Extending the LCDM model through spatial anisotropies

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Quantum fields and IR issues in de Sitter space
Natal - 2015

ΛCDM model ingredients

General Relativity

Extrapolate Einstein's equations to scales above 100 Mpc

+

Matter content

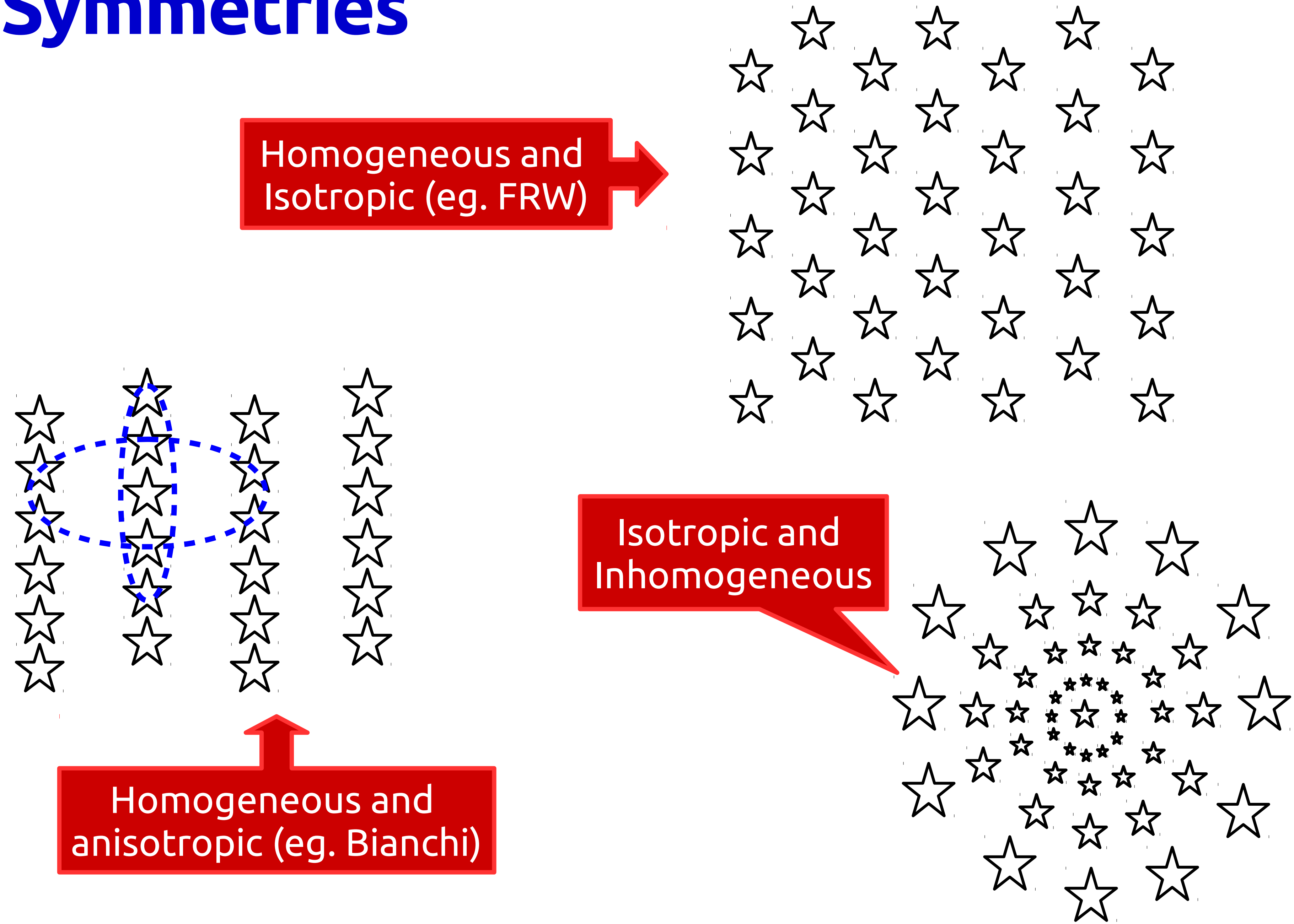
Standard model's particles + Λ + CDM

+

Cosmological Principle

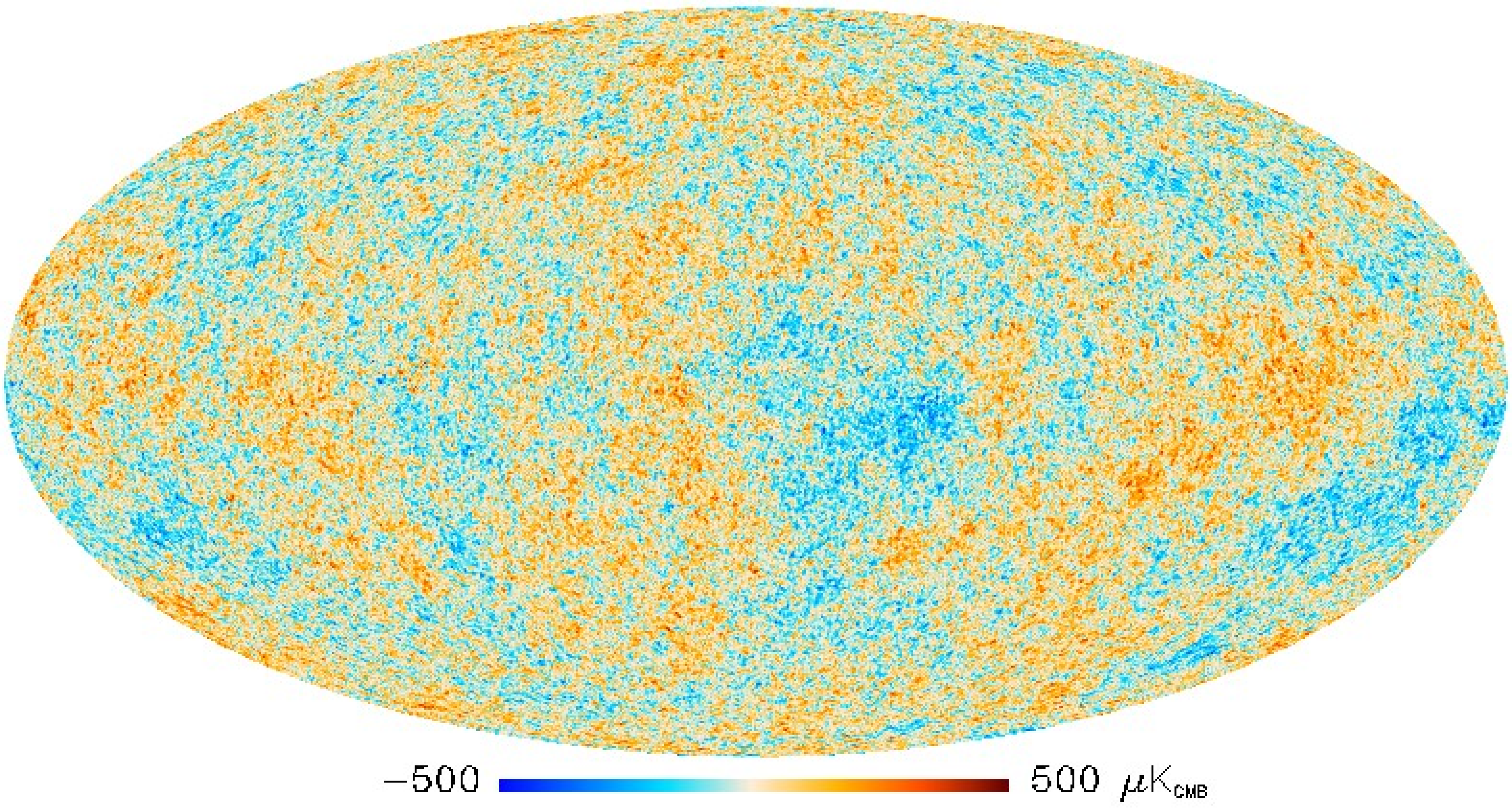
Universe's geometry and topology are as symmetric as possible

Symmetries


Homogeneous and
Isotropic (eg. FRW)
Isotropic and
Inhomogeneous
Homogeneous and
anisotropic (eg. Bianchi)

Early-time constraints: *CMB*

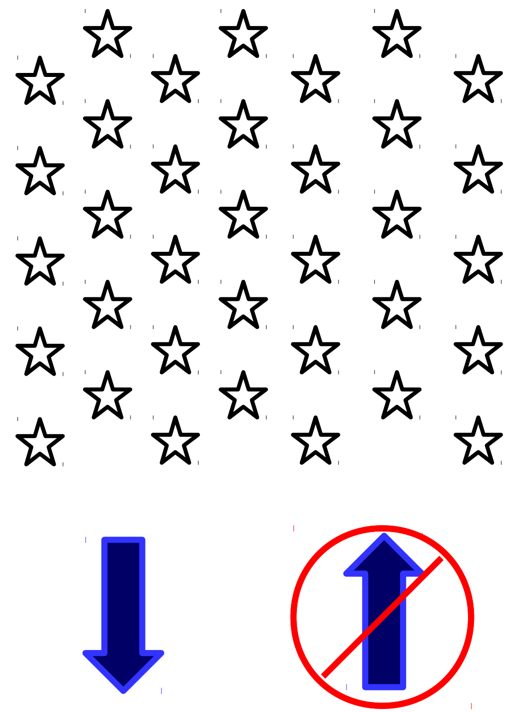
Planck 2013



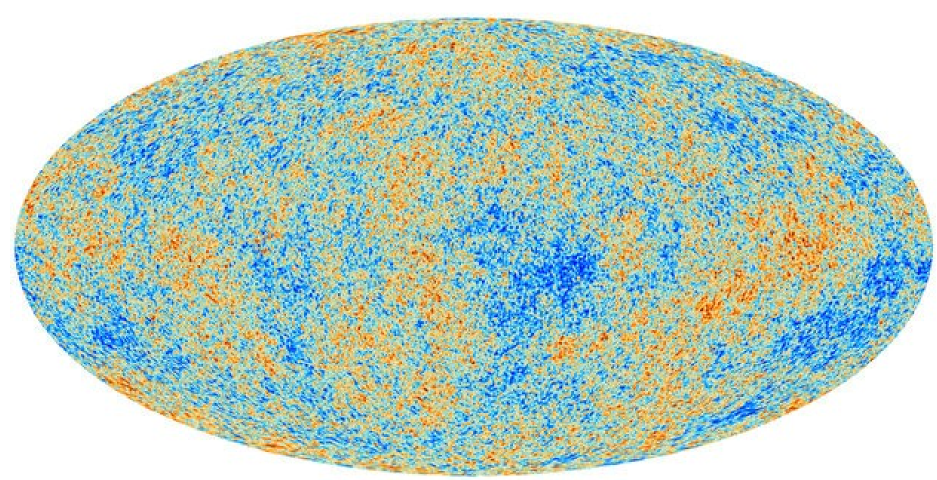
$$\frac{\Delta T}{T} \sim 0.001\% \quad @ \quad z \sim 1100$$

Early-time constraints: *CMB*

FLRW is assumed true at all redshifts...

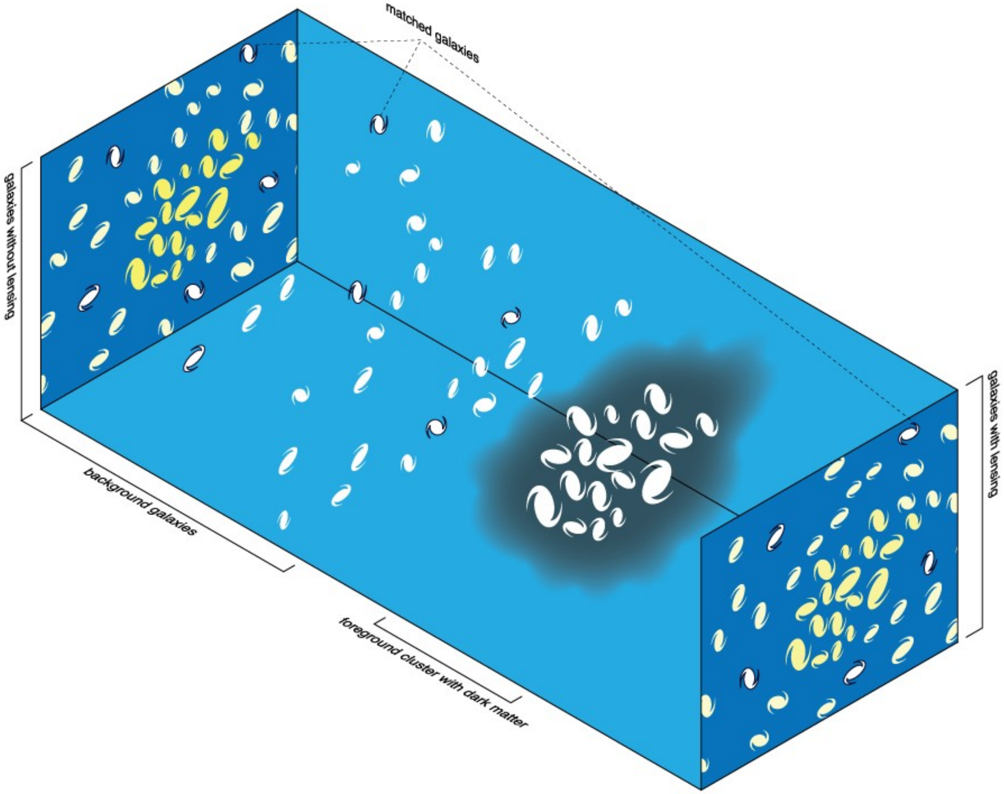


... but CMB is only observed at high redshifts



Late-time constraints: *Weak lensing*

Weak-lensing of large scale structure is a powerful tool to probe late-time geometry



(figure: en.wikipedia.org/wiki/Weak_gravitational_lensing)

Why care about spatial anisotropies?

- Because isotropy is a very strong hypothesis
  - Early-time anisotropy can be imprinted on CMB
  - Late-time anisotropy can result from backreaction
- Because we can!
  - Astronomical data has increased enormously, and is still growing...
- CMB anomalies
  - Though not strongly significant (~ 3σ), they are typical signatures of anisotropy
- Because it is fun! =)

Messages I plan to deliver

- Message 1
  - Homogeneous and anisotropic models are the simplest extension of FLRW
  - Perturbation theory in these models is feasible!
  - There is much more to anisotropy than anisotropic expansion.
- Message 2
  - There are ways to evade the isotropy of CMB
  - B-modes of cosmic shear is a direct tracer of late-time anisotropy.
- This talk
  - Part 1: Linear perturbation theory
  - Part 2: primordial anisotropies $\quad \rightarrow \quad T(t,k), \quad P(\vec{k})$
  - Part 3: late-time anisotropies $\quad \rightarrow \quad T(t,\vec{k}), \quad P(k)$
- Final Remarks

Part 1:
Linear Perturbation Theory

Linear Perturbation Theory
General requirements

1. Linear parameterization of perturbations

$$g_{\mu\nu} \to g_{\mu\nu} + \epsilon\delta g_{\mu\nu} + \mathcal{O}(\epsilon^2) \quad T_{\mu\nu} \to T_{\mu\nu} + \epsilon\delta T_{\mu\nu} + \mathcal{O}(\epsilon^2)$$

2. Appropriate mode decomposition:

- 1 + 3 → time + (Scalar+Vector+Tensor) (e.g.:FLRW)
- 1 + 2 + 1 → time + (Scalar+Vector) + (Scalar) (eg: LRS Bianchi)
- 1 + 1+1+1 → time + (Scalar)+(Scalar)+(Scalar) (eg: Bianchi I)

3. Complete basis of spatial eigenfunctions.

$$\nabla^2\phi = -q^2\phi \quad \to \quad f(\vec{x}) = \sum\!\!\!\!\!\!\int c_{\vec{q}}\, \phi(\vec{x}) d\vec{q}$$

4. This talk: Bianchi I, Bianchi III and Kantowski-Sachs

Linear Perturbation Theory
Bianchi I

Background:

$$\begin{aligned} ds^2 &= -dt^2 + X_1(t)^2 dx^2 + X_2(t)^2 dy^2 + X_3(t)^2 dz^2 \\ &= a(\eta)^2(-d\eta^2 + \gamma_{ij}(\eta) dx^i dx^j) \end{aligned}$$

Matter:

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu} + \Pi_{\mu\nu}$$

Dynamical equations:

$$\begin{aligned} 3H^2 &= \rho + \frac{1}{2}\sigma^2 \\ (\sigma^i{}_j)^\cdot &= -3H\sigma^i{}_j + \Pi^i{}_j \\ \dot{\rho}_{\rm de} &= -\sigma_{ij}\Pi^{ij} \end{aligned}$$

Formal solutions:

$$\begin{aligned} \Pi^i{}_j &\equiv \rho_{\rm de}\Delta\omega^i{}_j \\ \sigma^i{}_j &= \frac{1}{a^3}\left[C^i{}_j + \int \Pi^i{}_j a^2 \frac{da}{H}\right] \\ \rho_{\rm de} &= \rho^0_{\rm de} \exp\left[-\int \sigma^{ij}\Delta\omega_{ij}\frac{da}{aH}\right] \end{aligned}$$

Linear Perturbation Theory
Bianchi I

Perturbations are decomposed in Scalar-Vector-Tensor modes in the usual fashion

$$
\begin{aligned}
\varphi &\rightarrow \varphi \\
V_i &\rightarrow \partial_i V + \bar{V}_i \\
h_{ij} &\rightarrow C\left(\gamma_{ij} + \frac{\sigma_{ij}}{\mathcal{H}}\right) + \partial_i\partial_j E + \partial_{(i}\bar{E}_{j)} + \bar{E}_{ij}
\end{aligned}
$$

Subject to the TT conditions

$$
\partial^i \bar{V}_i = 0 = \partial^i \bar{E}_i \,, \quad \bar{E}^i_i = 0 = \partial^i \bar{E}_{ij}
$$

Price to pay for the SVT decomposition: dynamical mode-coupling!

Linear Perturbation Theory
Bianchi I

Formalism:

Plane waves decomposition is formally the same

$$f(x^i,\eta) = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} f(k_i,\eta) e^{\mathrm{i}k_i x^i}$$

But since the metric is time-dependent, we have

$$k_i = \text{constant}\,, \quad k^i = \gamma^{ij}(\eta) k_j\,, \quad k^2 = \gamma^{ij}(\eta) k_i k_j\,,$$

This leads to an intrinsic coupling between SVT modes.

$$\Phi_i = k_i \Phi + \bar{\Phi}_i\,, \qquad k^i \bar{\Phi}_i = 0$$

$$k^i\,(k_i\Phi)' + k^i(\bar{\Phi}_i)' \neq \left(k^i k_i \Phi\right)' + (k^i\bar{\Phi}_i)'$$

Linear Perturbation Theory
Bianchi I

Mukhanov-Sasaki variables:

There are three effective gauge-independent degrees of freedom, generalizing the Mukhanov-Sasaki variables:

$$v = a\left(\delta\varphi + \Psi\frac{\varphi'}{\mathcal{H}}\right), \quad \mu_+ = aE_+ \quad \mu_\times = aE_\times$$

Main characteristics:

- Vectors modes have no dynamics (if not sourced)
- Dynamical modes couple at linear order!
- Polarization modes with different dynamics

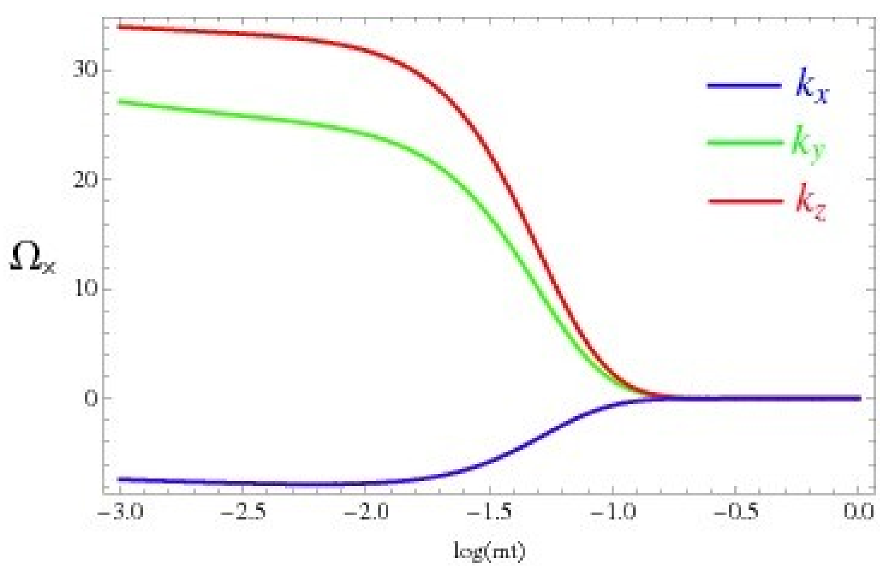
Linear Perturbation Theory
Bianchi I

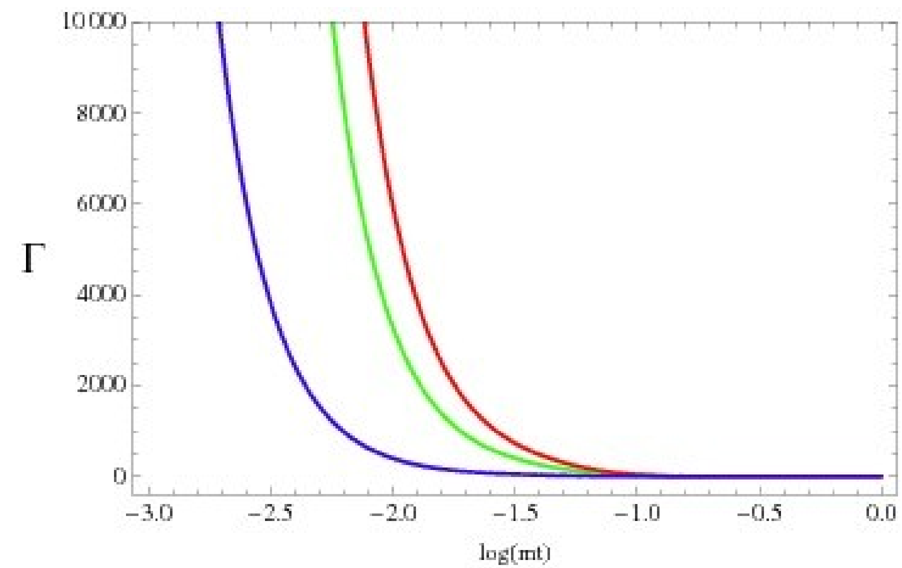
Dynamics:

Coupled oscillator-like equations:

$$\begin{pmatrix} v \\ \mu_+ \\ \mu_\times \end{pmatrix}'' + \begin{pmatrix} \omega_v^2 & -\Omega_+ & -\Omega_\times \\ -\Omega_+ & \omega_+^2 & -\Gamma \\ -\Omega_\times & -\Gamma & \omega_\times^2 \end{pmatrix} \begin{pmatrix} v \\ \mu_+ \\ \mu_\times \end{pmatrix} = 0$$

Numerical Solutions:



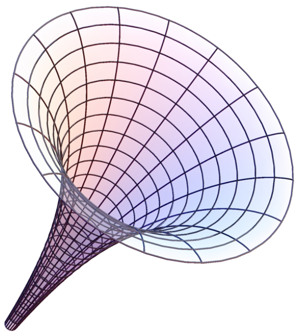


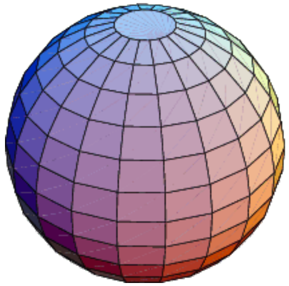
Linear Perturbation Theory
Bianchi III and Kantowski-Sachs

Anisotropic, homogeneous and spatially curved manifold

$$ds^2 = -dt^2 + a(t)^2\gamma_{ab}(x^c)dx^a dx^b + b^2(t)dz^2$$

Constant-time hypersurfaces:

Bianchi III:  $\times \mathbb{R}$

Kantowski Sachs:  $\times \mathbb{R}$

Since these universes are anisotropically curved,
can they expand isotropically?

Yes! Provide that the energy momentum tensor describes an **imperfect fluid**

Linear Perturbation Theory
Bianchi III and Kantowski-Sachs

Shear-free anisotropy:

Anisotropic models cannot simultaneously have perfect fluid content and shear-free timelike congruences. But since we assume that

$$\pi_{\mu\nu} \propto \sigma_{\mu\nu}$$

the vanishing of the shear implies a perfect fluid and FLRW dynamics...

...but this is not the only choice!

$$\dot{\theta} + \frac{1}{3}\theta^2 + 2\sigma^2 + \frac{1}{2}(\rho + 3p) = 0$$

$$\dot{\sigma}_{\mu\nu} + \sigma^{\rho}_{\mu}\sigma_{\rho\nu} + \frac{2}{3}\sigma_{\mu\nu}\theta - \frac{2}{3}\sigma^2 h_{\mu\nu} - \frac{1}{2}\pi_{\mu\nu} + E_{\mu\nu} = 0$$

$$\boxed{\sigma_{\mu\nu} = 0 \quad \rightarrow \quad \pi_{\mu\nu} = 2E_{\mu\nu}}$$

(Mimoso & Crawford, CQG, 1993)

Linear Perturbation Theory
Bianchi III and Kantowski-Sachs

Shear-free anisotropic metric:

$$ds^2 = a^2(\eta)(-d\eta^2 + \gamma_{ab}(x^c)dx^a dx^b + dz^2)$$

Isotropic expansion

Anisotropic spatial curvature

Matter content:

- Perfect fluid + Anisotropic Stress *(Mimoso&Crawford, CQG 1992)*
- Perfect fluid + Massless Scalar Field *(Carneiro&Mena-Marugán, PRD 2010)*
- Perfect fluid + Three-form *(Koivisto et. al., PRD 2010)*

Dynamics: FLRW background with curvature!

$$\mathcal{H}^2 = \frac{1}{3}a^2\rho - \frac{\kappa}{a^2} \qquad \Omega_\kappa \sim \frac{1}{a^2}$$

Linear Perturbation Theory
Bianchi III and Kantowski-Sachs

Geometry:

Natural 1+2 splitting at t = cte. Perturbations are decomposed in Scalar plus Scalar-Vector modes

2D-subspace

$$V_a \rightarrow D_a V + \bar{V}_a$$
$$h_{ab} \rightarrow 2S\gamma_{ab} + 2D_a D_b U + 2D_{(a}\bar{E}_{b)}$$

Real line

$$\phi\,,\ \psi\,,\ldots$$

Matter-content: $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} + \pi_{\mu\nu}$

Features:

- No nontrivial transverse and trace-free tensor modes.
- No dynamical coupling of modes!
- Perturbations to the stress tensor do not grow!.

Linear Perturbation Theory
Bianchi III and Kantowski-Sachs

Fourier analysis:

$$\nabla^2\phi = -q^2\phi, \qquad \Phi(t,\vec{x}) = \sum\!\!\!\!\!\!\int \Phi(t,\vec{q})\phi_{\vec{q}}(\vec{x})$$

Complete basis of eigenfunctions:

<u>Bianchi III</u>

$$\phi_{\vec{q}}(\vec{x}) \sim \mathcal{P}_\ell^m(\cosh\rho)e^{im\varphi}e^{ikz}$$

$$q^2 = \ell^2 + k^2 + 1/4$$

<u>Kantowski-Sachs</u>

$$\phi_{\vec{q}}(\vec{x}) \sim P_\ell^m(\cos\rho)e^{im\varphi}e^{ikz}$$

$$q^2 = \ell(\ell+1) + k^2$$

Common features:

- No supercurvature modes: $q^2 \geq |\text{curvature scale}|$
- Infinite curvature limit: $\phi_{\vec{q}} \sim J_m(\omega\rho)e^{im\varphi}e^{ikz}$

Linear Perturbation Theory
Bianchi III and Kantowski-Sachs

Mukhanov-Sasaki variables:

There are four effective gauge-independent degrees of freedom, generalizing the Mukhanov-Sasaki variables:

$$v'' - \left[\nabla^2 + \frac{1}{\eta^2}\left(2+6\epsilon-3\delta\right)\right] v = 0$$

$$\mu_+'' - \left[\nabla^2 - \frac{a''}{a} + 2\mathcal{H}^2 + 4\kappa\right]\mu_+ = 0$$

$$\mu_-'' - \left[\nabla^2 + \frac{2+3\epsilon}{\eta^2} + \kappa\right]\mu_- = 0$$

Gravitational waves

No dynamical couplings

and one decaying dynamical mode: $\delta\pi = \frac{\text{oscillating}}{a} \to 0$

- Scalar modes evolves as in standard FL with different eigenfunctions.
- Polarization of gravity waves with different dynamics!

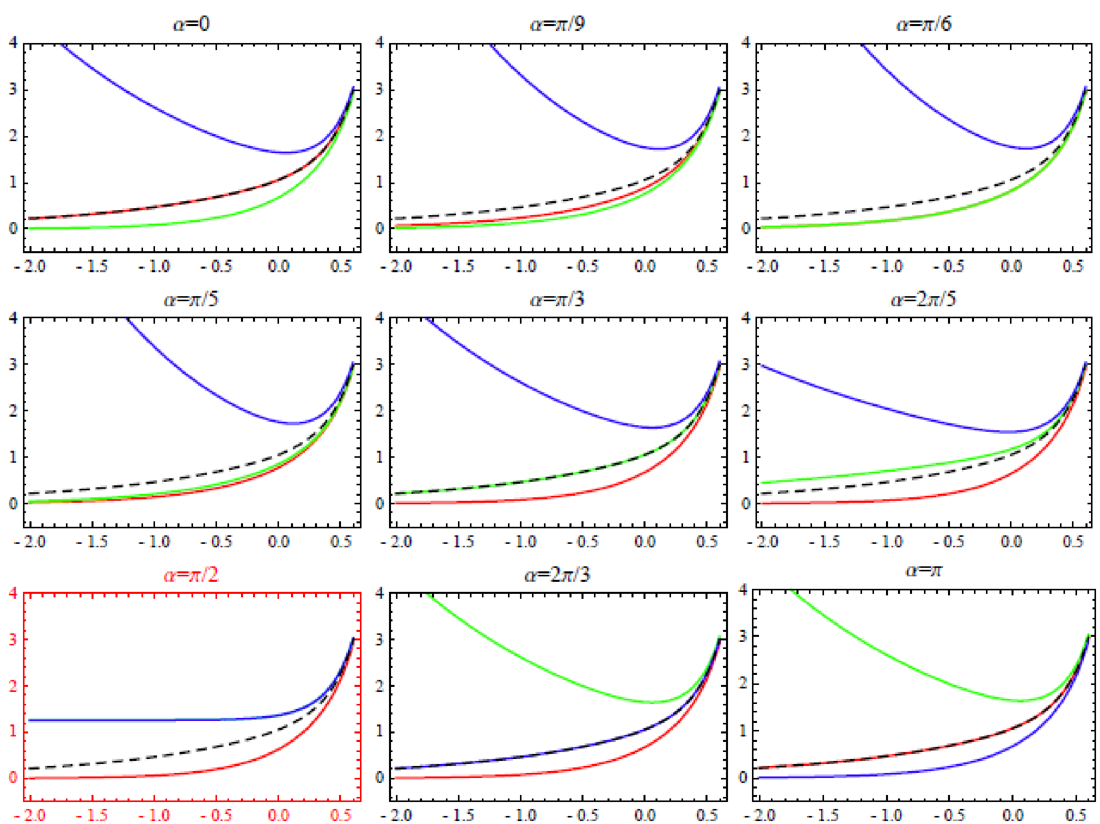
Part 2:
Primordial Anisotropies

Primordial Anisotropies
Bianchi I

Exact solution for a "de Sitter" phase:

$$X_i(t) = a_0 \left[\sinh\left(\frac{t}{t_0}\right)\right]^{\frac{1}{3}} \left[\tanh\left(\frac{t}{2t_0}\right)\right]^{\frac{2}{3}\sin\alpha_i} \quad , \quad \alpha_i = \alpha + \frac{2\pi}{3} i$$

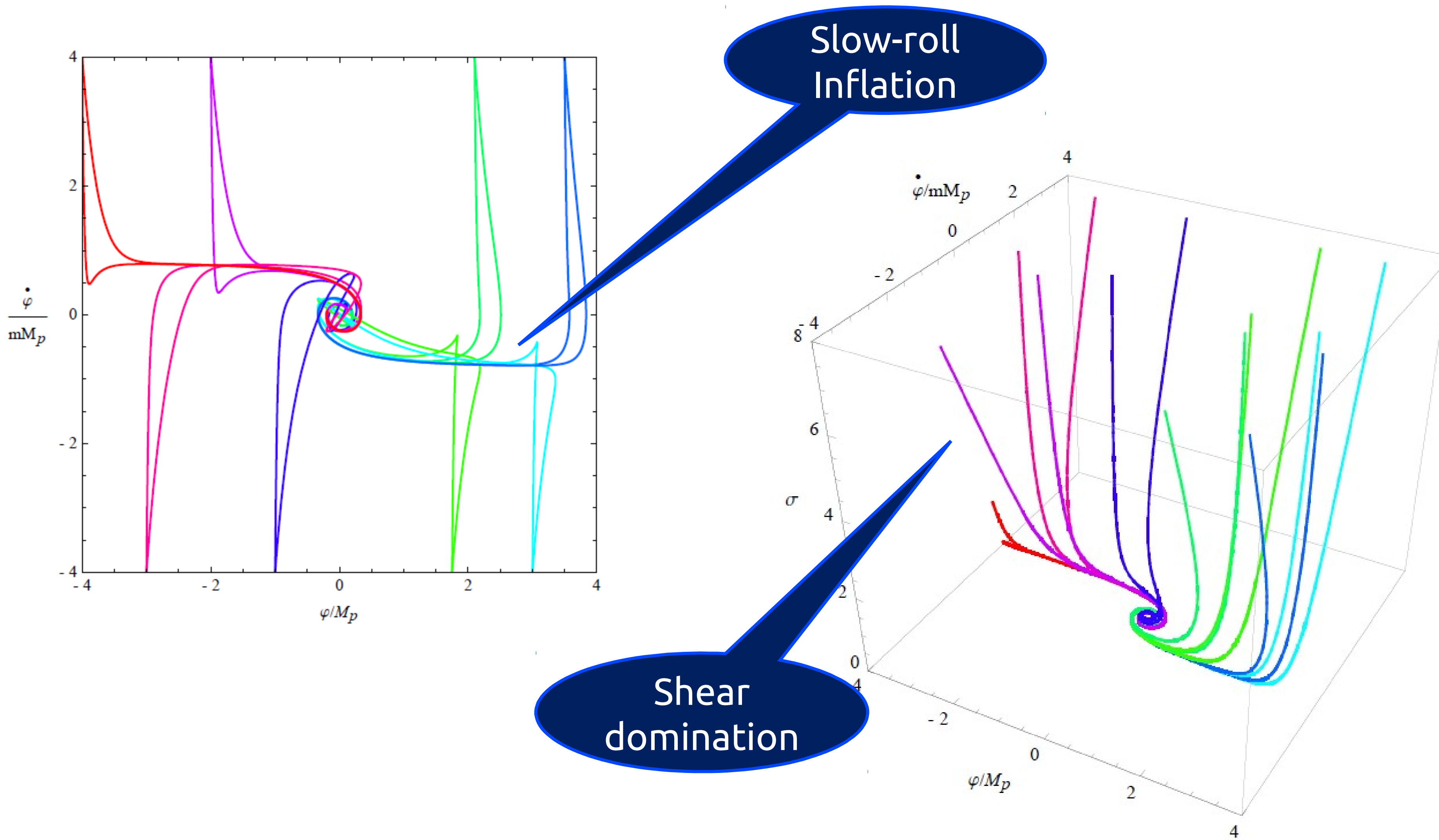
Planar symmetry
$X_1 = X_2 \neq X_3$



(*Pitrou et. al., JCAP 2008*)

Primordial Anisotropies
Bianchi I

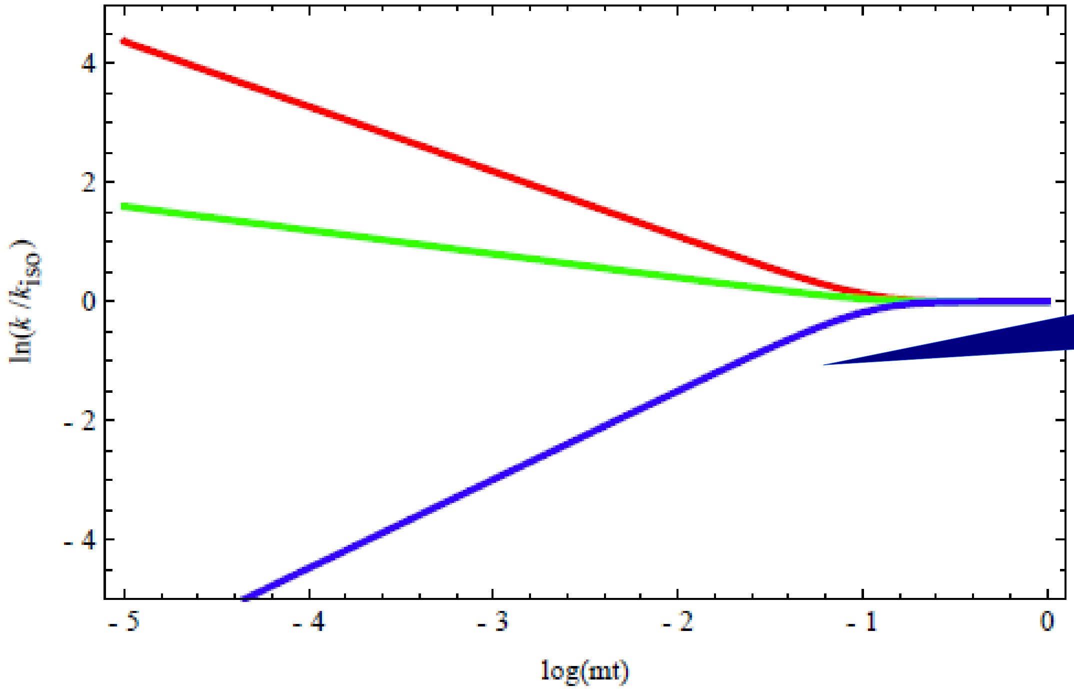
Application to chaotic inflation:



Primordial Anisotropies
Bianchi I

Quantization:

- FRW
  - Bunch-Davies vacuum at infinity past
  - WKB approximation is always possible at $t \to 0$
- Bianchi I
  - No Bunch-Davies vacuum at infinity past
  - WKB approximation does not always work



There is always one diverging mode in the infinite past

(*Pitrou, Pereira & Uzan, JCAP 2009*)

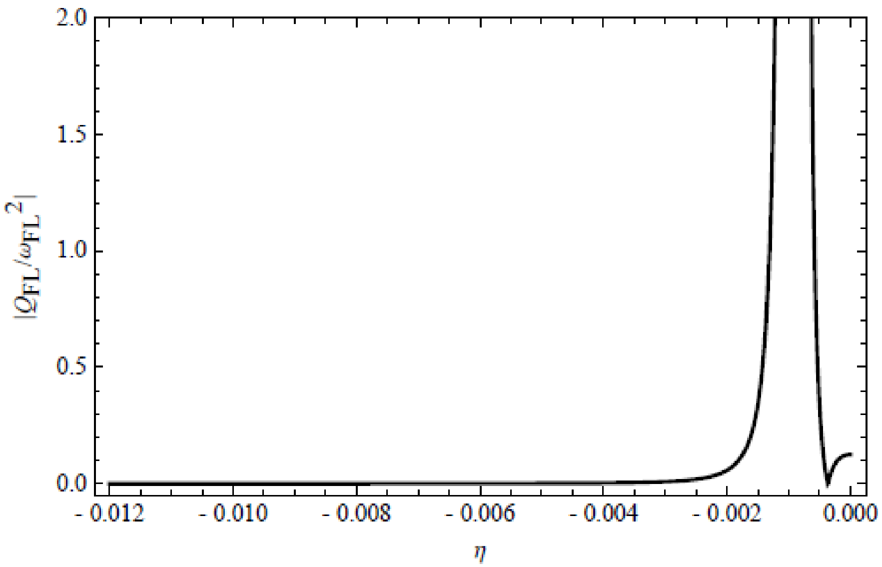
Primordial Anisotropies
Bianchi I

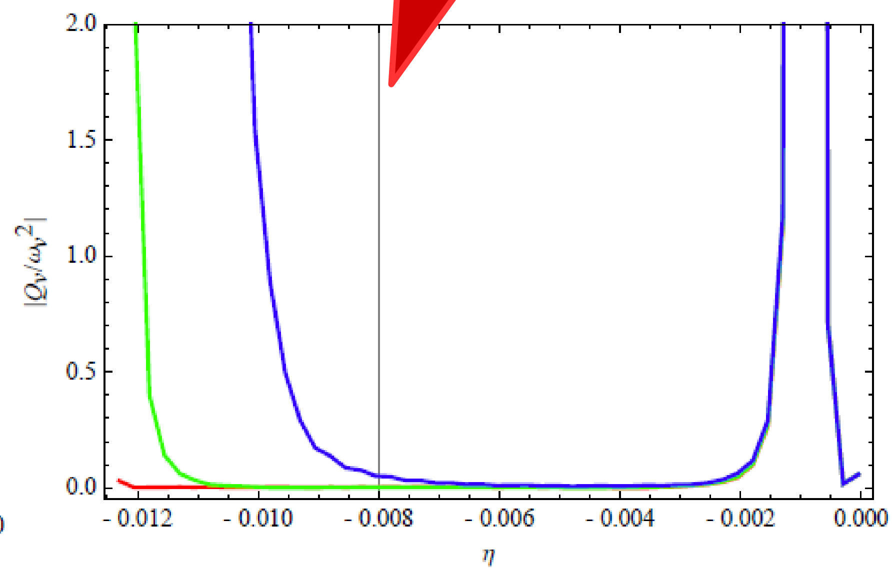
WKB approximation:

Plane-wave
Initial conditions can
be implemented
only in a small time
window

$$v_k'' + \left(\omega^2 - Q_{WKB}\right) v_k = 0$$

$$Q_{WKB} \equiv \frac{3}{4}\left(\frac{\omega'}{\omega}\right)^2 - \frac{1}{2}\frac{\omega''}{\omega}$$





Observational constraints from CMB give σ/H ≤ 1%

Primordial Anisotropies
Bianchi I

Spectrum of perturbations:

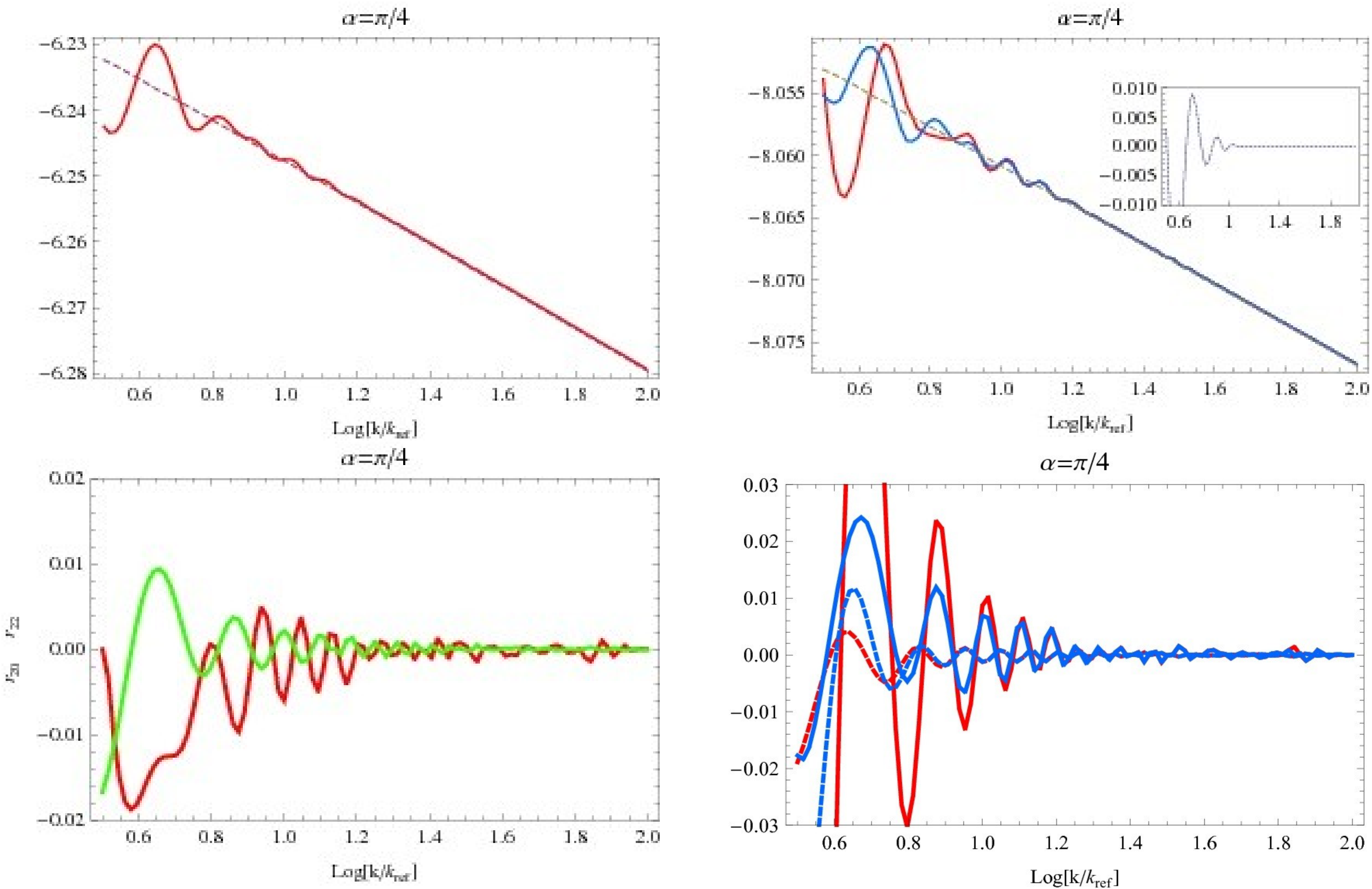
$$\langle \Phi(\vec{k})\Phi^*(\vec{q})\rangle = \mathcal{P}(\vec{k})\delta(\vec{k}-\vec{q}) \qquad \mathcal{P}(\vec{k}) = f(k)\sum_{\ell,m} r_{\ell m}(k) Y_{\ell,m}(\hat{k})$$

- $r_{\ell m} \to 0$ when $k \to \infty$ and $\ell \gg 1$
- Even perturbations: no odd-parity correlations are allowed!

$$\langle a_{2m} a^*_{3m}\rangle = 0$$

Side Effect: no quadrupole-octopole alignment from temperature perturbations!!

Primordial Anisotropies
Bianchi I



Primordial Anisotropies
Bianchi III and Kantowski-Sachs

Since there is no mode-coupling, we focus only on scalar modes:

$$ds^2 = a^2(\eta)\left[-(1+2\Phi)d\eta^2 - 2\Pi d\eta dz + (1+2\Psi)\gamma_{ab}dx^a dx^b + (1+2\Lambda)dz^2\right]$$

Scalar fields do not have stress:

$$\delta T^a_b = 0 \quad \rightarrow \quad D^a D_b(\Lambda + \Phi) = 0 \quad \rightarrow \quad \Lambda = -\Phi$$

During inflation, one has

$$\Pi = \text{const.} \times a^{-2} \quad \rightarrow \quad \Phi + \Psi = (a^2\Pi)' \rightarrow 0 \quad \rightarrow \quad \Psi = -\Phi$$

Thus:

$$\boxed{ds^2 = a^2(\eta)\left[-(1+2\Phi(\vec{x}))d\eta^2 + (1-2\Phi(\vec{x}))h_{ij}dx^i dx^j\right]}$$

(Pereira, Marugán, Carneiro, arXiv:1505.00794)

Corollaries:

- Time evolution of perturbations do not change
- Sachs-Wolfe effect is the same

Primordial Anisotropies
Bianchi III and Kantowski-Sachs

Large-angle temperature correlations

$$C(\hat{n},\hat{n}') = \langle \Delta T(\hat{n}) \Delta T(\hat{n}') \rangle \longrightarrow \Delta T(\hat{n}) = \frac{1}{3}\Phi(\vec{x})$$

$$\Phi(\vec{x}) = \sum\!\!\!\!\!\!\int \Phi(\vec{q})\phi_{\vec{q}}(\vec{x})$$

$$\langle \Phi(\vec{k})\Phi(\vec{q})\rangle = \mathcal{P}(\vec{q})\delta(\vec{k}+\vec{q})$$

...requires quantization in principle...

...But we can fix $\mathcal{P}(\vec{k})$ by demanding that the 2pcf agrees with the isotropic one when n → n'. This gives:

<u>B-III</u> $\mathcal{P}(\vec{q}) = (\tanh \pi\ell)^{-1}\mathcal{P}(q)$

<u>KS</u> $\mathcal{P}(\vec{q}) = 2\pi\mathcal{P}(q)$

$$q^3\mathcal{P}(q) = 2\pi^2 A q^{n_s - 1}$$

Primordial Anisotropies
Bianchi III and Kantowski-Sachs

General 2pcf: modifications results from geodesics and eigenfunctions

Isotropic case:

$$C(\hat{n}\cdot\hat{n}') = \int \omega d\omega \int dk \mathcal{P}(q) e^{ik\Delta z} J_0(\omega\Delta\rho) \qquad \Delta\rho^2 = \rho^2 + \rho'^2 - 2\rho\rho' \cos(\Delta\phi)$$

Bianchi III

$$C(\hat{n},\hat{n}') = \int \omega d\omega \int dk \mathcal{P}(q) e^{ik\Delta z} P_{-\frac{1}{2}+i\ell}(\cosh\Delta\rho)$$

$$\cosh\Delta\rho = \cosh\rho\cosh\rho' - \sinh\rho\sinh\rho'\cos(\Delta\phi)$$

Kantowski-Sachs

$$C(\hat{n},\hat{n}') = \sum_{\ell}\left(\ell+\frac{1}{2}\right)\int dk \mathcal{P}(q) e^{ik\Delta z} P_\ell(\cos\Delta\rho)$$

$$\cos\Delta\rho = \cos\rho\cos\rho' + \sin\rho\sin\rho'\cos(\Delta\phi)$$

Primordial Anisotropies
Bianchi III and Kantowski-Sachs

Large curvature limit: *L>>1*

$$C(\hat{n},\hat{n}') = C(\hat{n}\cdot\hat{n}') \pm \frac{1}{L^2}\left(\begin{array}{c}\text{geodesic}\\ \text{corrections}\end{array}\right) \mp \frac{1}{L^2}\left(\begin{array}{c}\text{eigenfunctions}\\ \text{corrections}\end{array}\right)$$

Correction linear in 1/L violate parity and thus they do not appear

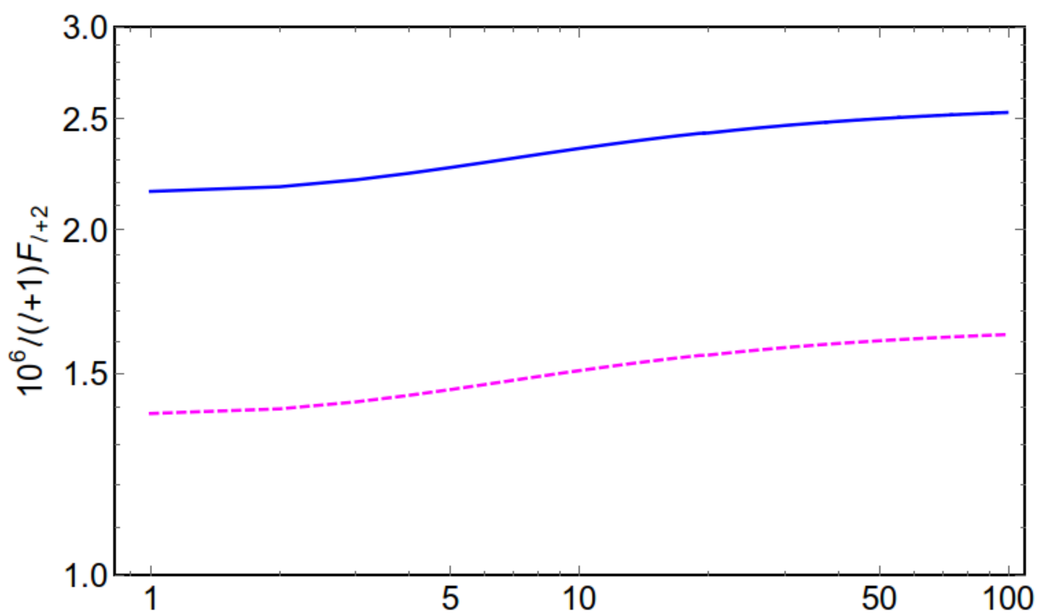
$$\langle a_{\ell m} a^*_{\ell' m'}\rangle = C_\ell \delta_{\ell\ell'}\delta_{mm'} + \mathcal{F}_{\ell m \ell' m'}$$

$$\mathcal{F}_{\ell+2} \equiv \frac{\sum_m |\langle a_{\ell m} a_{\ell+2 m}\rangle|}{2\ell+1}$$

CMB bounds from superhorizon perturbations suggest that

$$L \gtrsim 100 H^{-1}_{\text{today}}$$

(E.g., Erickcek, Carroll, Kamionkowski, PRD 2008)

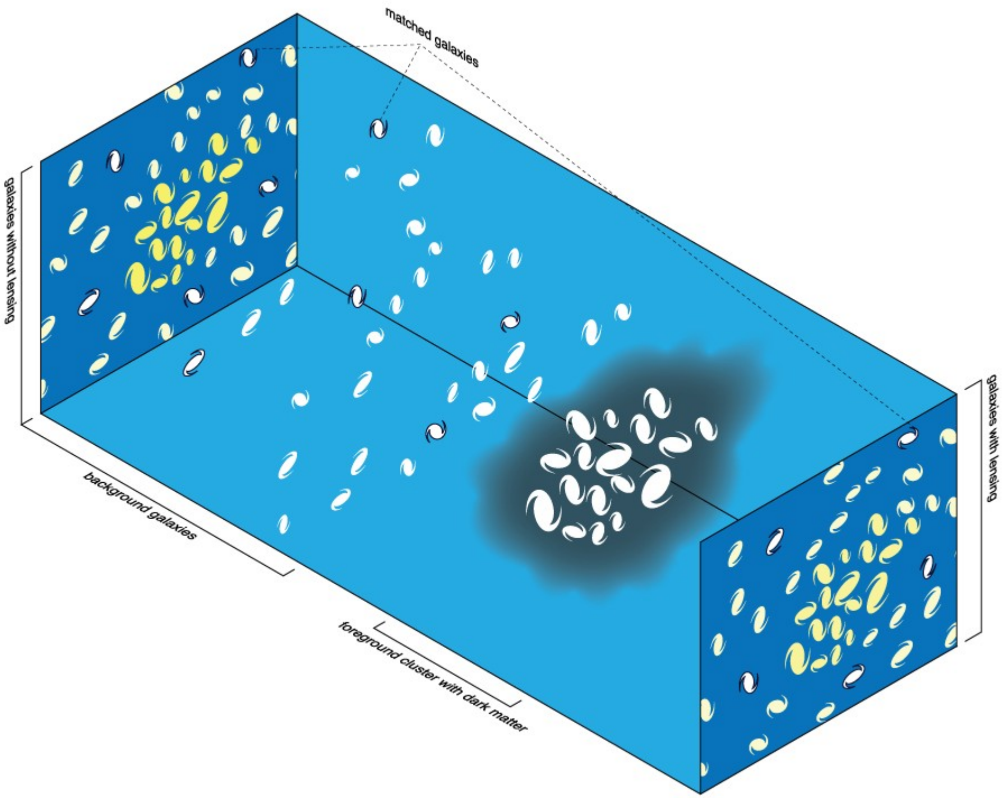


(Pereira, Marugán, Carneiro, arXiv:1505:00794)

Part 3:
Late-time Anisotropies

Weak Lensing

Weak-lensing of large scale structure is a powerful tool to probe late-time geometry



(figure: en.wikipedia.org/wiki/Weak_gravitational_lensing)

In the standard lore, B-modes are not cosmological

Weak-Lensing
geodesic deviation

Photons four-momenta are parallel-transported along null-geodesics:

$$\frac{Dk^\nu}{d\lambda} = 0$$

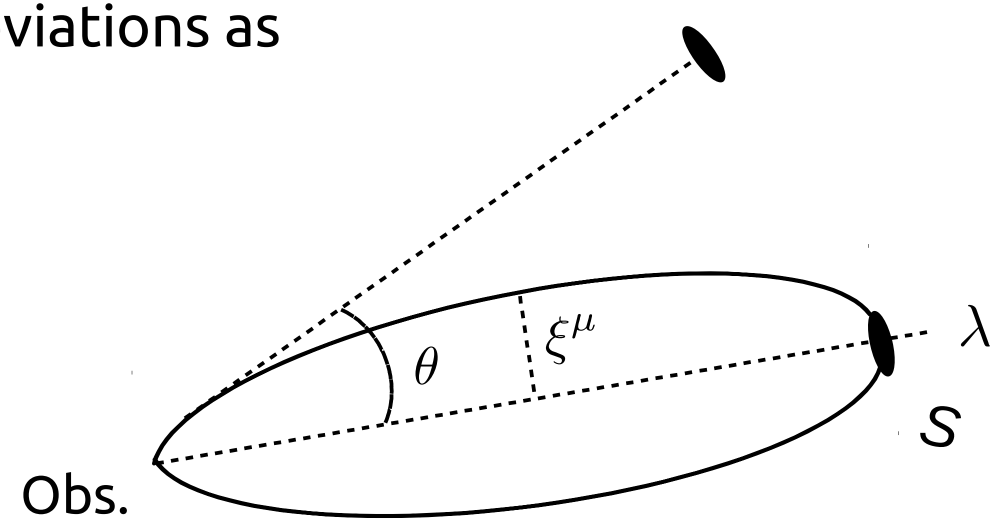
This vector can be decomposed as:

$$k^\mu = -u^\mu + n^\mu \qquad u^\mu n_\mu = 0 \qquad n^\mu n_\mu = 1$$

We parameterize small geodesic deviations as

$$x^\mu(\lambda) = \bar{x}^\mu(\lambda) + \xi^\mu(\lambda)$$

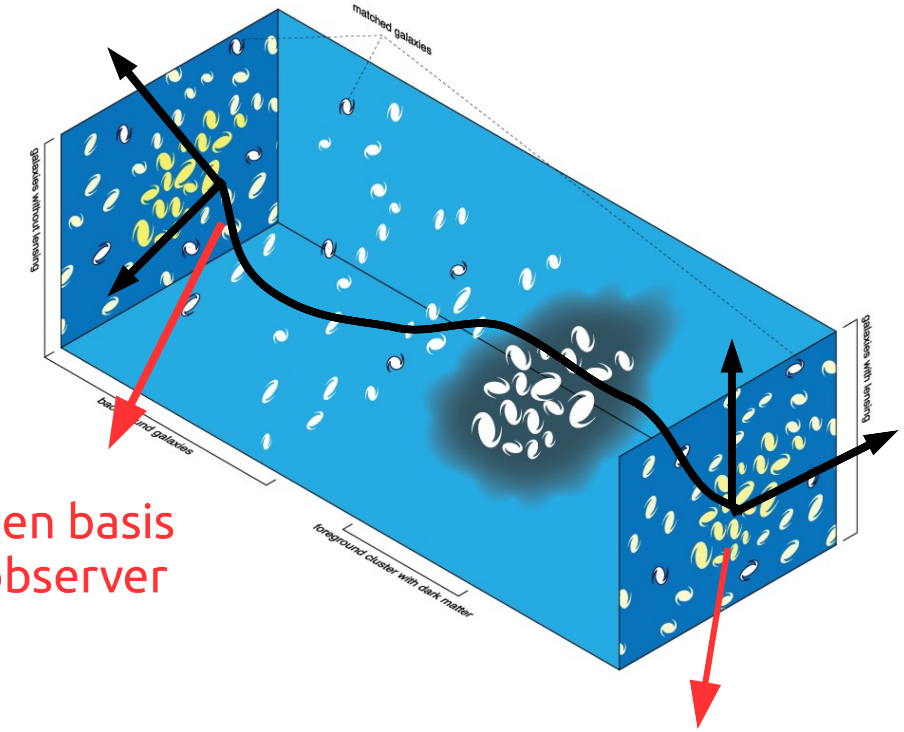
$$\frac{D^2 \xi^\mu}{d\lambda^2} = R^\mu_{\nu\alpha\beta} k^\nu k^\alpha \xi^\beta$$



Weak-Lensing
screen description

The integration of the geodesic equation gives us:

$$x^\mu(\hat{n}^o, \lambda) \quad \text{and} \quad n^\mu(\hat{n}^o, \lambda)$$



At the observer we define a 2D basis of vectors orthogonal to n°:

$$n_a{}^\mu \qquad a = 1, 2$$

$$n_a^\mu n_{b\mu} = \delta_{ab} \qquad n_a^\mu n_\mu = 0 = n_a^\mu u_\mu$$

This basis can be parallel-transported along the geodesics.

This allows us to define a helicity basis everywhere: $\hat{n}_\pm = \frac{1}{\sqrt{2}}(\hat{n}_1 \pm i\hat{n}_2)$

Weak-Lensing
screen description

We now project everything on the screen:

$$\mathcal{R}_{ab} = R_{\mu\nu\alpha\beta} k^\nu k^\alpha n_a^\mu n_b^\beta \qquad \xi_a = \xi_\mu n_a^\mu \qquad \frac{d^2 \xi_a}{d\lambda^2} = \mathcal{R}_{ab} \xi^b$$

Linearity gives:

$$\xi^a(\lambda) = \mathcal{D}(\lambda)^a_{\ b} \frac{d\xi^b(0)}{d\lambda}$$

And from this we get the Sachs equation:

$$\boxed{\frac{d^2 \mathcal{D}^a_b}{d\lambda^2} = \mathcal{R}^a_c(\lambda) \mathcal{D}^c_b(\lambda)}$$

with

$$\boxed{\mathcal{D}^a_b(0) = 0, \qquad \frac{d\mathcal{D}^a_b}{d\lambda}(0) = \delta^a_b}$$

Weak-Lensing
general prescription

Starting from

$$\boxed{\frac{d^2\mathcal{D}^a_b}{d\lambda^2} = \mathcal{R}^a_c(\lambda)\mathcal{D}^c_b(\lambda)}$$

we do the following:

1. Decompose the Riemann tensor in its trace (Ricci) and traceless (Weyl) part:

$$\mathcal{R}^a_{\ b} = \text{scalar} \ + \ \text{spin 2 field}$$

(Symmetric 2x2 matrix)

2. Decompose the Jacobi matrix in its irreducible pieces:

$$\mathcal{D}^a_{\ b} = \text{scalar} \ + \ \text{pseudo scalar} \ + \ \text{spin 2 field}$$

(General 2x2 matrix)

3. Derive coupled equations for the above quantities

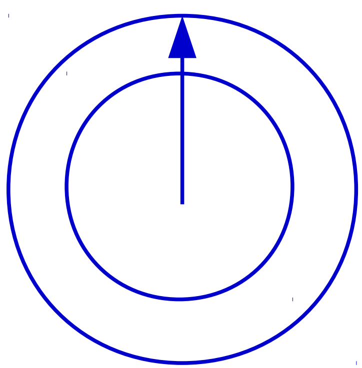
4. Harmonic decomposition → Power spectrum → Data analysis

Weak-Lensing *observables*

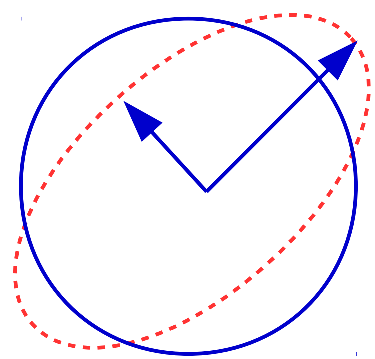
$$\mathcal{D}_{ab} = \kappa\delta_{ab} + V\epsilon_{ab} + \gamma_{ab}$$

Convergence Twist Shear

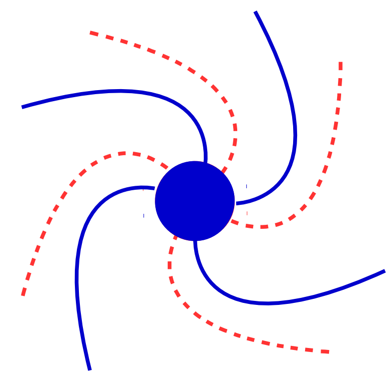
Main effects:



Convergence



Cosmic Shear



Twist

The evolution of k, V and γ are entirely determined by background metric and initial conditions!

Weak-Lensing *observables*

	Scalar	**Spin - 2**
Shape of the light-bundle	$\kappa(\hat{n}^{\rm o},\lambda),\ V(\hat{n}^{\rm o},\lambda)$	$\gamma^{\pm}(\hat{n}^{\rm o},\lambda)$
Properties of spacetime	$H_{\vert\vert}(\hat{n}^{\rm o},\lambda),\ \ \mathcal{R}(\hat{n}^{\rm o},\lambda)$	$\mathcal{W}^{\pm}(\hat{n}^{\rm o},\lambda)$

They are described by coupled equations:

$$\left(\frac{d^2}{d\lambda^2}+H_{\vert\vert}\frac{d}{d\lambda}-\mathcal{R}\right)\begin{pmatrix}\kappa\\ iV\\ \gamma^{\pm}\end{pmatrix}=-2\begin{pmatrix}\mathcal{W}^{(-}\gamma^{+)}\\ \mathcal{W}^{[-}\gamma^{+]}\\ \mathcal{W}^{\pm}\left(\kappa+iV\right)\end{pmatrix}$$

(Pitrou, Uzan & Pereira, PRD 2013)

The integration of this equation depends on the light-cone structure:

$$\hat{n}_a=\hat{n}_a(\hat{n}^{\rm o},\lambda),\ H_{\vert\vert}=H_{\vert\vert}(\hat{n}^{\rm o},\lambda)\ \ldots$$

Weak-Lensing
Harmonic Decomposition

We perform a harmonic decomposition of all variables

Scalar:

$$X(\hat{n}^{\rm o},\lambda) = \sum_{\ell,m} X_{\ell m} Y_{\ell m}(\hat{n}^{\rm o}) \qquad X = (\kappa,\, V,\, H_{||},\, \mathcal{R})$$

Spin-2:

$$\gamma^{\pm}(\hat{n}^{\rm o}) = \sum_{\ell,m} [E_{\ell m}(\lambda) + i B_{\ell m}(\lambda)] Y^{\pm 2}_{\ell m}(\hat{n}^{\rm o})$$

$$\mathcal{W}^{\pm}(\hat{n}^{\rm o},\lambda) = \sum_{\ell,m} [\mathcal{E}_{\ell m}(\lambda) + i \mathcal{B}_{\ell m}(\lambda)] Y^{\pm 2}_{\ell m}(\hat{n}^{\rm o})$$

- Resulting equations obey a Boltzmann-like hierarchy
- Equations valid for any spacetime
- Can be solved order by order in perturbations

Weak-Lensing
Consistency check: FLRW

At first order in perturbations, we have:

$$\mathcal{R}_{ab} = -D_a D_b(\Phi + \Psi) \longrightarrow \text{Pure "electric" part}$$

$$\hat{n}(\hat{n}^{\rm o}, \lambda) = \hat{n}^{\rm o} \longrightarrow \text{Trivial light-cone}$$

$$H_{||}(\hat{n}^{\rm o}, \lambda) = \text{const.} \longrightarrow \text{Isotropic expansion}$$

Cosmic shear E & B modes:

$$E^{(1)}_{\ell m} \to \left(\mathcal{R}^{(0)}_{00} - H^{(0)}_{00}\frac{d}{d\lambda}\right) E^{(1)}_{\ell m} - 2\kappa^{(0)}_{00}\,\mathcal{E}^{(1)}_{\ell m} \qquad B^{(1)}_{\ell m} \to \left(\mathcal{R}^{(0)}_{00} - H^{(0)}_{00}\frac{d}{d\lambda}\right) B^{(1)}_{\ell m}$$

Twist and convergence:

$$\kappa^{(1)}_{\ell m} \to \left(\mathcal{R}^{(0)}_{00} - H^{(0)}_{00}\frac{d}{d\lambda}\right) E^{(1)}_{\ell m} + \mathcal{R}^{(1)}_{\ell m}\kappa^{(0)}_{00} \qquad V^{(1)}_{\ell m} \to \left(\mathcal{R}^{(0)}_{00} - H^{(0)}_{00}\frac{d}{d\lambda}\right) V^{(1)}_{\ell m}$$

At first order in FLRW, E and k are sourced, but B and V are not! Thus

$$\boxed{E^{(1)}_{\ell m} \neq 0\,, \quad \kappa^{(1)}_{\ell m} \neq 0 \quad \text{and} \quad B^{(1)}_{\ell m} = V^{(1)}_{\ell m} = 0}$$

As expected...

Weak-lensing in Bianchi I Universes

Weak-Lensing in *BI*
Motivation

General setup:

$$ds^2 = -dt^2 + a(\eta)^2\gamma_{ij}(t)dx^i dx^j, \quad T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} + \Pi_{\mu\nu}$$

where:

$$\sigma_{ij} = \frac{1}{2}\frac{d}{dt}\gamma_{ij}(t), \qquad \Pi^i{}_j \propto (\Delta\omega)^i{}_j$$

The shear evolves as:

$$\sigma^i{}_j = \frac{1}{a^3}\left[\underbrace{C^i{}_j}_{\text{early}} + \underbrace{\int \Pi^i{}_j a^2 \frac{da}{H}}_{\text{late}}\right]$$

Thus, our motivations are twofold:

- To provide a new test of isotropy at late times
- Propose new observational tests on the anisotropy of dark energy

Weak-Lensing in *BI*
Perturbation scheme

Weak shear approximation:

$$\gamma_{ij}(\eta) \approx \delta_{ij} + 2\int_0^a \frac{\sigma_{ij}}{\mathcal{H}} \frac{da'}{a'}, \qquad \frac{\sigma_{ij}}{\mathcal{H}} \ll 1$$

We develop a perturbations scheme such that:

- Background FL quantities are of order {0,0}
- Background BI quantities are of order {1,0}
- Scalar metric perturbations are of order {0,1}
- Vector and tensor perturbations are of order {1,1}

The metric perturbations are:

$$ds^2 = a^2[-(1+2\Phi)d\eta^2 + 2\bar{B}_i d\eta dx^i + (\gamma_{ij} + h_{ij})dx^i dx^j]$$

$$h_{ij} = -2\left(\gamma_{ij} + \frac{\sigma_{ij}}{\mathcal{H}}\right)\Psi + 2E_{ij}$$

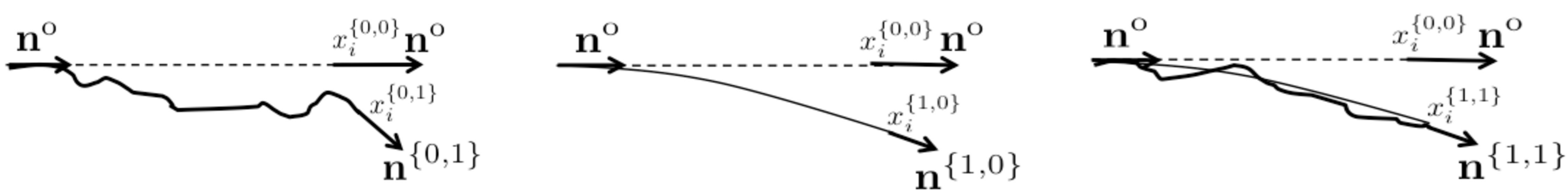
Weak-Lensing in *BI*
Central geodesic approximation

Main goal:

$$\frac{d^2}{d\chi^2}\mathcal{D}_{ab} = -\frac{1}{k^0}\frac{dk^0}{d\chi}\frac{d\mathcal{D}_{ab}}{d\chi} + \frac{1}{(k^0)^2}\mathcal{R}_{ac}\mathcal{D}_{cb} \longrightarrow \gamma_{ab}^{\{1,1\}} \longrightarrow C_\ell^{EE},\ C_\ell^{BB}$$

For that we need to:

- Determine the source position x^i and the geodesic direction **n** at relevant order
- Determine the screen basis $\mathbf{n}_a(\mathbf{n}_o, x)$ at order {1,1}
- Determine the source terms at order {1,1}
- Solve metric perturbations at order {0,1}
- Perform a multipolar expansion in quantities dependent on $\mathbf{n}_o$.



(Pereira, Pitrou, Uzan,arXiv:1503:01125)

Weak-Lensing in *BI*
Dominant effect

We are interested in the EE and BB correlations in the cosmic shear induced by the anisotropic expansion. At small scales the dominant contribution is

$$\gamma_{ab}^{\{1,1\}}(\chi, \mathbf{n}^{\mathrm{o}}) = -\int_0^{\chi} \frac{\chi - \chi'}{\chi'} \alpha^c(\mathbf{n}^{\mathrm{o}}, \chi) D_c D_{\langle a} D_{b\rangle} (\Phi + \Psi)(\mathbf{n}^{\mathrm{o}}, \chi) d\chi'$$

Deflection angle at order {1,0}

Dominant at small scales

Grav. Potentials at order {0,1}

We employ a gradient expansion (Hu, 2000), which allows us to get:

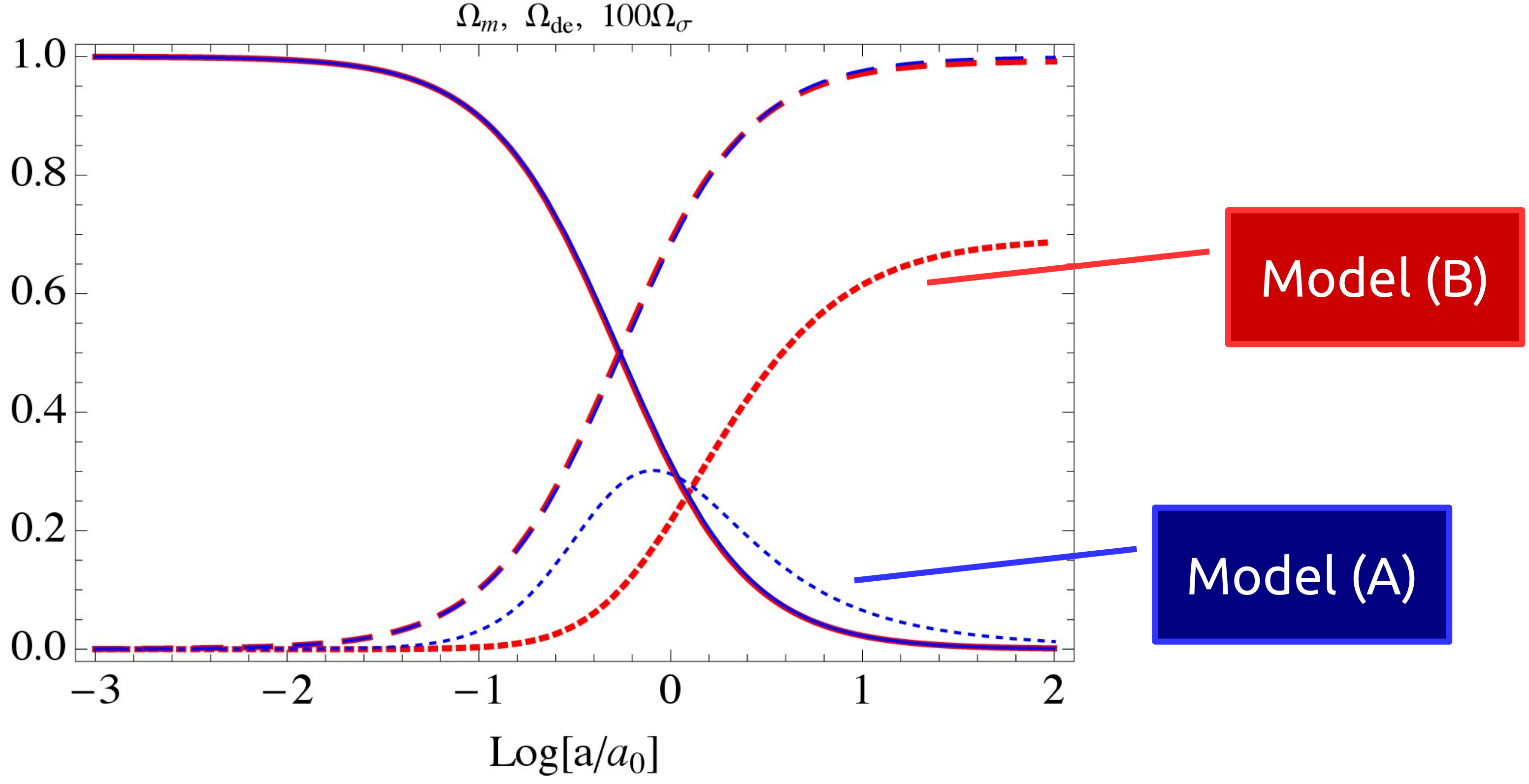
$$C_\ell^{BB} = \frac{2}{5\pi} \int_0^{\infty} k^2 dk\, P(k) \sum_{s=\pm 1} \frac{({}_2F_{\ell,2,\ell+s})^2}{2\ell+1} \sum_m \left| \int_0^{\infty} dN(z) \int_0^{\chi} dz' \alpha_{2m}(z') g_{\ell+s}(k, z, z') \right|^2$$

(Pereira, Pitrou, Uzan,arXiv:1503:01125)

Weak-Lensing in *BI Models*

We consider two phenomenological models for the anisotropic pressure:

$$(\mathrm{A}) \quad \Pi^i{}_j = \rho_{\mathrm{de}} \Delta\omega^i{}_j \qquad\qquad (\mathrm{B}) \quad \Pi^i{}_j = g(a) \Delta\omega^i{}_j$$



Weak-Lensing
Models

Source distribution:

Euclid: $N(z) = Az^2 \exp\left[-(z/z_0)^\beta\right]$,

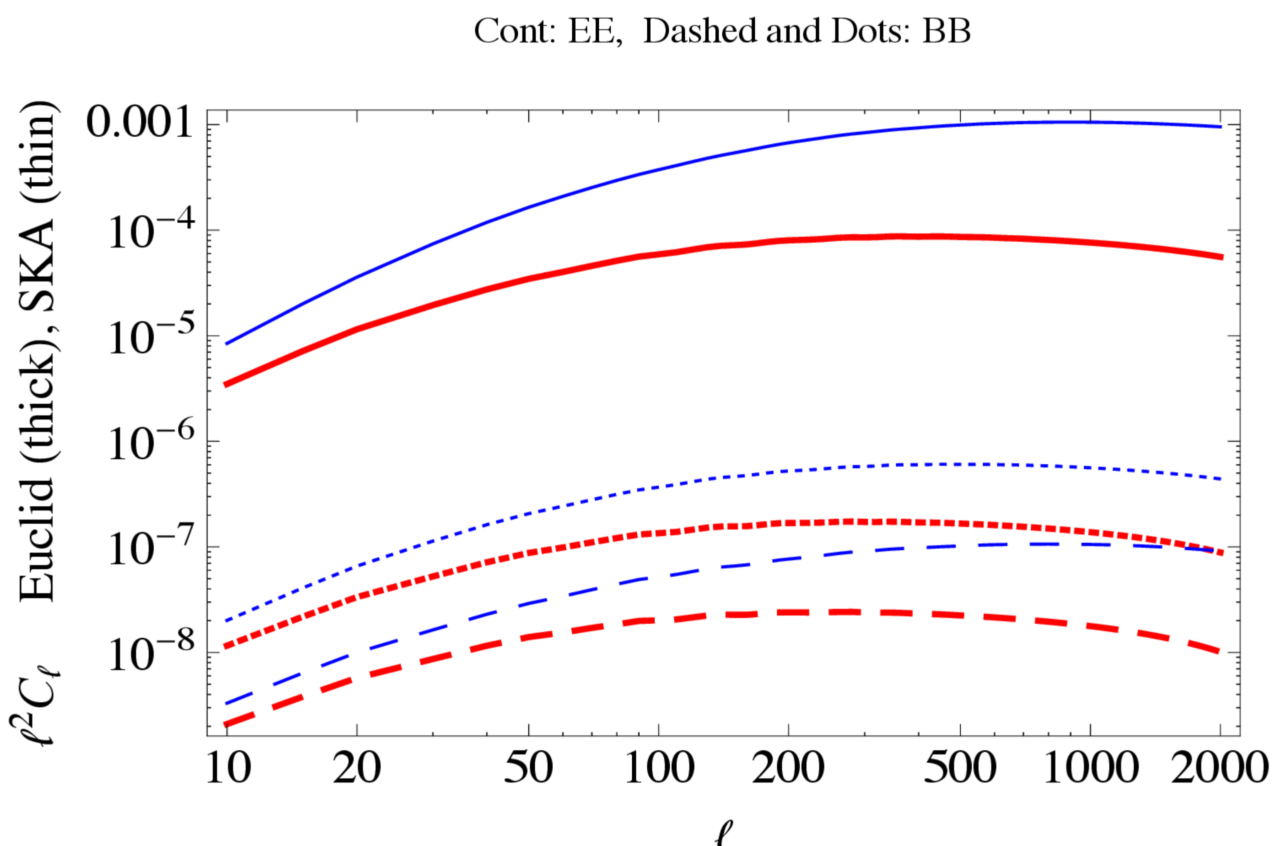
SKA: $N(z) = A\frac{z^n}{(1+z)^m}\exp\left[-\frac{(a+bz)^2}{(1+z)^2}\right]$

Free parameters:

A, z_0, β, a, b, m, n

CFHTLS: $\sigma/H_0 \leq 0.4$

Euclid $\sigma/H_0 \leq 0.008$



(Pereira, Pitrou, Uzan, arXiv:1503:01127)

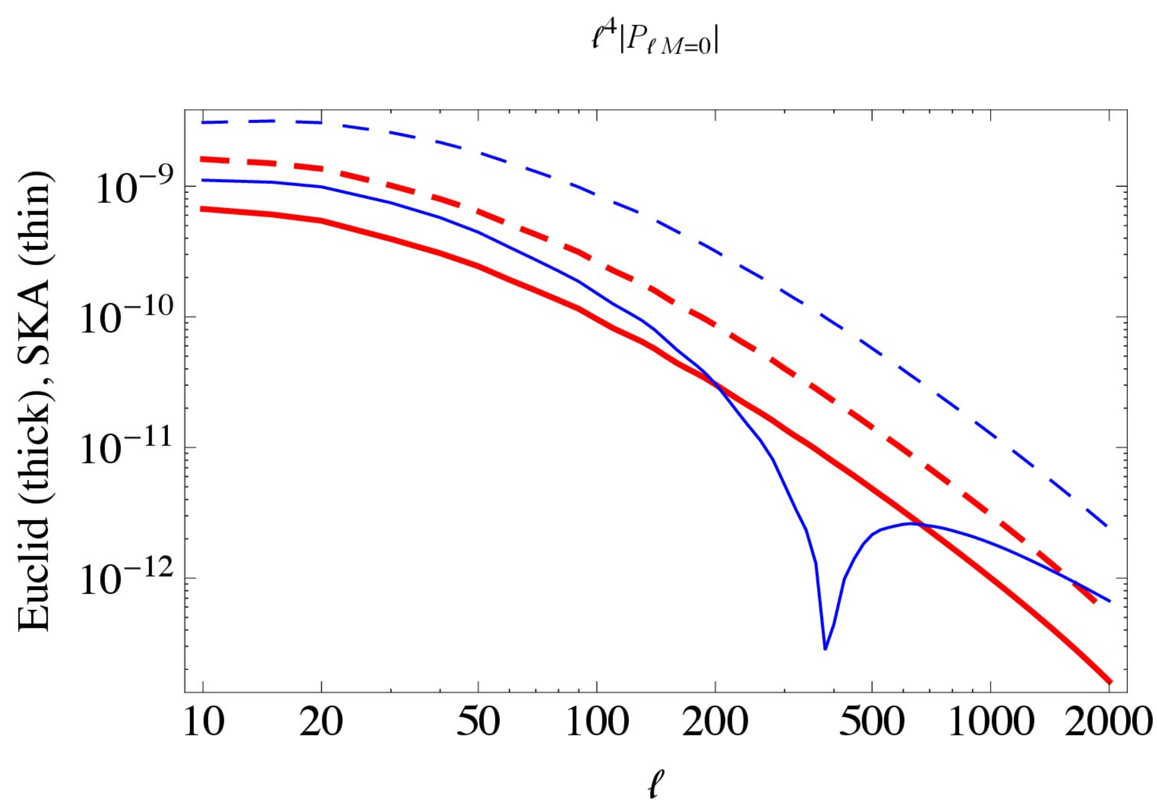
Weak-Lensing
Cross-correlations

The convergence, E- and B-modes cross-correlations are linear in σ_{ij}/H, and allow one to fully reconstruct the eigendirections of expansion.

$$\langle B_{\ell m} E^*_{\ell\pm1 m-M}\rangle$$
$$\langle B_{\ell m} \kappa^*_{\ell\pm1 m-M}\rangle$$
$$\langle E_{\ell m} E^*_{\ell\pm2 m-M}\rangle$$
$$\langle \kappa_{\ell m} \kappa^*_{\ell\pm2 m-M}\rangle$$
$$\langle E_{\ell m} \kappa^*_{\ell\pm2 m-M}\rangle$$

$$\mathcal{P}_{\ell M} = \int_0^\infty dk P(k) \alpha_{2M} \left|\text{transfer function}\right|^2$$

M = -2,-1,0,1,2
= 5 components
= N° of components in σ_{ij}



Final Remarks

- Homogeneous and spatially anisotropic models are straightforward extension of FLR cosmologies.
- Shear-free anisotropic models are very similar to FLRW, and yet richer
- Perturbation theory is possible:
  - Can be used to model anisotropic inflation...
  - ...But it's not clear that it is the origin of anomalies
  - Further advance depends strongly on existence of eigenfunctions.
- E- and B- modes of weak-lensing can be measured by future surveys such as Euclid and SKA
  - New tool to measure isotropy at low redshift
  - The eigendirections of expansions can be fully reconstructed from weak-lensing
  - This opens a window to new investigations of the anisotropy of dark energy.

Thank you!