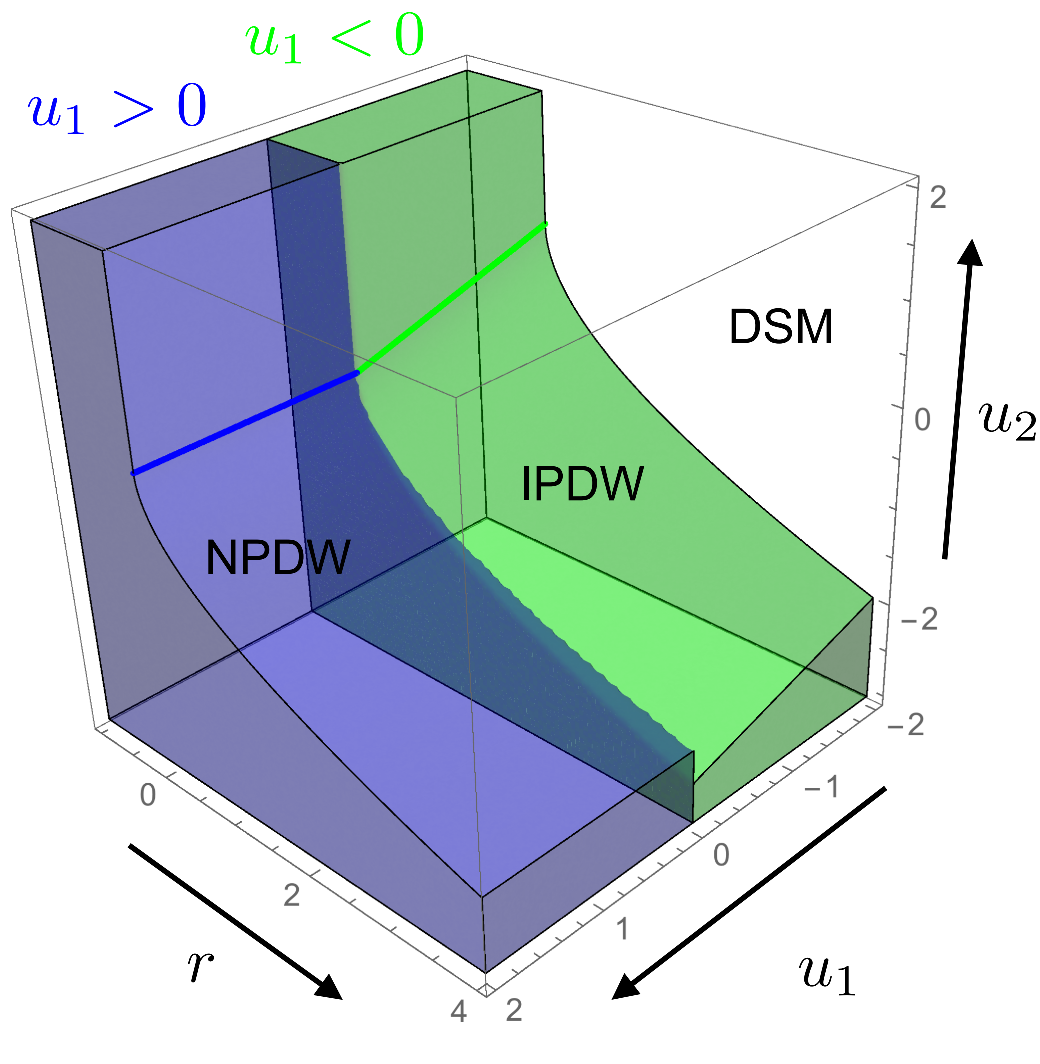
Isotropic PDW (IPDW): $\langle \phi_1 \rangle = \langle \phi_2 \rangle = \langle \phi_3 \rangle \neq 0$, 3 gapped Dirac cones

Nematic PDW (NPDW): $\langle \phi_1 \rangle \neq 0, \ \langle \phi_2 \rangle = \langle \phi_3 \rangle = 0$ & cyclic permutations,

2 gapped & 1 gapless Dirac cones: breaks C_3

Mean-field phase diagram (II)

- For $u_2 < 0$: must add sixth-order term $\sim w\left(\sum_i |\phi_i|^2\right)^3$ to stabilize the ground state energy



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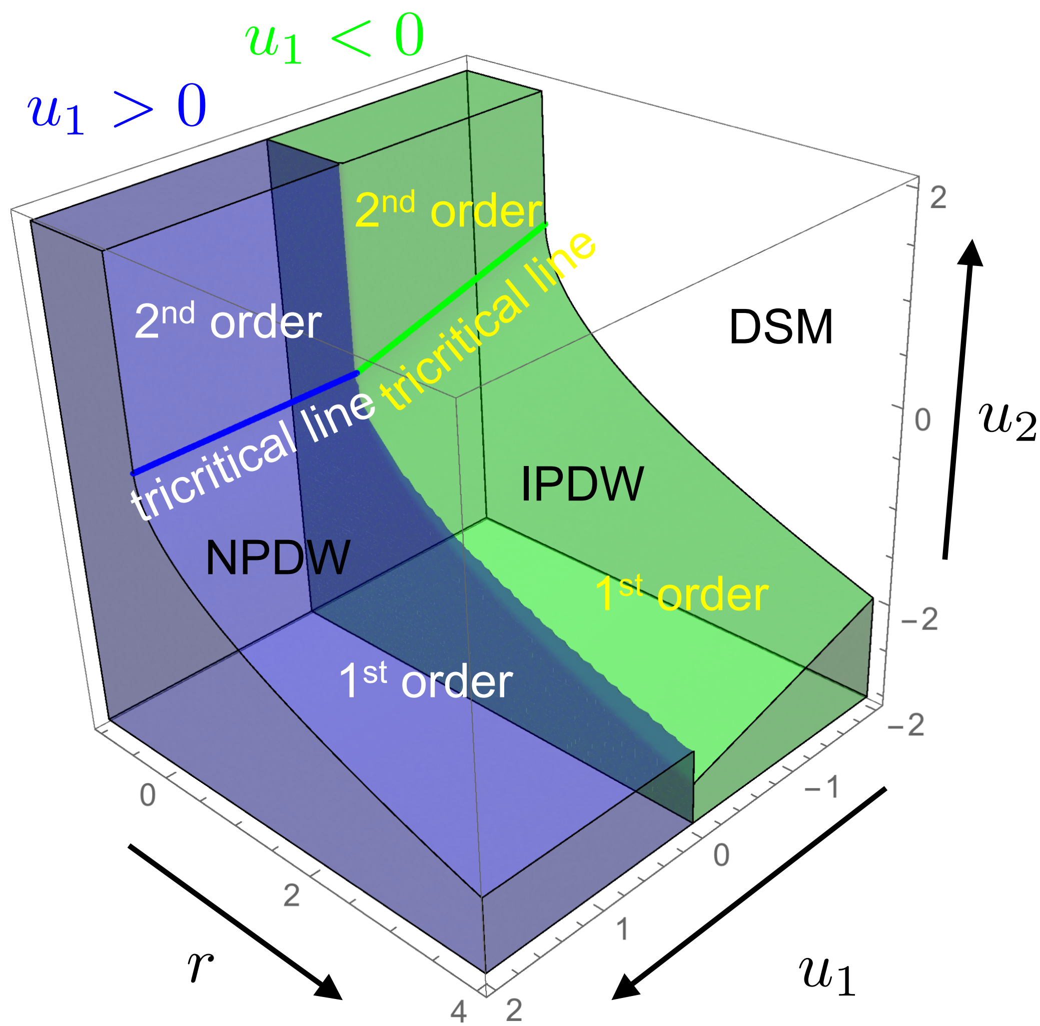
- NPDW-DSM and IPDW-DSM transitions go from continuous to first-order at **tricritical lines**

IPDW ($u_1 < 0$):

$$r = 0,\ u_2 = -u_1/3$$

NPDW ($u_1 > 0$):

$$r = 0,\ u_2 = 0$$



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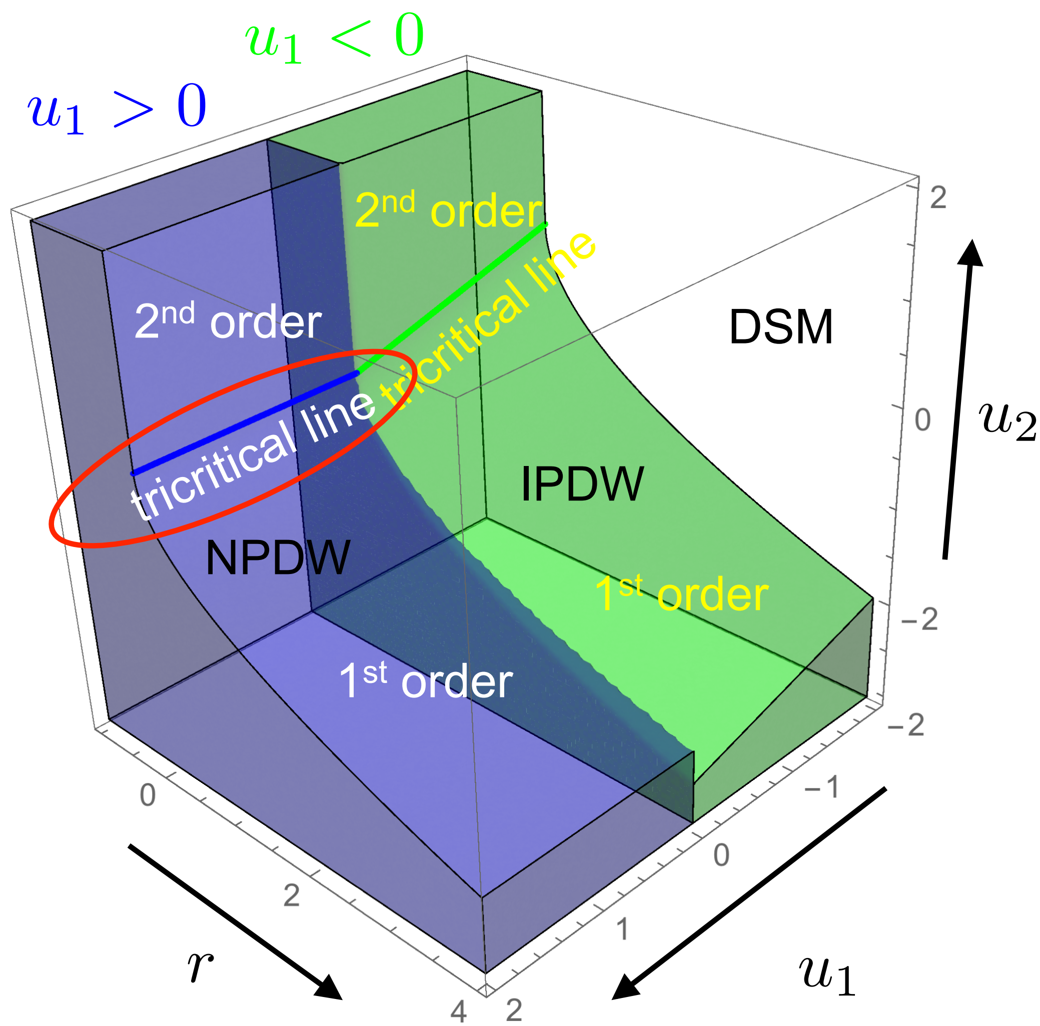


NPDW-DSM tricritical line (I)

- Low-energy effective theory of the NPDW-DSM tricritical line:

$$\mathcal{L} = \sum_i \bar{\psi}_i(\partial_\tau + h_i^f)\psi_i + \sum_i \phi_i^*(-\partial_\tau^2 + h_i^b)\phi_i$$

kinetic energy of Dirac fermion/Cooper pair

$$+r\sum_i |\phi_i|^2 + u_1 \sum_{i<j} |\phi_i|^2|\phi_j|^2 + u_1' \sum_{i<j}(\phi_i^{*2}\phi_j^2 + \text{h.c.})$$

$$+u_2\left(\sum_i |\phi_i|^2\right)^2$$

"classical" Landau energy

$$+g[(\phi_1^*\psi_2^T i\sigma^y\psi_3 + \text{c.p.}) + \text{h.c.}]$$

pair breaking

- Determine critical properties using Wilson and Fisher's ε-expansion (one-loop)

NPDW-DSM tricritical line (II)

- At low energies, fermion & boson velocities become **isotropic** and **equal** to each other: **emergent Lorentz invariance**
- Unstable fixed point with two relevant directions, r and u_2: NPDW-DSM tricritical line
- Fixed point couplings:

$$g^2 = u_1 = \frac{2\epsilon}{3\pi}, \; r = u_1' = u_2 = 0$$

- Fixed point Lagrangian:

$$\begin{aligned}\mathcal{L} = & \sum_i i\bar{\psi}_i \gamma_\mu \partial_\mu \psi_i + \sum_i |\partial_\mu \phi_i|^2 \\ & + g^2 (|\phi_1|^2|\phi_2|^2 + |\phi_2|^2|\phi_3|^2 + |\phi_3|^2|\phi_1|^2) \\ & + g(\phi_1^* \psi_2 \psi_3 + \phi_2^* \psi_3 \psi_1 + \phi_3^* \psi_1 \psi_2 + \text{h.c.})\end{aligned}$$

Emergent SUSY

- The fixed point Lagrangian on the NPDW-DSM tricritical line is a SUSY field theory known as the **XYZ model** (Aharony et al., NPB '97)

"fermionic" or "superspace" coordinates

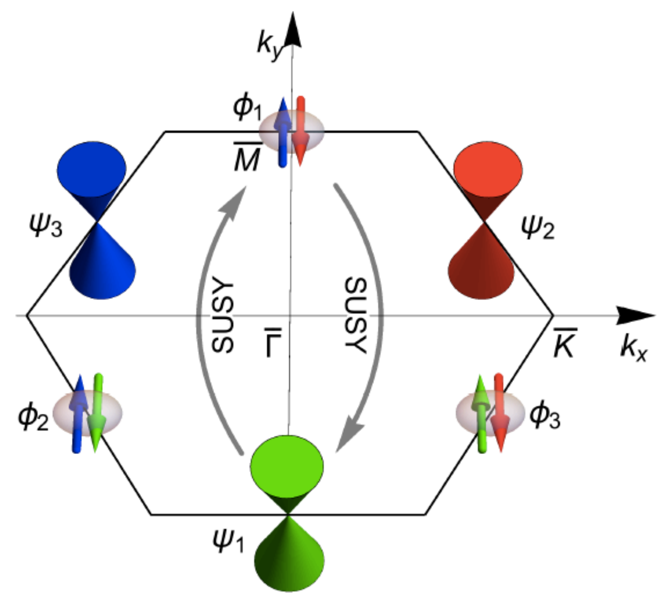
$$\mathcal{L} = \sum_{i=1}^{3} \int d^2\bar{\theta} d^2\theta \, \Phi_i^\dagger \Phi_i + g \left(\int d^2\theta \, \Phi_1 \Phi_2 \Phi_3 + \text{h.c.} \right)$$

SUSY kinetic term

"superpotential": SUSY interaction term

- Cooper pair ϕ_i and Dirac fermion ψ_i are superpartners, e.g., "components" of a single "superfield" Φ_i:

$$\Phi_i = \phi_i + \sqrt{2}\theta\psi_i + \ldots$$



XYZ tricritical line: critical properties

- As in the single Dirac case (N=2 WZ model), certain critical properties can be evaluated exactly even though the QCP is strongly interacting

- Dirac fermion/Cooper pair anomalous dimensions:

$$\eta_{\phi_i} = \eta_{\psi_i} = \frac{1}{3}, \; i = 1, 2, 3$$

- Universal T=0 optical conductivity:

$$\sigma(\omega) = \frac{15(16\pi - 9\sqrt{3})}{243\pi} \frac{e^2}{\hbar} \approx 0.681 \frac{e^2}{\hbar}$$

Mirror symmetry and $SQED_3$

- The XYZ model is interesting because it has an equivalent or “dual” description in terms of **N=2 supersymmetric quantum electrodynamics** in 2+1 dimensions (N=2 $SQED_3$) with a single “flavor” of matter fields
- This duality is known as **mirror symmetry** (Aharony et al., NPB ‘97) and can be understood as a SUSY version of **particle-vortex** duality (Dasgupta, Halperin, PRL ‘81)

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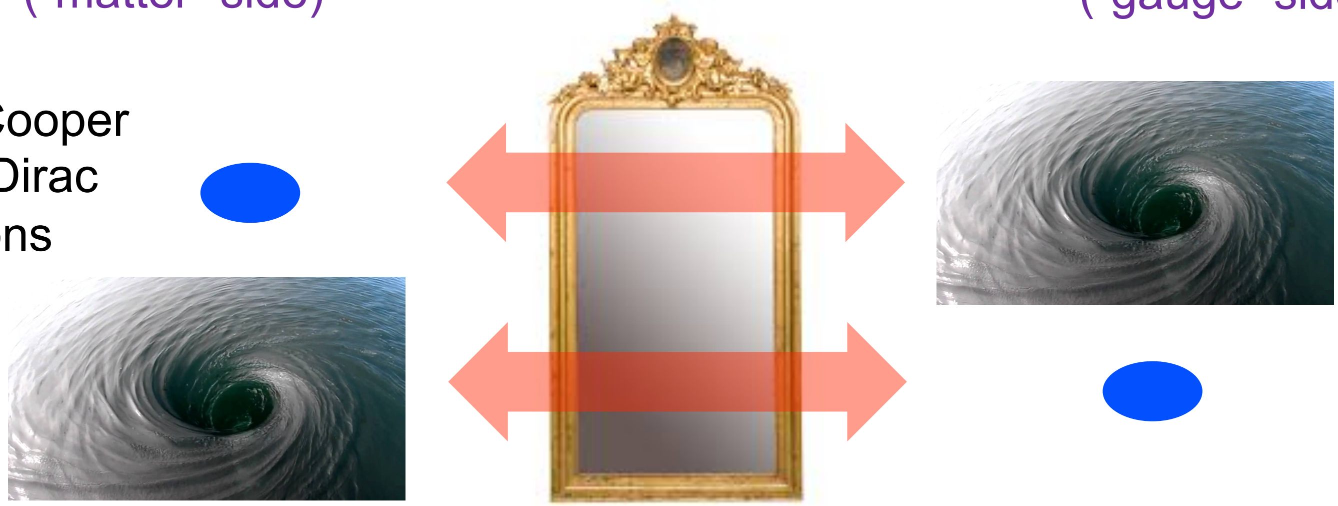

Summary

- At charge neutrality (Dirac point), the surface of 3D topological insulators can exhibit a **semimetal-superconductor quantum critical point** where gapless Dirac fermions and Cooper pairs interact strongly

- For a surface with one (three) Dirac cone(s), the QCP displays **emergent N=2 SUSY** of the Wess-Zumino (XYZ/$SQED_3$) type. Possible realization in Sb_2Te_3 (SmB_6) or other TI compounds?

- SUSY allows one to determine **exactly** certain response properties (optical conductivity, dynamical shear viscosity) of the QCP, despite strong correlations

- Realization of **mirror symmetry** in condensed matter: SUSY version of Son-Metlitski-Vishwanath-Senthil-… Dirac fermion/N_f=1 QED_3 duality