Fluid/Gravity Correspondence and Casadio-Fabbri-Mazzacurati solutions

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Strings at Dunes, IIP, Natal, **July 2016**

Outline

- Black strings hydrodynamics (Gregory-Laflamme $\Leftrightarrow$ Plateau-Rayleigh)
- Casadio-Fabbri-Mazzacurati CFM black strings
- Fluid/gravity correspondence
  - Kubo formula for fluid viscosity: CFM-AdS black branes
  - Shear viscosity-to-entropy density ratio: Kovtun-Son-Starinets
  - Theoretical prediction of the PPN parameter bound.

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Black strings

- “Low” energies: string action

  Callan, Friedan, Martinec, Perry, Nucl. Phys. B (1985).

- metric $g_{\mu\nu}$
  dilaton ϕ
  Maxwell field $F_{\mu\nu}$
  Kalb-Ramond $H_{\mu\nu\rho}$
- $H = dB - A \wedge F \quad \Rightarrow \quad dH = -F \wedge F$
- Action

$$S = \int d^D x \sqrt{-g}\, e^{-2\phi} \left[\Lambda + R + 4(\nabla\phi)^2 - F_{\mu\nu}F^{\mu\nu} - \frac{1}{12} H_{\mu\nu\rho}H^{\mu\nu\rho} \right]$$

e^{ϕ} is a coupling constant

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Black strings

- Equations of motion:

$$
\begin{aligned}
R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \phi - 2F_{\mu\lambda}F_\nu{}^\lambda - \frac{1}{4}H_{\mu\lambda\sigma}H_\nu{}^{\lambda\sigma} &= 0 \\
\nabla^\nu(e^{-2\phi}F_{\mu\nu}) + \frac{1}{12}e^{-2\phi}H_{\mu\nu\rho}F^{\nu\rho} &= 0 \\
\nabla^\mu(e^{-2\phi}H_{\mu\nu\rho}) &= 0 \\
4\nabla^2\phi - 4(\nabla\phi)^2 + \Lambda + R - F^2 - \frac{1}{12}H^2 &= 0
\end{aligned}
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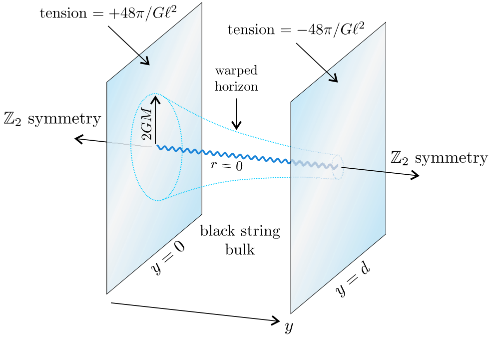
Black strings

Horowitz, Strominger, Nucl. Phys. B (1991)
Seahra, Clarkson, R. Maartens, *Phys. Rev. Lett.* (2005)
Chamblin, Reall, Hawking, *Phys. Rev. D* (2000)



- **Maartens**, LRR (2003),
  Casadio, PRD (2001)
  RdR, Hoff, PRD (2012) (including variable brane tension)
- Taylor expansion along the extra dimension y

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\begin{aligned}
g_{\mu\nu}^{\mathrm{BULK}} &= g_{\mu\nu} + \mathcal{L}_{\mathbf{n}} g_{\mu\nu}|_{y=0}\,|y| + \left(\mathcal{L}_{\mathbf{n}}\left(\mathcal{L}_{\mathbf{n}} g_{\mu\nu}\right)\right)|_{y=0}\,\frac{|y|^2}{2!} \\
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- **To probe** information about the bulk from the brane metric
- In Gaussian coordinates:

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\mathcal{L}_{\mathbf{n}} = \frac{\partial}{\partial\,(\text{extra}\ \ \text{dimension})}
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- Vacuum on the brane: $T_{\mu\nu} = 0$

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\begin{aligned}
g_{\mu\nu}(x^\mu, y) \quad = \quad & g_{\mu\nu} - \frac{1}{3}\kappa_5^2\lambda g_{\mu\nu}\,|y| \; + \left[-\mathcal{E}_{\mu\nu} + \left(\frac{1}{36}\kappa_5^4\lambda^2 - \frac{1}{6}\Lambda_5\right) g_{\mu\nu}\right] y^2 + \\
& + \left(\left(-\frac{193}{216}\lambda^3\kappa_5^6 - \frac{5}{18}\Lambda_5\kappa_5^2\lambda\right) g_{\mu\nu} + \frac{1}{6}\kappa_5^2\mathcal{E}_{\mu\nu} + \frac{1}{3}\kappa_5^2(\mathcal{E}_{\mu\nu} + R_{\mu\nu})\right) \frac{|y|^3}{3!} + \\
& + \left[\frac{1}{6}\Lambda_5\left(\left(R - \frac{1}{3}\Lambda_5 - \frac{1}{18}\lambda^2\kappa_5^4\right) + \frac{7}{324}\lambda^4\kappa_5^8\right) g_{\mu\nu} + \left(R - \Lambda_5 + \frac{19}{36}\lambda^2\kappa_5^4\right)\mathcal{E}_{\mu\nu}\right. \\
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- $\mathcal{E}^{\alpha\beta}$ is the electric part of the Weyl tensor.
- $\sqrt{g_{\theta\theta}(x^\mu, 0)}$: black holes horizon

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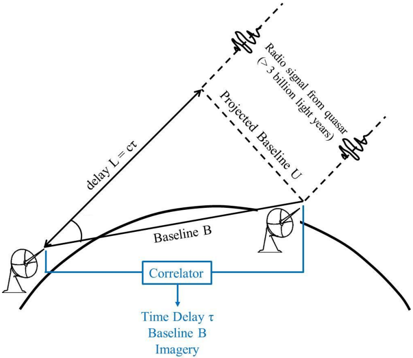
PPN parameter: experimental/observational bounds

C. Will, Living Rev. Rel. **9**, 3 (2006).

Very-long-baseline interferometry



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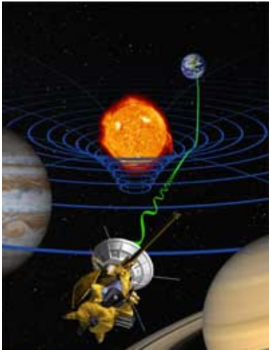
"*A light ray with passes the Sun at a distance d is deflected by an angle*

$$\Delta\theta = \frac{1+\beta}{2}\,\frac{4M_\odot}{d}\,\frac{1+\cos\Phi}{2}$$

where Φ is the angle between the Earth-Sun line and the incoming direction of the photon."

Cassini probe



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- **Casadio, Fabbri, Mazzacurati**, "New black holes in the brane world?," Phys. Rev. D **65** (2002) 084040.
- $ds^2 = -f(r)dt^2 + \frac{1}{A(r)}dr^2 + r^2\, d\Omega^2$:

  Why to take $f(r) = A(r)$?

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Black strings

5D Einstein equations
$\Downarrow$
Shiromizu-Sasaki-Maeda, PRD (2000)
$\Downarrow$
Effective 4D Einstein equations

$$\boxed{G_{\mu\nu} = 8\,\pi\,G\,T_{\mu\nu} - \frac{\Lambda_4}{2}\,g_{\mu\nu} + \frac{\kappa_5^4}{4}\left[\frac{g_{\mu\nu}}{2}\left(T^2 - T_{\alpha\beta}\,T^{\alpha\beta}\right) + T\,T_{\mu\nu} - T_{\mu\alpha}\,T^{\alpha}_{\ \nu}\right] - \mathcal{E}_{\mu\nu}}$$

$\kappa_5^2 = 8\,\pi\,G_5$
$G = \kappa_5^2\,\lambda/48\,\pi$ is the Newton constant
($\lambda \equiv$ brane tension)
$T_{\mu\nu}$: energy-momentum tensor of brane matter
$\mathcal{E}_{\mu\nu}$ is the Weyl tensor term.

Casadio-Fabbri-Mazzacurati (CFM) black strings

Solution I

- vacuum energy density = cosmological constant of our (4D) Universe

$$\boxed{ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \left(\frac{1 - \frac{3GM}{2c^2 r}}{\left(1 - \frac{2GM}{c^2 r}\right)\left(1 - \frac{GM}{2c^2 r}(4\beta - 1)\right)}\right) dr^2 + r^2 d\Omega^2}$$

$\boxed{\beta\text{: (PPN) }\textbf{post-Newtonian parameter}}$ $(|\beta| < 0.003)$.

- $\lim_{\beta\to 1}$ CFM = Schwarzschild.
- Hawking temperature

$$T_H = \frac{\sqrt{1 - 4(\beta - 1)}}{8\,\pi\,M}$$

- For $r = 0$ and (for $T_H \sim 0$) $r = \frac{3GM}{2c^2}$: (physical singularities) Kretschmann $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \to \infty$

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- GR $\mapsto$ **Newtonian potential** ($\approx$ weak field)
  $g_{00}^{(4D)} = -\left(1 + \frac{2GM}{R}\right)$.
- **Post-Newtonian** approximation:
  $g_{00}^{(4D)} = -\left(1 + \frac{2GM}{R} + (\beta - 1)\left(\frac{2GM}{R}\right)^2\right) + \cdots$

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  $g_{00}^{(4D)} = -\left(1 + \frac{2GM}{R}\right)$.
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Casadio-Fabbri-Mazzacurati (CFM) black strings

Solution II

- $$\boxed{ds^2 = -\left(1 - \frac{2GM}{c^2 r} + \frac{2G^2M^2}{c^4 r^2}(\beta - 1)\right) dt^2 + \left(\frac{1 - 3GM/2c^2 r}{\left(1 - \frac{2GM}{c^2 r}\right)\left(1 - \frac{GM}{2c^2 r}(4\beta - 1)\right)}\right) dr^2 + r^2 d\Omega^2}$$

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Classical Perturbation

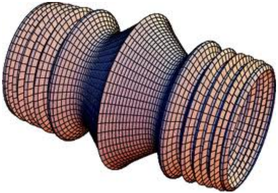
- Classical black string is unstable: (**Gregory, Laflamme**, *PRL* (1993))
- (at weak gravity:) corrections

$$h_{\mu\nu} = -\frac{2GM}{r}\left(1 + \frac{1}{3k^2r^2}\right)\delta_{\mu\nu}$$

- 5D Einstein equations:

$$\Delta h_{\mu\nu} + 2R_{\mu\lambda\nu\rho}h^{\lambda\rho} = 0$$

- Event horizon instability (Chamblin, Reall, Hawking *Phys. Rev. D* (2000))

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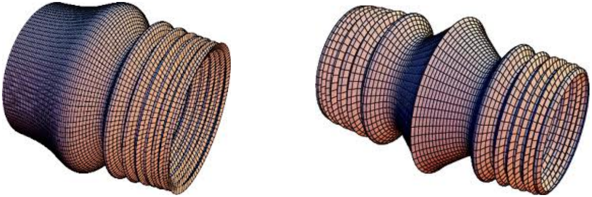

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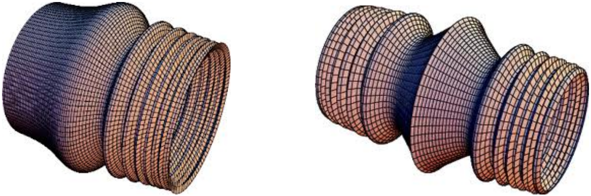

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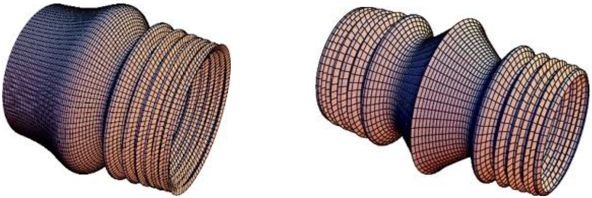

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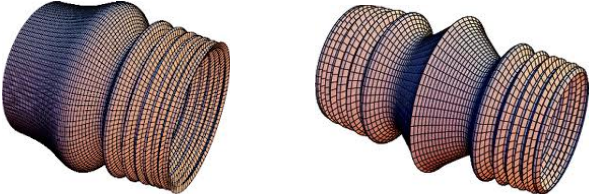
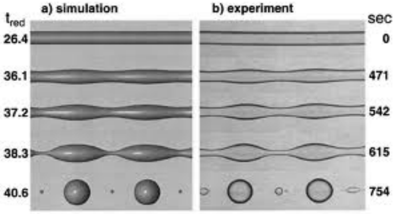


Black Strings (in the Kitchen)

Plateau-Rayleigh instability (1873): Jet of water pinches into drops **when** the wavelength is 3.18 times its diameter



Plateau-Rayleigh instability

What is the **final state** of a black string, after perturbations? It depends on the black string viscosity.



- **Lehner, Pretorius**, PRL (2010)
- **Wiseman**, Class. Quant. Grav. (2003)

Black string perturbations

Gregory-Laflamme instability

 $\longrightarrow$ perturbation $\rightarrow$ 

- **Lehner, Pretorius**, PRL (2010)

Final state of CFM (MGD) black strings

- **Kuerten, RdR**, Class. Quant. Grav. (2013)
  Casadio, Ovalle, RdR, Class. Quant. Grav. (2014)
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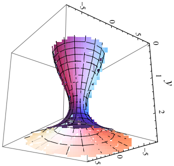

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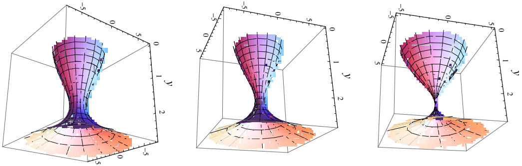

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Final state of CFM black strings

Transition regime occurs when $|\beta - 1| \lesssim 3 \times 10^{-1}$

$\Longleftrightarrow$

(preliminaries): **Casadio, Ovalle, RdR**, Class. Quant. Grav. (2014)

Fluid/Gravity Correspondence

- Black strings: temperature and entropy...
- ...and hydrodynamic features: viscosity, diffusion rates, diffusion constants and other transport coefficients.
- Bulk dynamics: specified by Einstein equations

$$R_{MN} - \frac{1}{2} R g_{MN} + \Lambda_5 g_{MN} = 0.$$

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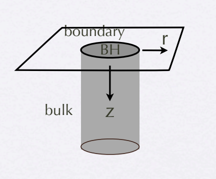

Bulk	Boundary
Collapse to black hole in gravity	thermalization in CFT
Stationary black hole	thermal equilibrium (at same T)
Quasinormal modes	approach to thermal equilibrium [Horowitz, Hubeny]
⋆ **Horizon response properties**	⋆ **transport coefficients in CFT** **[Kovtun, Son, Starinets]**
Long-wavelength, small frequency deformations	fluid flows
Einstein equations	relativistic Navier-Stokes equations (boundary conformal fluid).

KSS bound

- Bulk supergravity, $\mathcal{N} = 4$ supersymmetric $SU(N_c)$ Yang-Mills theory, in the regime $N_c \to \infty$ and large 't Hooft coupling $g^2 N_c$

  (**Buchel, Liu, Starinets**, Nucl. Phys. B (2005)).

$$\frac{\eta}{s} = \frac{1}{4\,\pi}\left[1 + \frac{135\,\zeta(3)}{8(2g^2 N_c)^{3/2}} + \cdots\right] ,$$

  ($\zeta(3)$ is the Apéry constant).

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PPN parameter: experimental/observational bounds

- CFM: Hawking temperature

$$T_H = \frac{\sqrt{1-4(\beta-1)}}{8\,\pi\,M} \geq 0$$

$\Rightarrow 1 < \beta < 1.25$ (Strongest theoretical bound).

- **Post-Newtonian** approximation: $g_{00}^{(4D)} = -\left(1 + \frac{2GM}{r} + (\beta-1)\left(\frac{2GM}{r}\right)^2\right)$
- Observational bound: $|\beta| \lesssim 1.003$
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Kubo formula

Green-Kubo formula

- Sources J^a, coupled to operators O^a $S \mapsto S + \int d^4x\, J_a(x)\, O^a(x)$

$$\langle O^a(x) \rangle = -\int dy\, G_R^{a|b}(x;y)\, J_b(y)\ ,$$

$G_R^{a|b}(x;y) = -i\theta(x^0 - y^0)\langle [O^a(x), O^b(y)] \rangle$ retarded Green function of O^a.

- interaction picture in QFT $\Rightarrow \delta\langle O^a(q) \rangle = -G_R^{a|b}(q)\, J_b(q)$

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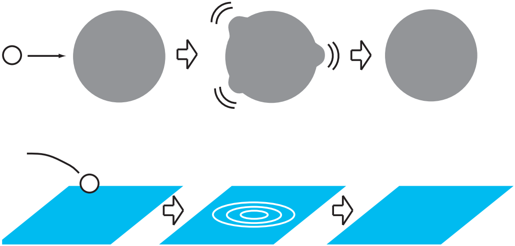

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$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \|h_{\mu\nu}\| \ll 1$	$\langle T^{\mu\nu} \rangle = \langle T^{\mu\nu}_{(0)} \rangle + \langle T^{\mu\nu}_{(1)} \rangle + \cdots$
Gravitational perturbations	Fluid energy-momentum tensor response
Horizon response properties	transport coefficients in CFT [Kovtun, Son, Starinets]
Long-wavelength, small frequency deformations	fluid flows
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(**Natsuume**, Lect. Notes Phys. **903** (2015))

Fluid response

Energy-momentum tensor

$$
\begin{aligned}
\langle T^{\mu\nu}(x)\rangle &= \langle T^{\mu\nu}\rangle_{h=0} - \tfrac{1}{2}\int d^4y\, G_{\mathrm{R}}^{\mu\nu|\rho\sigma}(x;y)\, h_{\rho\sigma}(y) \\
&\quad + \tfrac{1}{8}\int d^4y \int d^4z\, G_{\mathrm{R}}^{\mu\nu|\rho\sigma|\tau\zeta}(x;y,z)\, h_{\rho\sigma}(y)\, h_{\tau\zeta}(z) + \ldots \\
&\equiv \langle T^{\mu\nu}_{(0)}\rangle + \langle T^{\mu\nu}_{(1)}\rangle + \langle T^{\mu\nu}_{(2)}\rangle + \cdots ,
\end{aligned}
$$

$G_R^{\mu\nu|\cdots}$: **retarded *n*-point correlators**.

- Fluid response:
  – stress tensor conservation law $\nabla_\mu T^{\mu\nu} = 0$;
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Fluid response

Energy-momentum tensor

1^{st} order formalism $\Rightarrow$ 0^{th} order in derivatives:

$$T^{\mu\nu}_{(0)} = (\epsilon + P)\, u^{\mu}\, u^{\nu} + P\, \bar{g}^{\mu\nu} \ ,$$

u^{μ}: fluid 4-velocity;
ϵ: energy density;
P: pressure
$\bar{g}_{\mu\nu}$: 4D boundary unperturbed metric

Fluid response

Energy-momentum tensor: 1st-order

- **Son, Starinets**, Ann. Rev. Nucl. Part. Sci. (2007).

$$\langle T^{\mu\nu}(x)\rangle \sim \int dy\, G_R^{\mu\nu|\alpha\beta}(x;y)\, h_{\alpha\beta}(y)\ ,$$

for retarded Green function $G_R^{\mu\nu|\alpha\beta} = \langle T^{\mu\nu}(x)\, T^{\alpha\beta}(y)\rangle$.

$\Rightarrow$ $1^{\rm th}$ order in derivatives: dissipative terms, shear and bulk viscosities.

$$T^{\mu\nu}_{(1)} = -P^{\mu\alpha} P^{\nu\beta} \left[\eta \left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{2}{3}\, \bar{g}_{\alpha\beta}\, \nabla_\lambda u^\lambda\right) + \zeta\, \bar{g}_{\alpha\beta} \nabla_\lambda u^\lambda\right]\ ,$$

η: shear viscosity,
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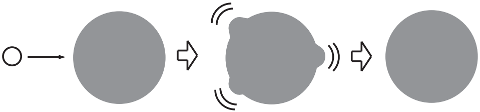

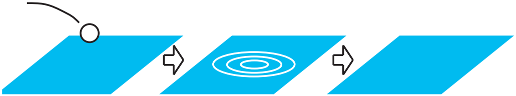

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Gravitational perturbations	Fluid energy-momentum tensor response

For

$$\boxed{g^{(0)}_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & h_{xy}(t) & 0 \\ 0 & h_{xy}(t) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

Fluid response

Energy-momentum tensor: 1st-order

- Fluctuations around thermal equilibrium are small $\Rightarrow$ the fluid has uniform temperature $\mathcal{T}(x^\mu) = \mathcal{T}_0$
- Kubo formula derivation (rest frame $u^\mu = (1, u^i = 0)$.)
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- **Emparan, Reall**, Living Rev. Rel. **11** (2008) 6

  "*It is expected that the Schwarzschild-AdS black hole is the unique, static, asymptotically AdS, black-hole solution of* **vacuum** *gravity with a negative cosmological constant, but this has not been proven.*"

CFM black branes

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$$\boxed{ds^2 = r^2\left(-f(r)dt^2 + \sum_{i=1}^{3}(dx^i)^2\right) + \frac{dr^2}{r^2 f(r)}} \qquad f(r) = 1 - \frac{r_+^4}{r^4} \qquad r_+ \text{ is the event horizon.}$$

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KSS bound: a controller Sheriff

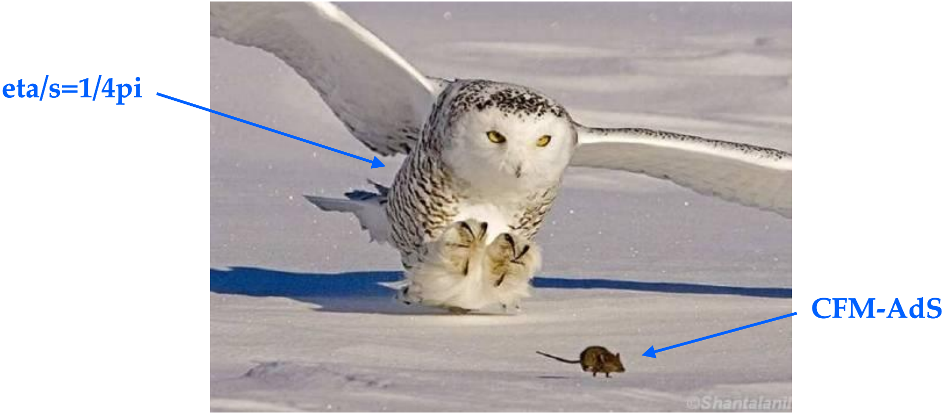

Figure: Shear viscosity-to-entropy density ratio $\times$ CFM-AdS solution.

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- $\Rightarrow$ mode equation $\partial_u \left(\sqrt{-g}\, g^{uu}\, \partial_u \phi\right) + \sqrt{-g}\, g^{\mu\nu}\, \partial_\mu \partial_\nu \phi = 0$
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- $\frac{d^2\Phi}{du^2} + \frac{V}{u}\frac{d\Phi}{du} + \left(1 - \frac{2\,\bar{M}}{R}\,u\right)\omega^2\,\Phi = 0$, where V is some potential.
- Green function $G_R(\omega, \vec{0}; \beta) = -\sqrt{-g}\,g^{uu}\,\Phi^*\,\frac{d\Phi}{du}\Big|_{u\to 0}$
- KSS bound: $\frac{\eta(\beta)}{s(\beta)} = -\frac{1}{s(\beta)}\lim_{\omega\to 0}\frac{\Im\,G_R(\omega,k=0;\beta)}{\omega} \geq \frac{1}{4\,\pi}$.

KSS bound

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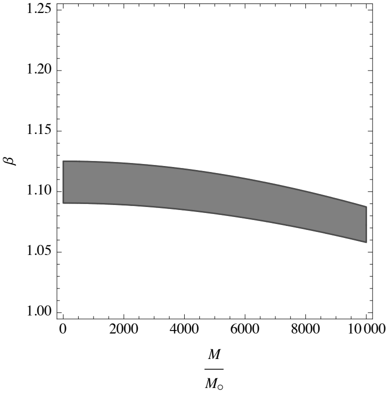

Figure: PPN parameter $\beta \times$ mass M, for CFM-AdS black branes: 1st-order corrections.

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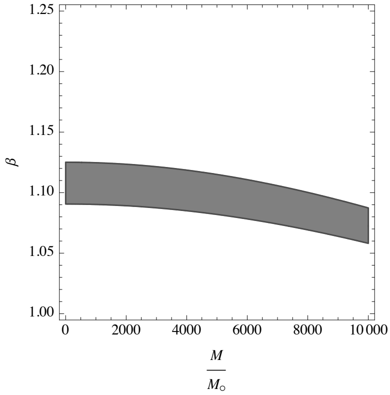

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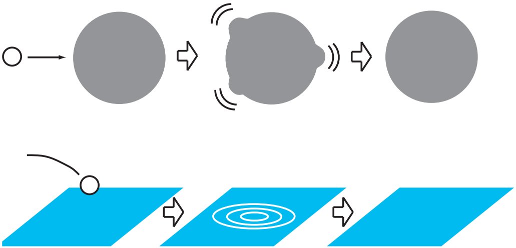
Kubo formula: 2nd order improvements



Casadio, Cavalcanti, RdR [arXiv:1601.03222 [hep-th]]

Bulk	Boundary
$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \|h_{\mu\nu}\| \ll 1$	$\langle T^{\mu\nu} \rangle = \langle T^{\mu\nu}_{(0)} \rangle + \langle T^{\mu\nu}_{(1)} \rangle + \langle T^{\mu\nu}_{(2)} \rangle \cdots$
Gravitational perturbations	Fluid energy-momentum tensor response

2nd order improvements

- **Bhattacharyya, Hubeny, Minwalla, Rangamani**, JHEP (2008).
  Arnold, Vaman, Wu, Xiao, JHEP (2011).
  Bu, Lublinsky, JHEP (2014).
  Grozdanov, Starinets, JHEP (2015).
- 2$^{\text{nd}}$-order (dissipative) stress-energy tensor

$$
\begin{aligned}
T^{\mu\nu}_{(2)} \quad = \quad & \eta\,\tau_\Pi \left[u^{\langle\rho}\nabla_\rho\sigma^{\mu\nu\rangle} + \frac{1}{2}\sigma^{\mu\nu}(\nabla\cdot u) \right] + \kappa \left(R^{\langle\mu\nu\rangle} - 2u_\rho u_\tau R^{\rho\langle\mu\nu\rangle\tau} \right) \\
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  κ: contributes to the **2-point Green's function**,
  $\sigma^{\alpha\beta} = P^{\alpha\mu}P^{\beta\nu}\nabla_{(\mu}u_{\nu)} - \frac{2}{3}P^{\alpha\beta}P^{\mu\nu}\nabla_\mu u_\nu$: **shear tensor**,
  $\Omega^{\alpha\beta} = P^{\alpha\mu}P^{\beta\nu}\nabla_{[\mu}u_{\nu]}$: **vorticity tensor**.
- How to calculate the (further) transport coefficients?
  (**Arnold, Vaman, Wu, Xiao**, JHEP (2011).
  Critelli, Finazzo, Zaniboni, J. Noronha, Phys. Rev. D (2014)).
  For example,

$$
\lambda_3 = -4 \lim_{\substack{k_1\to 0\\ k_2\to 0}} \frac{\partial}{\partial_{k_1}}\frac{\partial}{\partial_{k_2}} \lim_{\substack{\omega_1\to 0\\ \omega_2\to 0}} G^{xy|0x|0y}\ .
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KSS bound: 2nd order improvements

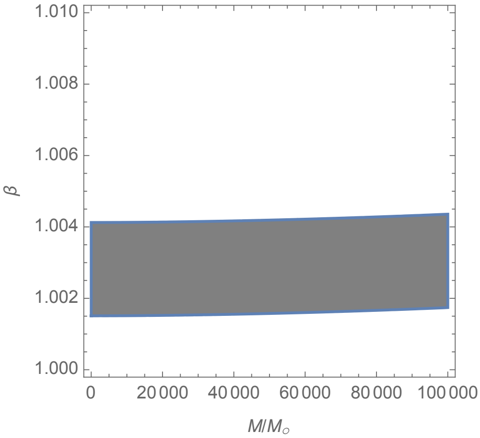

Figure: PPN parameter $\beta \times$ mass M, for CFM-AdS black branes: 2nd-order corrections.

Final remarks

- KSS bound $\Rightarrow$ PPN parameter bound, from fluid/gravity.
- KSS bound $\frac{\eta}{s} \geq \frac{1}{4\pi}$: shortcut/laboratory for experimental/observational quantities. (optimism/realism)
- Other black string/black brane solutions:
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Final remarks

1st claim proved!

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Final remarks: 2nd claim ...proved (?)

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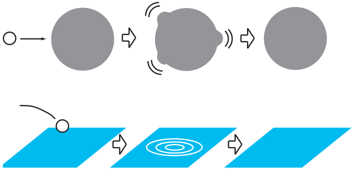

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CFM-AdS is (even more effectively) the Schwarzschild-AdS **if** KSS bound is imposed!

Final remarks

Thanks

2nd order improvements

- Arnold, Vaman, Wu, Xiao, JHEP (2011).

$$\begin{aligned}
\kappa &= \frac{N_c^2\,\mathcal{T}^2}{8}\,(1-10\,\gamma) \\
\tau_\Pi &= \frac{2-\ln 2}{2\pi\mathcal{T}} + \frac{375\gamma}{4\pi\mathcal{T}} + \ldots \\
\lambda_1 &= \frac{N_c^2\,\mathcal{T}^2}{16}\,(1+350\,\gamma) \\
\lambda_2 &= -\frac{N_c^2\,\mathcal{T}^2}{16}\,[2\,\ln 2 + 5\,(97+54\,\ln 2)\,\gamma + \ldots] \\
\lambda_3 &= \frac{25\,N_c^2\,\mathcal{T}^2}{2}\,\gamma + \ldots
\end{aligned}$$

where $\gamma = (g^2\,N_c)^{-3/2}\,\zeta(3)/8$.