Higher-derivative massive gravity on a two-dimensional brane in 4D Minkowski space

Francisco A. Brito

(UFCG)

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International Institute of Physics

UFRN-Natal

Introduction

1- Why low dimensional gravity ?

2- Why higher derivative gravity ?

3- Why massive gravity ?

4- Why localization of gravity on a 2-brane ?

Introduction

1- We shall see that these points are indeed connected with each other through a special mechanism of gravity localization called DGP (Dvali-Gabadadze-Porrati) scenario

2- Furthermore, the localization of gravity on a 2-brane is physically well-justified since we can see our real 3+1-dimensional universe as a stack of 2+1-dimensional branes

Gravity flutuations

$$\eta_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}$$

RS scenario: zero mode

$$H\psi = m^2\psi$$

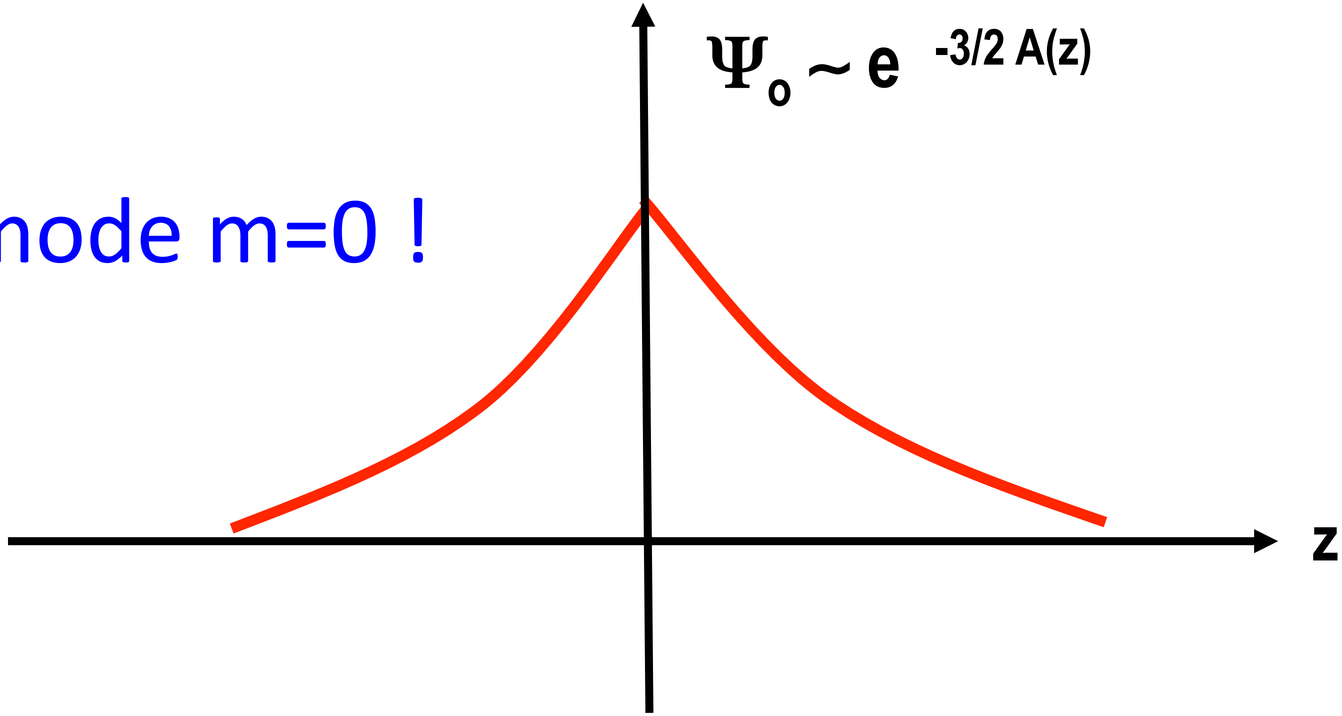
$$H = Q^\dagger Q$$

$$Q = \partial_z + \frac{3}{2}\partial_z A(z)$$

$$Q\psi_0 = 0 \longrightarrow \psi_0(z) = \exp\left(-\frac{3}{2}A(z)\right)$$

RS scenario: finite volume

Zero mode m=0 !



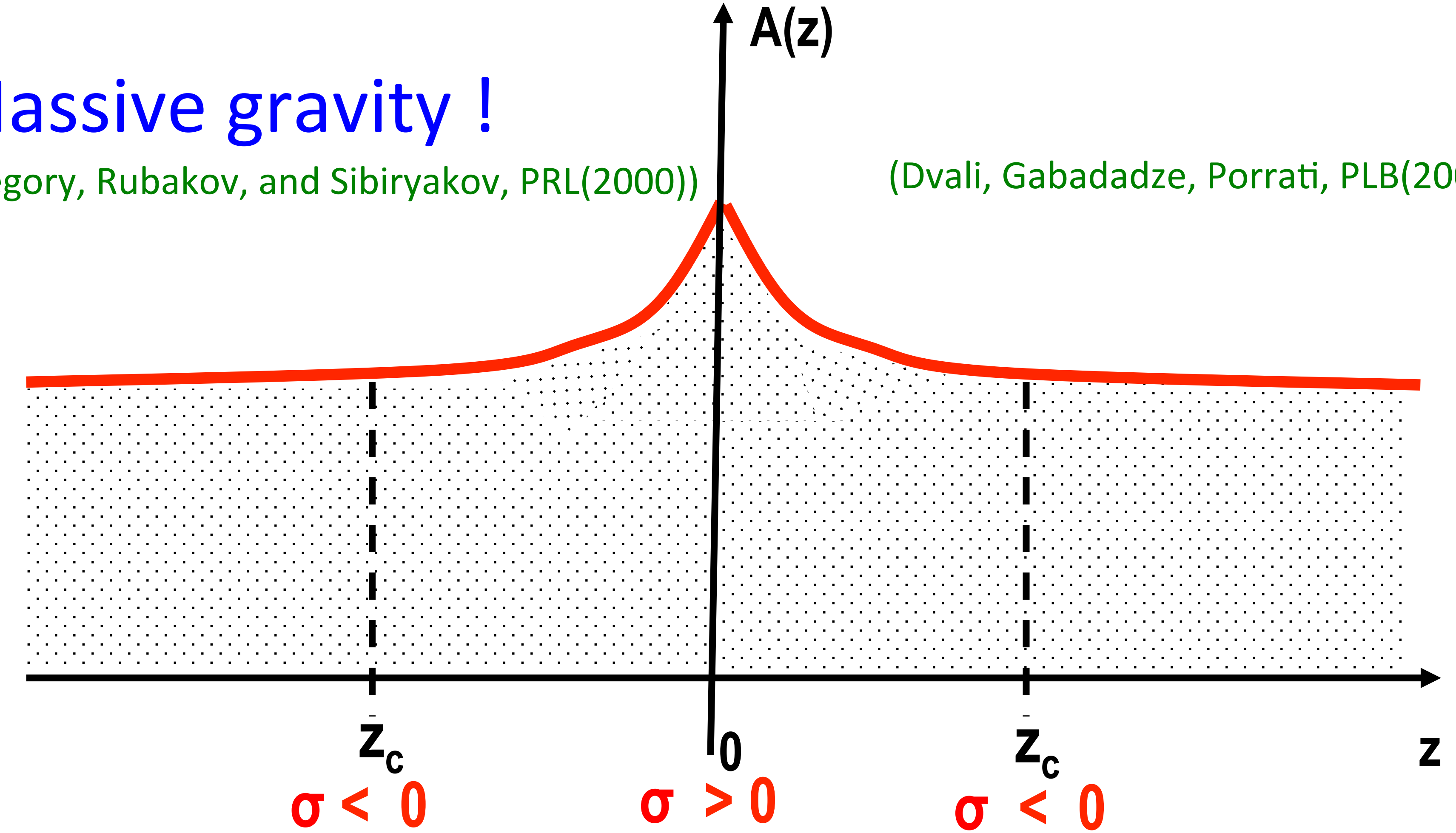
(Randall-Sundrum, PRL(1999))

GRS/DGP scenario: infinite volume

Massive gravity !

(Gregory, Rubakov, and Sibiryakov, PRL(2000))

(Dvali, Gabadadze, Porrati, PLB(2000))



Newtonian law between two unit masses on the brane

$$V(r) = \frac{M_5^{-3}}{4\, r\, f(k)\, k} \left[\sin\left(\frac{1}{2}\, \frac{f(k)\, r}{k}\right) \mathrm{Ci}\left(\frac{1}{2}\, \frac{f(k)\, r}{k}\right) - \cos\left(\frac{1}{2}\, \frac{f(k)\, r}{k}\right) \mathrm{Si}\left(\frac{1}{2}\, \frac{f(k)\, r}{k}\right) \right]$$

The crossover scale

Brito, Losano, Fonseca, JCAP(2012)

$$r_c = k/f(k) \qquad f\,(k) = \sqrt{\Lambda\, k^2 + 1}$$

Large *crossover* for *small cosmological const.*

$$r_c = 1/\sqrt{\Lambda} \qquad \Lambda k^2 >> 1$$

Cosmological horizon !

``Small'' distance (r<<rc)

$$V(r) \sim \frac{M_5^{-3}\pi}{8\, r\, f(k)\, k} \sim M_5^{-3}\frac{r_c}{k^2 r}$$

Large distance (r>>rc)

$$V(r) \sim \frac{M_5^{-3}}{2\, f(k)^2\, r^2} \sim M_5^{-3}\frac{r_c^2}{k^2 r^2}$$

Induced gravity on a 2-brane

We inspire ourselves with the five-dimensional *DGP scenario* to extend it to an analogous set up in four-dimensions

General Framework

$$S = S_{(D)} + S_{(D-1)}$$

$$S_{(D)} = \frac{M_D^2}{2} \int d^D x \sqrt{|g|} R^{(D)}$$

$$S_{(D-1)} = \frac{M_{D-1}}{2} \int d^{D-1} x \sqrt{|q|} R^{(D-1)}$$

$$q_{\mu\nu}(x) \equiv g_{\mu\nu}(x, w = 0)$$

$$m = \frac{M_{(D)}^2}{M_{(D-1)}} \equiv \frac{1}{r_c}$$

Dvali,Gabadadze & Porrati,PLB(2000)

Brito, Bazeia & Costa, PLB(2011)
Peng & Stojkovic, PRD(2014)

Brito, Bazeia & Costa, PLB (2015)

q is the induced metric on the brane and ***rc*** is the 'crossover' scale

Induced gravity on a 2-brane

We inspire ourselves with the five-dimensional *DGP scenario* to extend it to an analogous set up in four-dimensions

$$S_{(4)} = \frac{M_4^2}{2}\int d^4x\sqrt{|g|}R^{(4)}$$

$$S_{(3)} = \frac{M_3}{2}\int d^3x\sqrt{|q|}(\eta R^{(3)} + \gamma(R^{(3)})^2 + \delta R^{(3)\,\mu\nu}R^{(3)}{}_{\mu\nu})$$

$$m = \frac{M_{(4)}^2}{M_{(3)}} \equiv \frac{1}{r_c}$$

$$S = S_{(4)} + S_{(3)}$$

Higher derivative terms !

Brito, Bazeia & Costa, PLB (2015)

Induced gravity on a 2-brane

The particle content of this theory is obtained by expanding the total action to linear order around the flat space:

$$g_{ab} = \eta_{ab} + H_{ab};$$

$$q_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad N_\mu = n_\mu, \qquad N = 1 + n.$$

$$H_{\mu\nu} = h_{\mu\nu}, \qquad H_{\mu 4} = h_\mu, \qquad H_{44} = 2n.$$

Brito, Bazeia & Costa, PLB (2015)

Induced gravity on a 2-brane

After integrating out the bulk we obtain an effective 3D Lagrangian: $\mathcal{L}_{Grav} = \frac{1}{2} h_{\mu\nu} O^{\mu\nu,\alpha\beta} h_{\alpha\beta}$

In momentum space we have the operator:

$$O = \left[\delta \frac{M_3}{4} k^4 + \eta \frac{M_3}{4} k^2 - M_3 m k\right] P^{(2)} - M_3 m k P^{(1)}$$

$$+ \left[\left(\frac{8\gamma + 3\delta}{4}\right) M_3 k^4 - \eta \frac{M_3}{4} k^2 + M_3 m k\right] P^{(0-s)}$$

$$+ \sqrt{2} M_3 m k (P^{(0-sw)} + P^{(0-ws)})$$

Brito, Bazeia & Costa, PLB (2015)

Induced gravity on a 2-brane

After integrating out the bulk we obtain an effective 3D Lagrangian:

$$\mathcal{L}_{Grav} = \frac{1}{2} h_{\mu\nu} O^{\mu\nu,\alpha\beta} h_{\alpha\beta}$$

The operator *O* is written in terms of the Barnes-Rivers operators $P^{(2)}$,$P^{(1)}$,$P^{(0\text{-}s)}$, … which are spin projection operators that in three spacetime dimensions form a complete set and are defined as:

Brito, Bazeia & Costa, PLB (2015)

Induced gravity on a 2-brane

$$P^{(2)}_{\mu\nu,\kappa\lambda} = \frac{1}{2}(\theta_{\mu\kappa}\theta_{\nu\lambda} + \theta_{\mu\lambda}\theta_{\nu\kappa} - \theta_{\mu\nu}\theta_{\kappa\lambda}),$$

$$P^{(1)}_{\mu\nu,\kappa\lambda} = \frac{1}{2}(\theta_{\mu\kappa}\omega_{\nu\lambda} + \theta_{\mu\lambda}\omega_{\nu\kappa} + \theta_{\mu\nu}\omega_{\kappa\lambda}),$$

$$P^{(0-s)}_{\mu\nu,\kappa\lambda} = \frac{1}{2}\theta_{\mu\nu}\theta_{\kappa\lambda}, \quad P^{(0-w)}_{\mu\nu,\kappa\lambda} = \frac{1}{2}\omega_{\mu\nu}\omega_{\kappa\lambda},$$

$$P^{(0-sw)}_{\mu\nu,\kappa\lambda} = \frac{1}{\sqrt{2}}\theta_{\mu\nu}\omega_{\kappa\lambda}, \qquad P^{(0-ws)}_{\mu\nu,\kappa\lambda} = \frac{1}{\sqrt{2}}\omega_{\mu\nu}\theta_{\kappa\lambda},$$

$$\theta_{\mu\kappa} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}, \qquad \omega_{\mu\nu} = \frac{k_\mu k_\nu}{k^2},$$

transverse and the longitudinal operators

1. The propagator structure of the full action on the 2-brane

$$O^{-1} = \frac{1}{\delta \frac{M_3}{4} k^4 + \eta \frac{M_3}{4} k^2 - M_3 m k} P^{(2)} - \frac{1}{M_3 m k} P^{(1)}$$

$$+ \frac{(8\gamma + 3\delta) M_3 k^4 - \eta M_3 k^2 - 4 M_3 m k}{8 M_3^2 m^2 k^2} P^{(0-w)}$$

$$+ \frac{4\sqrt{2} M_3 m k (P^{(0-sw)} + P^{(0-ws)})}{8 M_3^2 m^2 k^2}.$$

1. The propagator structure of the full action on the 2-brane

The potential along the 2-brane is given in by

$$V(r) \sim \int \frac{d^3\mathbf{k}}{2\pi^3} O^{-1} e^{i\mathbf{k}\cdot\mathbf{r}}$$

Taking only the spin-2 modes gives

$$V(r) \sim \int \frac{d^3\mathbf{k}}{2\pi^3} \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\delta \frac{M_3}{4}\mathbf{k}^4 + \eta \frac{M_3}{4}\mathbf{k}^2 - M_3 m \mathbf{k}}$$

Here $\mathbf{k}^2$ denotes the square of an Euclidean three-momentum.

1. The propagator structure of the full action on the 2-brane

The `cascading gravity'

According to intermediate scales of energy or (distance *r*) we may find the following cascading 4D→3D →2D behavior:

$$V(r) \sim \int \frac{d^3\mathbf{k}}{2\pi^3} \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\delta \frac{M_3}{4}\mathbf{k}^4 + \eta \frac{M_3}{4}\mathbf{k}^2 - M_3 m \mathbf{k}}$$

$$V(r) \sim \frac{1}{r} \qquad \longrightarrow \qquad \ln(r) \qquad \longrightarrow \qquad r$$

(IR) k dominance → $\mathbf{k}^2$ dominance → $\mathbf{k}^4$ dominance (UV)

1. The propagator structure of the full action on the 2-brane

The `vanishing dimension'

This is also in the direction of the recently proposed "vanishing dimension" scenario where at high energy (or short scales) the physics appears to be lower dimensional !

Stojkovic, MPLA, (2013)
Horava, PRD (2009)

$$V(r) \sim \frac{1}{r} \qquad \longrightarrow \qquad \ln(r) \qquad \longrightarrow \qquad r$$

(IR) **k** dominance → $\mathbf{k}^2$ dominance → $\mathbf{k}^4$ dominance (UV)

2. Graviton propagator with generalized mass term

$$m^2 \rightarrow m^2(\Box).$$

2. Graviton propagator with generalized mass term

$$O^{-1} = \frac{1}{\delta \frac{M_3}{4} k^4 + \eta \frac{M_3}{4} k^2 - M_3 m^2(\Box)} P^{(2)} - \frac{2}{2 M_3 m^2(\Box)} P^{(1)}$$

$$+ \frac{(8\gamma + 3\delta) M_3 k^4 - \eta M_3 k^2 - 4 M_3 m^2(\Box)}{8 M_3^2 (m^2(\Box))^2} P^{(0-w)}$$

$$+ \frac{4\sqrt{2} M_3 m^2(\Box)(P^{(0-sw)} + P^{(0-ws)})}{8 M_3^2 (m^2(\Box))^2}.$$

2. Graviton propagator with generalized mass term

$$m^2(\Box) = L^{2(\alpha-1)}\Box^{\alpha},$$

with *L* being a length scale and α being a constant

2. Graviton propagator with generalized mass term

$$m^2(\Box) = L^{2(\alpha-1)}\Box^{\alpha},$$

α = 1/2 recovers the *DGP model*

$$m^2(\Box) = mk$$ *(m=1/L)*

2. Graviton propagator with generalized mass term

$$m^2(\Box) = L^{2(\alpha-1)}\Box^{\alpha}$$

$\alpha = 0$ $(m^2(\Box) = 1/L^2)$, $\eta = -1$, $\delta = 1/M^2$, $\gamma = -3/8M^2$
gives the *Pauli-Fierz* mass term added to the *new massive gravity* in three dimensions Bergshoeff, Hohm & Townsend, PRL (2009)

$$O^{-1} = \frac{4}{M_3}\frac{1}{\left(\frac{1}{M^2}k^4 - k^2 - \frac{4}{L^2}\right)}P^{(2)} - \frac{L^2}{M_3}P^{(1)}$$

$$-\frac{L^4}{8M_3}\left(k^2 + \frac{4}{L^2}\right)P^{(0-w)} + \frac{\sqrt{2}L^2}{2M_3}(P^{(0-sw)} + P^{(0-ws)}).$$

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- There are massive poles in the sector of spin-2 graviton modes.

- One is the unitary mode of positive mass and positive norm.

- The other is the ghost of tachyonic mass and negative norm.

So both unitarity and causality are violated because of the existence of this tachyonic ghost !

2. Graviton propagator with generalized mass term

$$m^2(\Box) = L^{2(\alpha-1)}\Box^{\alpha}; \qquad \alpha \to 1 \qquad m^2(\Box) \approx \Box = k^2.$$

$\delta = 1/M^2$, $\gamma = -3/8M^2$ gives

Bazeia, Brito & Costa, PLB (2015)

$$O^{-1} = \frac{4M^2}{M_3}\frac{1}{[k^2-(4-\eta)M^2]\,k^2}P^{(2)} - \frac{1}{M_3k^2}P^{(1)}$$

$$-\frac{(4-\eta)}{M_3k^2}P^{(0-w)} + \frac{\sqrt{2}}{2M_3k^2}(P^{(0-sw)} + P^{(0-ws)}).$$

The condition for the absence of tachyons is satisfied for both 'right' and 'wrong' η signs.

2. Graviton propagator with generalized mass term

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$$O^{-1} = \frac{4}{(4-\eta)M_3}\frac{m_2^2}{\left(k^2 - m_2^2\right)k^2}P^{(2)} - \frac{1}{M_3k^2}P^{(1)}$$

$$-\frac{(4-\eta)}{M_3k^2}P^{(0-w)} + \frac{\sqrt{2}}{2M_3k^2}\left(P^{(0-sw)} + P^{(0-ws)}\right).$$

Here $m_2^2 = (4-\eta)M^2$ is the massive spin-2 mode.

Conclusions

1- We consider the problem of localizing gravity in a 2-brane embedded in a 4D Minkowski space to address induction of high derivative massive gravity.

2- The structure of propagators shows a well-behaved higher-derivative massive gravity induced on the 2-brane.

3- We consider a special case in the generalized mass term of the graviton propagator, which ends up with a consistent higher order gravity.

4- The condition for the absence of tachyons is satisfied for both ‘right’ and ‘wrong’ signs of the Einstein-Hilbert term on the 2-brane.

5- By properly choosing the parameters of the theory one can find the Pauli-Fierz mass term added to the *new massive gravity* and recover the low dimensional DGP model.

6- It would be interesting to consider this study to pursue possible new aspects of the following cascading gravity: 4D → 3D → 2D

Thanks a lot !