Soft Pomeron in Holographic QCD

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Alfonso Ballon Bayona
IFT - UNESP

In collaboration with Robert Carcassés Quevedo, Miguel Costa and Marko Djurić (University of Porto)

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Summary

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- String approach : BPST Pomeron
- Improved Holographic QCD (IHQCD)
- 5-d Regge theory in IHQCD and the Soft Pomeron
- Conclusions

4-d Regge theory and the Soft Pomeron

Consider the scattering process $1+2 \to 3+4$. The Mandelstam variables associated with this process are

$$s = (k_1 + k_2)^2 \quad , \quad t = (k_1 - k_3)^2 \quad , \quad u = (k_1 - k_4)^2 \,. \tag{1}$$

They satisfy the identity $s + t + u = \sum_{i=1}^{4} m_i^2$.

Since the u variable can be written in terms of s and t this process is described by the scattering amplitude $\mathcal{A}(s,t)$

When the masses are equal, i.e. $m_i = m$, the Mandelstam variables take the form

$$s = 4(|\vec{k}|^2 + m^2) \quad , \quad t = -2|\vec{k}|^2(1 - \cos\theta_s)\,, \tag{2}$$

where $|\vec{k}|$ and θ_s are the momentum and scattering angle in the center of mass frame.

Crossing symmetry implies that

$$\mathcal{A}_{1+2\to 3+4}(s,t) = \mathcal{A}_{1+\bar{3}\to\bar{2}+4}(t,s) \tag{3}$$

Partial-wave amplitudes

At fixed s the momentum transfer t varies linearly with

$$z_s = cos\theta_s \,. \tag{4}$$

Explicitly we have that

$$t = \frac{1}{2}(s - 4m^2)(z_s - 1)\,. \tag{5}$$

The amplitude can be expanded in the partial-wave series

$$\mathcal{A}(s,t) = 16\pi \sum_{\ell=0}^{\infty}(2\ell+1)\mathcal{A}_\ell(s)P_\ell(z_s)\,, \tag{6}$$

where $P_\ell(z)$ is the Legendre polynomial of the first kind, of order ℓ. The quantities $\mathcal{A}_\ell(s)$ are the so called partial-wave amplitudes.

Similarly, in the t-channel t we can also expand the amplitude as

$$\mathcal{A}(s,t) = 16\pi \sum_{J=0}^{\infty}(2J+1)\mathcal{A}_J(t)P_J(z_t)\,, \tag{7}$$

where

$$z_t = \cos\theta_t = 1 + \frac{2s}{t-4m^2}\,. \tag{8}$$

The contribution at each J can be interpreted in terms of the t-channel exchange of a resonance of spin J.

In the Regge limit, i.e. $s \to \infty$ with fixed t, each spin J resonance would contribute to the amplitude as

$$A(s,t) \sim f_J(t) s^J \,. \tag{9}$$

Using the optical theorem we find that the total cross section for the scattering process $1 + 2 \to X$ takes the form

$$\sigma^{\rm Tot}(s) \sim s^{J-1} \,. \tag{10}$$

However, experimental data for $\bar{p}p$ total cross section suggest the following behaviour

$$\sigma^{\rm Tot}(s) \sim s^{J_0 - 1} \quad , \quad J_0 \approx 1.08 \,. \tag{11}$$

The interpretation is the following : The spin J resonances contribute simultaneously to the scattering amplitude in groups of **Regge trajectories**. Regge theory is the mathematical tool to add all these families of resonances.

The Sommerfeld-Watson transform

Extend J to the complex plane and define functions $A(J,t)$ such that

$$\mathcal{A}(J,t) = \mathcal{A}_J(t) \quad J = 0, 1, 2, \dots \tag{12}$$

It is convenient to separate the contributions from even and odd spin defining $A^{\pm}(J,t)$.

The partial-wave series can be rewritten as the contour integral

$$\mathcal{A}^{\pm}(s,t) = 8\pi i \int_{\mathcal{C}} dJ(2J+1)\mathcal{A}^{\pm}(J,t) \frac{P_J(-z_t) \pm P_J(z_t)}{\sin(\pi J)} \,. \tag{13}$$

The contour $\mathcal{C}$ is described in Figure 1.

It surrounds the poles at non-negative integer J.

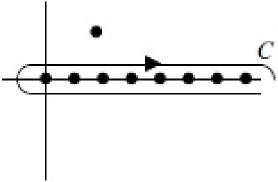

Figure 1 : Contour $\mathcal{C}$ in the complex J plane.

The contour $\mathcal{C}$ can be deformed as shown in Figure 2.

As the contour passes the poles we must pick up their residues.

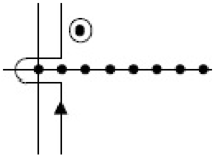

Figure 2 : Deforming the contour $\mathcal{C}$.

Pushing the left side of the contour to $\mathrm{Re}\, J = -1/2$ and taking the Regge limit we find that the amplitude is described entirely by the Regge poles.

In particular, the amplitude for the even sector takes the form

$$\mathcal{A}^{+}(s,t) = \sum_{n} \Pi(j_n(t))\, s^{j_n(t)}\,. \tag{14}$$

where $j_n(t)$ are the Regge poles representing families of Regge resonances (Regge trajectories). The first Regge pole is known as the **Soft Pomeron**.

String approach : BPST Pomeron

Brower, Polchinski, Strassler and Tan 2006

Consider closed strings in $AdS_5 \times S^5$ with the AdS_5 metric given by

$$ds^2 = e^{2A_0(z)} \left(dz^2 + dX_{1,3}^2\right) \quad , \quad A_0(z) = \log(\frac{L}{z}) \, , \qquad (15)$$

where L is the AdS radius.

Consider $2 \to 2$ scattering of scalars in 4-d. The 4-d scattering amplitude $\mathcal{A}(s,t)$ is obtained from closed string scattering in $AdS_5 \times S_5$.

When $\sqrt{\lambda} \gg \log s$, a local approximation can be used to obtain

$$\mathcal{A}(s,t) = V \int dz \sqrt{-g} \phi_1(z) \phi_3(z) \Pi(\alpha' \tilde{t}) (\alpha' \tilde{s})^{2 + \frac{\alpha'}{2} \tilde{t}} \phi_2(z) \phi_4(z) \, , \qquad (16)$$

where g_{mn} is the AdS_5 metric (15) and

$$V = \int d^4 x \, e^{i x \cdot (k_1 + k_2 + k_3 + k_4)} = (2\pi)^4 \delta^4 (\sum_i k_i) \, ,$$

$$\tilde{s} = e^{-2A_0} s \, , \; \tilde{t} = e^{-2A_0} t \quad , \quad \Pi(\alpha' \tilde{t}) = 2\pi \frac{\Gamma(-\frac{\alpha' \tilde{t}}{4})}{\Gamma(1 + \frac{\alpha' \tilde{t}}{4})} e^{-i\pi \frac{\alpha' \tilde{t}}{4}} \qquad (17)$$

When $\sqrt{\lambda} \sim \log s$ the local approximation is not valid and a formal string calculation leads to

$$\begin{aligned} \mathcal{A}(s,t) &= \int d^4x\, dz \sqrt{-g}\, e^{ix.(k_1+k_3)} \phi_1(z)\phi_3(z) \Pi(\alpha' \Delta_2)(\alpha' \tilde{s})^{2+\frac{\alpha'}{2}\Delta_2} \\ &\times \; e^{ix.(k_2+k_4)} \phi_2(z)\phi_4(z)\,, \end{aligned} \tag{18}$$

where $\Delta_2 = e^{2A_0} \nabla_0^2 e^{-2A_0}$ and ∇_0^2 is the covariant Laplacian of a scalar in AdS_5.

The scattering amplitude can be rewritten as

$$\begin{aligned} \mathcal{A}(s,t) &= g_0^2 V \int dz\, dz' \sqrt{g(z)} \phi_1(z)\phi_3(z) K(s,t,z,z') \\ &\times \; \sqrt{g(z')} \phi_2(z')\phi_4(z')\,, \end{aligned} \tag{19}$$

where $K(s,t,z,z')$ is interpreted as the **Pomeron kernel**.

The Regge approach

Brower, Strassler and Tan 2007 , Cornalba, Costa and Penedones 2007

Rewrite the Pomeron kernel as an inverse Mellin transform of a complex J kernel

$$K(s,t,z,z') = -\int \frac{dJ}{2\pi i}\left[\frac{\hat{s}^J + (-\hat{s})^J}{\sin \pi J}\right] K(j,t,z,z') \,, \qquad (20)$$

where $\hat{s} = R^2 e^{-A(z)} e^{-A(z')} s$. It turns out that $K(J,t,z,z')$ satisfies the equation

$$L\Big\{ e^{-2A_0}\left[e^{-3A_0}\partial_z(e^{3A_0}\partial_z) + t\right] - m_J^2 \Big\} K(J,t,z,z') = e^{-5A_0}\,\delta(z-z')\,, \qquad (21)$$

with

$$m_J^2 = \frac{2}{\alpha'}(J-2) = 2\frac{\sqrt{\lambda}}{L^2}(J-2)\,. \qquad (22)$$

Then $K(J,t,z,z')$ is interpreted as the 5-d propagator of higher spin field with masses m_J obeying a **Regge behaviour**.

The J Kernel eq. (21) can be written in a Schrodinger form

$$L\Big\{\partial_z^2 + t - V(z)\Big\}\bar{G}(J,t,z,z') = \delta(z-z')\,, \tag{23}$$

with

$$V(z) = \frac{3}{2}\ddot{A}_0 + \frac{9}{4}\dot{A}_0^2 + e^{2A_0}m_J^2 = \frac{1}{z^2}\left[\frac{15}{4} + m_J^2 L^2\right]\,. \tag{24}$$

This implies a continuum spectrum (expected for AdS).

The Green's function $\bar{G}(J,t,z,z')$ can be expanded as

$$\bar{G}(J,t,z,z') = \int_0^\infty \frac{dE}{2\pi\sqrt{E}}\,\frac{\psi_{J,E}(z)\psi^\star_{J,E}(z')}{E-t}\,, \tag{25}$$

where

$$\psi_{J,E}(z) = \quad (\pi\sqrt{E}z)^{1/2} J_{i\nu_J}(\sqrt{E}z)\,, \tag{26}$$

with

$$i\nu_J = \sqrt{4 + m_J^2 L^2} = \sqrt{4 + 2\sqrt{\lambda}(J-2)}\,. \tag{27}$$

Recalling that $m_J^2 L^2 = \Delta(\Delta - 4)$ where Δ is the conformal dimension of the dual operator we find

$$J = J_0(\lambda) + D(\lambda)(\Delta - 2)^2\,, \tag{28}$$

where

$$J_0 = 2 - \frac{2}{\sqrt{\lambda}} \quad , \quad D(\lambda) = \frac{1}{2\sqrt{\lambda}}\,. \tag{29}$$

This relation describes the anomalous dimension of spin J operators of the form $\mathrm{Tr}(F_{\mu_1\nu} D_{\mu_2} \dots D_{\mu_{J-1}} F^{\nu}_{\ \mu_J})$.

It has the same quadratic behaviour at strong coupling (BPST) and weak coupling (BFKL).

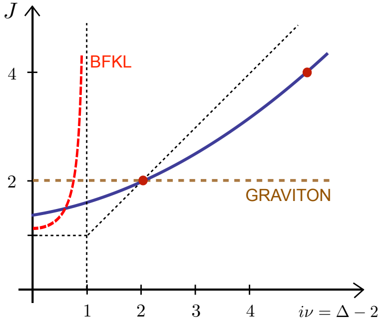

Figure 3 : J as a function of $\Delta - 2$

Improved Holographic QCD (IHQCD)

Gursoy, Kiritsis and Nitti 2007

Consider 5-d Dilaton-Gravity in the Einstein-Frame

$$S = M^3 N_c^2 \int d^5x \sqrt{-g} \left[R - \frac{4}{3} g^{mn} \partial_m \Phi \partial_n \Phi + V[\Phi] \right] , \tag{30}$$

In the String-Frame the action takes the form

$$S = M^3 N_c^2 \int d^5x \sqrt{-g_s} e^{-2\Phi} \left[R_s + 4 g_s^{mn} \partial_m \Phi \partial_n \Phi + V_s[\Phi] \right] , \tag{31}$$

where

$$g^s_{mn} = e^{\frac{4}{3}\Phi} g_{mn} \quad , \quad V_s[\Phi] = e^{-\frac{4}{3}\Phi} V[\Phi]. \tag{32}$$

The Dilaton-Gravity equations arising from (1) are

$$R_{mn} = \frac{4}{3} \partial_m \Phi \partial_n \Phi - \frac{1}{3} g_{mn} V , \tag{33}$$

$$\frac{4}{3} \nabla^2 \Phi = -\frac{1}{2} \frac{dV}{d\Phi}. \tag{34}$$

Consider the following family of backgrounds :

$$\begin{array}{rcl} \mathrm{ds}^2 & = & e^{2A(z)}\left(\mathrm{dz}^2 + dX_{1,3}^2\right) \\ \Phi & = & \Phi(z). \end{array} \tag{35}$$

For these backgrounds the Ricci tensor takes the form

$$\begin{array}{rcl} R_{\mathrm{zz}} & = & -4\ddot{A} \quad , \quad R_{\mathrm{z}\mu}=0 \ , \\ R_{\mu\nu} & = & -\left[\ddot{A} + 3\dot{A}^2\right]\eta_{\mu\nu} \, . \end{array} \tag{36}$$

Then the Dilaton-Gravity equations become the ordinary differential equations :

$$\begin{array}{rcl} e^{2A}V & = & 12\ddot{A} + 4\dot{\Phi}^2 \, , \\ e^{2A}V & = & 3\ddot{A} + 9\dot{A}^2 \, , \\ -\frac{3}{8}e^{2A}\frac{\mathrm{dV}}{\mathrm{d}\Phi} & = & \ddot{\Phi} + 3\dot{A}\dot{\Phi} \, . \end{array} \tag{37}$$

The system (37) have only two independent equations. They can be written as

$$\begin{aligned} V &= e^{-2A}\left(3\ddot{A}+9\dot{A}^2\right)\,, \\ \ddot{A} &= \dot{A}^2-\frac{4}{9}\dot{\Phi}^2\,. \end{aligned} \tag{38}$$

Introducing a superpotential W[Φ] the equations (38) become the first order differential equations

$$\dot{\Phi} = \frac{\mathrm{dW}}{\mathrm{d}\Phi}e^{A}\,, \tag{39}$$

$$\dot{A} = -\frac{4}{9}We^{A}\,, \tag{40}$$

with

$$V=\frac{64}{27}W^2-\frac{4}{3}\left(\frac{\mathrm{dW}}{\mathrm{d}\Phi}\right)^2\,. \tag{41}$$

An important quantity in these backgrounds is

$$X:=\frac{\dot{\Phi}}{3\dot{A}}=-\frac{3}{4}\frac{d\mathrm{LogW}}{\mathrm{d}\Phi}\,. \tag{42}$$

Holographic map

$$\begin{aligned} \log E &= A(z)\,, \\ \bar{\lambda}(E) &= c_0\, e^{\Phi(z)} =: c_0 \lambda(z)\,. \end{aligned} \tag{43}$$

The energy scale E is the dual of the warp factor $A(z)$ whereas the 't Hooft coupling $\bar{\lambda}(E)$ is determined by the dilaton $\Phi(z)$.

A nice consequence of the holographic map is

$$X = \frac{\mathrm{d}\Phi}{3\mathrm{dA}} = \frac{\mathrm{d}\lambda}{(3\lambda)\mathrm{LogE}} = \frac{\beta}{3\lambda}\,. \tag{44}$$

The field $X(z)$ maps to the beta function of the dual theory. For large-N QCD the two-loop perturbative beta function gives

$$\bar{\beta} = -\bar{b}_0 \bar{\lambda}^2 - \bar{b}_1 \bar{\lambda}^3\,, \quad \bar{b}_0 = \frac{2}{3}\frac{11}{(4\pi)^2}\,, \quad \frac{\bar{b}_1}{\bar{b}_0^2} = \frac{51}{121}\,. \tag{45}$$

This fixes the **UV behaviour** of the superpotential $W[\Phi]$.

In order to guarantee confinement the **IR behaviour** of the warp factor should be the following

$$A(z \gg 1) = -Cz^{\alpha} + \dots , \tag{46}$$

where $\alpha \geq 1$ and $C > 0$.

Background I : A simple superpotential that interpolates between the UV and IR behaviours is the following

$$W[\lambda] = W_0 \left(1 + \frac{2}{3} b_0 \lambda\right)^{\frac{2}{3}} \left(1 + \frac{\left(2b_0^2 + 3b_1\right) \mathrm{Log}\left[1 + \lambda^2\right]}{18a}\right)^{\frac{4}{3}a} , \tag{47}$$

where

$$\begin{aligned} W_0 &= \frac{9}{4L} \quad , \quad a = \frac{3}{8} \frac{\alpha - 1}{\alpha} \, , \\ b_0 &= c_0 \bar{b}_0 \quad , \quad \frac{b_1}{b_0^2} = \frac{\bar{b}_1}{\bar{b}_0^2} = \frac{51}{121} \, , \end{aligned} \tag{48}$$

and L is the AdS radius. A good agreement with lattice QCD results is obtained for

$$\alpha = 2 \quad , \quad b_0 = 4.2 \, . \tag{49}$$

For $N_c = 3$ the QCD running coupling can be identified with $\alpha_s = \bar{\lambda}/(12\pi)$. Figure 4 shows how α_s runs with the energy scale in the model.

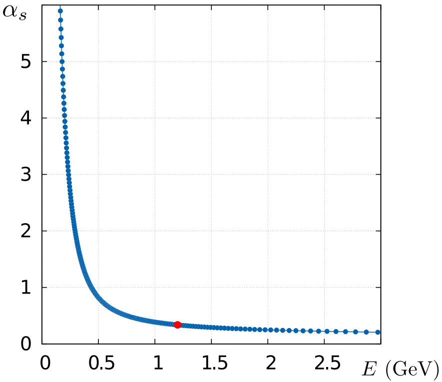

Figure 4 : Running coupling α_s vs. energy scale. The red point is $\alpha_s(1.2\,\mathrm{GeV}) = 0.34$.

Spin 2 glueballs in IHQCD

Consider the fluctuations h_{mn} and φ defined by

$$g_{mn} + h_{mn} \,, \qquad \Phi + \varphi \,. \tag{50}$$

The metric perturbations h_{mn} are decomposed according to the $SO(1,3)$ global symmetry of the background ,i.e.

$$\begin{aligned} &h_{\alpha\beta} = h^{TT}_{\alpha\beta} + \partial_{(\alpha} h^{T}_{\beta)} + \left(4\partial_\alpha\partial_\beta - \eta_{\alpha\beta}\partial^2\right)\bar{h} + \eta_{\alpha\beta} h \,, \\ &h_{zz} \,, \qquad h_{z\alpha} = v^T_\alpha + \partial_\alpha s \,. \end{aligned} \tag{51}$$

The spin 2 glueball spectrum is obtained by solving the equation for $h^{TT}_{\alpha\beta}$. In IHQCD it takes the form

$$\left(\nabla^2 + 2\dot{A}^2 e^{-2A(z)}\right) h^{TT}_{\alpha\beta} = 0 \,. \tag{52}$$

In the String Frame the eq. (52) becomes

$$\left(\nabla^2 - 2e^{-2A(z)}\dot{\Phi}\nabla_z + 2\dot{A}^2 e^{-2A(z)}\right) h^{TT}_{\alpha\beta} = 0 \,. \tag{53}$$

5-d Regge theory in IHQCD and the Soft Pomeron

Consider the operators $\mathrm{Tr}(F_{\mu_1\nu}D_{\mu_2}\dots D_{\mu_{J-1}}F^{\nu}{}_{\mu_J})$. The dual of those operators are higher spin fields $h^{TT}_{\alpha_1\dots\alpha_J}$.

In AdS_5 higher spin fields dual to operators of dimension Δ satisfy the eq.

$$[\nabla^2_{AdS_5} - \frac{\Delta(\Delta-4)-J}{L^2}]h^{TT}_{\alpha_1\dots\alpha_J} = 0\,. \tag{54}$$

Costa, Gonçalves and Penedones 2014

At weak coupling we expect $\Delta = J + 2 + \gamma_J$.

Our proposal for spin J fields in Dilaton-Gravity backgrounds :

$$\left(\nabla^2 - 2\,e^{-2A}\dot{\Phi}\nabla_z - \frac{\Delta(\Delta-4)}{L^2} + J\dot{A}^2 e^{-2A}\right)h_{\alpha_1\dots\alpha_J} = 0\,, \tag{55}$$

For $J = 2, \Delta = 4$ this eq. reduces to eq. (53) for the metric perturbation. For $A(z) = \ln(L/z)$ and $\Phi = 0$ we recover the spin J AdS equation (54).

In the the region of $J \leq 2$ we use the diffusion approximation

$$\frac{\Delta(\Delta-4)}{L^2} \approx \frac{2}{\alpha'}\,(J-2)\,, \tag{56}$$

where $l_s = \sqrt{\alpha'}$ is the string length.

In AdS_5 we have that $\alpha' = L^2/\sqrt{\lambda}$. In our model we take l_s as a phenomenological parameter to be fixed by data.

The diffusion approximation misses the UV expectation that $\Delta = J + 2$. However, for IR processes this approximation is very useful.

The eq. for the spin J fields (55) can be put in a Schrödinger form

$$\left(-\frac{d^2}{dz^2} + U(z)\right)\psi(z) = t\,\psi(z)\,, \tag{57}$$

$$U(z) = \frac{3}{2}\left(\ddot{A} - \frac{2}{3}\ddot{\Phi}\right) + \frac{9}{4}\left(\dot{A} - \frac{2}{3}\dot{\Phi}\right)^2 + \frac{\Delta(\Delta-4)}{L^2}\,e^{2A(z)}\,, \tag{58}$$

The energy spectrum for each J quantises $t = t_n(J)$. For $J = 2$ they correspond to the spin 2 glueball masses.

The effective potential for different values of spin J is shown in Figure 5.

This includes the first two spin 2 glueball states $2^{++}, 2^{++*}$.

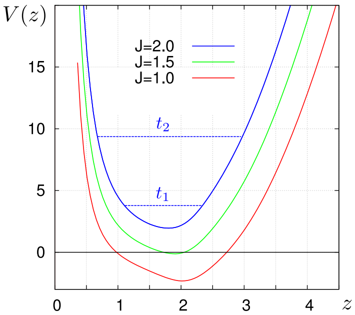

Figure 5 : Effective potential for different values of spin J

t-channel spin J exchange

Consider the elastic scattering of scalar hadronic states of equal masses m. We write the incoming momenta in light-cone coordinates

$$k_1 = \left(\sqrt{s}, \frac{m^2}{\sqrt{s}}, 0\right), \quad k_3 = -\left(\sqrt{s}, \frac{m^2 + q_\perp^2}{\sqrt{s}}, q_\perp\right), \tag{59}$$

$$k_2 = \left(\frac{m^2}{\sqrt{s}}, \sqrt{s}, 0\right), \quad k_4 = -\left(\frac{m^2 + q_\perp^2}{\sqrt{s}}, \sqrt{s}, -q_\perp\right),$$

where we consider the Regge limit $s \gg t = -q_\perp^2$.

Each hadron is described by a norm. mode $\Upsilon_i(z,x) = e^{ik_i \cdot x_i} \upsilon(z)$ with a coupling to the spin J field given by

$$\kappa_J \int d^5x \sqrt{-g}\, e^{-\Phi} h_{a_1 \cdots a_J} \Upsilon \nabla^{a_1} \cdots \nabla^{a_J} \Upsilon\,. \tag{60}$$

In the Regge limit we spin J exchange is described by

$$\mathcal{A}_J(k_i) = -\kappa_J^2 \int d^4x dz d^4x' dz' \sqrt{-g(z)} \sqrt{-g(z')} e^{-\Phi(z) - \Phi(z')}$$

$$\left(\Upsilon_1 \partial_-^J \Upsilon_3\right) \Pi^{-\cdots-,+\cdots+}(x,z,x',z') \left(\Upsilon_2' \partial_+'^J \Upsilon_4'\right). \tag{61}$$

After some algebra the scattering amplitude takes the form

$$\mathcal{A}(s,t) = iV \sum_{J=(2,4,\dots)} \frac{\kappa_J^2}{(-2)^J} \int dzdz' \sqrt{g(z)}\sqrt{g(z')} e^{-\Phi(z)-\Phi(z')} |\upsilon(z)|^2 |\upsilon(z')|^2 \left(s\, e^{-A(z)-A(z')}\right)^J K(J,t,z,z'), \quad (62)$$

where the spin J propagator $K(J,t,z,z')$ satisfies the eq.

$$L\{e^{-2A}\left[e^{2\Phi-3A}\partial_z(e^{3A-2\Phi}\partial_z) + t\right] - m_J^2\} K(J,t,z,z') = e^{-5A} e^{2\Phi} \delta(z-z'). \quad (63)$$

Mapping this eq. to a Schrödinger form and using the completeness relation $\sum_n \psi_n(z)\psi_n^*(z') = \delta(z-z')$ we find that the spin J kernel can be expanded as

$$K(J,t,z,z') = e^{\Phi(z)-\frac{3}{2}A(z)+\Phi(z')-\frac{3}{2}A(z')} \sum_n \frac{\psi_n(z)\psi_n^*(z')}{t_n(J)-t}. \quad (64)$$

The Soft Pomeron in IHQCD

The sum over spin can be converted to a Sommerfeld-Watson integral in the complex J plane

$$\frac{1}{2}\sum_{J\geq 2}\left(s^J+(-s)^J\right) \to -\frac{\pi}{2}\int \frac{dJ}{2\pi i}\frac{s^J+(-s)^J}{\sin(\pi J)}\,, \tag{65}$$

Then the amplitude becomes

$$\mathcal{A}(s,t) = iV\int dz\,dz'\,e^{3(A(z)+A(z'))}|\upsilon(z)|^2|\upsilon(z')|^2\sum_n \chi_n(z,z',s,t)\,, \tag{66}$$

where

$$\chi_n(z,z',s,t) = -\frac{\pi}{2}\int \frac{dJ}{2\pi i}\frac{s^J+(-s)^J}{\sin(\pi J)}\frac{\kappa_J^2}{2^J}e^{-(J-\frac{1}{2})(A(z)+A(z'))}\frac{\psi_n(z)\psi_n^*(z')}{t_n(J)-t}\,. \tag{67}$$

We assume the J-plane integral can be deformed from the poles at even values of J, to the poles $J=j_n(t)$ defined by $t_n(J)=t$.

In the scattering domain of negative t the Regge poles are along the real axis for $J < 2$. Thus we can write the amplitude as

$$\mathcal{A}(s,t) = V \sum_n s^{j_n(t)} \Pi(j_n(t)) \,, \tag{68}$$

where

$$\begin{aligned} \Pi(j_n(t)) &= \frac{\pi}{2}\left(1 - i\cot\frac{\pi j_n(t)}{2}\right) \frac{\kappa^2_{j_n(t)}}{2^{j_n(t)}} \frac{dj_n(t)}{dt} \\ &\times \left(\int dz\, e^{3A(z)} e^{-(j_n(t)-\frac{1}{2})A(z)} \upsilon^2(z)\psi_n(z)\right)^2 . \end{aligned} \tag{69}$$

Figure 6 shows the Regge trajectories $j_n(t)$.

As J decreases the energy levels cross the zero energy value. This is the intercept for the n-th Regge trajectory.

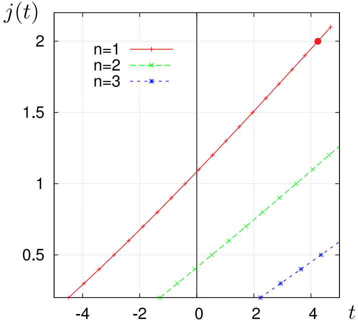

Figure 6 : Regge trajectories.

Results

First Regge trajectory :

- **Scenario I :** For $l_s = 0.178\ \mathrm{GeV}^{-1}$ and $\Lambda_{QCD} = 0.265$ we obtain an approximate linear trajectory consistent with the Soft Pomeron

$$J(t) \approx 1.08 + 0.25t\,. \tag{70}$$

Donnachie and Landshoff 1992

- **Scenario II :**For $l_s = 0.192\ \mathrm{GeV}^{-1}$ and $\Lambda_{QCD} = 0.292$ we find an approximate linear trajectory in agreement with the lattice $SU(3)$ result.

$$J(t) \approx 0.93 + 0.25t\,. \tag{71}$$

Meyer and Teper 2004

Second Regge trajectory :

$$\begin{aligned} J(t) &\approx 0.43 + 0.21t \qquad (\mathrm{Scenario\,I})\,, \\ J(t) &\approx 0.17 + 0.2t \qquad (\mathrm{Scenario\,II})\,. \end{aligned} \tag{72}$$

Fit to $p\,\bar{p}$ total cross section

The contribution from the first two Regge poles in our model takes the form

$$\sigma = g_0 S^{J_1(0)-1} + g_1 S^{J_2(0)-1}\,. \tag{73}$$

Using g_0 and g_1 as parameters we ran fits to $p\,\bar{p}$ total cross section data in the region $\sqrt{s} > 10\,\text{GeV}$ Olive et al (Particle Data Group) 2014 .

Our fit is shown in Figure 7. The blue line is the fit of two Regge poles and the green line is a fit with just one Regge pole.

A second Regge pole inside the range $\approx 0.35 - 0.55$ is necessary to get $\chi^2_{d.o.f.} \leq 1$.

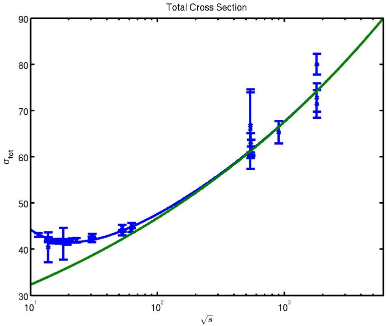

Figure 7 : A fit to $p\,\bar{p}$ total cross section data using two Regge poles.

Conclusions

Using IHQCD we have developed a 5-d Regge approach to scattering process dominated by Soft Pomeron exchange.

Although inspired by the BPST approach, our model describe a discrete set of Regge poles in constrast to the branch cut obtained in the BPST approach.

Our models lacks of an UV description of spin J fields that should match with perturbative QCD analysis of the dual operators. This would allow a unified description of Soft and Hard Pomeron.

Analysis of the $p\bar{p}$ differential cross sections could lead to a better understanding of the coefficients $\Pi(j_n(t))$.

Deep inelastic Scattering (DIS) is an interesting laboratory to investigate the competition between Soft and Hard Pomeron at very low Bjorken variable x (work in progress).