Non perturbative renormalisation group for quantum field theory in de Sitter space

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de Sitter Poincaré patch

$$ds^2 = a^2(\eta)(-d\eta^2 + d\mathbf{X}^2) \quad with \quad a(t) = \frac{-1}{H\eta}$$

$D = d + 1$ dimensions

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Scalar field

$$S[\varphi] = -\int_x \left(V(\varphi) + \frac{1}{2}(\nabla\varphi)^2 \right), \quad V(\varphi) \sim \frac{m^2}{2}\varphi^2 + \frac{\lambda}{4!}\varphi^4$$

Introduction : QFT in de Sitter space

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In-In formalism : closed time path

Schwinger '61

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Generating functional for expectation values

$$Z[J] = e^{iW[J]} = \int \mathscr{D}\varphi \; \exp\left(iS[\varphi] + i\int_x J\varphi \right)$$

scalar field in de Sitter

$$(-\Box + m^2)u = 0$$
$$\implies u \propto H_\nu(p)$$

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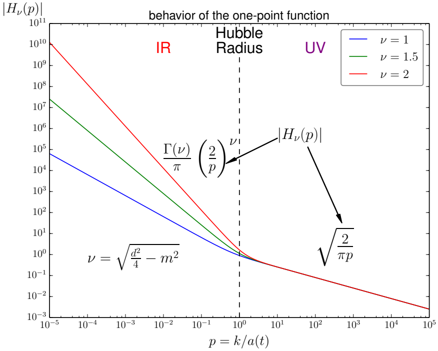

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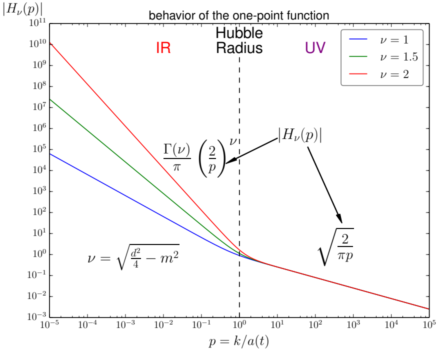
mass $\Longrightarrow$ IR regulation

small mass :
non perturbative physics



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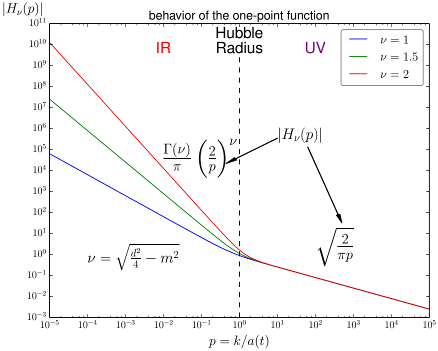

Perturbative QFT

- Sort contributions as powers of λ (small parameter)
- each contribution contains fluctuations of all size

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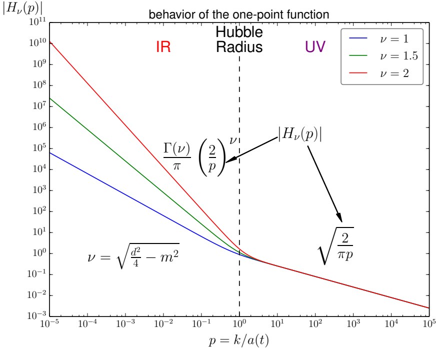

Perturbative QFT

- Sort contributions as powers of λ (small parameter)
- each contribution contains fluctuations of all size

NPRG

- Sort contributions by fluctuation size.
- Add contributions progressively

Overview

1. The Non-Perturbative Renormalisation Group
2. Onset of gravitational effects
3. Stochastic approach and Euclidean de Sitter space
4. NPRG flows and symmetry restoration
5. Conclusions

Overview

NPRG : general framework

Idea : regularize the theory at the action level :

$$S_k = S + \Delta S_k, \qquad \Delta S_k[\varphi] = \frac{1}{2}\int_{x,y} \varphi(x) R_k(x,y) \varphi(y)$$

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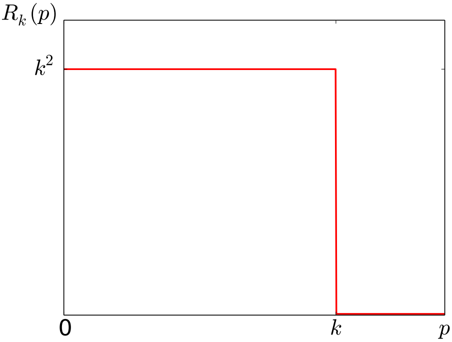

- $R_k(p) \underset{p\to 0}{\to} k^2$
- $R_k(p) \underset{p\to\infty}{\to} 0$

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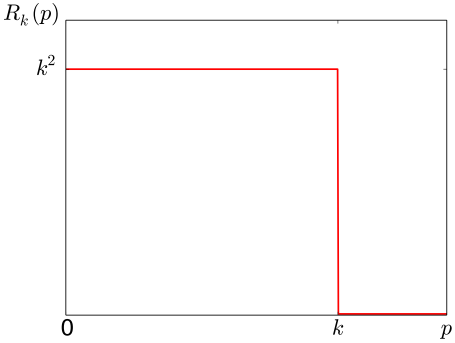

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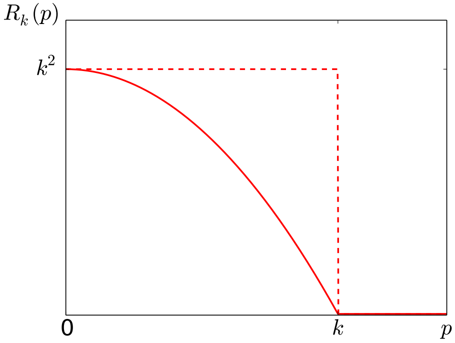

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Litim regulator : $R_k(p) = (k^2 - p^2)\theta(k^2 - p^2)$ Litim '01

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UV limit : $\Gamma_k[\phi] \underset{k\to\Lambda}{\rightarrow} S[\phi]$

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Flow equation

$$\dot{\Gamma}_k = \frac{i}{2}\mathrm{Tr}\left\{\dot{R}_k\left(\Gamma_k^{(2)} + R_k\right)^{-1}\right\}$$

Wetterich '93, Berges Mesterházy '12, Gasenzer Pawlowski '08

$t = \log(k/H)$: flow time

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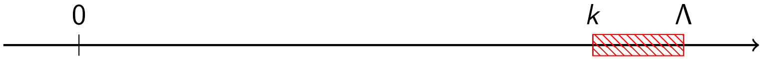

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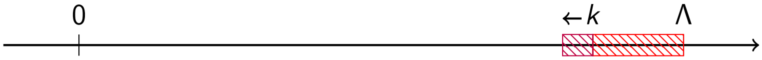

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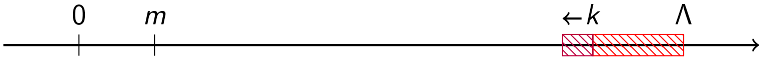

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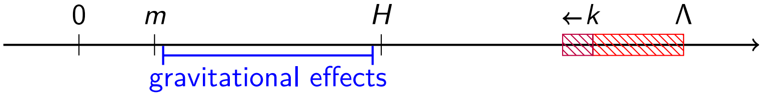

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NPRG : Local Potential Approximation

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- Extract information on large scale behavior

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complicated...

- Extract information on large scale behavior
- Derivative expansion :

$$\Gamma_k[\phi] = -\int_x \left(V_k(\phi) + \frac{Z_k(\phi)}{2}(\nabla\phi)^2 + \frac{Z_k^{(2)}(\phi)}{4!}(\nabla\phi)^4 + \ldots\right)$$

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NPRG : flow of the local potential

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$$B_d(\nu, k) = e^{-\pi \mathrm{Im}(\nu)}\left\{ \left(d^2 - 2\nu^2 + 2k^2\right)|H_\nu(k)|^2 + 2k^2|H_\nu'(k)|^2 - 2dk\,\mathrm{Re}\left[H_\nu^*(k)H_\nu'(k)\right]\right\}$$

Kaya '13

Serreau '14

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Heavy and/or UV regime $\rightarrow$ Minkowski

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No gravitational effects

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light IR regime

$$\beta(V'', k) \propto \frac{k^{d+2-2\nu_k}}{k^2 + V_k''}(d + 2\nu_k)\Gamma^2(\nu_k)$$

Onset of gravitational effects : Crossing the horizon

Minkowski

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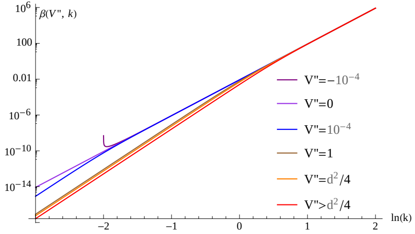

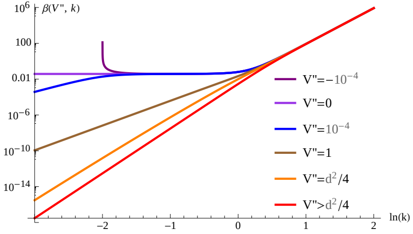

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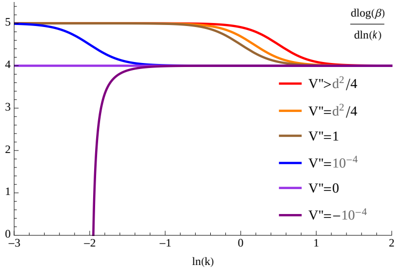

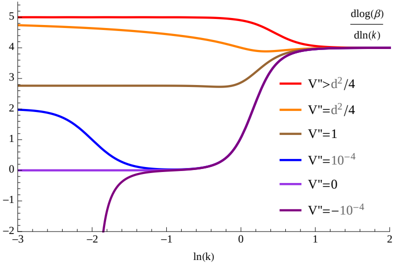

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$$\Omega_{D+1} = \frac{2\pi^{\frac{D+1}{2}}}{\Gamma(\frac{D+1}{2})}$$

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Onset of gravitational effects : dimensional reduction

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Correspondence

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- Stochastic approach
- QFT on Euclidean de Sitter

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Starobinsky and Yokoyama : effective theory for light fields on superhorizon scales

Relation to the stochastic approach

Starobinsky and Yokoyama : effective theory for light fields on superhorizon scales

- Large amplitude of fluctuations on superhorizon scales
  $\implies$ classical stochastic behavior.

Starobinsky '86, Starobinsky Yokoyama '94

Relation to the stochastic approach

Starobinsky and Yokoyama : effective theory for light fields on superhorizon scales

- Large amplitude of fluctuations on superhorizon scales
  $\Longrightarrow$ classical stochastic behavior.
- Spatial size $>$ causal horizon & slow evolution
  $\Longrightarrow$ single degree of freedom $\varphi(t)$.

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  $\Longrightarrow$ classical stochastic behavior.
- Spatial size $>$ causal horizon & slow evolution
  $\Longrightarrow$ single degree of freedom $\varphi(t)$.
- Stationary gravitational redshift
  $\Longrightarrow$ stochastic sourcing by the subhorizon modes.

Starobinsky '86, Starobinsky Yokoyama '94

Relation to the stochastic approach

Starobinsky and Yokoyama : effective theory for light fields on superhorizon scales

- Langevin equation for the effective dynamics :

$$\partial_t \varphi(t) + \frac{1}{d} \frac{\partial V_{\text{eff}}(\varphi)}{\partial \varphi(t)} = \xi(t)$$

Lazzari Prokopec '13

Relation to the stochastic approach

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- Stationary attractor solution at late times (equilibrium state) :

$$\mathcal{P}(\varphi) \propto e^{-\Omega_{D+1} V_{\text{eff}}(\varphi)}$$

Lazzari Prokopec '13

Relation to the stochastic approach

Starobinsky and Yokoyama : effective theory for light fields on superhorizon scales

- Langevin equation for the effective dynamics :

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- Stationary attractor solution at late times (equilibrium state) :

$$\mathcal{P}(\varphi) \propto e^{-\Omega_{D+1} V_{\text{eff}}(\varphi)}$$

- Identical to zero-dimensional generating functional, e.g. :

$$\langle \varphi^2 \rangle = \frac{\int d\varphi\, \varphi^2\, \mathcal{P}(\varphi)}{\int d\varphi\, \mathcal{P}(\varphi)} = \frac{1}{\Omega_{D+1}} \frac{\partial^2 \mathcal{W}_{k=0}(J)}{\partial J^2}\bigg|_{J=0}$$

Lazzari Prokopec '13

QFT on the D-sphere

Decomposing on the spherical harmonics :
$$\varphi(x) = \sum_{\vec{L}} \varphi_{\vec{L}} Y_{\vec{L}}(x) = \underbrace{\varphi_0 Y_0}_{\bar{\varphi}} + \hat{\varphi}(x)$$

Beneke Moch '12,
Benedetti '14

Relation to Euclidean de Sitter space

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$$e^{-\Omega_{D+1}\mathcal{W}_k(J)} = \int d\bar{\varphi}\, e^{-\Omega_{D+1}\left[V_{\text{eff}}(\bar{\varphi}) + \frac{k^2}{2}\bar{\varphi}^2 + J\bar{\varphi}\right]}$$

Beneke Moch '12,
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Overview

Generalisation to $O(N)$ symmetry

- Define : $V_k(\phi) = NU_k(\rho)$ with $\rho = \dfrac{\phi_a\phi_a}{2N}$

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- Large N limit : $\dot{U}_k = \beta\left(U_k'(\rho), k\right)$ first order equation!

Symmetry restoration in the large N limit

potential minimum

- $\dot{U}_k = \beta\left(U'_k(\rho), k\right)$
- $U'_k(\bar{\rho}_k) = 0$
- $\dot{\bar{\rho}}_k = -\partial_W \beta(W, k)|_{W=0}$

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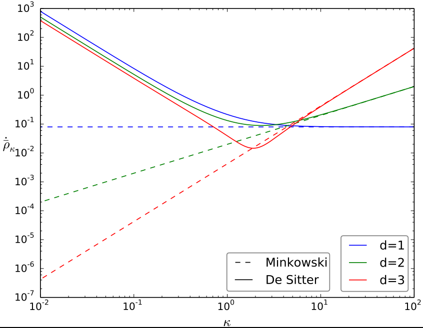

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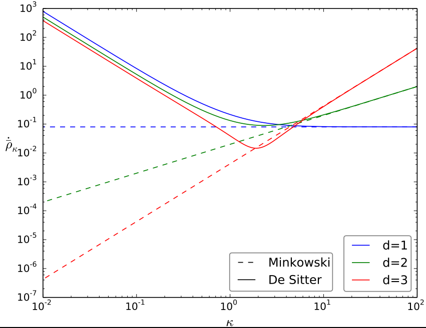

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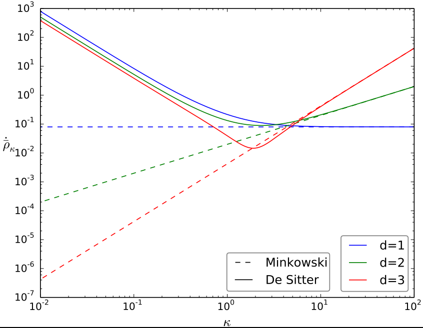

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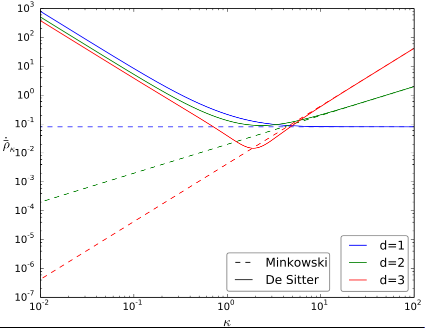

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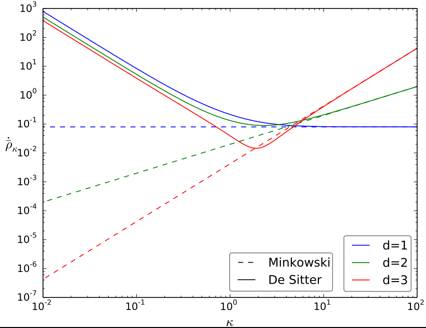

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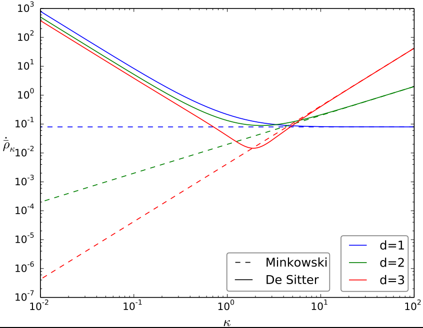

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IR regime

$\dot{\bar{\rho}}_k \propto k^{0-2}$ $\qquad D_{\text{eff}} = 0$

Flow in the IR : large N limit

exact solution to the flow

$$U_k(\rho) = U_k(0) - k\rho + \frac{M_k^4(\rho) - M_k^4(0)}{2\lambda_{k_0}} + \frac{1}{2\Omega_{D+1}}\left(1 - \frac{k^2}{k_0^2}\right)\ln\frac{M_k^2(\rho)}{M_k^2(0)}$$

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Serreau '11

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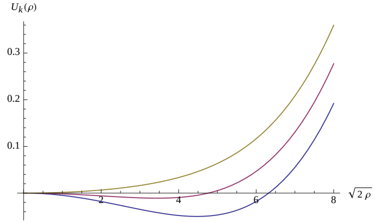

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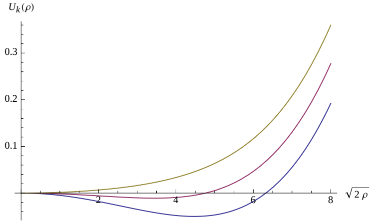
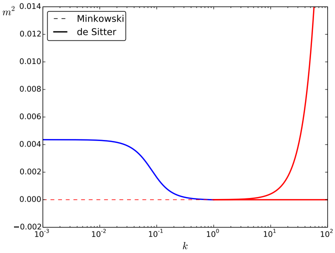


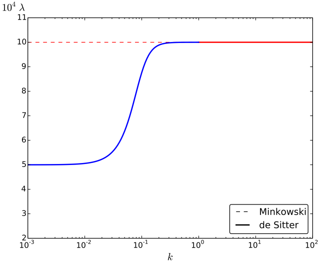
Effective mass and coupling

$$m_{k=0}^2 = \frac{m_{k_0}^2}{2} + \sqrt{\frac{m_{k_0}^4}{4} + \frac{\lambda_{k_0}}{2\Omega_{D+1}}}$$

$$\lambda_{k=0} = \lambda_{k_0}\left(1 + \frac{\lambda_{k_0}}{2\Omega_{D+1} m_{k=0}^4}\right)^{-1}$$

From UV to IR : mass regeneration close to criticality



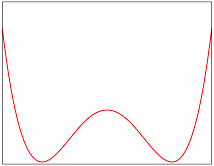


Symmetry restoration at the horizon

- $\bar{\rho}_\Lambda = \bar{\rho}_c$
- $k_0 \approx H,\ m_{k_0} \to 0$
- effective mass $m^2_{k=0} \approx \sqrt{\dfrac{\lambda_{k_0}}{2\Omega_{D+1}}}$
- effective coupling $\lambda_{k=0} \approx \dfrac{\lambda_{k_0}}{2}$

Convexity

- $\beta(V'', k)$ is decreasing with V''
  $\implies$ convexification



Symmetry restoration for finite N

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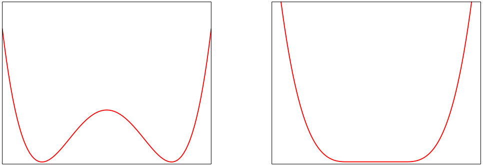

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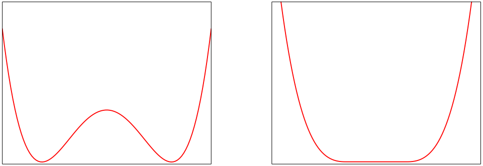

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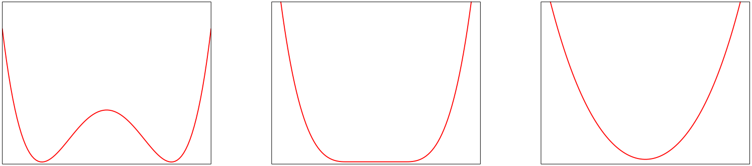

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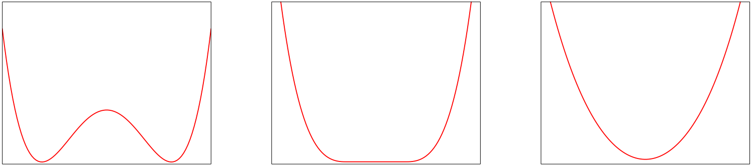

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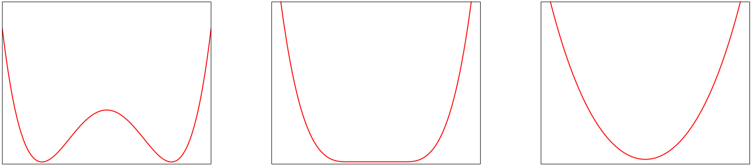

Symmetry restoration and mass regeneration

- $m^2_{k=0} = \mathcal{A}(N)\sqrt{\dfrac{\lambda_{k_0}}{2\Omega_{D+1}}}$
- $\dfrac{\lambda_{k=0}}{\lambda_{k_0}} = \dfrac{N\mathcal{A}^2(N)}{2}\left(1 - \dfrac{\mathcal{A}^2(N)}{1+2/N}\right)$

$$\mathcal{A}(N) = \frac{\sqrt{N}}{2}\frac{\Gamma\left(\frac{N}{4}\right)}{\Gamma\left(\frac{N+2}{4}\right)}$$

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LPA'

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momentum dependance

$$\dot{\Gamma}_k = \frac{i}{2} \mathrm{Tr} \left\{ \dot{R}_k \left(\Gamma_k^{(2)} + R_k \right)^{-1} \right\}$$


Quantum fields and IR Issues in de Sitter Space
Workshop: July 20-31, 2015, Natal, Brazil
Thank you!
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Atsushi Higuchi
University of York (York, United Kingdom)
Shun-Pei Miao
National Cheng Kung University (Tainan, Taiwan)
Ian Morrison
McGill University (Montreal, Canada)
Emil Mottola
Los Alamos National Laboratory (Los Alamos, USA)
Thiago Pereira
Londrina State University (Parana, Brazil)
Tomislav Prokopec
Utrecht University (Utrecht, Netherlands)
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Takahiro Tanaka
Kyoto University (Kyoto, Japan)
Yaser Tavakoli
Federal University of Espírito Santo (Vitória, Brazil)
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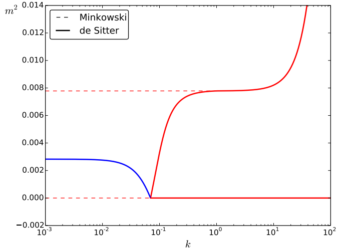
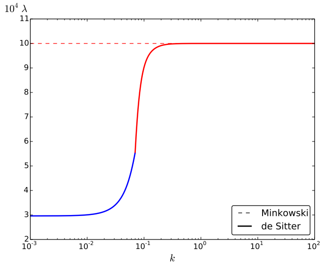
Early registration deadline May 20, 2015
For registration visit www.iip.ufrn.br
or contact us at events@iip.ufrn.br

From UV to IR : mass regeneration





Symmetry restoration at the horizon

- $k_0 \approx 1,\ m_{k_0} < 0$
- effective mass $m^2_{k=0} \approx \frac{\lambda^{\text{eff}}_{k_0}}{2} |m^2_{k_0}|$
- effective coupling
  $\lambda_{k=0} \approx \frac{\lambda^{\text{eff}}_{k_0}}{2} \lambda_{k_0}$