Many-body localization in the presence of a single particle mobility edge

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Indian Institute of Science

Workshop on Quantum Non-Equilibrium Phenomena

International Institute of Physics

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Ranjan Modak, IISc.

Ranjan Modak and Subroto Mukerjee, Phys. Rev. Lett. 115, 230401 (2015), arXiv:1602.02067 (2016)

Acknowledgments: Rahul Nandkishore, David Huse, Ehud Altman, Shriram Ganeshan and Diptiman Sen

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Localization in the presence of interactions

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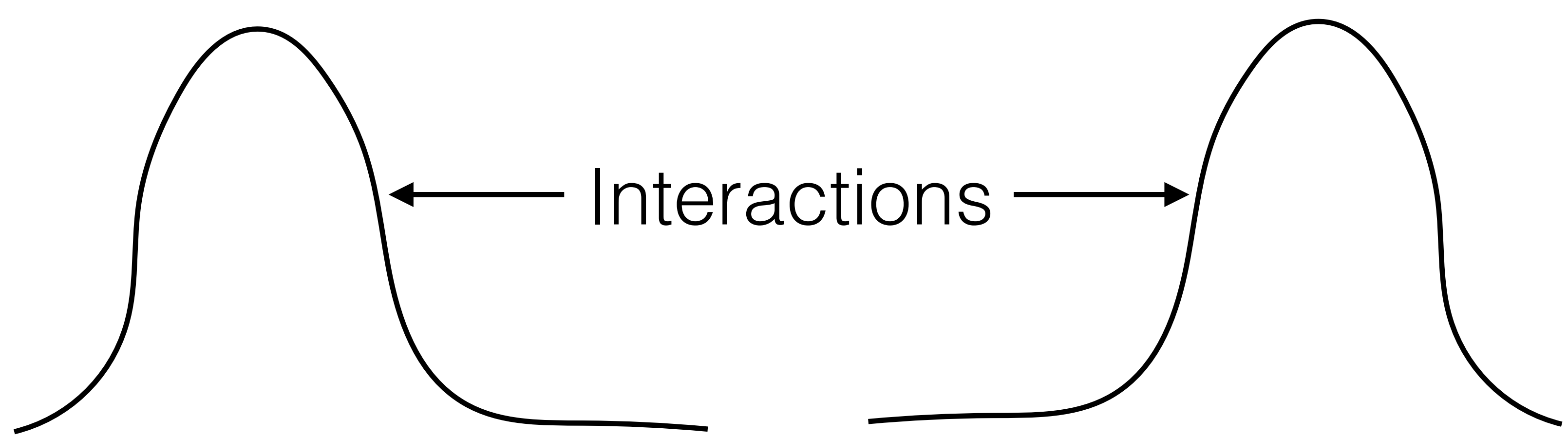

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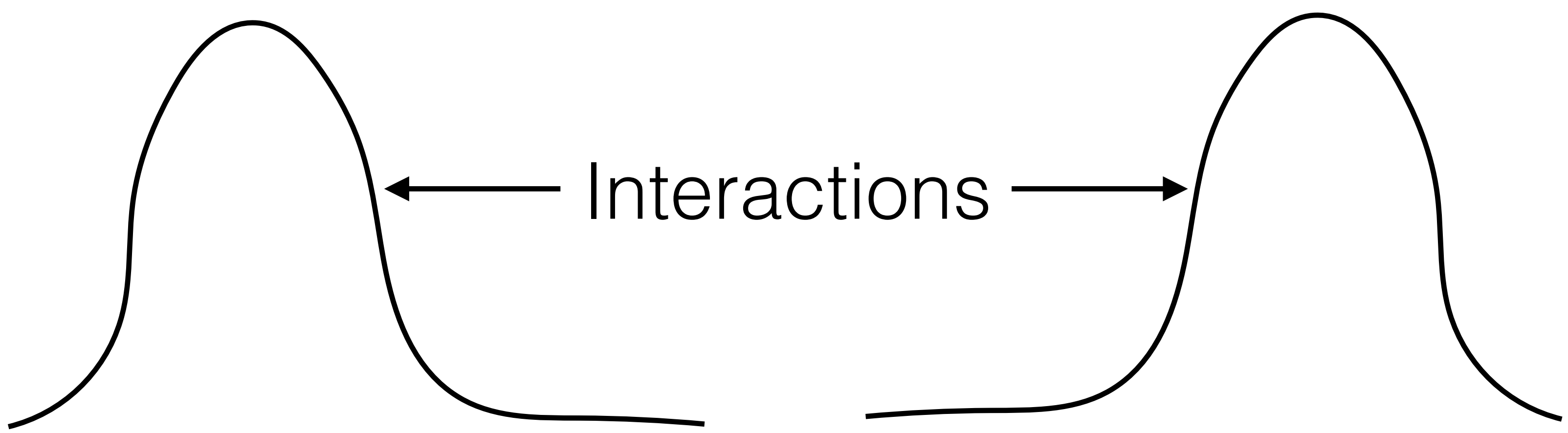

Interactions also tend to cause thermalization in isolated systems

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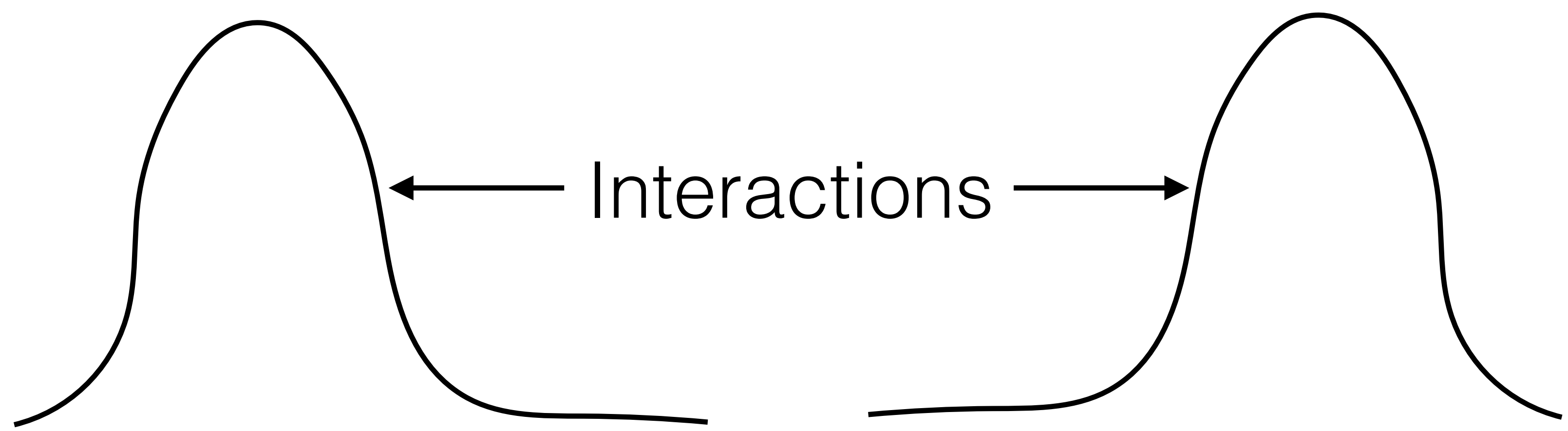

Interactions also tend to cause thermalization in isolated systems

Thus, one would generally expect interacting thermal systems to be delocalized and thermal (diffusive)

Many Body Localization

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Many-Body Localized systems remain athermal even in the presence of interactions

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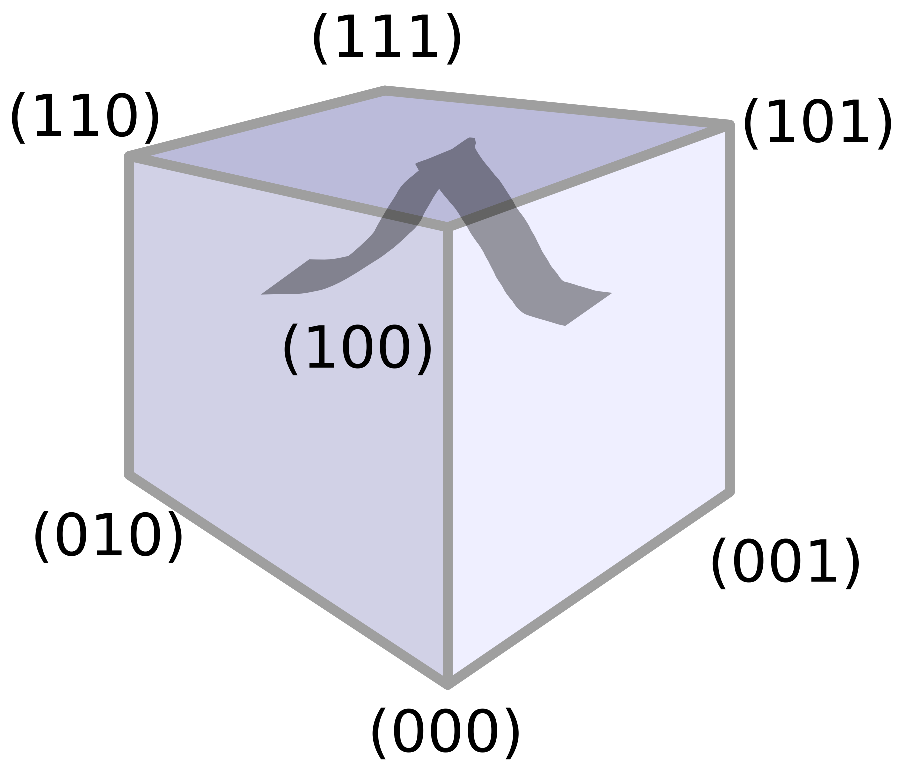
Many-Body energy eigenfunctions are localized in Fock space



Basko, Aleiner and Altshuler, Ann. Phys. 321, 1126 (2006)

Memory of initial many-body state remains under Hamiltonian evolution

Many Body Localization

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Many-body localized systems

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Do not obey the Eigenstate Thermalization Hypothesis (ETH)

ETH - Deustch, PRA 43 2146 (1991); Srednicki, PRE 50 888 (1994); Rigol, Djunko & Olshanii, Nature 452 854 (2008)

Quantum Stat. Mech. does not apply !

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In thermal systems, a typical state has volume law entanglement

Many-body localized systems have an infinite number of conservation laws

Many Body Localization

Thermal phase	Single-particle localized	Many-body localized
Memory of initial conditions 'hidden' in global operators at long times	Some memory of local initial conditions preserved in local observables at long times	Some memory of local initial conditions preserved in local observables at long times.
ETH true	ETH false	ETH false
May have non-zero DC conductivity	Zero DC conductivity	Zero DC conductivity
Continuous local spectrum	Discrete local spectrum	Discrete local spectrum
Eigenstates with volume-law entanglement	Eigenstates with area-law entanglement	Eigenstates with area-law entanglement
Power-law spreading of entanglement from non-entangled initial condition	No spreading of entanglement	Logarithmic spreading of entanglement from non-entangled initial condition
Dephasing and dissipation	No dephasing, no dissipation	Dephasing but no dissipation

Nandkishore and Huse, Annual Review of Condensed Matter Physics, Vol. 6: 15-38 (2015)

Many Body Localization

Many Body Localization

Model Hamiltonian

1D spinless fermions

$$H = -t\sum_j \left(c_j^\dagger c_{j+1} + \text{h.c.}\right) + \sum_j \epsilon_j n_j + V\sum_j n_j n_{j+1}$$

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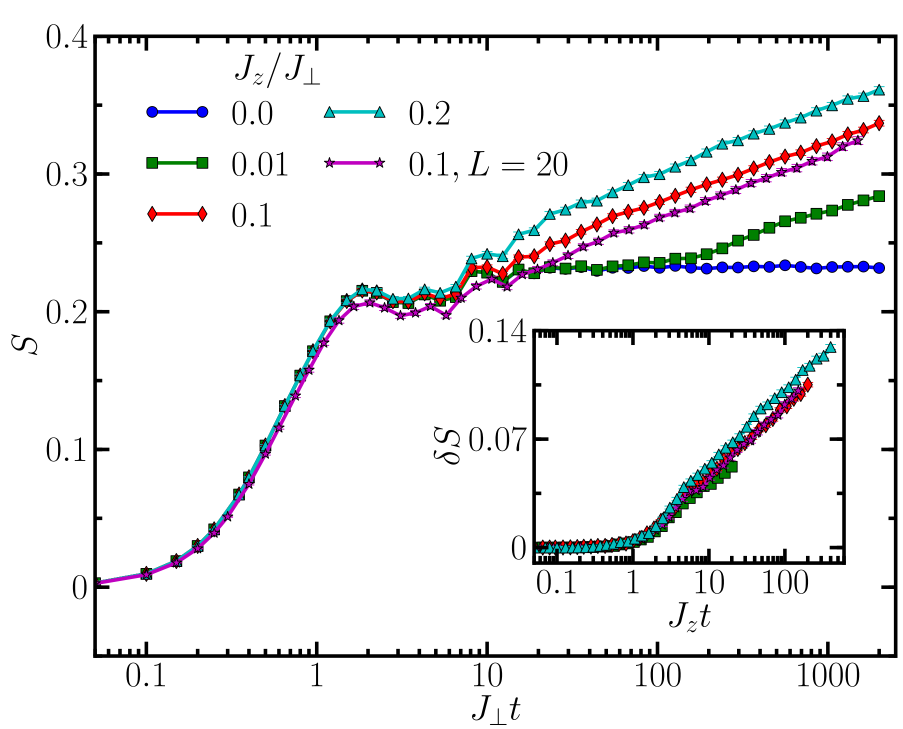
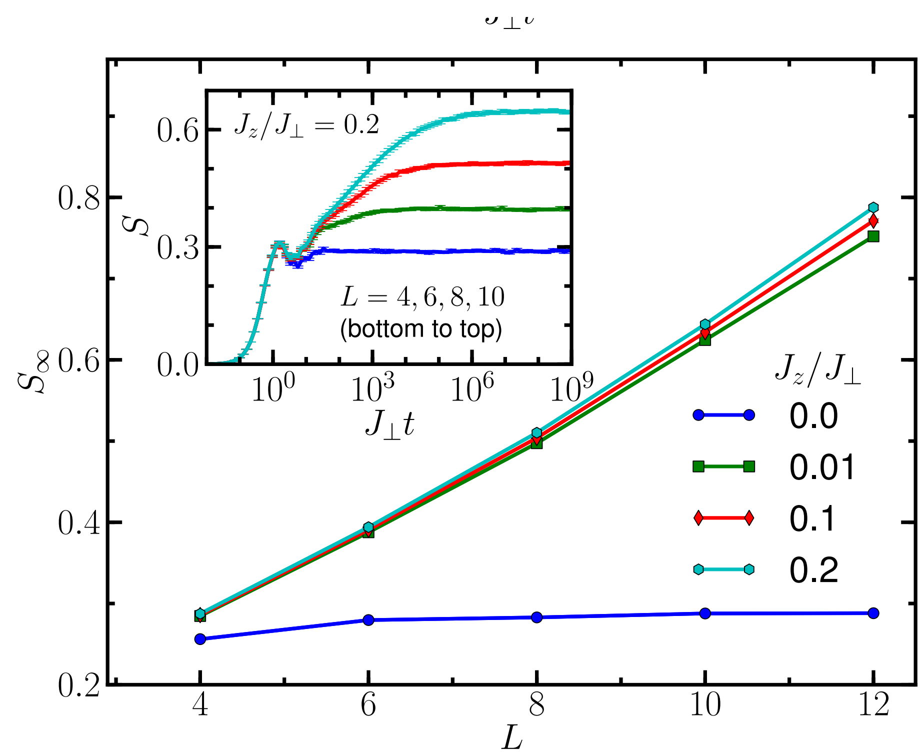
TEBD (short times) and numerical ED (long times)





Bardarson, Pollmann and Moore, PRL 109, 017202 (2012)

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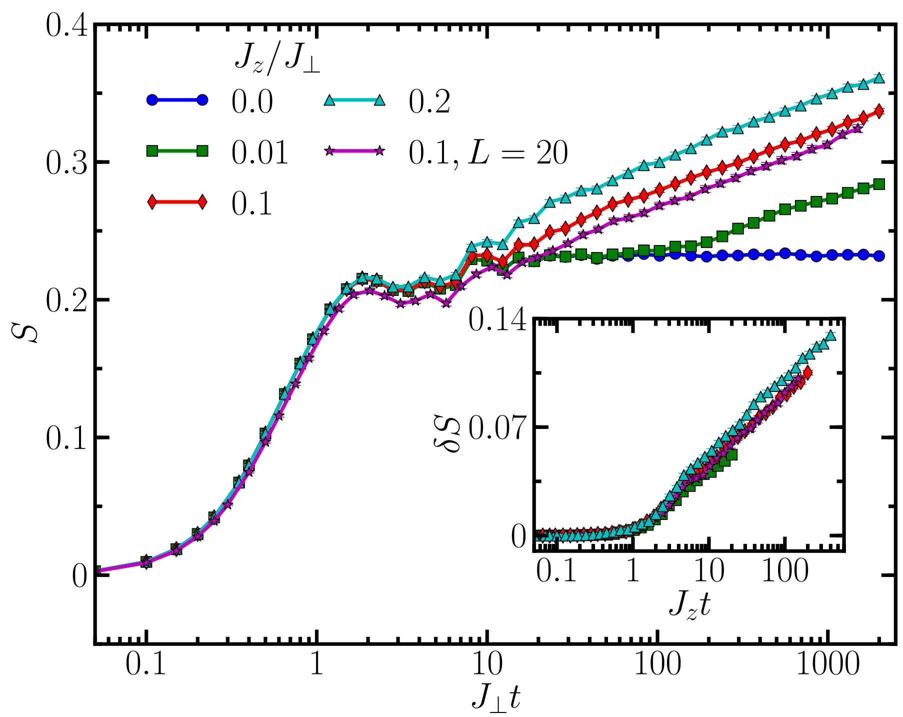

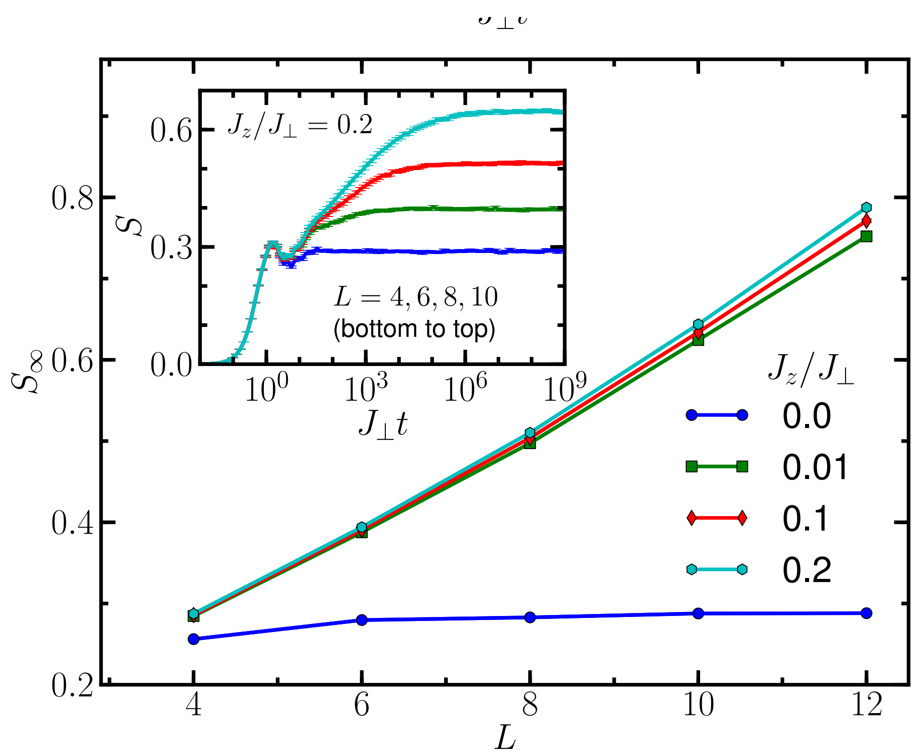

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Entropy increases logarithmically with time and saturates to an extensive subthermal value for a many-body localized system

MBL vs. Thermalization

Weak interactions

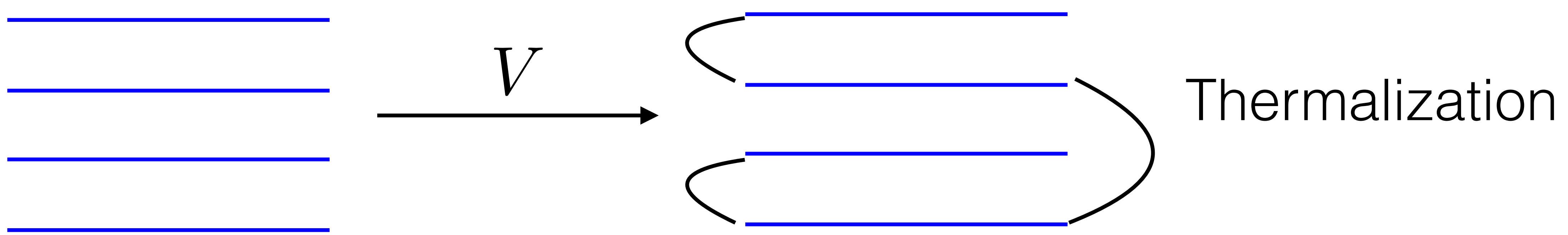
MBL vs. Thermalization

Weak interactions

Ergodic system

Non-interacting limit

All states extended



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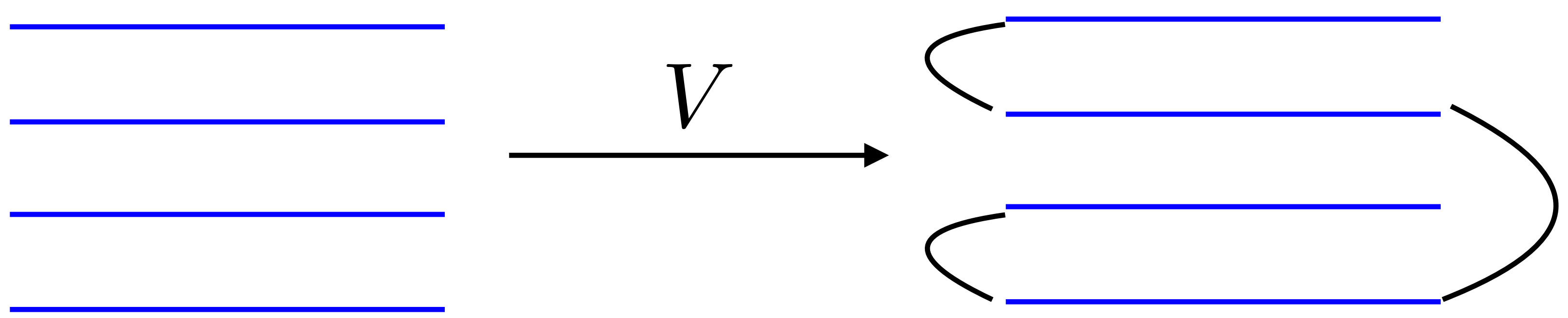
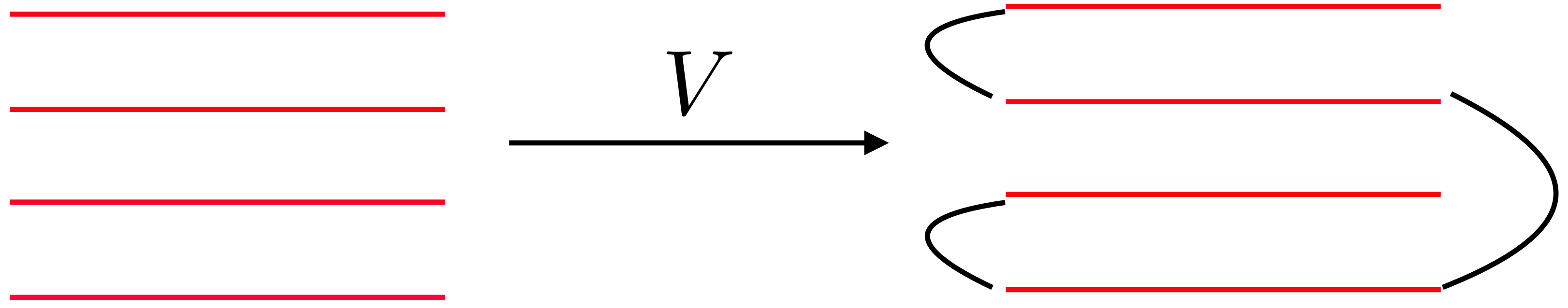


Thermalization

MBL system

Non-interacting limit

All states localized



Localized

MBL with single particle mobility edge

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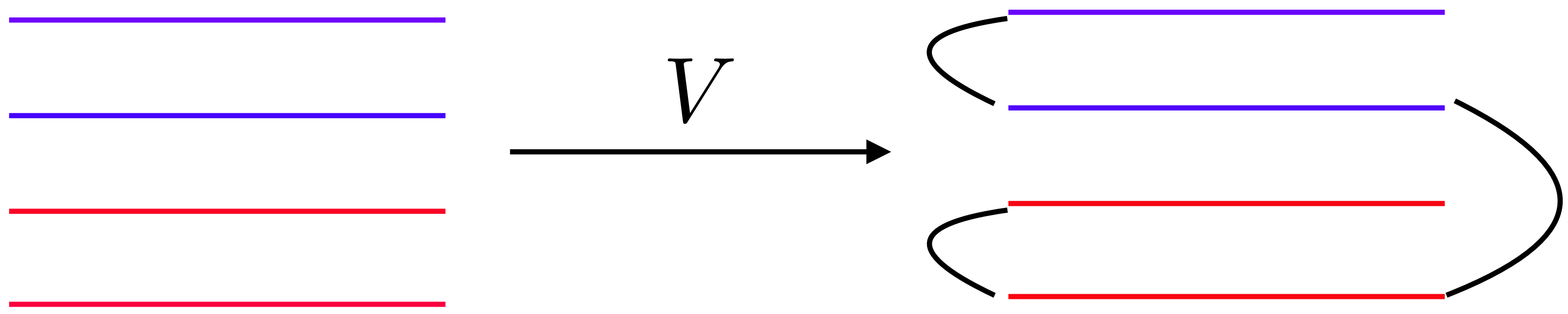
Non-interacting limit

Localized and extended states



?????

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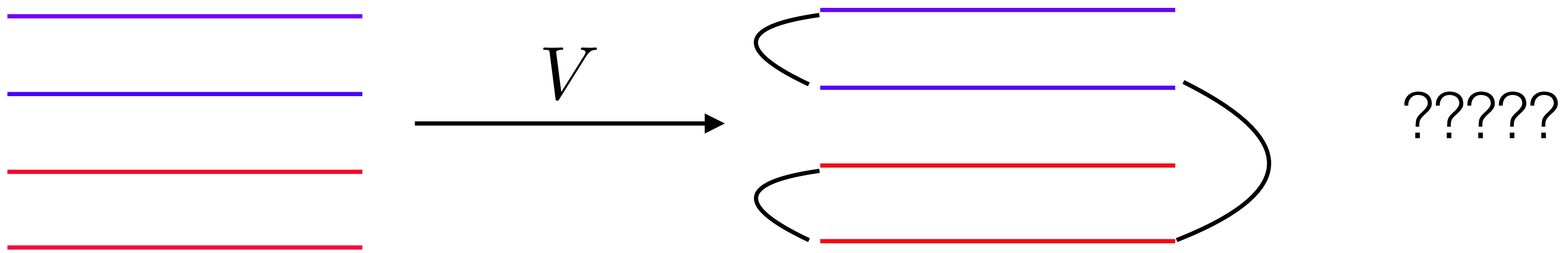
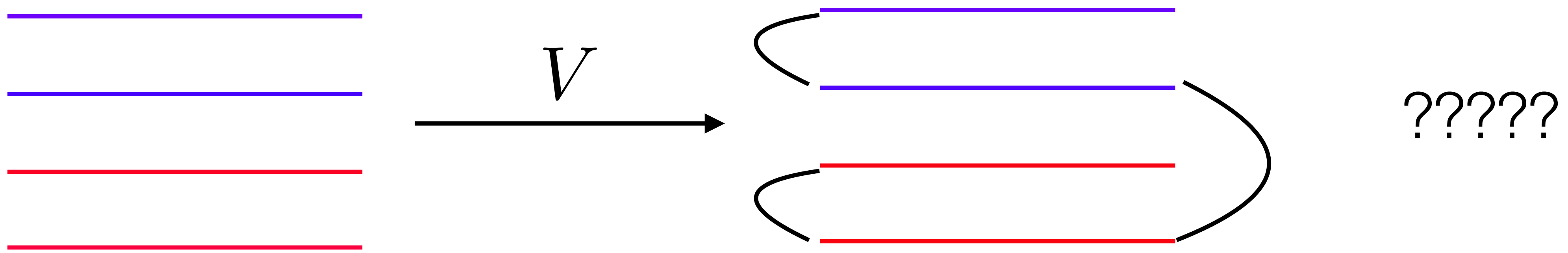


Why is this interesting?

MBL with single particle mobility edge

Non-interacting limit

Localized and extended states



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Even a single **protected** delocalized state can thermalize a localized system coupled to it

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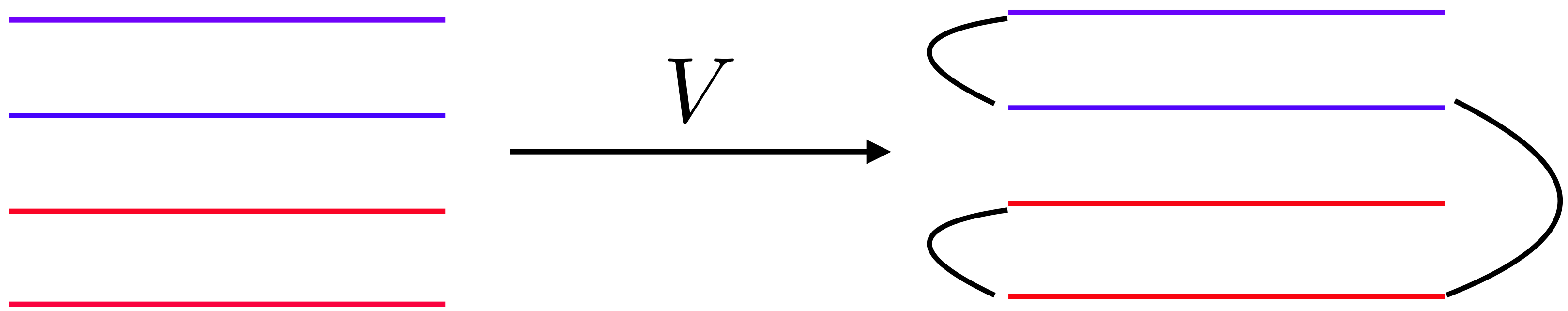

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Nandkishore and Potter, PRB 90 195115 (2014)

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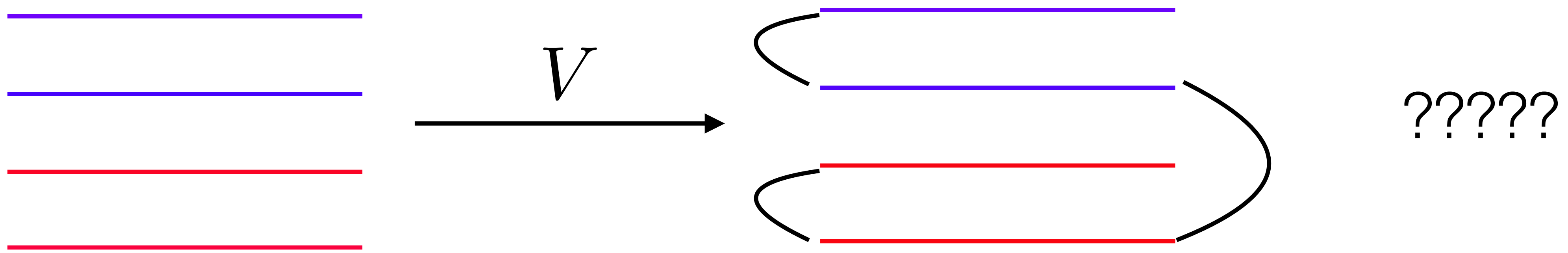

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Nandkishore and Potter, PRB 90 195115 (2014)

If delocalized states are unprotected, they can be localized by the localized states, the “many-body proximity effect”

Nandkishore, Phys. Rev. B 92, 245141(2015)

MBL with single particle mobility edge

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How do we get a single particle mobility-edge in 1D?

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How do we buck Anderson localization?

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Completely correlated across sites but aperiodic

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Completely correlated across sites but aperiodic

In 3D uncorrelated disorder produces mobility edges generically

Models

Models

Aubry-Andre model

Aubry and Andre, Ann. Israel. Phys. Soc. 3, 1 (1980)

$$H = -t \sum_j \left(c_j^\dagger c_{j+1} + \text{h.c.} + \epsilon_j n_j \right)$$

$\epsilon_j = h \cos(2\pi \alpha j)$ α irrational

Quasi-periodic potential

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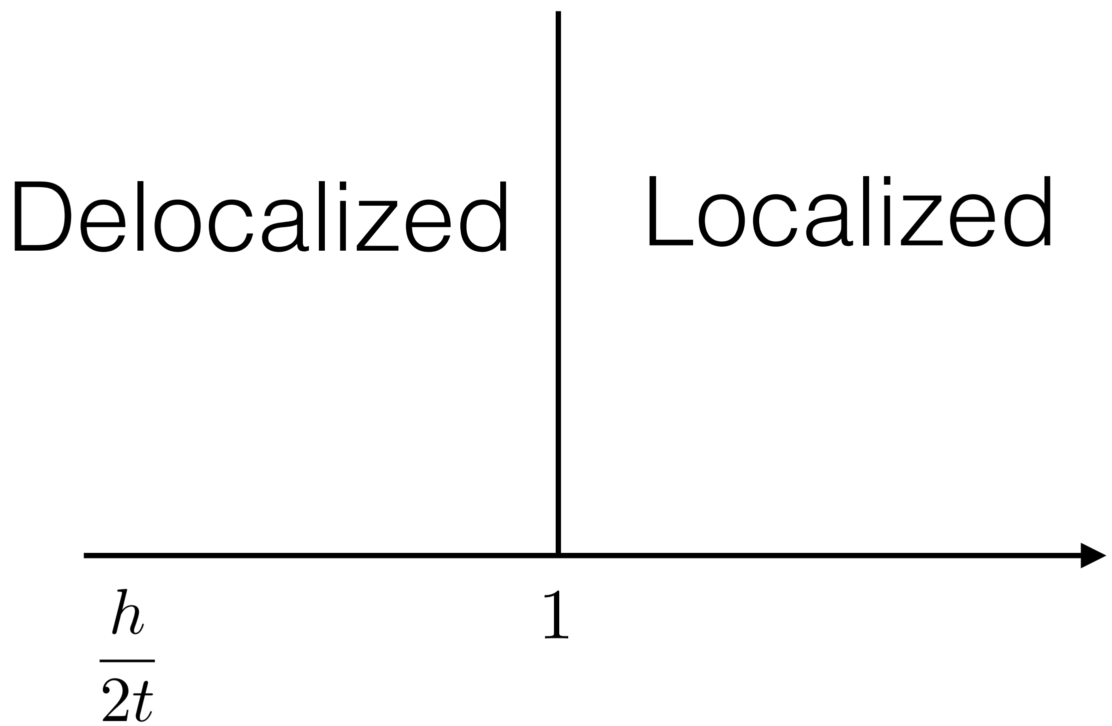
Quasi-periodic potential



Localization-delocalization transition

Models

MBL in the Aubrey-Andre model

spinless fermions

$$H = -t \sum_j \left(c_j^\dagger c_{j+1} + \text{h.c.} + \epsilon_j n_j + {\color{blue} V n_j n_{j+1}} \right)$$

Iyer, Oganesyan, Refael and Huse, PRB 87, 134202 (2013)

Experimental realization

cold atoms ^{40}K- spinful fermions

Schreiber et. al., Science, 349 842 (2015)

Models

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Modified Aubry-Andre models with mobility edges

Model I: $\epsilon_j = h\cos(2\pi\alpha j^\nu)$ $\quad 0 < \nu < 1$

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Delocalized

$2t - h$

Localized

Griniasty and Fishman, PRL 60 1334 (1988)
Das Sarma, He and Xie, PRB 41 5544 (1990)

Position of mobility edge independent of ν

Models

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Model II: $$\epsilon_j = h\frac{1-\cos(2\pi j\alpha)}{1+\beta\cos(2\pi j\alpha)}$$

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Delocalized

$(2t-h)/\beta$

Localized

Ganeshan, Pixley and Das Sarma, PRL 114 144601 (2015)

Mobility edge can be tuned as a function of β

Diagnostics and technique

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- Level spacing statistics
- Entanglement entropy: Growth and saturation value
- Optical conductivity
- Return probability

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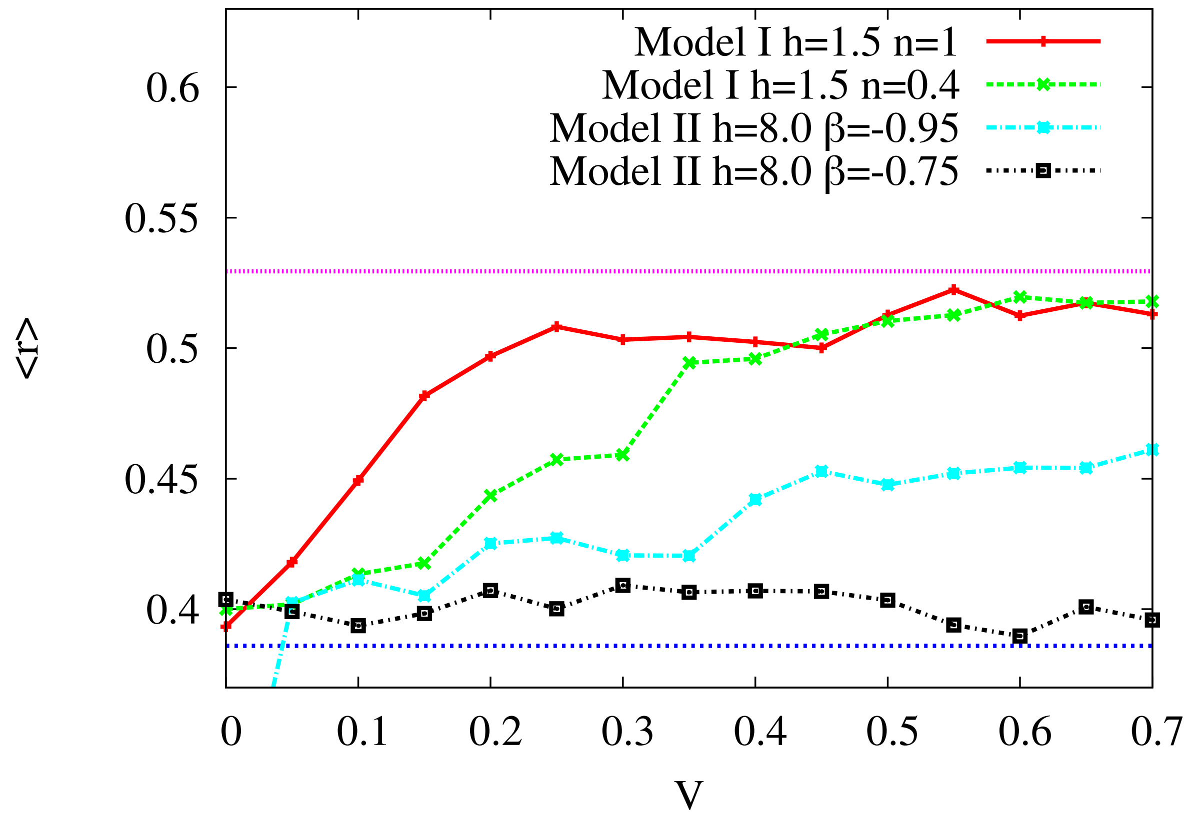
Technique

Numerical exact diagonalization on systems up to $L = 16$

Average over offset angle for better statistics

Results

Level spacing distribution



$$\langle r \rangle = \frac{min\left(\delta_n, \delta_{n+1}\right)}{max\left(\delta_n, \delta_{n+1}\right)} \qquad \delta_n = E_n - E_{n-1}$$

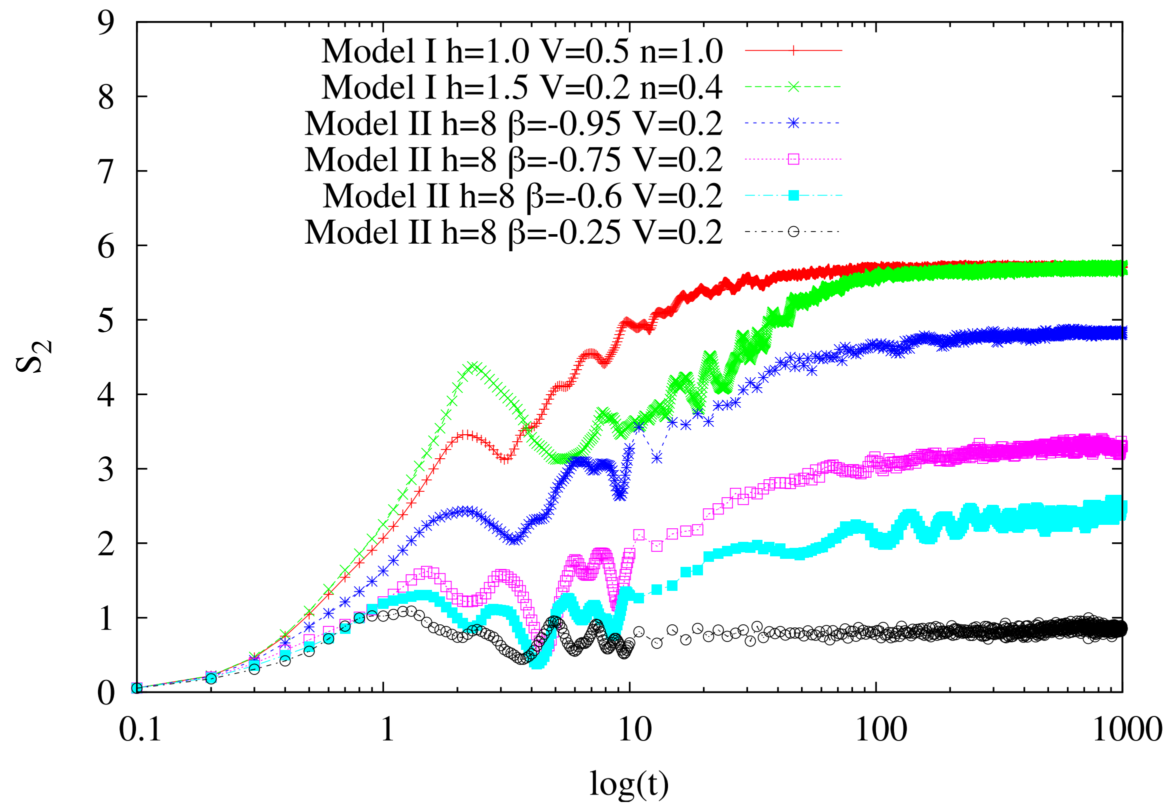
$\langle r \rangle = 0.386$ Poissonian distribution (localized)

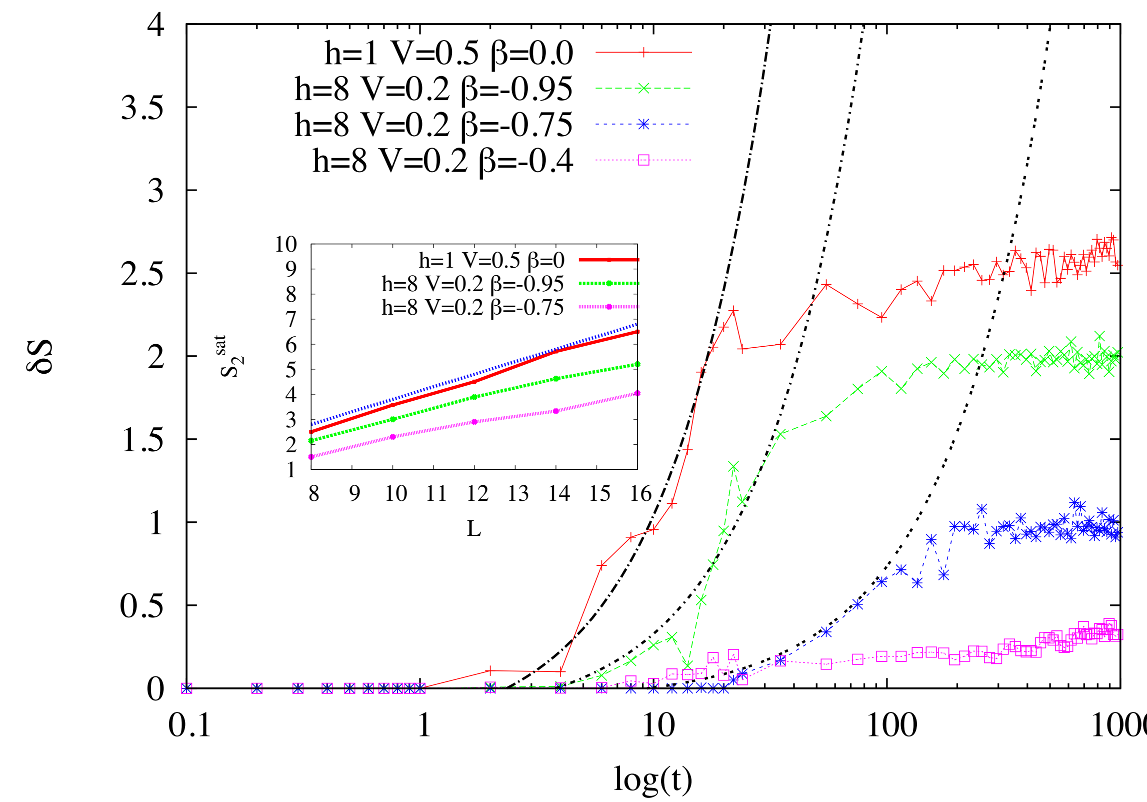
$\langle r \rangle = 0.523$ Wigner-Dyson (thermal)

Modak and Mukerjee, Phys. Rev. Lett. 115, 230401 (2015)

Results

Entanglement entropy





S saturates to thermal (subthermal) value indicates thermalization (localization)

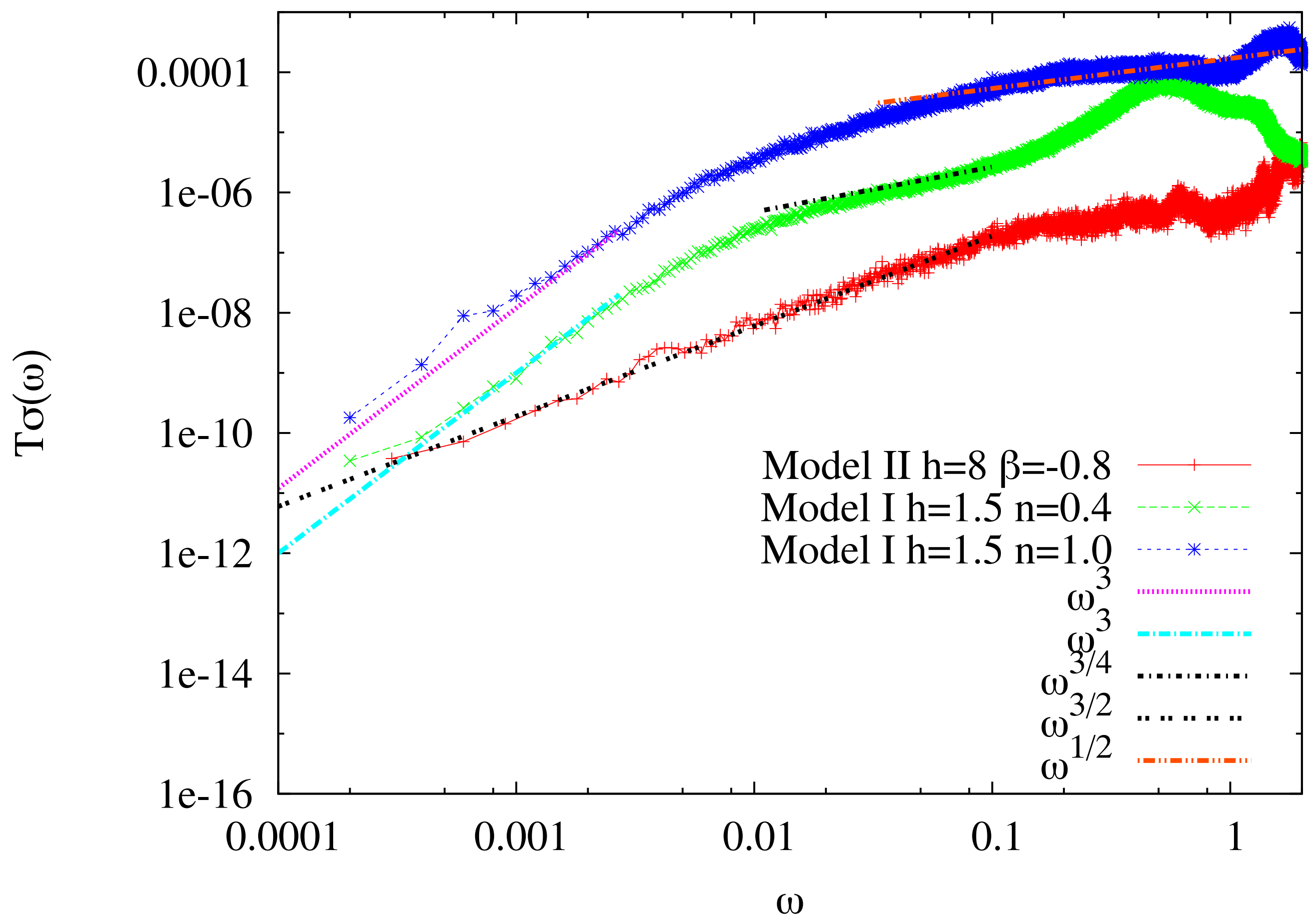
$S(t)$ not logarithmic in time even for localized system!

Length scale $L(t) \sim t^{\alpha}$

Modak and Mukerjee, Phys. Rev. Lett. 115, 230401 (2015)

Results

Optical conductivity as $T \to \infty$



$$T\sigma(\omega) \sim \omega^a \begin{cases} 0 < a < 1 & \text{for thermal} \\ 1 < a < 2 & \text{for localized,} \end{cases}$$

after appropriate subtraction

Modak and Mukerjee, Phys. Rev. Lett. 115, 230401 (2015)

Results

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Model I appears to thermalize

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Model I appears to thermalize

Model II does not thermalize

However, non-ergodicity of model II is not like for MBL: the entropy increases faster than logarithmically with time

Consistent with the existence of non-ergodic metal proposed in these systems

Also Li, Ganeshan, Pixley and Das Sarma, Phys. Rev. Lett. 115, 186601 (2015)

Criterion for Non-ergodicity

What decides if a given model with a single particle mobility edge displays thermalizes upon the introduction of weak interactions?

Ans: How strongly localized the localized states are relative to how strongly delocalized the delocalized ones are.

Criterion for Non-ergodicity

Model I thermalizes but model II does not

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How do we quantify this?

Modak and Mukerjee, arXiv:1602.02067 (2016)

Criterion for non-ergodicity

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$$\epsilon = \frac{\eta(1 - MPR_D/L)}{(MPR_L - 1)}$$

η ratio of # of localized to delocalized states

MPR_D mean participation ratio of delocalized states

MPR_L mean participation ratio of localized states

L system size

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$\epsilon > 1(\mathrm{MBL})$ $\epsilon < 1(\mathrm{Thermal})$

Modak and Mukerjee, arXiv:1602.02067 (2016)

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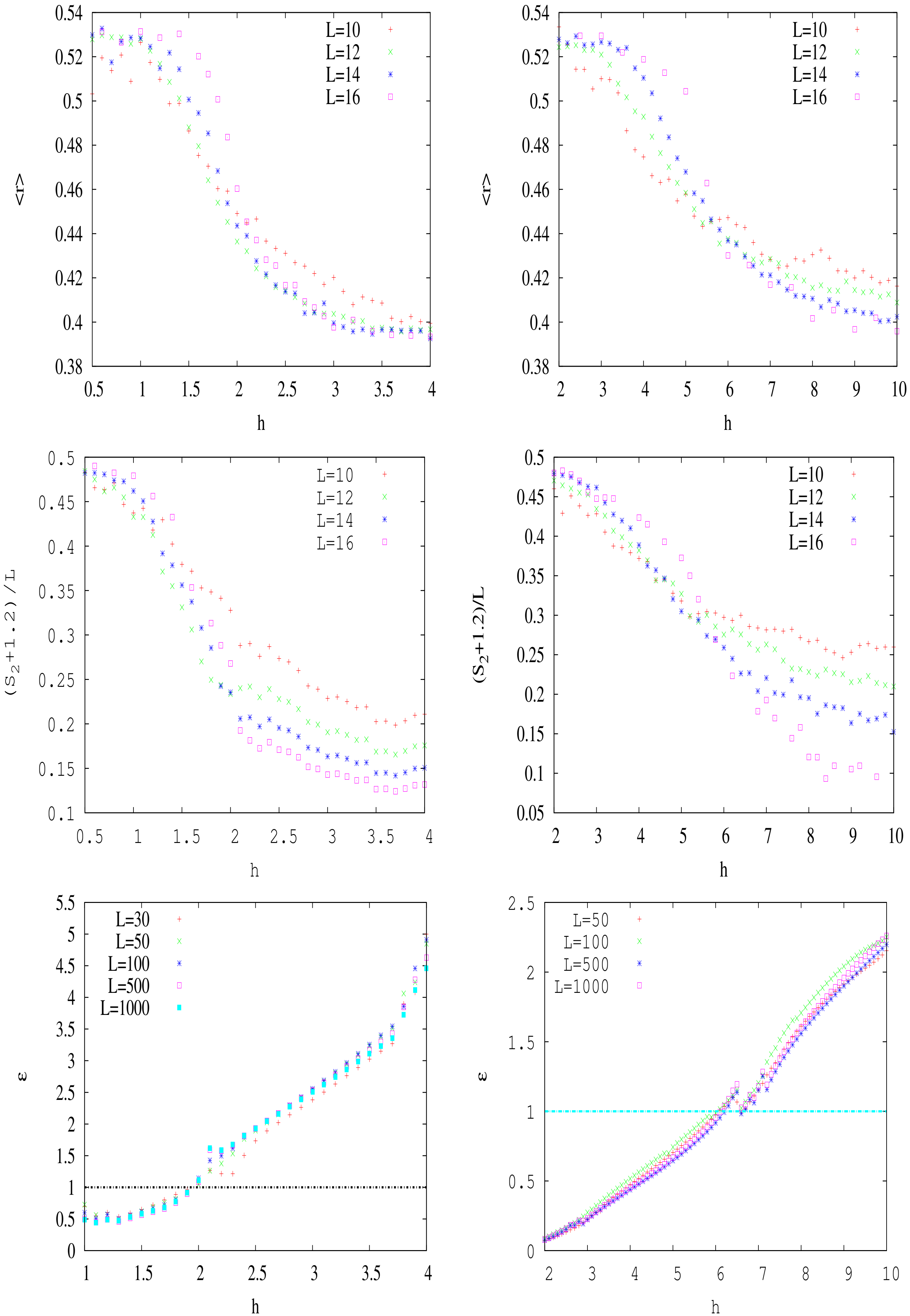

Modak and Mukerjee, arXiv: 1602.02067 (2016)

Criterion for non-ergodicity

Model	Non-ergodic phase	Ergodic phase	ν	ϵ
Model I	Yes	Yes	< 1	> 1 (Non-ergodic phase) and < 1 (Ergodic phase)
Model II	Yes	Yes	< 1	> 1 (Non-ergodic phase) and < 1 (Ergodic phase)
Model III	No	Yes	1	< 1
Model IV	No	Yes	< 1	< 1
Model V	No	Yes	> 1	< 1

$$\epsilon > 1(\mathrm{MBL}) \qquad \epsilon < 1(\mathrm{Thermal})$$

Modak and Mukerjee, arXiv:1602.02067 (2016)

Non-ergodicity and localization

	Ergodic conductor	Non-ergodic conductor	Non-ergodic insulator
ETH	Yes	No	No
Eigenstate entanglement	$\sim L$ (thermal)	$\sim L$ (sub-thermal)	$\sim L^0$
Energy level statistics	Level repulsion	No level repulsion	No level repulsion
$S(t)$	Linear growth	Linear growth	Logarithmic growth
$S(t \to \infty)$	Thermal	Sub-thermal	Sub-thermal
Integrals of motion	None	Non-local (???)	Local

Non-ergodic conductor shares features with traditional integrable systems

Also

Li, Ganeshan, Pixley and Das Sarma, Phys. Rev. Lett. 115, 186601 (2015)

Li, Pixley, Deng, Ganeshan and Das Sarma, Phys. Rev. B 93, 184204 (2016)

Conclusions and questions

- Non-ergodic physics can occur in the presence of a single particle mobility edge but not always

- Criterion for occurrence of the non-ergodicity for weak interactions can be quantified using the single particle spectrum

- How do the local degrees of freedom interact?