Disappearance of Mott oscillations in the scattering of identical charged particles

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Quantum theory of scattering - discernible particles:

\[
\left[ -\frac{\hbar^2}{2\mu} \nabla^2 + V(r) \right] \psi^{(+)}(r) = E \psi^{(+)}(r)
\]

asymptotic boundary condition: \( \psi^{(+)}(r) \to A [ \psi_{\text{in}}(r) + \psi_{\text{scatt}}(r) ] \)

Cross section: \( \sigma(\theta) \equiv \frac{d\sigma(\theta)}{d\Omega} = |f(\theta)|^2 \)
identical bosons $\rightarrow$ symmetric wave function:

\[ P \Leftrightarrow T \equiv r \rightarrow -r \implies \theta \rightarrow \pi - \theta \]

\[ \psi(r) = \psi(-r) \implies \psi(r) \rightarrow \frac{1}{\sqrt{2}} [\psi(r) + \psi(-r)] \]

Consequence:

\[ f(\theta) \rightarrow f(\theta) + f(\pi - \theta) \]

And thus,

\[ \sigma(\theta) = \left| f(\theta) + f(\pi - \theta) \right|^2 \]

symmetry around $\pi/2$ !
The cross section can be written,
\\[ \sigma(\theta) = \sigma_{\text{class}}(\theta) + \Xi_{\text{int}}(\theta) \]

Classical term,
\\[ \sigma_{\text{class}}(\theta) = \left| f(\theta) \right|^2 + \left| f(\pi - \theta) \right|^2 \]

Interference (Q.M.) term,
\\[ \Xi_{\text{int}}(\theta) = 2 \Re \left\{ f^*(\theta) \times f(\pi - \theta) \right\} \]
Symmetry around $\pi/2 \rightarrow \sigma(\pi/2)$ is a maximum or a minimum.

Which one? Look at data!

Is this a general result?
An problem with exact solution: Coulomb scattering

- discernible particles (Rutherford scattering)

\[ f_C(\theta) = -\frac{a}{2} e^{2i\sigma_0} e^{-i\eta \ln(\sin^2 \theta/2)} \]

\( \sigma_0 \) is the Coulomb phase shift for \( l = 0 \)

\( \eta = q_p q_T / \hbar v \) is the Sommerfeld parameter

\( a \) is half the distance of c. a. in a head–on collision

Rutherford cross section:

\[ \sigma_R(\theta) = \left| f_C(\theta) \right|^2 = \frac{a^2}{4} \left[ \frac{1}{\sin^4 (\theta/2)} \right] \]
- identical particles (Mott scattering)

\[ \sigma_R(\theta) = \left| f_C(\theta) + f_C(\pi - \theta) \right|^2 \]

classical term (renormalized) – system & energy independent:

\[ \bar{\sigma}_{\text{class}}(\theta) \equiv \frac{\sigma_{\text{class}}(\theta)}{\sigma_{\text{class}}(\pi/2)} = \frac{1}{16} \left[ \frac{1}{\sin^4 (\theta/2)} + \frac{1}{\cos^4 (\theta/2)} \right] \]

exchange term (renormalized) – system & energy dependent:

\[ \bar{\Xi}_{\text{int}}(\theta) \equiv \frac{\Xi_{\text{int}}(\theta)}{\Xi_{\text{int}}(\pi/2)} = \frac{1}{16} \left[ 2 \cos \left[ 2\eta \ln (\tan(\theta/2)) \right] \right] \]

\[ \bar{\Xi}_{\text{int}}(\pi/2) = 0 \]
Behavior of $\sigma_M(\theta)$ around $\pi/2$

$$\bar{\sigma}_{\text{class}}(\pi/2) \rightarrow \text{minimum (constant curvature)}$$

$$\bar{\Xi}_{\text{int}}(\pi/2; \eta) \rightarrow \text{maximum (curvature grows with } \eta)$$

$$\bar{\sigma}(\pi/2) \rightarrow \text{competition : } \bar{\sigma}_{\text{class}} \times \bar{\Xi}_{\text{int}}$$

- minimum for small $\eta$
- maximum for large $\eta$
\( \eta = 0.2 \) (\( \sigma_{\text{class}} \) dominates)

\( \eta = 4.0 \) (\( \Xi_{\text{int}} \) dominates)

- Minimum at \( \eta = 0.2 \rightarrow \left[ \frac{d^2 \bar{\sigma}(\theta; \eta)}{d\theta^2} \right]_{\theta=\pi/2, \eta=0.2} > 0 \)

- Maximum at \( \eta = 0.2 \rightarrow \left[ \frac{d^2 \bar{\sigma}(\theta; \eta)}{d\theta^2} \right]_{\theta=\pi/2, \eta=0.2} < 0 \)
A transition takes place at $\eta = \eta_0$, such that:

$$\left. \frac{d^2 \sigma(\theta; \eta)}{d\theta^2} \right|_{\theta=\pi/2, \eta=\eta_0} = 0$$

That is:

$$\frac{d^2}{d\theta^2} \left[ \frac{1}{\sin^4(\theta/2)} + \frac{1}{\cos^4(\theta/2)} + 2 \frac{\cos[2\eta \ln(\tan(\theta/2))]}{\sin^2(\theta/2) \cos^2(\theta/2)} \right] = 0$$

Very complicated equation ….. but can be solved analytically by programs like Maple
The solution is $\eta_0 = \sqrt{2}$

Thus, for this Sommerfeld parameter, the cross section should be very flat. (Transverse isotropy - TI)
Experimental investigation of TI

Nucleus-nucleus collisions are good candidates

\[ V_N(r) \sim 0 \quad \Rightarrow \quad V_{\text{tot}} \sim V_C \]

Is this condition satisfied?
Ideal system: $^4\text{He} + ^4\text{He}$, $E_{\text{c.m.}} = 397$ keV
$^4\text{He} - ^4\text{He}$ elastic scattering

data at $E_{c.m.} = E_\alpha/2 \geq 1.0 \text{ MeV}$

- no data at $E_{c.m.} \sim 0.4 \text{ MeV}$
- TI at $E_{c.m.} \sim 1.0 \text{ MeV}$
- TI at $E_{c.m.} \sim 1.5 \text{ MeV}$

Are there other $\eta_0$ solutions for

$$\left[ \frac{d^2\sigma(\theta; \eta)}{d\theta^2} \right]_{\theta=\pi/2, \eta=\eta_0} = 0$$

Abdullah et al. NPA 775 (2006) 1
Second derivative for pure Coulomb potential

\[ \sigma''(90^\circ) \] (barn/rad^2)

\[ E_{c.m.} \] (MeV)

\[ V = V_C \rightarrow \text{T.I. only for } E_{c.m.} = 0.397 \text{ MeV} !! \]
Effect of nuclear forces?

Akŷuz-Winther interaction (popular in HI collisions)

\[ V_C \rightarrow V_C + V_N^{A.W.} \quad \implies \quad V_B = 0.87 \text{ MeV} \]

Data at \( E_{\text{c.m.}} = 1 \text{ MeV} > V_B \)

Nuclear potential is not negligible!

\[ \left[ \frac{d^2 \sigma(\theta; \eta)}{d\theta^2} \right]_{\theta=\pi/2, \eta=\eta_0} = 0 \]

Must be taken into account in the condition,
$V_C + V_N^{A.W.}$

- $E_{c.m.} < 0.4 \text{ MeV} : \sigma'' < 0 \rightarrow \text{maximum at 90°}$
- $E_{c.m.} = 0.4 \text{ MeV} : \sigma'' = 0 \rightarrow \text{flat(TI)}$
- $0.4 < E_{c.m.} < 1.1 : \sigma'' > 0 \rightarrow \text{minimum at 90°}$
- $E_{c.m.} = 1.1 \text{ MeV} : \sigma'' = 0 \rightarrow \text{flat(TI)}$
- $E_{c.m.} > 1.1 \text{ MeV} : \sigma'' < 0 \rightarrow \text{maximum at 90°}$
Theory

- $E_{\text{c.m.}} < 0.4 \text{ MeV}$: $\rightarrow$ maximum
- $E_{\text{c.m.}} = 0.4 \text{ MeV}$: $\rightarrow$ flat (TI)
  Cannot be checked

- $0.4 < E_{\text{c.m.}} < 1.1$: $\rightarrow$ minimum
- $E_{\text{c.m.}} = 1.1 \text{ MeV}$: $\rightarrow$ flat
  About right

- $E_{\text{c.m.}} > 1.1 \text{ MeV}$: $\rightarrow$ maximum
  Wrong! minimum at 1.92 MeV

Experiment
A more realistic potential for $^4\text{He} + ^4\text{He}$ scattering

(AW is a Woods-Saxon. For $^4\text{He}$, gaussians are more realistic)

The BFWC potential (sum of attractive and repulsive gaussians):

$$V_{N}^{\text{FBWC}} = -V_A e^{-r^2/R_A^2} + V_R e^{-r^2/R_R^2}, \quad \text{with} \quad V_A = 123 \text{ MeV}, \quad R_A = 2.13 \text{ fm}$$
$$V_R = 3.0 \text{ MeV}, \quad R_R = 2.0 \text{ fm}$$
Theory (BWFC potential)

- $E_{c.m.} = 1.0 \text{ MeV}$
- $E_{c.m.} = 1.5 \text{ MeV}$
- $E_{c.m.} = 1.92 \text{ MeV}$
- $E_{c.m.} = 2.63 \text{ MeV}$
- $E_{c.m.} = 3.24 \text{ MeV}$
- $E_{c.m.} = 3.74 \text{ MeV}$
- $E_{c.m.} = 4.44 \text{ MeV}$
- $E_{c.m.} = 4.94 \text{ MeV}$
- $E_{c.m.} = 5.44 \text{ MeV}$
- $E_{c.m.} = 5.94 \text{ MeV}$
- $E_{c.m.} = 6.15 \text{ MeV}$
- $E_{c.m.} = 7.60 \text{ MeV}$

Fully consistent !!
Concluding remarks

• Mott scattering is a good testing vehicle for effects arising from different contributions, such as vacuum polarization, dipole polarization, color Van der Waals contribution etc. These are very small contributions to the predominantly Coulomb, nucleus-nucleus interaction, but their effects are quite visible in the interference term of the cross section.

• The Transverse Isotropy is a purely EM effect, but the correction arising from the very weak nuclear interaction at sub-barrier energies is found to be very important to accurately account for the available data.

• Missing, is a measurement of the $^4\text{He} + ^4\text{He}$ cross section at $E_{\text{c.m.}}$ about 0.4 MeV. We are hoping that the measurement is forthcoming.