Synthetic Tensor Modes

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Heads-Up

- Ingredients:
  - Axion-like particles
  - Axial coupling with gauge fields
- Phenomenology:
  - Gauge-field production
  - Tensor Modes

Motivation

- Observations of large scale magnetic fields suggest some inflationary enhancement of gauge fields

  [Turner et al. 88'; Ratra 92'; Neronov et al. 10'; **RZF** et al. 13',14'; ...]

- Axion-like particles (pseudo-scalars) (ϕ) appear in many different contexts (CP problem, String Theory, BSM, etc.).

- Pseudo Goldstone boson of an (explicitly) broken global symmetry.

- In inflation they appear, e.g., in UV completions of large field models as a protection mechanism for radiative corrections (Natural Inflation)

  [Freese, Frieman and Olinto '90]

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)^2 - \Lambda^4\left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

  where f is the decay constant of the axion and Λ is some scale at which the mass term is generated.

- However, slow-roll inflation requires a superplanckian f which is, normally, problematic.

Motivation

Ways-out:

- Kim-Nilles-Peloso mechanism: 2 axions (ϕ_i) with anomalous couplings to 2 gauge groups and decay constants (f_i, g_i)

  [Kim, Nilles, Peloso '04]

$$V(\phi_1, \phi_2) = \Lambda_1 \cos\left(\frac{\phi_1}{f_1} + \frac{\phi_2}{g_1}\right) + \Lambda_2 \cos\left(\frac{\phi_1}{f_2} + \frac{\phi_2}{g_2}\right)$$

- N-flation: N axions [Dimopoulos et al. '05]

$$V(\phi_i) = \sum_i \Lambda_i \cos\left(\frac{\phi_i}{f_i}\right)$$

- Monodromy [McAllister et al. '08]

Axial Coupling with gauge fields

- We expect axions (ϕ) to couple to gauge fields as

$$\mathcal{L}_{\text{int}} = -\frac{\alpha\phi}{4f} F^{a}_{\mu\nu}\tilde{F}^{\mu\nu}_{a}$$

where α is a dimensionless coefficient.

- We consider the case of a coupling to a $U(1)$ gauge field (A_μ) (although couplings with $SU(N)$ should share some features)
- ϕ is assumed to be a homogeneous field during inflation (gets a VEV)

Equations of Motion

- For an FRLW metric:

$$ds^2 = a^2(\tau)\left(-d\tau^2 + d\vec{r}^{\,2}\right),$$

the equation of motion for A_μ in the Coulomb gauge and in the basis of circular polarization vectors $(\vec{e}_\pm)$ is [Anber and Sorbo 06']

$$A_\pm(\tau,k)'' + \left(k^2 \pm \frac{\alpha k \phi'}{M_p}\right) A_\pm(\tau,k) = 0,$$

where $H = \dot{a}/a$ is the Hubble parameter.

- For an inflationary background where $\tau \simeq -(aH)^{-1}$ it becomes

$$A_{\pm}(\tau, k)'' + \left(k^2 \pm \frac{2k\xi}{\tau}\right) A_{\pm}(\tau, k) = 0,$$

where the coupling ξ is

$$\xi \equiv \frac{\alpha\dot{\phi}}{2fH} = \frac{\alpha M_p}{f}\sqrt{\frac{\epsilon_\phi}{2}}$$

and

$$\epsilon_\phi \equiv \frac{\dot{\phi}}{2M_p^2 H^2}$$

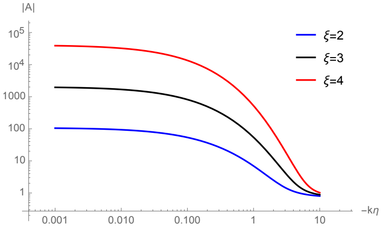
is the first slow-roll parameter. Therefore, for a light axion, $m_\phi^2 \ll H^2$, $\xi \simeq$ const during inflation.

- Initial conditions: Plane waves at past infinity.
- Then, the solution for the eom with can be expressed in terms of Coulomb functions:

$$A_+(\tau, k) = \frac{1}{2k}\left(G_0(\xi, -k\tau) + iF_0(\xi, -k\tau)\right).$$

- The axial coupling leads to a tachyonic enhancement of one polarization around the time each mode is crossing the horizon crossing ($k_{\text{phys}} \simeq H$):

$$(8\xi)^{-1} \lesssim \; -k\tau \lesssim 2\xi.$$



- Gauge field mode function oscillates, is enhanced at horizon crossing and then freezes. Around horizon crossing the solution is: [Anber and Sorbo 06']

$$A((8\xi)^{-1} \lesssim -k\tau \lesssim 2\xi) \simeq \left(\frac{-\tau}{2^3 k\xi}\right)^{1/4} e^{\pi\xi - 2\sqrt{-2\xi k\tau}}.$$

Tachyonic enhancement is very sensitive to ξ and classicalizes the gauge field.

Generation of quantum fluctuations

- Gauge fields are strongly produced, how do they affect the generation of anisotropies (scalar and tensor perturbations)?

- After expanding the fields around its background values:

$$\begin{aligned} \phi(t,\vec{x}) &= \phi(t) + \delta\phi(t,\vec{x}) \\ g_{ij} &= g_{ij} + \delta g_{ij} \end{aligned}$$

  we look for the interactiong Lagrangian with gauge fields.

- The axial coupling introduces the interaction

$$\mathcal{L}_{\text{int}} = -\frac{\alpha\delta\phi}{4f} F_{\mu\nu}\tilde{F}^{\mu\nu} \ .$$

- Relevant gauge invariant observable quantities:
  - Adiabatic curvature perturbation:

$$\mathcal{R} \simeq \psi + H\frac{\delta\chi}{\dot{\chi}}$$

    where χ is the adiabatic perturbation, parallel to the background trajectory and ψ the gravitaitonal potential.

  - Isocurvature perturbation between 2 scalar fields X, Y

$$\mathcal{S}_{XY} = H\left(\frac{\delta X}{\dot{X}} - \frac{\delta Y}{\dot{Y}}\right)$$

  - Transverse and traceless tensor perturbation (h_{ij})

Axion as the inflaton

- If the axion is the inflaton then curvature perturbation interacts directly with gauge-fields

$$\mathcal{L}_{\text{int}}^{\text{scalar}} = 2\xi a^3 \mathcal{R}\, \vec{E} \cdot \vec{B}, \qquad F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\vec{E} \cdot \vec{B}.$$

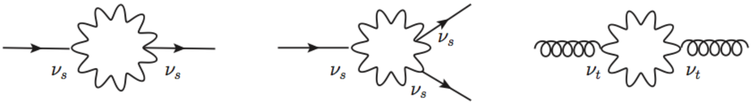
Interaction is parametrically stronger than the gravitational coupling.

- No new interaction with tensor perturbations is generated

$$\mathcal{L}_{\text{int}}^{\text{tensor}} = T_{\mu\nu}^{\text{EM}} \delta g^{\mu\nu}$$

because the axial coupling does not contribute to $T_{\mu\nu}^{\text{EM}}$.

- The tachyonic enhancement of gauge fields affects the expectation value of the n-point correlation function of $\mathcal{R}$ and h_{ij} due to inverse decay processes. [Barnaby and Peloso 11']



- Schematically,

$$\begin{aligned}
\delta\phi &= \delta\phi^0 + \delta\phi^p \\
&\rightarrow \langle\mathcal{R}\mathcal{R}\rangle = \langle\mathcal{R}\mathcal{R}\rangle_0 + \left\langle(\mathcal{R}\vec{E}\cdot\vec{B})(\mathcal{R}\vec{E}\cdot\vec{B})\mathcal{R}\mathcal{R}\right\rangle \\
&\rightarrow \langle\mathcal{R}\mathcal{R}\mathcal{R}\rangle = \langle\mathcal{R}\mathcal{R}\mathcal{R}\rangle_0 + \left\langle(\mathcal{R}\vec{E}\cdot\vec{B})(\mathcal{R}\vec{E}\cdot\vec{B})(\mathcal{R}\vec{E}\cdot\vec{B})\mathcal{R}\mathcal{R}\mathcal{R}\right\rangle
\end{aligned}$$

- To compute this effect one can use the in-in formalism or, as the gauge fields are approximately classical, simply the equations of motion with a source: [Barnaby and Peloso 10', Sorbo 11']

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi + a^2\frac{dV}{d\phi} = a^2\frac{\alpha}{f}\vec{E}\cdot\vec{B}$$

$$\frac{1}{2a^2}\left(\partial_\tau^2 + \frac{2a'}{a}\partial_\tau - \nabla^2\right)h_{ij} = \frac{1}{M_p^2}\left(-E_iE_j - B_iB_j\right)^{TT}$$

- Power spectrum is changed to (γ_α are "small" numerical coefficients)

$$P_{\mathcal{R}} \simeq \mathcal{P}\left(1 + \gamma_s\frac{\mathcal{P}}{\xi^6}e^{4\pi\xi}\right), \qquad \mathcal{P}^{1/2} = \frac{H^2}{2\pi\dot{\phi}}$$

$$P_{\rm GW} \simeq 16\epsilon\mathcal{P}\left(1 + \gamma_t\frac{\epsilon\mathcal{P}}{\xi^6}e^{4\pi\xi}\right);$$

Note: Generation of tensor modes is (ϵ) suppressed in comparison to scalar modes.

- 3-point function (non-gaussianities):

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle^{\text{one-loop}} = (2\pi)^3 \delta^{(3)}(\sum_i \vec{k}_i) f(k_1, k_2, k_3) \frac{\mathcal{P}^3}{\xi^9} e^{6\pi\xi};$$

Peaks on the equilateral configuration $(k_1 = k_2 = k_3)$ [Barnaby et al. 11']

$$f_{NL}^{\text{equi}} = \gamma_{NG} \frac{\mathcal{P}}{\xi^9} e^{6\pi\xi},$$

Anisotropies are exponentially sensitive to ξ.

Observational constraints (on the largest scales): [Planck '13]

- Power Spectrum amplitude: $P_{\mathcal{R}} \simeq 10^{-9}$;
- Non-Gaussian parameter: $f_{NL}^{\text{equi}} < -42 \pm 75$;

Consequences:

- $\xi \lesssim 3$ or, equivalently, a non-trivial lower bound on the axion decay constant: $f \gtrsim \frac{\alpha}{6}\frac{\dot{\phi}}{H} M_p$. [Barnaby et al. 11', Meerburg and Pajer 12', Linde et al. 12']
- Contributions to scalar and tensor power spectrum from the axial coupling have to be smaller than the vacuum contribution if the axion is the inflaton [Barnaby et al. 11', Sorbo 11']

$$P_{\text{GW},\mathcal{R}}^{\text{one-loop}} < P_{\text{GW},\mathcal{R}}^{0}$$

Axion **Not** the inflaton

- *What if the pseudo-scalar (σ) in the axial coupling is not the inflaton but instead some other pseudo-scalar, irrelevant for the inflationary evolution?* [Barnaby et al. 12', Shiraishi et al. 13', Cook , Sorbo 13', Mukohyama et al. 14', **RZF**, Sloth 14']

$$\mathcal{L}_{\text{int}} = -\frac{\alpha\sigma}{4f} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- Tensor production is unaffected because $T^{\text{EM}}_{\mu\nu}$ remains the same.
- There is no direct interaction with the inflaton, beside the gravitational, so adiabatic curvature perturbations might be suppressed and the constraints on ξ would relax (?).

- Proposed as a mechanism for generating GW larger than the vacuum.
- Consequences:
  - Observation of tensor modes would not tell us the energy scale of inflation.
  - Tensor modes would be parity violating and nongaussian
- Problem: $\delta\sigma$ is not a gauge invariant quantity. What happens when we rewrite the interaction in terms of gauge invariant quantities?

Interaction term

- In the comoving gauge ($\delta\phi = 0$, $h_{ij} = a^2 e^{2\mathcal{R}} [e^{\gamma}]_{ij}$) and using the ADM formalism we decompose the metric as:

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt).$$

The leading interaction Hamiltonian to third order involving gauge fields is ($T^{\mu\nu} \equiv T^{\mu\nu}_{EM}$) [Chaicherdsakul 06']

$$H_I(t) \;=\; \int d^3x a^3 \left(-\frac{1}{2} T^{\mu\nu} \delta g_{\mu\nu} + \frac{\alpha\delta\sigma}{4f} F_{\mu\nu} \tilde{F}^{\mu\nu} \right),$$

The first term can be simplified (up to slow-roll corrections)

$$-a^3 \int d^3x \frac{1}{2} T^{\mu\nu} \delta g_{\mu\nu} \;\; = \;\; \int d^3x \left[a^3 \frac{\mathcal{R}}{H} \nabla_\mu T^{\mu 0} - \frac{1}{H} \partial_t \left(a^3 \mathcal{R} T^{00} \right) \right]$$

Due to the axial coupling $T_{\mu\nu}$ is not conserved:

$$\nabla_\mu T^{\mu 0} = -\frac{\alpha \dot{\sigma}}{f} \vec{E} \cdot \vec{B}.$$

On the other hand, in this gauge $\delta\sigma = \dot{\sigma} \mathcal{S}_{\sigma\phi}/H$. Therefore, the interaction Hamiltonian is [**RZF**, Sloth 14']

$$H_I(\tau') \;\; = \;\; -2\xi a^3 \int d^3x \left(\mathcal{R} + \mathcal{S}_{\sigma\phi} \right) \vec{E} \cdot \vec{B}$$

The same coupling with adiabatic fluctuations!

The same result can be seen in the spatially flat gauge by simply rewriting

$$\delta\sigma = \frac{\dot{\sigma}}{H}\left(\mathcal{S}_{\sigma\phi} + \mathcal{R}\right).$$

Implications: [**RZF**, Sloth 14']

- Axions couple universally to adiabatic perturbations in terms of ξ;
- Non-gaussianities should constraint ξ in the same way, independently of whether the axion is the inflaton or not.
- Adiabatic and isocurvature modes are equally generated;

Superhorizon Evolution of Curvature Perturbations

- Axion does not decay during inflation:
  - Generated adiabatic curvature perturbation passes to radiation at the end of inflation.
  - Isocurvature perturbations (very much constrained by Planck) could lead to even stronger constraints on ξ, but it depends on what how and when the axion decays.
- Axion becomes massive and decays during inflation: [Mukohyama et al. 14']
  - Curvature and isocurvature perturbation cancel each other:

$$\delta\sigma \to 0 \quad \Rightarrow \quad \mathcal{R} + \mathcal{S}_{\sigma\phi} \to 0$$

Superhorizon Evolution of Curvature Perturbations

However, due to isocurvature perturbations $\mathcal{R}$ is not conserved outside the horizon so there is still an extra enhancement [Linde et al. 04']

$$\mathcal{R}'_\phi = -\left(\frac{\dot\sigma}{\dot\phi}\right)^2 \mathcal{R}'_\sigma$$

where $\mathcal{R}_\phi \simeq \mathcal{R}$ and $\mathcal{R}_\sigma$ is, in this case, enhanced by the axial coupling at horizon crossing. Therefore,

$$\mathcal{R}(\tau) = \mathcal{R}^* - \int_{\tau_*}^{\tau_f} \left(\frac{\dot\sigma}{\dot\phi}\right)^2 \mathcal{R}'_\sigma \, d\tau \qquad * \equiv \text{horizon crossing}$$

The first term corresponds to the usual vacuum contribution and the second term is the extra sourcing.

In order to compute $\mathcal{R}'_\sigma$ we need to solve the system of equations of motion for $\delta\phi$ and $\delta\sigma$ which are not mass eigenstates due to the gravitational coupling: [Sasaki 86', Mukhanov 88]

$$\delta\ddot{\phi}_I + 3H\delta\dot{\phi}_I + \frac{k^2}{a^2}\delta\phi_I + \sum_J \left[V_{IJ} - \frac{1}{a^3}\frac{d}{dt}\left(\frac{a^3}{H}\dot{\phi}_I\dot{\phi}_J\right)\right]\delta\phi_J = S_I,$$

where the source term is

$$S_I = \begin{pmatrix} 0 \\ \frac{\alpha}{f}\vec{E}\cdot\vec{B} \end{pmatrix}.$$

Before axion decay, the mixing matrix is constant at first order in slow-roll parameters and we can decouple the system into the mass eigenstates $\{v_1, v_2\}$

The mass eigenstates $\{v_1, v_2\}$ are related to the original fields as

$$\begin{aligned} a\,\delta\phi &= v_1 - \Theta\, v_2 \\ a\,\delta\sigma &= \Theta\, v_1 + v_2\,, \end{aligned}$$

where $\Theta \propto \dot{\sigma}_*/\dot{\phi}_* \ll 1$, and satisfy the equations of motion

$$v_I'' + \left[k^2 - \frac{1}{\tau^2}\left(\mu_I^2 - \frac{1}{4}\right)\right] v_I = a^3 \frac{\alpha}{f} \vec{E}\cdot\vec{B} \begin{pmatrix}\Theta \\ 1\end{pmatrix},$$

where $\mu_I \equiv 3/2 + \lambda_I$ and λ_I is related with the slow-roll parameters of ϕ and σ [Barnes, Wands 06']

$$\lambda_{1,2} \simeq \frac{1}{2}\left(4\epsilon_\phi - \eta_\phi + \eta_\sigma \pm |2\epsilon_\phi - \eta_\phi + \eta_\sigma|\right), \quad \eta_{\phi_i} = \frac{m_{\phi_i}^2}{3H^2}.$$

Given that $v_2 \gg \Theta\, v_1$, then $a\,\delta\sigma \simeq v_2$.

As usual, the fluctuations of light scalar fields during inflation are frozen on superhorizon scales up to slow-roll corrections stored in λ_I:

$$v_2 \simeq \frac{C}{\sqrt{k}}(-k\tau)^{-1-\lambda_2},$$

Thus,

$$\mathcal{R}'_\sigma(\tau^* < \tau < \tau_{osc}) = \frac{d}{d\tau}\left(H\frac{\delta\sigma}{\dot{\sigma}_*}\right) \simeq -aH\mathcal{R}^*_\sigma\,(2\,\epsilon_\phi - \lambda_2)\,.$$

At $\tau \simeq \tau_{osc}$ the axion oscillates and decays, usually like matter, as

$$\begin{aligned}\dot{\sigma} &= \dot{\sigma}_* \cos(m_\sigma(t - t_{osc}))\left(\frac{a_{osc}}{a}\right)^{3/2} \\ \Rightarrow &\quad \mathcal{R}'_\sigma(\tau_{osc} < \tau) = -\epsilon_\phi\, aH\mathcal{R}^*_\sigma.\end{aligned}$$

Putting everything together we finally get the total enhancement of $\mathcal{R}_\phi \simeq \mathcal{R}$ [**RZF**, Sloth 14']

$$\mathcal{R}(\tau) = \mathcal{R}_* + \left(\frac{\dot{\sigma}_*}{\dot{\phi}_*}\right)^2 \mathcal{R}_\sigma^* \left[\Delta N \left(2\epsilon_\phi - \lambda_2\right) + \frac{\epsilon_\phi}{6}\right]$$

where $\Delta N = \log\left(\tau^*/\tau_{osc}\right)$ is the duration from horizon crossing until the decay of the axion in e-folds.

Although the enhancement is suppressed by $(\dot{\sigma}_*/\dot{\phi}_*)^2$, $\mathcal{R}_\sigma^*$ can be very large and there is an extra ΔN factor.

Overall results [**RZF**, Sloth 14']

- Axial coupling to curvature perturbation has an Universal form.
- If the axion decays during inflation the contribution to scalar curvature perturbation is erased but there is still an extra sourcing on superhorizon scales which leads to:
  - Scalar Power Spectrum

$$P_{\mathcal{R}} \simeq \mathcal{P}\left(1+\gamma_s\,\Delta N^2\epsilon^2\frac{\mathcal{P}}{\xi^6}e^{4\pi\xi}\right) \ .$$

    If $\Delta N > 2.2$ the non-gaussian contribution to the power spectrum is larger than the tensor spectrum.

  - Non-Gaussian parameter: $f^{\mathrm{eq}}_{\mathrm{NL},\sigma} \simeq \epsilon^3\Delta N^3\,\gamma_{NG}\mathcal{P}\frac{e^{6\pi\xi}}{\xi^9}.$

- Given that the generation of GW remains unchanged and applying the non-gaussian constraints one gets for the tensor to scalar ratio

$$r = \frac{P_{\rm GW}}{P_{\mathcal{R}}} < \frac{10^{-2}}{\Delta N^2} \left(f_{\sf NL}^{\sf eq}\right)^{2/3} .$$

  A large (observable) value of r on the large scales is only possible if the axion decays right after the largest scales left the horizon but it is still allowed on the smallest scales.

- Even if the curvature perturbation is generated by some other mechanism like the curvaton, the amount of tensor modes is still suppressed by non-gaussian constraints ($r \lesssim 10^{-5}$).

Implications

- A constraint on the ξ can be translated in a lower bound on the decay constant of all axions with axial couplings with $U(1)$ gauge fields:

$$\xi \lesssim 3 \quad \Rightarrow \quad f_i \gtrsim \frac{\alpha_i}{6}\frac{\dot{\phi}_i}{H} M_p \quad \forall i$$

- For example, if the axial coupling is with the Standard Model $U(1)$ then $f_i \gtrsim 5 \times 10^{-2}(\dot{\phi}_i/H)\, M_p$;

- Even though the effective decay constant can be superPlanckian (as in Natural Inflation) each decay constant has to satisfy this bound.

- The presence of N axions (with similar couplings to the gauge fields) could make the constraint N times stronger.

Conclusion

- The presence of axion-like particles is very natural in many frameworks.
- The axial coupling with gauge fields have been studied is many contexts. It leads to a tachyonic enhancement which could generate significant anisotropies.
- The coupling between adiabatic curvature perturbation and gauge fields has an Universal form independently of the role of the axion during inflation.
- In order to generate large GW the axion has to decay quickly after horizon crossing of the largest scales. However, the mechanism remains unconstrained for small GW (but larger than vacuum) on large scales or large GW on small scales.