Matter loop corrections and the linearised Weyl tensor as an observable in cosmological spacetimes

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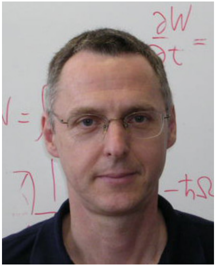

Enric Verdaguer
Barcelona

Albert Roura
Ulm

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- It turns out that
  $\mathcal{O} = \{F_{\mu\nu}\} \otimes (\{\text{electric charges}\} \oplus \{\text{Aharonov-Bohm phases}\})$
  Becker/Schenkel/Szabo '14

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- $\mathcal{O} = \{ f(F^a_{\mu\nu}, \nabla_\rho F^a_{\mu\nu}, \ldots) \}$ for suitable f (e.g., smooth & trace)
  Barnich/Brandt/Henneaux '00

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- Further advantage: $C^{(1)}_{\mu\nu\rho\sigma}$ and $R^{(1)\mu\nu}{}_{\rho\sigma}$ depend locally on $h_{\mu\nu}$

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- Riemann correlator including loop corrections from conformal fields is IR-finite MBF/Roura/Verdaguer '12-'15

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$S[h_{\mu\nu}, \phi] = \kappa^{-2} S_{\mathsf{EH}}[h_{\mu\nu}] + S_{\mathsf{HD}}[h_{\mu\nu}] + \mathrm{i} S_{\mathsf{M}}[h_{\mu\nu}, \phi]$

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- Integrate out matter fields, renormalise (same counterterms as in flat space!), obtain effective action for gravitons $S_{\mathsf{eff}} = S_{\mathsf{G}}[h^+_{\mu\nu}] - S_{\mathsf{G}}[h^-_{\mu\nu}] + \Sigma[h^\pm_{\mu\nu}]$ with non-local part $\Sigma[h^\pm_{\mu\nu}] = 1/4 \iint h^\pm_{\mu\nu}(x) \left\langle T^{\mu\nu}_\pm(x) T^{\rho\sigma}_\pm(x') \right\rangle_{\mathsf{c}} h^\pm_{\rho\sigma}(x') \sqrt{-g}\, \mathrm{d}^4 x \sqrt{-g}\, \mathrm{d}^4 x'$

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- Make most general ansatz for $\left\langle \tilde{C}^{\mu\nu}{}_{\rho\sigma}(x) \tilde{R}^{\rho'}{}_{\sigma'}(x') \right\rangle_{\mathsf{c}}$

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- Evaluating Bianchi identities gives dS-invariant result!

General matter loop corrections

$$\begin{aligned}\left\langle T_{\mu\nu}(x) T_{\rho'\sigma'}(x') \right\rangle_{\mathrm{c}} &= g_{\mu\nu} g_{\rho'\sigma'} {}^{(1)}\mathcal{S}(Z) \\ &+ \left(g_{\mu\nu} n_{\rho'} n_{\sigma'} + g_{\rho'\sigma'} n_\mu n_\nu \right) {}^{(2)}\mathcal{S}(Z) + n_\mu n_\nu n_{\rho'} n_{\sigma'} {}^{(3)}\mathcal{S}(Z) \\ &+ 4 n_{(\mu} g_{\nu)(\rho'} n_{\sigma')} {}^{(4)}\mathcal{S}(Z) + 2 g_{\mu(\rho'} g_{\sigma')\nu} {}^{(5)}\mathcal{S}(Z)\end{aligned}$$

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\end{aligned}
$$

$$
\begin{aligned}
&\left\langle \tilde{C}^{\mu\nu}{}_{\rho\sigma}(x) \tilde{R}^{\rho'}{}_{\sigma'}(x') \right\rangle_{\mathrm{c}} \\
&= \frac{\kappa^4}{30} \Bigg[2\delta^\mu_{[\rho} \delta^\nu_{\sigma]} \left(\delta^{\rho'}_{\sigma'} - n^{\rho'} n_{\sigma'} \right) - 6 \delta^{[\mu}_{[\rho} n^{\nu]} n_{\sigma]} \delta^{\rho'}_{\sigma'} \\
&\qquad - 3 \delta^{[\mu}_{[\rho} \left(n^{\nu]} g^{\rho'}_{\sigma]} n_{\sigma'} + n^{\nu]} g_{\sigma]\sigma'} n^{\rho'} + n_{\sigma]} g^{\nu]\rho'} n_{\sigma'} + n_{\sigma]} g^{\nu]}_{\sigma'} n^{\rho'} \right) \\
&\qquad - 3 \left(\delta^{[\mu}_{[\rho} - 2 n^{[\mu} n_{[\rho} \right) \left(g^{\nu]\rho'} g_{\sigma]\sigma'} + g^{\nu]}_{\sigma'} g^{\rho'}_{\sigma]} \right) \Bigg] \times \\
&\quad \times \left[- {}^{(2)}\mathcal{S}(Z) - {}^{(3)}\mathcal{S}(Z) - (4 - 11Z) {}^{(4)}\mathcal{S}(Z) + (1 - Z)(7 - 4Z) {}^{(5)}\mathcal{S}(Z) \right]
\end{aligned}
$$

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- Again, make most general ansatz for the Weyl-Weyl correlator compatible with invariance under spatial rotations and translations
- Evaluating Bianchi identities gives again dS-invariant result!
- $\left\langle \tilde{C}^{ab}{}_{cd}(x) \tilde{C}^{m'n'}{}_{p'q'}(x') \right\rangle_c = \sum_{k=1}^{3} {}^{(k)}\mathcal{C}^{[ab]}{}_{[cd]}{}^{[m'n']}{}_{[p'q']} {}^{(k)}\mathcal{D}(Z(x,x'))$

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- ${}^{(k)}\mathcal{D}_{abcdm'n'p'q'}$ are maximally symmetric bitensors incorporating all symmetries and tracelessness, e.g., ${}^{(1)}\mathcal{D}_{abcdm'n'p'q'} = g_{ac} g_{bd} g_{m'p'} g_{n'q'} - 6 g_{ac} g_{m'p'} g_{b(n'} g_{q')d} + 4 g_{a(m'} g_{p')c} g_{b(n'} g_{q')d}$

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- ${}^{(k)}\mathcal{D}(Z)$ are functions of the stress tensor components and function ${}^{(2)}\mathcal{C}(Z)$, given by

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- Fall-off behaviour similar to conformal matter case: ${}^{(k)}\mathcal{D}(Z) \sim \kappa^2 H^6 Z^{-2} + \kappa^4 H^8 \left(Z^{-2} + Z^{-3} \ln Z \right)$ MBF/Roura/Verdaguer '15

Friedmann-Lemaître-Robertson-Walker (FLRW) spacetimes

- Conformally flat $g_{\mu\nu} \, \mathrm{d}x^\mu \, \mathrm{d}x^\nu = - \mathrm{d}t^2 + a^2(t) \, \mathrm{d}\boldsymbol{x}^2$

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- For large enough ϵ, Bunch-Davies vacuum probably not physical: create more regular states by, e.g., matching from spacetimes without IR divergences Janssen/Prokopec '09

Metric perturbations

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- On irreducible components: $\delta s_1 = 2\xi_0' + 2Ha\xi_0\,, \quad \delta s_2 = \xi_0 + \frac{\partial_k}{\triangle}\xi_k'\,, \quad \delta s_3 = 2\frac{\partial_k}{\triangle}\xi_k\,, \quad \delta s_4 = \frac{2}{(n-1)}\partial_k\xi_k - 2Ha\xi_0\,, \quad \delta v_k^{\mathrm{T}1} = \xi_k' - \frac{\partial_k\partial_l}{\triangle}\xi_l'\,, \quad \delta v_k^{\mathrm{T}2} = \xi_k - \frac{\partial_k\partial_l}{\triangle}\xi_l\,, \quad \delta h_{kl}^{\mathrm{TT}} = 0$

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- Four gauge-invariant combinations: $H_{kl} = h_{kl}^{\mathrm{TT}}$, $V_k = v_k^{\mathrm{T1}} - v_k^{\mathrm{T2}\prime}$,
  $S = s_1 - (2s_2 - s_3')' - Ha(2s_2 - s_3')$,
  $\Sigma = s_4 - \frac{1}{n-1} \triangle s_3 + Ha(2s_2 - s_3')$

Linearised Weyl tensor

- Linearised Weyl tensor only involves gauge-invariant combinations $\left(\Pi_{kl} = \delta_{kl} - (n-1)\frac{\partial_k \partial_l}{\triangle}\right)$:

$$
\begin{aligned}
2(n-2) C^{0j}{}_{0l} &= (n-3) H^{j\,\prime\prime}_{l} + \triangle H^{j}_{l} - (n-3)\left(\partial^j V_l' + \partial_l V^{j\,\prime}\right) \\
&\quad - \frac{n-3}{n-1} \Pi^{j}_{l} \triangle (S + \Sigma) \,, \\
C^{0j}{}_{kl} &= \partial_{[k} H^{j\,\prime}_{l]} - \partial^j \partial_{[k} V_{l]} + \frac{1}{n-2} \delta^{j}_{[k} \triangle V_{l]} \,, \\
C^{ij}{}_{kl} &= -2 \partial^{[i} \partial_{[k} H^{j]}_{l]} + \frac{2}{n-2} \delta^{[i}_{[k} \left(\partial^2 H^{j]}_{l]} + \partial^{j]} V_{l]}' + \partial_{l]} V^{j]\prime}\right) \\
&\quad + \frac{2}{(n-1)(n-2)} \Pi^{[i}_{[k} \delta^{j]}_{l]} \triangle (S + \Sigma)
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&\quad + \frac{2}{(n-1)(n-2)} \Pi^{[i}_{[k} \delta^{j]}_{l]} \triangle (S + \Sigma)
\end{aligned}
$$

- Can be inversed, e.g.
$\triangle^2 H_{kl} = 2 \triangle C^{0k}{}_{0l} - 2 \partial_j \partial_{(k} C^{0j}{}_{0l)} - \partial_j C^{0k\prime}{}_{jl} - \partial_j C^{0l\prime}{}_{jk} - 2 \partial_i \partial_j C^{ik}{}_{jl}$

Weyl tensor correlator

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- Result ($P_{kl} = \eta_{kl} - \frac{\partial_k \partial_l}{\triangle}$):
  $\langle C_{0i0j}(x) C_{0k0l}(x') \rangle = \left[P_{k(i} P_{j)l} \triangle^2 - \frac{1}{2} P_{kl} P_{ij} \triangle^2 \right] I_2 + \Pi_{ij} \Pi_{kl} \triangle^2 \, I_1$

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- $I_1 = \kappa^4/(64\pi^2)\epsilon H^2/(27r) \Big[(r + \eta - \eta')^3 \ln\left(\mathrm{i}(r + \eta - \eta')\right) + (r - \eta + \eta')^3 \ln\left(-\mathrm{i}(r - \eta + \eta')\right) \Big]$

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- Result ($P_{kl} = \eta_{kl} - \frac{\partial_k \partial_l}{\triangle}$):
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- $I_1 = \kappa^4/(64\pi^2)\epsilon H^2/(27r) \Big[(r + \eta - \eta')^3 \ln\left(\mathrm{i}(r + \eta - \eta')\right) + (r - \eta + \eta')^3 \ln\left(-\mathrm{i}(r - \eta + \eta')\right) \Big]$
- $I_2 = \kappa^4/(16\pi^2) H(\eta') H(\eta) \Big[(1 - Z)^{-1} + \epsilon/(1 + Z) \ln\left(\frac{1-Z}{2}\right) + \epsilon \left(\eta - \eta'\right)/r \ln\left((\eta - \eta' + r)/(\eta - \eta' - r)\right) \Big]$ with
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- Other components are similar, give correct dS limit

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- Reconstruction of tensor part from Weyl tensor is not worse:
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- Gauge-invariant parts of metric perturbation can be reconstructed from Weyl tensor, but are non-local and need boundary conditions, while Weyl tensor is local

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- Solution is given by $a = (-H\eta)^{-1}$ with $H = \sqrt{\Lambda_{\text{eff}}/3}$ – de Sitter

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- Similar equations for vector and scalar parts of metric perturbation

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- Eliminate again unphysical solutions by order reduction: $h''^{\mathrm{TT}}_{ij} - 2/\eta\,(1-\nu)\,h'^{\mathrm{TT}}_{ij} - (1-2\nu)\,\triangle\,h^{\mathrm{TT}}_{ij} = \mathcal{O}\left(\kappa^4\right)$ with $\nu = 3\alpha\kappa^2 H^2$

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- At late times $\eta \to 0$, we get $g^{\pm} = \mathrm{const} + \mathcal{O}(\omega\eta)$, $(g^{\pm})' = \mathrm{const} + \mathcal{O}(\omega\eta)$

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- Solution $h^{\mathrm{TT}}_{ij}(\eta, \boldsymbol{p}) = e^{\pm}_{ij}(\boldsymbol{p}) g^{\pm}(\eta, \boldsymbol{p})$ with $g^{\pm} = (-\omega\eta)^{\frac{3}{2}-\nu}\left[C^{\pm}_1 J_{\frac{3}{2}-\nu}(-\omega\eta) + C^{\pm}_2 Y_{\frac{3}{2}-\nu}(-\omega\eta)\right]$ and $\omega^2 = (1-2\nu)\boldsymbol{p}^2$
- At late times $\eta \to 0$, we get $g^{\pm} = \mathrm{const} + \mathcal{O}(\omega\eta)$, $(g^{\pm})' = \mathrm{const} + \mathcal{O}(\omega\eta)$
- However, linearised Riemann tensor involves $\eta(g^{\pm})'$ and $\eta^2 g^{\pm}$ and thus decays at late times

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- Scalar and vector parts are constrained and have vanishing solutions

Perturbed initial state

- Generate a large class of non-vacuum initial states at $\eta = \eta_0$ by evolving an asymptotic BD state in a fixed perturbed geometry, asymptotically dS as $\eta \to -\infty$ and smoothly connected to the dynamics at $\eta = \eta_0$ Pérez-Nadal/Roura/Verdaguer '08

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- Corrections due to δT_{ij} decay faster than in the vacuum case! (similar for vector and scalar)
- Poincaré patch is thus stable for arbitrary linear perturbations due to conformal fields, including arbitrary, spatial inhomogeneous (but sufficiently small) perturbations of initial state
  Fröb/Papadopoulos/Roura/Verdaguer '13

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- Status of graviton self-interactions (or internal graviton lines) unclear: no good observables known
- Poincaré patch stable under quantum perturbations (terms and conditions apply)

Thanks for your attention

Questions?

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References: arXiv:1205.3097, arXiv:1301.5261, arXiv:1403.3335, arXiv:1409.7964, arXiv:1509.?????