Lectures on Strings and Phenomenology

Anamaría Font V.

Universidad Central de Venezuela

Outline

I. The Standard Model
II. Strings
III. Overview of String Phenomenology
IV. Heterotic model building
V. D-Brane constructions
VI. Flux compactifications and moduli stabilization

Bibliography

1. *String Theory and Particle Physics: An Introduction to String Phenomenology,*
   L.E. Ibáñez and A.M. Uranga, CUP 2012.
2. *Basic Concepts of String Theory*, R. Blumenhagen, D. Lüst and S. Theisen, Springer 2013.
3. *String Theory, Vols. I, II*, J. Polchinski, CUP 1998.
4. *String Theory and M-theory: A Modern Introduction*, K. Becker, M. Becker and J.H. Schwarz, CUP 2007.

I. The Standard Model

Basics

The SM describes electromagnetic, weak and strong interactions.
It is a quantum field theory with gauge group

(force carriers)
gauge bosons $\Longrightarrow$

$$\underbrace{SU(3)}_{\text{gluons } G^a_\mu} \times \underbrace{SU(2) \times U(1)_Y}_{W^\pm_\mu,\ Z_\mu,\ A_\mu}$$

$W^\pm$ and Z massive due to spontaneous symmetry breaking, $m_{EW} \sim 10^2$ Gev

Matter particles: quarks + leptons in 3 families

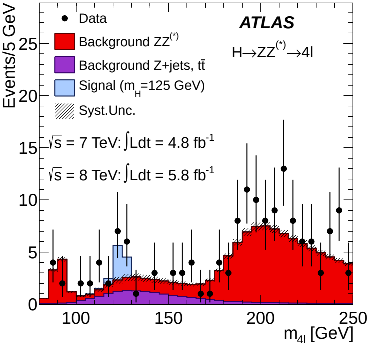
$Q^i_L = \begin{pmatrix} U^i_L \\ D^i_L \end{pmatrix}$	D^i_R	U^i_R	$L^i = \begin{pmatrix} \nu^i_L \\ E^i_L \end{pmatrix}$	E^i_R
$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$(\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$	$(\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}$	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$(\mathbf{1},\mathbf{1})_1$

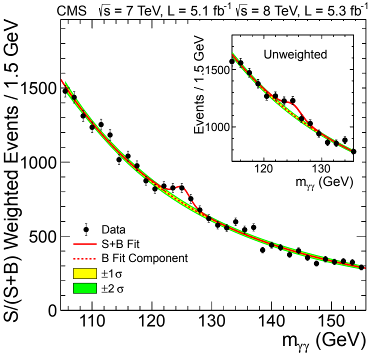
$i = 1, 2, 3$

left-handed
Weyl spinors

Higgs scalar $\quad H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix} \quad (\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$

Higgs found at LHC, July 2012





$m_H \sim 125$ GeV

SM Lagrangian

Schematically

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}^{i\,\dagger}\bar{\sigma}^{\mu}D_{\mu}\psi^{i}$$

$$+ |D_{\mu}H|^2 - V(H) \qquad \text{Higgs}$$

$$+ Y_{ij}\psi^{i}\psi^{j}H \qquad \text{Yukawa interactions}$$

only terms of mass dimension $\leq 4 \;\Rightarrow\;$ conservation of B and L

Features of the SM

- $\langle H\rangle \neq 0 \;\Rightarrow\;$ electroweak spontaneous symmetry breaking (EW SSB)

$$SU(2)\times U(1)_Y \xrightarrow{\langle H\rangle} U(1)_{\mathrm{EM}}$$

- the fermionic spectrum is chiral, i.e. left-handed and right-handed fermions have different $SU(2)\times U(1)_Y$ quantum numbers
- chiral fermions $\Rightarrow$ Dirac masses $m\,\bar{f}_R f_L +$ h.c. not gauge invariant
- fermion masses due to EW SSB and Yukawa couplings

$$\mathcal{L}_{\mathrm{Yuk}} \;=\; Y^L_{ij} L^i E^j_R H \;+\; Y^D_{ij} Q^i_L D^j_R H \;+\; Y^U_{ij} Q^i_L U^j_R H^* + \mathrm{h.c.}$$

$$\mathcal{L}_{\mathrm{Yuk}} \xrightarrow{\langle H\rangle} m^L_{ij}\, L^i E^j_R \;+\; m^D_{ij}\, Q^i_L D^j_R \;+\; m^U_{ij}\, Q^i_L U^j_R + \mathrm{h.c.}$$

$$m \;=\; Y\langle H\rangle \xrightarrow{V_L m V_R^\dagger} \mathrm{diag}(m_1, m_2, m_3)$$

- couplings of $W^\pm$ to U- and D-quarks given by

$$V_{CKM} = V^U_L V^{D\,\dagger}_L \qquad \text{Cabbibo-Kobayashi-Maskawa matrix}$$

Neutrino masses

In the SM $m_\nu = 0$

but observed neutrino oscillations require non-zero tiny $m_\nu \sim 10^{-6} m_e$.

It can be explained introducing right-handed neutrinos ν_R

transforming as $(\mathbf{1},\mathbf{1})_0$ under $SU(3)\times SU(2)\times U(1)_Y$

and implementing the *see-saw* mechanism via

$$\mathcal{L}_{\text{Yuk}} \supset Y^\nu_{ij} L^i \nu^j_R H^* + M_{ij} \nu^i_R \nu^j_R + \text{h.c.}$$

with $M \gg Y^\nu \langle H \rangle$

Alternatively, without ν_R, it can be explained allowing

lepton-number violating terms $\frac{h_{ij}}{M} L^i L^j H^* H^* + \text{h.c.}$

More open questions

- ▷ Many free parameters, e.g. three coupling constants, quark and lepton masses.
  In particular there is a flavor puzzle

observed values

quarks: $(m_u, m_c, m_t) \sim (0.003, 1.3, 170)$ GeV ; $(m_d, m_s, m_b) \sim (0.005, 0.1, 4)$ GeV

leptons: $(m_e, m_\mu, m_\tau) \sim (0.0005, 0.1, 1.8)$ GeV

$$|V_{CKM}| \sim \begin{array}{c} \begin{array}{ccc} \text{d} & \text{s} & \text{b} \end{array} \\ \left|\begin{array}{ccc} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.04 \\ 0.008 & 0.04 & 0.99 \end{array}\right| \end{array} \begin{array}{c} \\ \text{u} \\ \text{c} \\ \text{t} \end{array}$$

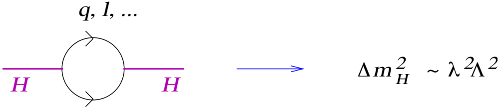
$*$ large hierarchies $m_3 \ggg m_2 \gg m_1$

$*$ small mixings $V_{su} \sim \epsilon, \quad V_{bc} \sim \epsilon^2, \quad V_{bu} \sim \epsilon^3$

More open questions

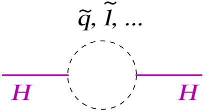
- EW hierarchy problem: Why is the Higgs mass m_H not modified by loop corrections ?

The problem is due to radiative corrections



and the cutoff scale Λ could be as large as the Planck mass.

Supersymmetry gives a solution. For every fermion $q, l, \cdots$ there is a scalar $\tilde{q}, \tilde{l}, \cdots$ and the above loop diagram is cancelled by



MSSM

Minimal Supersymmetric Standard Model:

extension of the SM with one additional Higgs and supersymmetric partners (gauginos, squarks, sleptons, Higgsinos).

There are dim 4 couplings violating B and L, e.g. $U_R D_R \tilde{D}$, $LL\tilde{E}$. Such couplings lead to fast proton decay. They can be forbidden imposing R-parity, a $\mathbb{Z}_2$ symmetry under which the SM particles are even and the partners are odd. R-parity ensures that the lightest supersymmetric particle is stable and is then a candidate for dark matter.

Since the superpartners have not been detected, supersymmetry must be broken above the electroweak scale but so far no evidence has been found the LHC.

More open questions

- Why $G_{\rm SM} = SU(3) \times SU(2) \times U(1)_Y$ and the specific matter representations ?

  Some simplification is achieved in Grand Unified Theories (GUTs).

  The idea is that there is a bigger symmetry group $G_{\rm GUT} \supset G_{\rm SM}$ manifest at high energy scales $M_{GUT} \sim 10^{16}$ Gev.

  The GUT idea is supported by the unification of gauge couplings g_a, obtained extrapolating the lower scale experimental values using the renormalization group equations,

$$\frac{4\pi}{g_a^2(Q^2)} = \frac{4\pi}{g_a^2(M^2)} + \frac{b_a}{4\pi} \log \frac{M^2}{Q^2}$$

  The one-loop β-function coefficients b_a depend on the group and the matter content, e.g. for $SU(3)$ $b_3 = -11 + \frac{4}{3} N_{\rm gen}$.

Gauge coupling unification

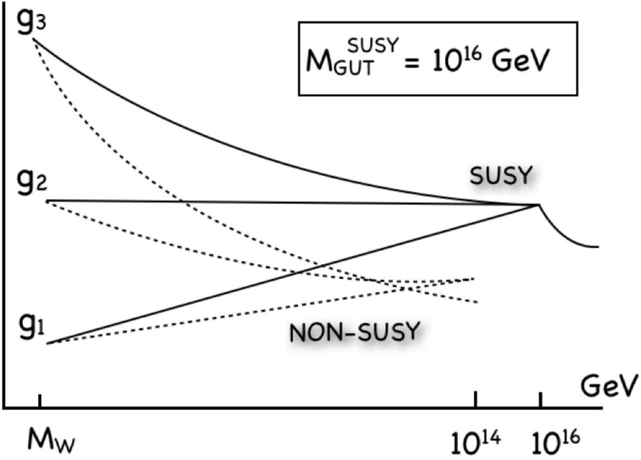


Figure from *String Theory and Particle Physics: An Introduction to String Phenomenology* L.E.Ibáñez, A.M. Uranga

GUTs

$G_{\mathrm{GUT}} = SU(5)$

1 family $= \mathbf{10} + \overline{\mathbf{5}}$

$SU(5) \supset SU(3) \times SU(2) \times U(1)_Y$

$$\mathbf{10} = (\mathbf{3}, \mathbf{2})_{\frac{1}{6}} + (\overline{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}} + (\mathbf{1}, \mathbf{1})_1$$

$$\overline{\mathbf{5}} = (\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} + (\mathbf{2}, \mathbf{1})_{-\frac{1}{2}}$$

$SU(5)$ broken to G_{SM} by Higgs in the adjoint **24**.

For EW SSB the Higgs is also in $\overline{\mathbf{5}}$. Quark and lepton masses from Yukawa couplings: $\mathbf{10} \cdot \mathbf{10} \cdot \overline{\mathbf{5}}$, $\quad \mathbf{10} \cdot \overline{\mathbf{5}} \cdot \overline{\mathbf{5}}$

The triplets in the Higgs $\overline{\mathbf{5}}$ can mediate proton decay so they must be much more massive than the doublets.This is the doublet-triplet splitting problem.

Other GUTs

$G_{\text{GUT}} = SO(10)$

1 family + ν_R= **16**

$SO(10) \supset SU(5) \times U(1)$

$\mathbf{16} = \mathbf{10} + \overline{\mathbf{5}} + \mathbf{1}$

$G_{\text{GUT}} = E_6$

1 family + ν_R + exotics= **27**

$E_6 \supset SO(10) \times U(1)$

$\mathbf{27} = \mathbf{16} + \mathbf{10}_V + \mathbf{1}$

$E_6 \supset SU(3) \times SU(3) \times SU(3)$

$\mathbf{27} = (\mathbf{3}, \overline{\mathbf{3}}, \mathbf{1}) + (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{3}) + (\mathbf{1}, \mathbf{3}, \overline{\mathbf{3}})$

More open questions

- How to include gravity ?

The scale at which gravitational interactions become important is the Planck mass

$$M_P = \sqrt{\frac{\hbar c}{G_N}} \sim 10^{19}\,\text{Gev}$$

G_N is the fundamental constant in Newton's law $F_{grav} = G_N \frac{m_1 m_2}{r^2}$.

Since $m \sim E$, $G_N \sim 1/M_P^2$, the effective gravitational coupling is

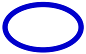
$\alpha_{grav} = (E/M_P)^2$ which grows quadratically with energy.

The perturbative expansion of gravity diverges.

An ultraviolet (UV) completion is needed $\Longrightarrow$ Strings ?!

II. Strings

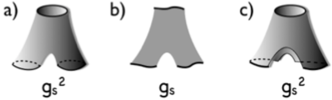
Quick look/reminder

▷ extended objects  open strings  closed strings

string tension: $T = 1/2\pi\alpha'$ string length, mass: $\ell_s \sim \sqrt{\alpha'}$, $M_s \sim 1/\sqrt{\alpha'}$

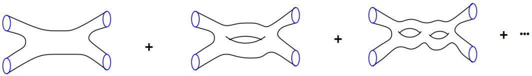
▷ vibration modes $\Rightarrow$ particles graviton $G_{\mu\nu}$ always in spectrum

▷ interactions

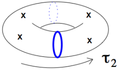


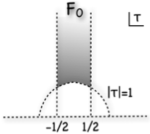
$g_s = e^{\langle\varphi\rangle}$, φ :dilaton

▷ perturbative expansion



no UV divergences

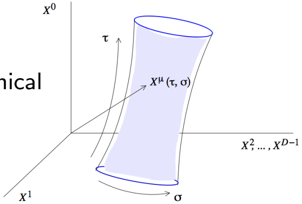




$\tau_2 \to 0$ excluded by modular invariance

Bosonic strings 1

action $S \propto T \cdot$ area of world-sheet



light-cone quantization: $X^0 \pm X^{D-1}$ non-dynamical

(due to reparametrization invariance)

$X^i(\tau,\sigma), i = 1, \cdots, D-2,$ satisfy wave equation

$$\frac{\partial^2 X^i}{\partial \tau^2} = \frac{\partial^2 X^i}{\partial \sigma^2} \Longrightarrow X^i(\tau,\sigma) = X^i_L(\tau+\sigma) + X^i_R(\tau-\sigma)$$

mode expansions: $$X^i_L = \frac{x^i}{2} + \frac{p^i}{2p^+}(\tau+\sigma) + i\sum_{n\neq 0}\frac{\alpha^i_n}{n}e^{-in(\tau+\sigma)}$$

$$X^i_R = \frac{x^i}{2} + \frac{p^i}{2p^+}(\tau-\sigma) + i\sum_{n\neq 0}\frac{\tilde{\alpha}^i_n}{n}e^{-in(\tau-\sigma)}$$

$[\alpha^i_m, \alpha^j_n] = [\tilde{\alpha}^i_m, \tilde{\alpha}^j_n] = m\,\delta^{ij}\,\delta_{m,-n}, \quad [\alpha^i_m, \tilde{\alpha}^j_n] = 0$ infinite sets of harmonic oscillators

anhilation ops. $\alpha^i_n, \tilde{\alpha}^i_n,\ n > 0,$ creation ops. $\alpha^i_{-n}, \tilde{\alpha}^i_{-n},\ n > 0$

Bosonic strings 2

vacuum: $|0\rangle$, $\alpha_n^i|0\rangle = \tilde{\alpha}_n^i|0\rangle = 0$, $n > 0$, $\quad i = 1, \cdots, (D-2)$

excited states: chains of $\alpha_{-n}^i, \tilde{\alpha}_{-n}^i$, $n > 0$, acting on $|0\rangle$

osc. numbers: $N = \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i$, $\tilde{N} = \sum_{n=1}^{\infty} \tilde{\alpha}_{-n}^i \tilde{\alpha}_n^i$, $\quad$ level-matching: $N = \tilde{N}$

vacuum energies: $E_0 = \tilde{E}_0 = (D-2)\frac{1}{2}\sum_{n=1}^{\infty} n \xrightarrow{\text{regularization}} -\frac{(D-2)}{24}$

mass formula: $\alpha' M^2 = 2(N + \tilde{N} + E_0 + \tilde{E}_0)$

spectrum: level 0: $|0\rangle$, $\alpha' M^2 = 4E_0$, level 1: $\alpha_{-1}^i \tilde{\alpha}_{-1}^j |0\rangle$, $\alpha' M^2 = 4(E_0 + 1)$, ...

Lorentz invariance $\Longrightarrow E_0 = -1, \ D = 26$

so at level 1, $M = 0$ and $(D-2)^2$ states fill rep. of massless little group $SO(D-2)$

$\alpha_{-1}^i \tilde{\alpha}_{-1}^j |0\rangle \longrightarrow$ graviton $G_{\mu\nu}$, dilaton φ, anti-symmetric tensor $B_{\mu\nu}$

$|0\rangle \longrightarrow$ tachyon

Open strings and D-branes

$\delta S = 0 \Rightarrow$ boundary conditions: $\delta X_\mu \partial_\sigma X^\mu \Big|_0^\pi = 0$

Lorentz inv. in D dim. $\Rightarrow$ Neumann (N) b.c. $\partial_\sigma X^\mu \big|_{0,\pi} = 0, \;\; \mu = 0, \cdots, D-1$

b.c. mix L- and R-movers: $\alpha^i_n = \tilde{\alpha}^i_n, \;\; i = 1, \cdots, D-2$

spectrum: level 0: $|0\rangle$, $\alpha' M^2 = E_0$, level 1: $\alpha^i_{-1}|0\rangle$, $\alpha' M^2 = (E_0 + 1)$, ...

Lorentz inv. $\Rightarrow E_0 = -1, \;\; D = 26$, so $\alpha^i_{-1}|0\rangle$ massless, vector of $SO(D-2)$

$\alpha^i_{-1}|0\rangle \longrightarrow$ gauge vector A^μ

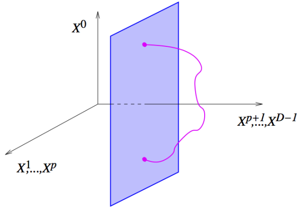
can consider fixed ends or Dirichlet (D) b.c. in some directions, i.e.

$X^0, \cdots, X^p$ (N) *space-time* $\quad X^{p+1}, \cdots, X^{25}$ (D)

string endpoints lie on a Dp-brane

$\alpha^i_{-1}|0\rangle, i = 1, \cdots, p-1$ massless vectors

$\alpha^t_{-1}|0\rangle, t = p+1, \cdots, 25$ massless scalars



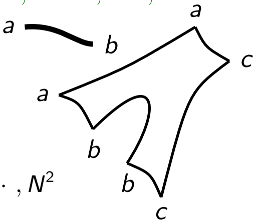
Dp-branes required by T-duality

Chan-Paton labels and D-branes

non-dynamical degrees of freedom at endpoints

consistent with symmetries and interactions

$a, b = 1, \cdots, N$



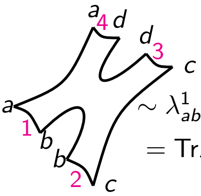
states carry extra labels, e.g. $|0\rangle$ becomes $|ab\rangle$

massless vectors: $\alpha^i_{-1}|ab\rangle \to A^\mu_{ab}$ gauge fields

Chan-Paton factors: $N \times N$ matrices $\lambda^\flat_{ab}$, $\flat = 1, \cdots, N^2$

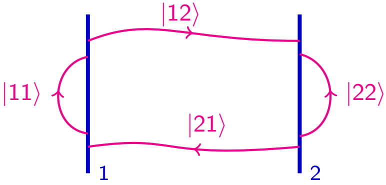
$$\sum_{a,b} \lambda^\flat_{ab}\, \alpha^i_{-1}|ab\rangle \longrightarrow A^\flat_\mu$$

$U(N)$ gauge fields, for oriented strings



$$\sim \lambda^1_{ab}\lambda^2_{bc}\lambda^3_{cd}\lambda^4_{da} = \mathrm{Tr}\lambda^1\lambda^2\lambda^3\lambda^4$$

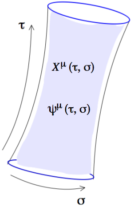
a,b label D-branes at endpoints



Superstrings 1

extra world-sheet d.o.f. $\psi^\mu(\tau,\sigma)$ 2d fermions, $\mu = 0, 1, \cdots, D-1$

$\psi^\mu(\tau,\sigma+2\pi) = \mp\psi^\mu(\tau,\sigma)$; $-$ Neveu-Schwarz(NS), $+$ Ramond



$$\partial\!\!\!/\psi^i(\tau,\sigma) = 0 \;\Rightarrow\; \psi^i(\tau,\sigma) = \begin{pmatrix} \psi^i_R(\tau-\sigma) \\ \psi^i_L(\tau+\sigma) \end{pmatrix}, \quad i = 1, \cdots, D-2$$

$$\psi^i_L = \sum_r b^i_r e^{-ir(\tau+\sigma)}, \quad \{b^i_r, b^j_s\} = \delta^{ij}\delta_{r,-s},$$ NS: $r \in \mathbb{Z} + \frac{1}{2}$, Ramond: $r \in \mathbb{Z}$

Lorentz invariance $\Longrightarrow$ $D = 10$ massless little group $SO(8)$

NS states: $|0\rangle,\ b^i_{-\frac{1}{2}}|0\rangle$ massless $\mathbf{8}_v, \cdots$

Ramond states: $|S\rangle, |C\rangle$ massless $\mathbf{8}_s, \mathbf{8}_c, \cdots$ $\{b^i_0, b^j_0\} = \delta^{ij}$, Clifford algebra

GSO projection: $(-1)^F = 1,\ F =$ world-sheet fermion number

$|0\rangle, |S\rangle$ projected out, full spectrum is supersymmetric in $D = 10$

GSO projection is required by modular invariance

Superstrings 2 massless spectra

IIB left-right symmetric

$$[\mathbf{8}_v \oplus \mathbf{8}_c]_L \otimes [\mathbf{8}_v \oplus \mathbf{8}_c]_R = (\mathbf{1} \oplus \mathbf{35}_v \oplus \mathbf{28}_v) \oplus (\mathbf{1} \oplus \mathbf{28}_c \oplus \mathbf{35}_c)$$
$$\oplus(\mathbf{8}_s \oplus \mathbf{56}_s \oplus \mathbf{8}_s \oplus \mathbf{56}_s)$$

massless fields of $\mathcal{N} = 2$ IIB supergravity (chiral)

$$\{\varphi, G_{\mu\nu}, B_{\mu\nu}\} + \underbrace{\{a, C_{\mu\nu}, C_{\mu\nu\alpha\beta}\}}_{\text{RR 0-,2-,4-forms}} + \{\Psi_1, \Psi_1^\mu, \Psi_2, \Psi_2^\mu\}$$

anomaly-free

IIA

$$[\mathbf{8}_v \oplus \mathbf{8}_c]_L \otimes [\mathbf{8}_v \oplus \mathbf{8}_s]_R = (\mathbf{1} \oplus \mathbf{35}_v \oplus \mathbf{28}_v) \oplus (\mathbf{8}_v \oplus \mathbf{56}_v)$$
$$\oplus(\mathbf{8}_s \oplus \mathbf{56}_s \oplus \mathbf{8}_c \oplus \mathbf{56}_c)$$

massless fields of $\mathcal{N} = 2$ IIA supergravity (non-chiral)

$$\{\varphi, G_{\mu\nu}, B_{\mu\nu}\} + \underbrace{\{C_\mu, C_{\mu\nu\alpha}\}}_{\text{RR 1-,3-forms}} + \left\{\Psi^+, \Psi^+_\mu, \Psi^-, \Psi^-_\mu\right\}$$

IIA supergravity can be obtained from circle compactification of 11-dimensional supergravity with fields $\{G_{MN}, C_{MN}, \Psi_M\}$

massless fields of M-theory

Type I and IIB orientifolds closed sector

- ▷ type I is a theory of *unoriented* closed and open superstrings,
it can be described as a quotient of IIB by world-sheet parity Ω

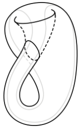
- ▷ $\Omega : \sigma \to (2\pi - \sigma)$, exchanges left and right modes, reverses orientation

- ▷ IIB is symmetric under Ω, can take quotient type IIB$/\Omega$ "orientifold"

- ▷ projection $\frac{1}{2}(1 + \Omega)$ gives invariant states, introduces unoriented topologies

unoriented world-sheets

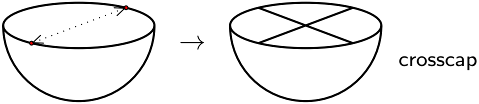
e.g. Klein bottle



generic world-sheet is sphere with g handles, n_b boundaries and n_c crosscaps

Euler characteristic: $\chi_E = 2 - 2g - n_b - n_c$

crosscap = disk with opposite sides identified = $\mathbf{RP}_2$



Klein bottle is sphere with two crosscaps, has $\chi_E = 0$, appears at 1-loop

Type I and IIB orientifolds closed sector

$\triangleright$ type I is a theory of *unoriented* closed and open superstrings, it can be described as a quotient of IIB by world-sheet parity Ω

$\triangleright$ $\Omega : \sigma \to (2\pi - \sigma)$, exchanges left and right modes, reverses orientation

$\triangleright$ IIB is symmetric under Ω, can take quotient type IIB$/\Omega$ "orientifold"

$\triangleright$ projection $\frac{1}{2}(1+\Omega)$ gives invariant states, introduces unoriented topologies

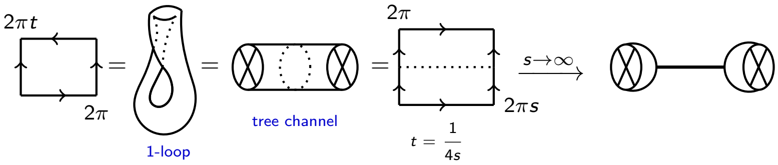
$\triangleright$ massless states: NS-NS: $\mathbf{8}_v \otimes \mathbf{8}_v \big|_S \longrightarrow \varphi, G_{\mu\nu}$, R-R: $\mathbf{8}_c \otimes \mathbf{8}_c \big|_A \longrightarrow C_{\mu\nu}$

NS-R + R-NS: $\mathbf{8}_v \otimes \mathbf{8}_c + \mathbf{8}_c \otimes \mathbf{8}_v \longrightarrow \Psi, \Psi_\mu$

$\mathcal{N} = 1, D = 10$ supergravity has gravitational anomaly

Type I and IIB orientifolds closed sector

▷ at string level anomaly due to divergence in 1-loop Klein bottle amplitude



$t \to 0$ in 1-loop $\equiv s \to \infty$ in tree channel

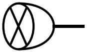
divergence due to tadpole of massless states

tadpoles

▷ in field theory:  1-point vertex

particle appears/disappears from/into the vacuum, so at momentum $k^\mu = 0$

 $\sim \dfrac{1}{k^2 + M^2} \;, \xrightarrow[M=0]{k^\mu=0} \infty$ divergence due to tadpole of massless particle

▷ divergence in Klein bottle (KB) amplitude due to crosscap tadpole 

▷ by Lorentz invariance the emitted massless states are the NS-NS $G_{\mu\nu}$ or φ,

or the R-R 10-form C_{10}, which is non-propagating since $dC_{10} = 0$ in $D = 10$

▷ by supersymmetry the KB amplitude is zero but R-R tadpoles must cancel

in effective action C_{10} enters *only* in $Q_{crosscap} \displaystyle\int_{M_{10}} C_{10}$

eq. of motion for C_{10} would imply $Q_{crosscap} = 0$,

but divergence in the R-R piece of the amplitude means $Q_{crosscap} \neq 0$

solution: C_{10} has other sources, naturally D9-branes $\longrightarrow$ open strings

Type I and IIB orientifolds open sector

a — b

$a, b = 1, \cdots, N$

▷ Neumann boundary conditions in all directions $\longrightarrow$ N D9-branes

▷ massless states: NS: $b^i_{-\frac{1}{2}}|ab\rangle\lambda_{ab}$, R: $|C, ab\rangle\lambda_{ab}$, $\quad \mathbf{8}_v \oplus \mathbf{8}_c \longrightarrow A_\mu, \chi$

$\mathcal{N} = 1, D = 10$ super Yang-Mills, gauge group $U(N)$ for oriented open strings
anomalous

▷ projection $\frac{1}{2}(1+\Omega)$ gives invariant states, introduces unoriented topologies

▷ Ω action: $b^i_{-\frac{1}{2}}|ab\rangle \to -b^i_{-\frac{1}{2}}|ab\rangle, \quad \lambda \to \gamma_\Omega \lambda^T \gamma_\Omega^{-1}, \quad a \leftrightarrow b$

γ_Ω is embedding of Ω in Chan-Paton factors, $\quad \Omega^2 = 1 \Rightarrow \gamma_\Omega^T = \pm\gamma_\Omega$

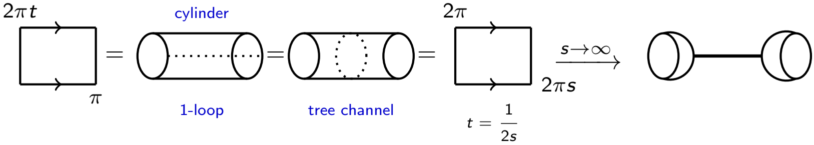
▷ massless invariant states: if $\gamma_\Omega = \gamma_\Omega^T = \mathbf{1}_N \Rightarrow \lambda^T = -\lambda \Rightarrow SO(N)$ gauge group

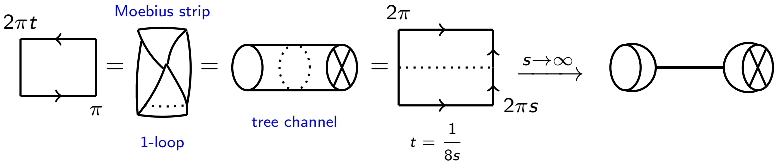
if $\gamma_\Omega = -\gamma_\Omega^T = i \begin{pmatrix} 0 & \mathbf{1}_{\frac{N}{2}} \\ -\mathbf{1}_{\frac{N}{2}} & 0 \end{pmatrix} \Rightarrow USp(N)$ gauge group (N even)

in any case, $\mathcal{N} = 1, D = 10$ super Yang-Mills anomalous

Type I and IIB orientifolds open sector

▷ anomaly due to divergences in 1-loop cylinder and Moebius strip amplitudes





$t \to 0$ in 1-loop $\equiv s \to \infty$ in tree channel

divergences due to tadpoles of massless states

cylinder, Moebius and Klein tadpoles cancel for $\gamma_\Omega^T = \gamma_\Omega, \; N = 32$

massless fields of $\mathcal{N} = 1, D = 10$ supergravity $+ \; SO(32)$ super Yang-Mills

anomaly cancelled by Green-Schwarz mechanism

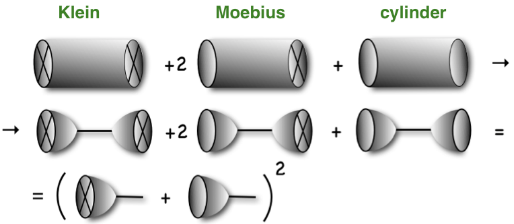
tadpole cancellation

- ▷ divergences due to crosscap and disk R-R tadpoles

- ▷ in effective action $\left(Q_{crosscap} + NQ_{disk}\right)\int_{M_{10}} C_{10}$

eq. of motion for $C_{10} \Longrightarrow Q_{crosscap} + NQ_{disk} = 0$

- ▷ charges computed from amplitudes



$$\left(Q_{crosscap} + NQ_{disk}\right)^2 = 32^2 - 64\,\mathrm{Tr}\,\gamma_\Omega^{-1}\gamma_\Omega^T + N^2 = 0 \Rightarrow \quad \gamma_\Omega^T = \gamma_\Omega, \; N = 32$$

$$Q_{disk} = 1, \quad Q_{crosscap} = -32, \quad \text{i.e.} \quad Q_{\mathrm{D9}} = 1, \quad Q_{\mathrm{O9}} = -32$$

orientifold 9-plane

Heterotic strings 1

world-sheet degrees of freedom (fermionic formulation $SO(32)$)

right R

$X_R^\mu \quad , \quad \psi^\mu$ $2d$ fermions

$\mu = 0, \cdots, 9 \quad , \quad i = 1, \cdots, 8$ light-cone

left L

$X_L^\mu \quad , \quad \lambda^A$ $2d$ fermions

$A = 1, \cdots, 32$

$\psi^\mu(\tau, \sigma + 2\pi) = \mp\psi^\mu(\tau, \sigma) \quad ; \quad \lambda^A(\tau, \sigma + 2\pi) = \mp\lambda^A(\tau, \sigma) \quad ;$ $-$ Neveu-Schwarz(NS), $+$ Ramond

massless states $|R\rangle \otimes |L\rangle$

NS $\otimes$ NS

e.g. $b^i_{-\frac{1}{2}}\,|0\rangle \;\otimes\; \lambda^A_{-\frac{1}{2}}\,\lambda^B_{-\frac{1}{2}}\,|0\rangle$

$$M_R^2 = \tilde{N}_X + \tilde{N}_\psi - \tfrac{1}{2}$$
$$M_L^2 = N_X + N_\lambda - 1$$

496 gauge vectors of $SO(32)$ in $10d$ (gauginos in Ramond $\otimes$ NS)

Heterotic strings 2

Full massless spectrum

$R \otimes L$ in light cone

$$[(\mathbf{8}_v \oplus \mathbf{8}_s, \mathbf{1})]_R \otimes [(\mathbf{8}_v, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{496})]_L$$

$$= (\mathbf{1} \oplus \mathbf{35}_v \oplus \mathbf{28}_v + \mathbf{8}_c + \mathbf{56}_s, \mathbf{1}) \ \oplus \ (\mathbf{8}_v \oplus \mathbf{8}_s, \mathbf{496})$$

massless fields

$$\{\varphi, G_{\mu\nu}, B_{\mu\nu}, \Psi, \Psi_\mu\} \ \oplus \ \left\{A_k^\mu, \chi_k\right\} \quad k = 1, \ldots, \dim G_{\text{het}}$$

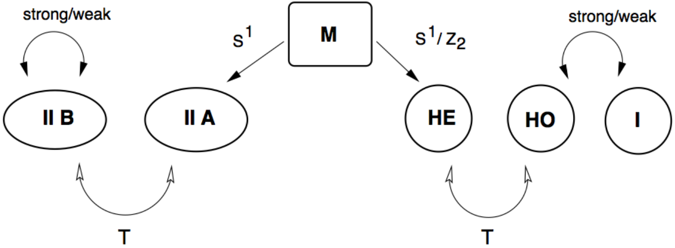
$D = 10$, $\mathcal{N} = 1$ supergravity $\oplus$ super Yang-Mills G_{het}

$G_{\text{het}} = E_8 \times E_8, SO(32)$

Gauge and gravitational anomalies are cancelled

by the Green-Schwarz mechanism

Dualities



III. String Phenomenology overview

Aim

- Study how to embed the SM in string/M-theory and address the open questions.
  - Identify classes of constructions that realize characteristic features: chirality, family replication, EW SB, flavor structure, ...
  - Extract generic properties and look for mechanisms behind.
  - Obtain and analyze explicit models.

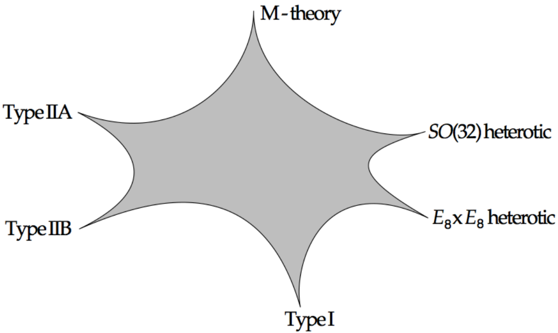
A main difference with conventional model building is that after specifying the starting setup, for instance the internal space or the D-brane content, the particle spectrum and the interactions are fixed.

String/M-theory

To begin we have the 10d string theories: $E_8 \times E_8$ heterotic, $SO(32)$ heterotic , type I, type IIA and type IIB.

There is also the 11d M-theory.

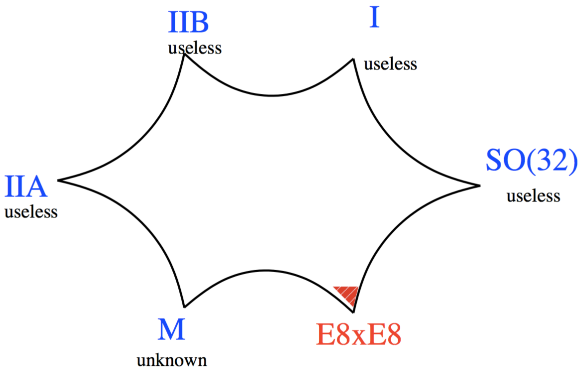
They are now thought to be all manifestations of one theory.



A brief history

In the period 1985-1995 attention mostly focused on compactifications of the $E_8 \times E_8$ heterotic.

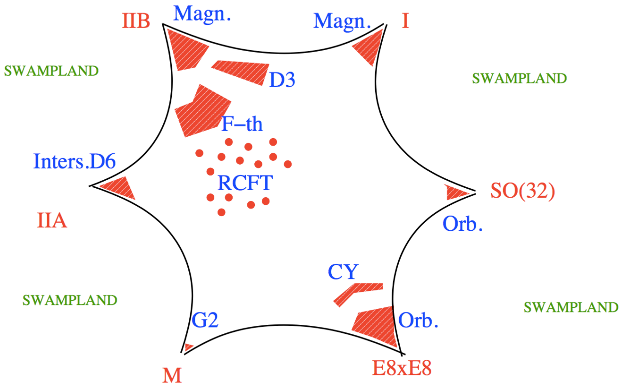
In this theory gauge multiplets are already present in 10d and give rise to e.g. E_6 GUTs and chiral fermions in 4d.



A brief history

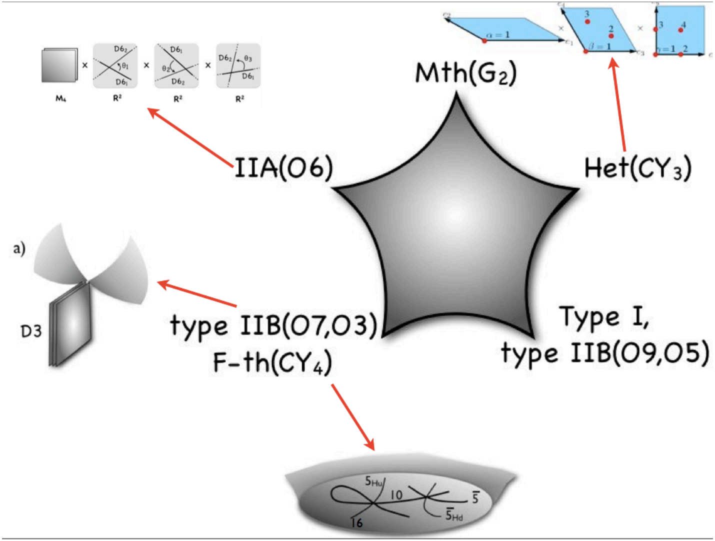
After the advent of D-branes in 1995 it was understood how the SM could be reproduced in the context of type I and type II strings.

At present all corners of the underlying theory are being explored.



Figures from *Sumary Talk, String Pheno 2014* by L.E.Ibáñez

Classes of models



Preview

In these lectures we will study realizations of the SM via:

- Compactification of the heterotic string on orbifolds and Calabi-Yau (CY) manifolds.
- D-brane constructions.

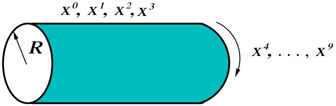
Some generic properties that are found:

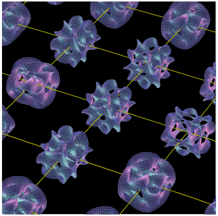
- Chiral fermionic spectrum.
- Family replication.
- Gauge coupling unification, with or without GUT.
- Existence of moduli, i.e. massless scalars whose undetermined vacuum expectation values (vevs) give coupling constants.

Compactifications of the heterotic string

1985

Kaluza-Klein idea: $\mathcal{M}_{10} = \mathcal{M}_4 \times K_6$





Gauge vectors in $10d$: A_M^k,

$M = 0, \ldots, 9, \quad k = 1, \ldots, \dim G_{\rm het}, \quad G_{\rm het} = E_8 \times E_8$ or $SO(32)$

Compactifying on $K_6 = \mathrm{T}^6$ gives fields in $4d$:

$A_\mu^k, \;\; \mu = 0, \ldots, 3$ gauge vectors $\oplus$ $A_m^k, \;\; m = 4, \ldots, 9$ 6 charged scalars

$10d$ gauginos give susy partners in $4d$

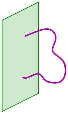
$\mathcal{N} = 4$ theory, non-chiral fermions

This problem is avoided if K_6 has $SU(3)$ holonomy as in CYs and orbifolds.

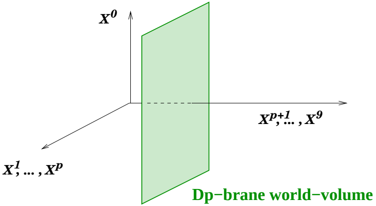
D-branes and gauge theories

1995

degrees of freedom:



open superstrings



massless states:
gauge multiplet, charged multiplets

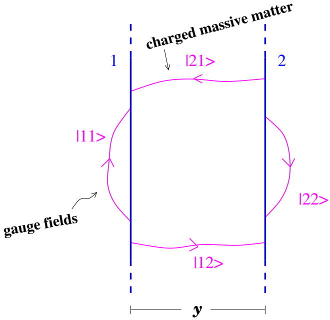
example: susy Yang-Mills in D7-branes

$b^i_{-\frac{1}{2}}|0\rangle$, $i = 1, \cdots, 6$, $b^8_{-\frac{1}{2}}|0\rangle$, $b^9_{-\frac{1}{2}}|0\rangle$ massless Neveu-Schwarz states

fields A_M, Φ, en 8 dim (Φ : complex scalar $\sim$ transverse degrees of freedom)

gauginos λ $\Leftarrow$ massless Ramond states

similar: $4d, \mathcal{N} = 4$ susy Yang-Mills in D3-branes

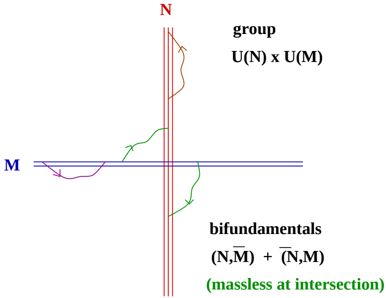


$$U(1) \times U(1) \xrightarrow{y=0} U(2)$$

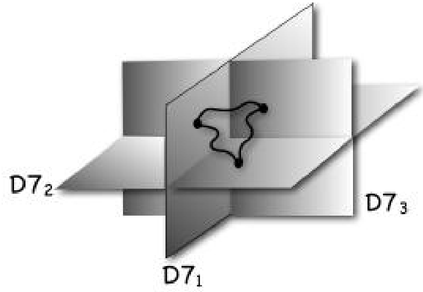
Higgs mechanism = brane separation

$\Phi \sim y$ (transverse d.o.f.)

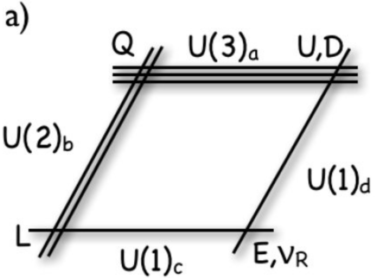
$$\langle \Phi \rangle \neq 0 \iff y \neq 0$$



Intersecting D-brane models

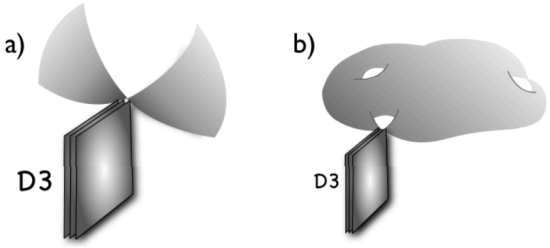


Madrid model



Global vs. local models

- Heterotic models are global. Full knowledge of the internal space is needed. All phenomenological questions have to be addressed at once.
- D-branes allow for localized SM. Questions like gauge group, chiral spectrum, Yukawa couplings, can be addressed one by one, i.e. in a bottom-up approach. In the end it is necessary to embed in full compactification.



The string scale $M_s = 1/\sqrt{\alpha'}$ and $M_P \sim 10^{19}\mathrm{GeV}$

In perturbative heterotic

effective action in $10d$

$$S_{10} \sim M_s^8 \int d^{10}x \sqrt{-G}\, e^{-2\varphi} \left(\mathcal{R} \;+\; M_s^{-2} F_{MN}^2\right) + \cdots$$

compactification $\mathcal{M}_{10} = \mathcal{M}_4 \times K_6$ gives effective action in $4d$

$$S_4 \sim \int d^4x \sqrt{-g} \left(M_P^2 \mathcal{R}_4 \;+\; \frac{1}{g_{\mathrm{YM}}^2} F_{\mu\nu}^2 \right) + \cdots$$

$$M_P^2 \sim \frac{M_s^8 V_6}{g_s^2} \qquad ; \qquad \frac{1}{g_{\mathrm{YM}}^2} \sim \frac{M_s^6 V_6}{g_s^2}$$

$V_6 = \mathrm{Vol}(K_6)$, $g_s = e^{\langle\varphi\rangle}$

$$M_s \sim g_{\mathrm{YM}} M_P \sim 10^{18}\mathrm{GeV}$$

In D-brane constructions

Recall that on a Dp-brane gauge fields propagate only on the $(p+1)$-dim world-surface, so they must wrap only a $(p-3)$-cycle in K_6. The relation between g_{YM} and M_s involves only the volume of this cycle. E.g. for a D3-brane

$$\frac{1}{g_{\text{YM}}^2} \sim \frac{1}{g_s}$$

As before $M_P^2 \sim \dfrac{M_s^8 V_6}{g_s^2}$. Then, for a D3-brane

$$M_s^8 \sim \frac{M_P^2\, g_{\text{YM}}^4}{V_6}$$

Now it is possible $M_s \ll M_P$ by having large extra dimensions transverse to the brane.

IV. Heteroitc model building

Compactification on Calabi-Yau (CY) manifolds

Recall fields in $10d$

$$\{\varphi, G_{MN}, B_{MN}, \Psi, \Psi_M\} \;\oplus\; \left\{A_a^M, \chi_a\right\}, \quad a = 1, \ldots, \dim G_{\text{het}} \qquad G_{\text{het}} = E_8 \times E_8, SO(32)$$

Compactification $\mathcal{M}_{10} = \mathcal{M}_4 \times K_6 \qquad ds^2 = G_{\mu\nu} dx^\mu dx^\nu + G_{mn} dx^m dx^n$

Supersymmetry in $4d$ $\Rightarrow$ $D_M \epsilon = 0$ $\Rightarrow R_{MN} = 0 \quad R_{\mu\nu} = 0, R_{mn} = 0$

K_6 Ricci-flat $\Rightarrow$ holonomy $SU(3) \quad D_m \eta = 0 \qquad K_6$ is CY

Furthermore, K_6 is Kähler (complex with special property of the metric)

$x^m \longrightarrow z^i, \bar{z}^{\bar{i}} \qquad$ k-forms: $\omega_{m_1 \ldots m_k} \qquad$ (p,q)-forms: $\omega_{i_1 \ldots i_p \bar{j}_1 \ldots \bar{j}_q}$

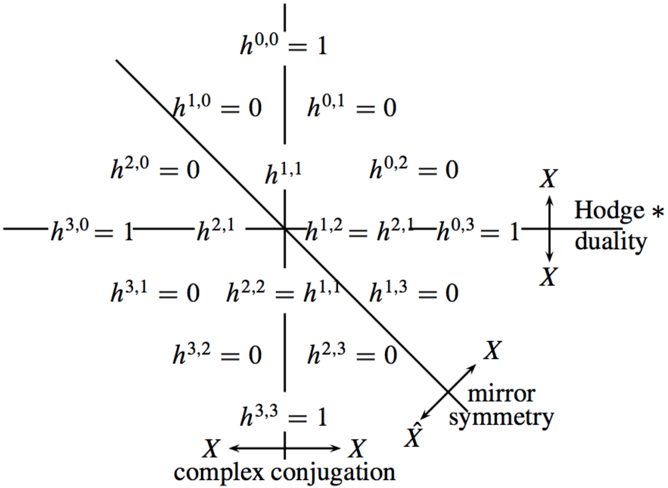
Betti numbers $b_k = \#$ closed (mod exact) k-forms $= \#$ harmonic k-forms

Hodge numbers

$h^{p,q} = \#$ closed (mod exact) (p,q)-forms $= \#$ harmonic (p,q)-forms

$$b_k = \sum_{p=0}^{k} h^{p,k-p}$$

Hodge diamond of a CY X



$\chi = 2(h^{1,1} - h^{1,2})$ Euler characteristic of X

Hodge plot

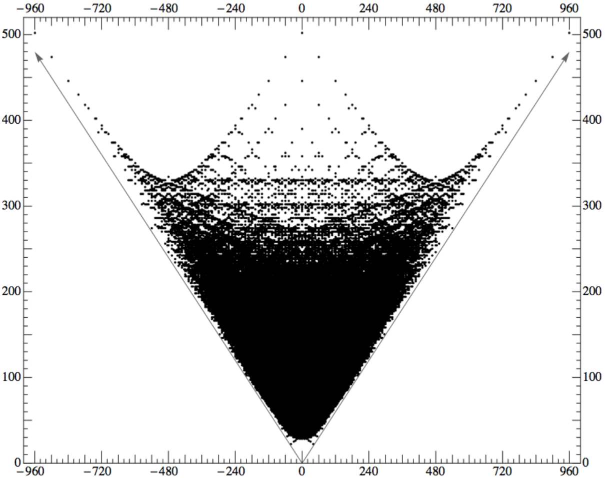


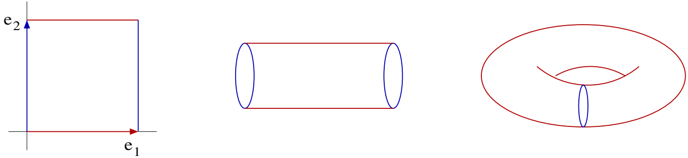
Figure 1: *The Hodge plot for the list or reflexive 4-polytopes. The Euler number* $\chi = 2\left(h^{1,1} - h^{1,2}\right)$ *is plotted against the height* $y = h^{1,1} + h^{1,2}$. *The oblique axes correspond to* $h^{1,1} = 0$ *and* $h^{1,2} = 0$.

Taken from arXiv:1207.4792, based on the Kreuzer-Skarke list

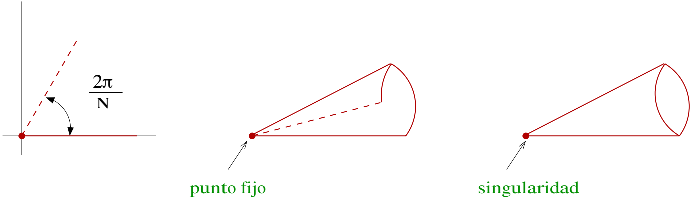
Orbifolds

$\mathcal{O} = \mathcal{M}/\Gamma \quad ; \quad \Gamma =$ discrete group of isometries of $\mathcal{M}$

$\mathbf{T}^2 = \mathbf{R}^2 / \Lambda$



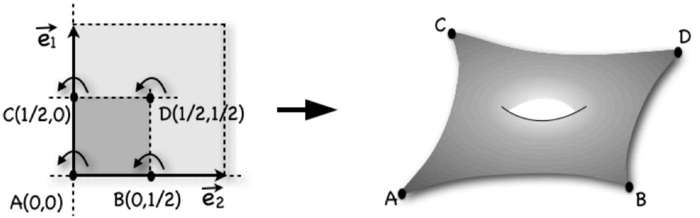
cono $\quad \mathbf{R}^2 / \mathbf{Z_N}$



$T^2/\mathbb{Z}_2$ orbifold

$T^2 = \mathbb{R}^2/\Lambda, \quad \Lambda = SO(4)$ root lattice

$\mathbb{Z}_2 = \{\mathbf{1}, \theta\}, \quad \theta =$ rotation by π

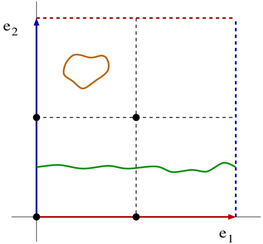
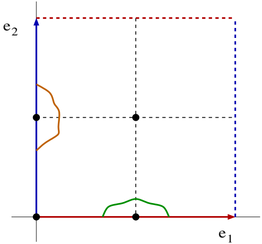


$\bullet$: fixed points $\longrightarrow$ singularities

Closed strings on orbifolds $\mathcal{M}/\Gamma$

Dixon, Harvey, Vafa, Witten

*

untwisted sector	twisted sector
	
$\vec{X}(\tau, \sigma + 2\pi) = \vec{X}(\tau, \sigma) + n_i \vec{e_i}$	$\vec{X}(\tau, \sigma + 2\pi) = g\vec{X}(\tau, \sigma) + n_i \vec{e_i}$
	$g \in \Gamma$ admits fixed points

* Orbifold projection: physical states are invariant under Γ

Both conditions are required by modular invariance

Toroidal orbifolds $\mathrm{T}^6/\mathbb{Z}_N$

$$\mathrm{T}^6 = \mathbb{R}^6/\Lambda, \quad \mathbb{Z}_N = \{\mathbf{1}, \theta, \ldots, \theta^{N-1}\},\ \theta \in SO(6)$$

crystallographic action: $W \in \Lambda,\ \theta W \in \Lambda$

$\theta^N = \mathbf{1} \Rightarrow \theta$ has eigenvalues $e^{\pm 2\pi i v_i}$, $v_i = \dfrac{k_i}{N}$, $k_i \in \mathbb{Z}$, $i = 1, 2, 3$

complex internal coordinates $\quad Z^i = \frac{1}{\sqrt{2}}\left(X^{2i+2} + iX^{2i+3}\right), \quad \theta Z^i = e^{2\pi i v_i} Z^i$

$\theta = \exp 2\pi i(v_1 J_{12} + v_2 J_{34} + v_3 J_{56})$, $J_{2i-1,2i}$: $SO(6)$ Cartan generator

action on spinor representation

$$\theta|\pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}\rangle = e^{i\pi(\pm v_1 \pm v_2 \pm v_3)}|\pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}\rangle$$

supersymmetry $\Rightarrow$ $\pm v_1 \pm v_2 \pm v_3 =$ even

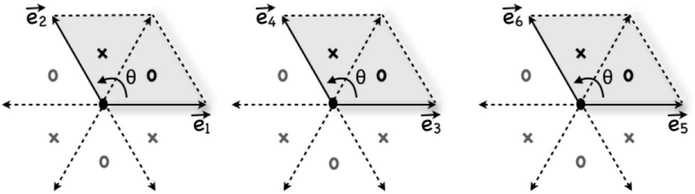
$\theta^N = \mathbf{1}$ acting on fermions $\Rightarrow$ $N(v_1 + v_2 + v_3) =$ even

Example: $T^6/\mathbb{Z}_3$ orbifold

$T^6 = \mathbb{R}^6/\Lambda, \quad \Lambda =$ product of three $SU(3)$ root lattices

$\mathbb{Z}_3 = \{\mathbf{1}, \theta, \theta^2\}, \quad \theta =$ rotation by $\frac{2\pi}{3}$ in each sub-lattice

$(v_1, v_2, v_3) = (\frac{1}{3}, \frac{1}{3}, \text{-}\frac{2}{3})$



$\bullet, \circ, \times$: fixed points altogether $3 \times 3 \times 3 = 27$ fixed points

Action on $2d$ fermions

right movers

θ-twisted sector

$$\Psi^i = \frac{1}{\sqrt{2}}\left(\psi^{2i+2} + i\psi^{2i+3}\right), \qquad \Psi^i(\tau, \sigma + 2\pi) = \mp e^{2\pi i v_i}\Psi^i(\tau, \sigma + 2\pi)$$

left movers   give gauge degrees of freedom

$$\lambda^A_\pm = \frac{1}{\sqrt{2}}\left(\lambda^{2A-1} \pm i\lambda^{2A}\right), \qquad A = 1, \ldots, 16$$

for $E_8 \times E_8$ divide in two groups: $\lambda^A_\pm,\ \lambda'^A_\pm,\ A = 1, \ldots, 8$

Modular invariance requires that λ^A transforms under θ. The action can be realized by rotation γ with eigenvalues $e^{\pm 2\pi i V_A},\ V_A = \frac{K_A}{N},\ K_A \in \mathbb{Z}$.

$\gamma^N = \mathbf{1} \Rightarrow\ N(V_1 + \ldots + V_{16}) = \text{even}$

Gauge shift vector: $V = (V_1, \ldots, V_8) \times (V'_1, \ldots, V'_8),\ \ E_8 \times E_8$

Modular invariance (level-matching) further requires $N(V^2 - v^2) = \text{even}$

Standard Embedding: $V = (v_1, v_2, v_3, 0, \ldots, 0) \times (0, \ldots, 0)$

V. D-brane constructions

D-branes and the SM

- D-branes support gauge theories with gauge group $U(N)$ so its natural to use them to realize the SM group $SU(3) \times SU(2) \times U(1)$.

- Chirality can be obtained via:

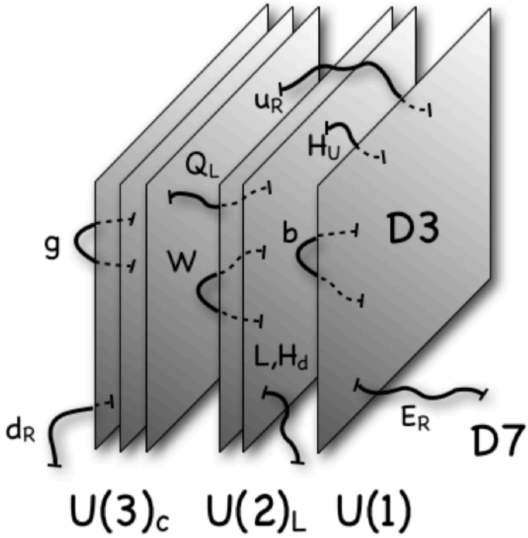
⋆ Branes at singularities.

⋆ Intersecting branes.

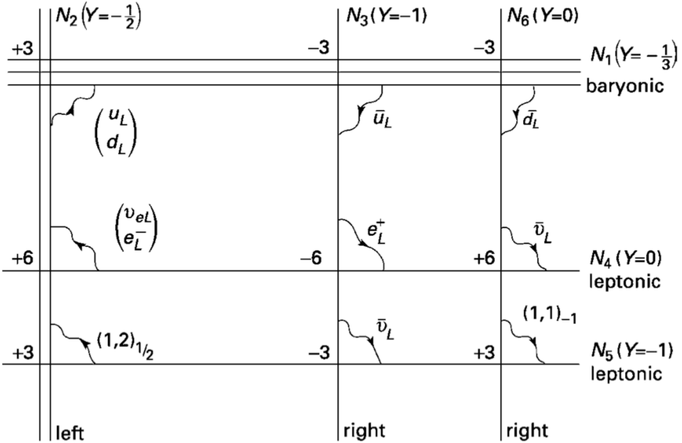
⋆ Magnetized branes.

In these lectures we will describe two simple examples that can be worked out using basic tools.

Example 1: D3-branes at a $\mathbb{C}^3/\mathbb{Z}_3$ singularity



Example 2: Intersecting D6-branes



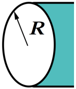
Chap. 21, *A First Course in String Theory, B. Zwiebach, CUP 2009.*

VI. Flux compactifications and moduli stabilization

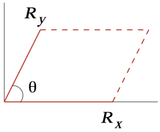
Moduli

Moduli are free parameters of the compactification that change the size and shape of the internal manifold but not its topology. E.g.:

In circle compactification the radius R is a modulus.

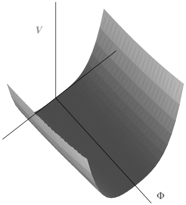


In T^2 compactification there is one Kähler modulus (T) and one complex structure modulus (U).



$$U = -i\frac{R_y}{R_x}e^{i\theta}, \quad T = R_x R_y \sin\theta + iB_{xy} \quad \text{(in heterotic)}$$

In $4d$ the moduli correspond to massless scalars Φ, also called moduli, with a flat potential.



Another important example is the dilaton φ.

(Recall that the effective action is invariant under $\varphi \to \varphi + \text{const.}$)

Stabilization

It is necessary to generate a potential and give masses to the moduli.

- Massless moduli would mediate unphysical long-range fifth forces.

- Vacuum expectation values (vevs) of moduli must be fixed or "stabilized". Coupling constants in the low-energy theory depend on these vevs.

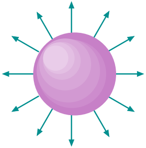
Moduli stabilization can be achieved in flux compactifications

Fluxes

Fluxes are non-trivial backgrounds for YM, NSNS and RR field strengths.

e.g. Maxwell flux

$$\int_{\Pi_2} \langle \mathcal{F}_2 \rangle = g \neq 0$$



e.g. NSNS flux

$$\int_{\Pi_3} \langle H_3 \rangle = h \neq 0 \qquad (H_3 = dB_2)$$

Fluxes thread non-trivial cycles Π_n in the extra dimensions.

Flux induced potentials

$$S_{10} = M_s^8 \int d^{10}x \, \sqrt{-G} \left\{ e^{-2\varphi} \left[\mathcal{R} - H_{MNP}^2 \right] + \cdots \right\}$$

$$\int_{\Pi_3} \langle H_3 \rangle = h \qquad \Rightarrow \qquad V \sim \frac{h^2 e^{2\varphi}}{R^{12}} \quad \text{potential in } 4d$$

$$\text{vol}(K_6) = R^6$$