Hawking radiation inspired toy model for inflation and baryogenesis*

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Quantum Fields and IR Issues in de Sitter Space

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*S.K. Modak and D. Singleton
arXiv:1205.3404 [gr-qc] *IJMPD **21**, 124202 (2012)*
arXiv:1207.0230 [gr-qc] *PRD **86**, 123515 (2012)*
arXiv:1410.6785 *EJPC **75**, 200 (2015)*

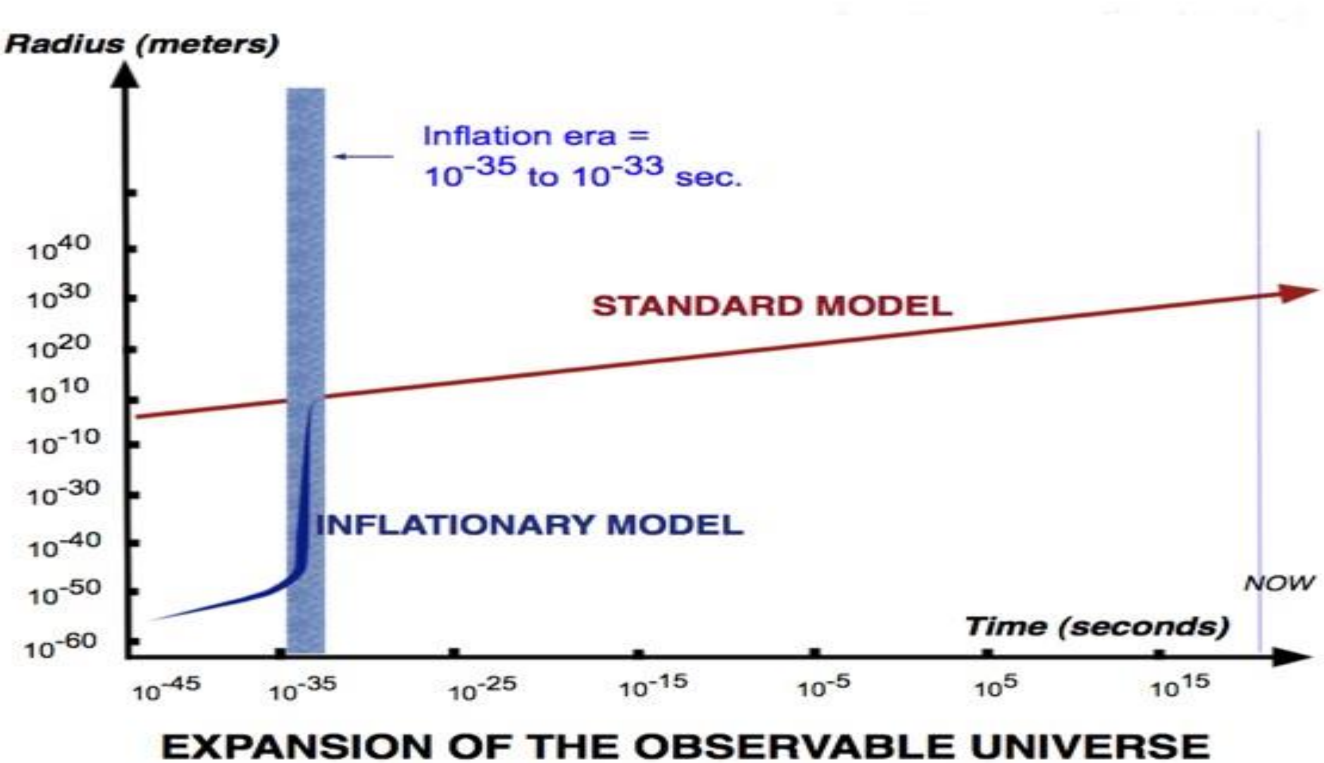
Outline of Talk

- Particle Creation in Cosmology
- Modified FRW Equations and Consequences
- Baryogenesis
- Summary

Inflation basic details

•Rapid expansion in the early Universe. Addresses Horizon, Flatness Monopole Problems.

•Questions: how does inflation start/stop ? What is mechanism for inflation?

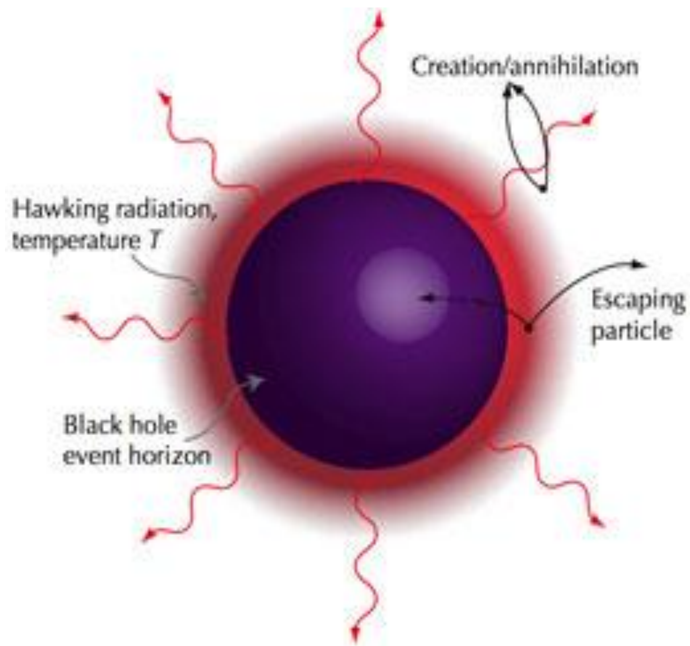


Influence of particle production/decay on cosmology

- Idea that particle production influences cosmological evolution goes back to Erwin Schrödinger *[Physica **6**, 899 (1930)]*

- Our approach is similar to Prigogine, *et al.* in *[GRG **21**, 767 (1989)].* See also works by JAS Lima *et al.* starting in 1995 and onward

- Prigogine *et al.* considered *generic* particle creation. We consider <u>Hawking-like radiation</u> as the particle creation mechanism.



Advantage of generic particle creation inflation

- Prigogine *et al* emphasized two advantages of their particle creation model of inflation:

(1) Explains the enormous entropy production in the early Universe via the *irreversible energy flow from the gravitational field to the created particles.*

(2) Since matter creation is an irreversible process the initial singularity is avoided. The Universe begins from an *instability of the vacuum* instead of a singularity.

Hawking radiation of FRW space-time

- We consider the cosmological particle creation model of Prigogine *et al* with Hawking radiation of FRW.

- FRW emits Hawking-like radiation [T. Zhu *et al,* IJMPD **19**, 159 (2010)] with temperature

$$T = \frac{\hbar c \kappa}{2\pi k_B} = \frac{\hbar c}{2\pi k_B}\left(\frac{1}{\tilde{r}_A}\right)\left|1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right| = \frac{\hbar\sqrt{H^2 + kc^2/a^2}}{2\pi k_B} \approx \frac{\hbar H}{2\pi k_B}.$$

- Where $\tilde{r}_A = \dfrac{c}{\sqrt{H^2 + \frac{kc^2}{a^2}}}$ and $H = \dfrac{\dot{a}}{a}$.

- Various approximations (k~0 and d$\check{r}_A$/dt ~0).

FRW equations for matter creation

- In the presence of matter creation the equations of the standard FRW metric become

$$3\frac{\dot{a}^2}{a^2} + 3\frac{kc^2}{a^2} = \frac{8\pi G\rho}{c^2}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = -\frac{8\pi G}{c^2}(p - p_c) \qquad p_c = \frac{\rho + p}{3nH}\psi$$

$$\frac{\dot{n}}{n} + 3\frac{\dot{a}}{a} = \frac{\psi}{n} \; .$$

- 1st Friedmann equation unaltered; 2nd Friedmann equation has creation pressure term; 3rd, new equation is particle number density (n) and Ψ =generic creation rate.

Thermodynamics and Particle Creation

- From the first law of thermodynamics

$$\frac{dQ}{dt} = \frac{d}{dt}(\rho V) + p\frac{dV}{dt}$$

- Usual assumption – universe is *closed, adiabatic* system dQ=0. Hard to explain large entropy production TdS=dQ=0.

- Due to Hawking radiation of FRW space-time we take from the Stephan-Boltzmann Law (A_H is horizon area)

$$P = +\frac{dQ}{dt} = \sigma A_H T^4$$

Thermodynamics and Particle Creation

- Combining this dQ/dt with the 1st law of thermodynamics

$$\frac{\dot{\rho}}{\rho} + 3(1+\omega)\frac{\dot{a}}{a} = 3\omega_c(t)\frac{\dot{a}}{a} \quad \rightarrow \quad \psi_{FRW}(t) = \frac{3nH\omega_c(t)}{(1+\omega)}$$

- With $\omega_c(t) = \alpha\rho(t)$, where $\alpha = \frac{\hbar G^2}{45c^7} = 4.8 \times 10^{-116}(J/m^3)^{-1}$

- Setting ω=1/3 (relativistic matter) and solving the above gives

$$\rho = \frac{D_0 a^{-3(1+\omega)}}{1 + (\frac{\alpha D_0}{1+\omega})a^{-3(1+\omega)}} \rightarrow \frac{D_0}{a^4 + \frac{3\alpha D_0}{4}}$$

Particle Creation Rate

- For a generic particle creation model one need *$\psi=3nH$* for *$a(t)=e^{Ht}$* *[JAS Lima et al. PRD 1995]*

- The creation rate for the above is

$$\psi_H(t) = \frac{3nH\omega_c(t)}{(1+\omega)}$$

- *$\psi=3nH$* and *$a(t)=e^{Ht}$* occur when $\omega_c \sim (1+\omega)$ or $\omega_c = \alpha\rho(t) \sim 1$

Scale factor a(t)

- For this ρ(t) and ψ(t) one can find an exact, analytical expression for a(t)

$$\sqrt{\alpha D_0+\frac{4}{3}a^4}+\sqrt{\alpha D_0}\ln\left[\frac{a^2}{2\sqrt{3}\left(\sqrt{\alpha D_0}+\sqrt{\alpha D_0+\frac{4}{3}a^4}\right)}\right]=\frac{8}{3}\sqrt{\frac{2\pi G D}{c^2}}t-(K_0-1)\sqrt{\alpha D_0}$$

Very early Universe limit

- For very early universe one has $\alpha D_0 \gg a^4$ and $\rho \sim 4\alpha/3$
- The *inflationary* solution for a(t) is now (K_0 is integration constant)

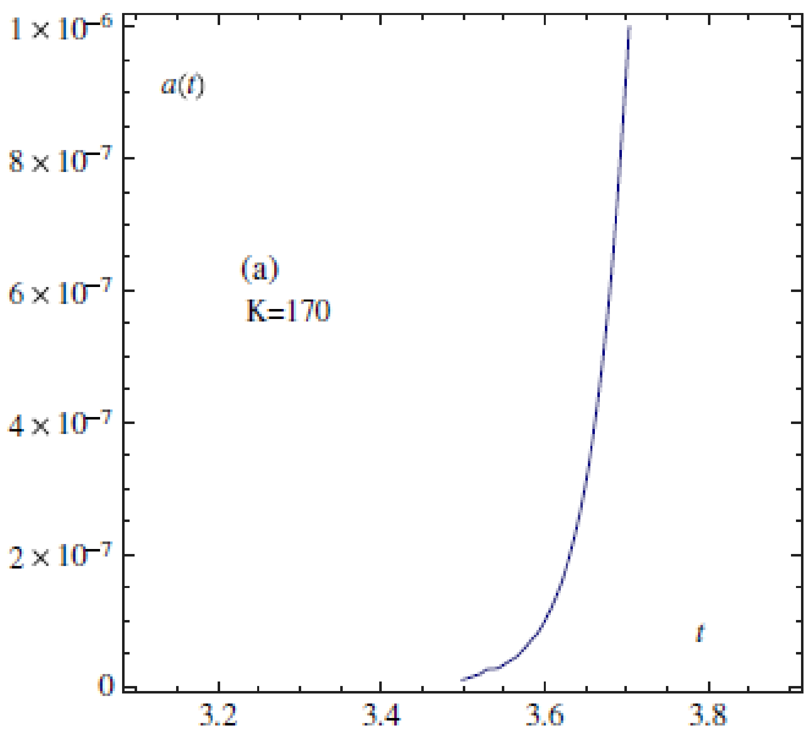
$$a(t) = 2(3\alpha D_0)^{\frac{1}{4}} \exp\left[\sqrt{\frac{32\pi G}{9c^2\alpha}}\, t - \frac{K_0}{2}\right]$$

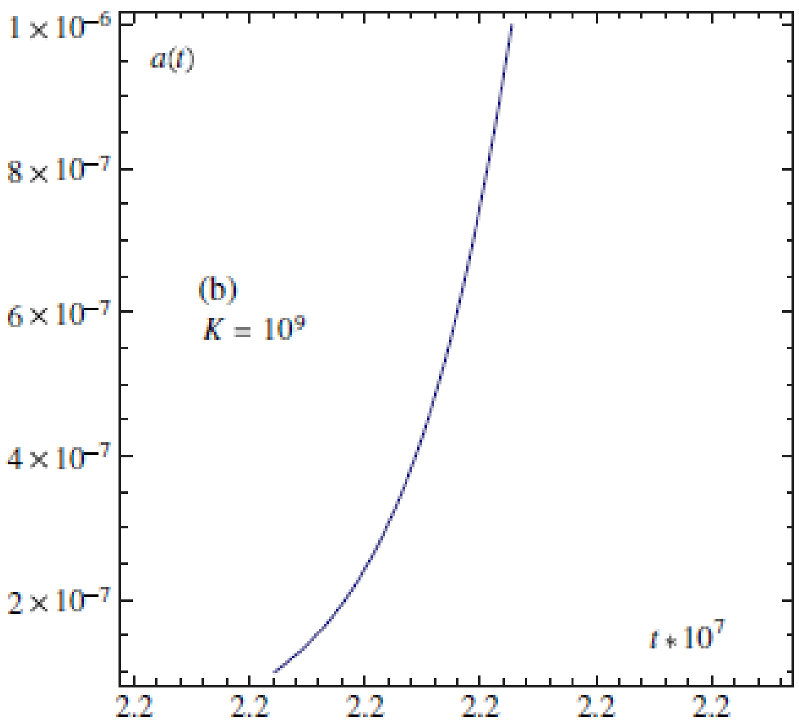
- The Hubble constant is enormous $H = \frac{\dot{a}}{a} = \sqrt{\frac{32\pi G}{9c^2\alpha}} \approx 10^{45}\frac{1}{sec}$

- One sees that $H^2 \gg kc^2/a^2$ is valid. Also since a(t) ~Exp[..t] one has $d\check{r}_A/dt \sim 0$.

Very early Universe limit

- Exponential expansion





Early Universe limit

- For early universe one has $a^4 >> \alpha D_0$ and $\rho \sim D_0 / a^4$

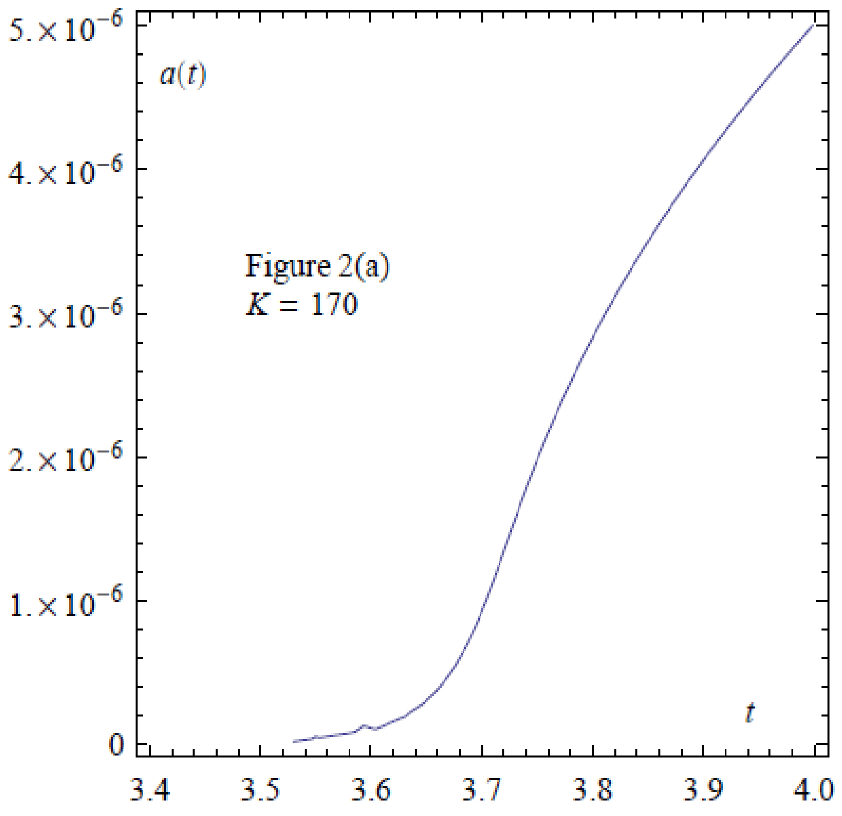
- The solution now is that of radiation

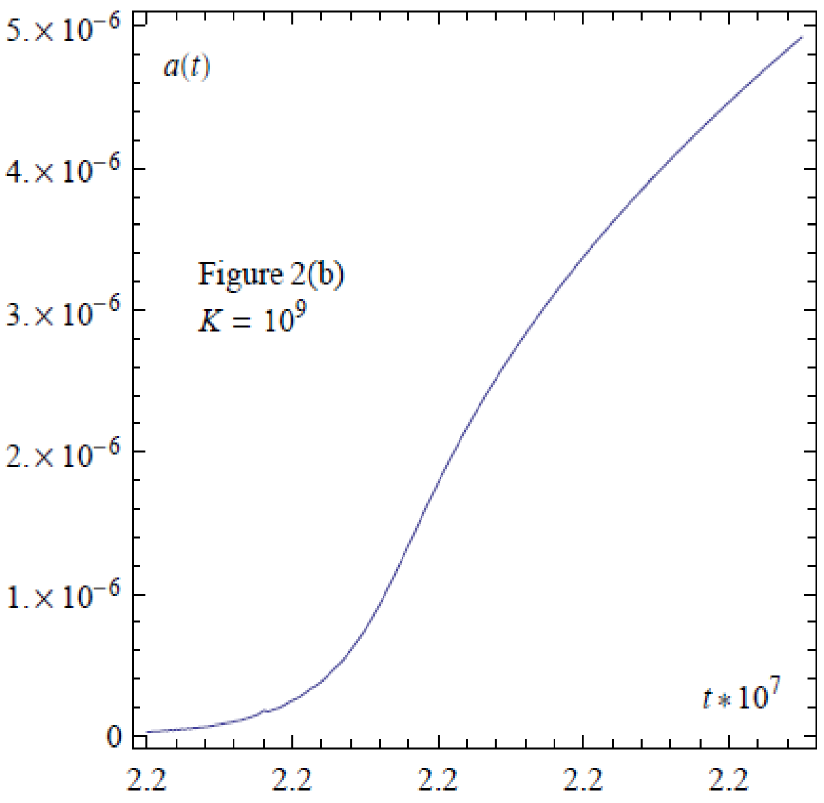
$$a(t) \approx \left(\frac{32\pi G D_0}{3c^2}\right)^{1/4} t^{1/2}$$

- This give a natural transition from Exp[..] expansion to radiation power law expansion.

Exit from inflation

- Time is in Planck time and different values of K_0. Transition from a(t)~Exp[…] to $a(t) \sim t^{1/2}$ behavior.





Entrance to inflation

- Here inflation is driven by Hawking radiation/particle creation. Reverse of BH evaporation.

- It is speculated that in the quantum gravity regime Hawking radiation turns off [e.g. *P. Nicolini, IJMPA **24**, 1229 (2010)*].

(i) The Universe expands according to $t^{1/2}$ until one exits the QG regime

(ii) Hawking-like radiation turns on and drives inflation until the Hawking temperature drops below a critical value

(iii) At which point the expansion becomes $t^{1/2}$ again.

Comparison with standard inflation

Standard Inflation

- Size increase by a factor of 10^{26}
- a(t) goes from $\sim 10^{-27}$ m to $\sim 10^{-1}$ m
- Lasts for a time $\Delta t \sim 10^{-32}$ to 10^{-33} sec
- Scale of inflation =< 10^{16} GeV

Hawking FRWL Inflation

- Size increase by a factor of ~ 10^{26}
- a(t) goes from $\sim 10^{-32}$ m to ~ 10^{-6} m
- Lasts for a time $\Delta t \sim 10^{-43}$ sec
- Scale of inflation around Planck scale

Comparison with standard inflation

- In bare form incompatible with observations
- Some “nice” features – no scalar field, turn-on/turn-off mechanism.
- Possible fixes:

(i) Running of G(p) with energy *a la* QFT.

(ii) The dimensionality of the early Universe is different so G at present is an effective G different from G_0>G in the early Universe [*ADD-scenario PLB 1998 or Mureika Stojkovic PRL 2011]*. Makes α larger.

$$\alpha = \frac{\hbar G^2}{45 c^7}$$

Baryogenesis

- The Universe contains more matter than anti-matter
- Characterized by $\eta = \dfrac{n_B - n_{\bar{B}}}{s} \approx 6 \times 10^{-10}$
- Sakharov gave three conditions for baryogenesis

(i) Violation of baryon number

(ii) C and CP violation

(iii) Departure from thermal equilibrium

Gravitational B-L

- A string theory inspired gravitational B-L interaction *[H. Davoudiasl et al PRL (2004)]*

$$\frac{\hbar^3}{M_*^2 c}\int d^4x\sqrt{-g}(\partial_\mu \mathcal{R})J^\mu_{B-L}$$

- Allows on to define a chemical potential which distinguishes particle species (charge q_i) based on B-L charge

$$\mu_i = \frac{q_i \hbar^3}{c^2}\frac{\dot{\mathcal{R}}}{M_*^2}$$

Gravitational B-L plus thermal background

- In a thermal bath the rate per area of B-L creation is *[A. Hook PRD (2014) here using primordial BHs]*

$$\frac{d(\Delta N_{B-L})}{d(Area)(cdt)} = \int \frac{d^3p}{(2\pi\hbar)^3}\frac{1}{e^{(pc-\mu)/k_BT}+1} - \int \frac{d^3p}{(2\pi\hbar)^3}\frac{1}{e^{(pc+\mu)/k_BT}+1}$$

- Above has Pauli-Dirac statistics since in SM only <u>fermions</u> carry B and L numbers

- Integrating the above and assuming μ<<T gives

$$\frac{d(\Delta N_{B-L})}{d(Area)(cdt)} \approx \frac{\mu^3}{6\pi^2(\hbar c)^3} + \frac{\mu k_B^2 T^2}{6(\hbar c)^3} \approx \frac{\mu k_B^2 T^2}{6(\hbar c)^3}$$

Gravitational B-L plus thermal background

- Summing over different fermions with degrees of freedom g_i and charge q_i

$$\frac{d(\Delta N_{B-L})}{dt} = \frac{k_B^2 T^2 \times Area}{6 M_*^2 c^4} \sum_i g_i q_i^2 \dot{\mathcal{R}}$$

- Now need *Area*, *T* and d*R*/dt to get the baryon excess.

Gravitational B-L: Area and T parts

- For the area we use effective horizon radius

$$Area = 4\pi r_{FRW}^2 = \frac{4\pi c^2}{H^2}$$

- For the temperature we take FRW temperature

$$T \approx \frac{\hbar H}{2\pi k_B}$$

Gravitational B-L: d*R*/dt part

- The change is the Ricci scalar is given at tree level by

$$\dot{\mathcal{R}} = -9(1-3\omega)(1+\omega)\frac{H^3}{c^2}$$

- This is zero for ω=1/3 , -1 (radiation and dS)

- To one-loop (g=SU(3) coupling, N_c =3 , N_F =6)

$$1-3\omega = \frac{5}{6\pi^2}\frac{g^4}{(4\pi\hbar c)^2}\frac{(N_c+\frac{5}{4}N_f)(\frac{11}{3}N_c-\frac{2}{3}N_f)}{2+\frac{7}{2}[N_cN_f/(N_c^2-1)]}$$

N_{B-L}

- We can now calculate N_{B-L} and its density n_B
- No baryogenesis during inflation since $1+\omega=0$.
- Baryogenesis during radiation phase $1-3\omega \sim 10^{-2}$ and $H \sim 1/t$

$$\frac{d(\Delta N_{B-L})}{dt} = -\frac{0.13}{8\pi M_*^2 c^4}\frac{\hbar^2}{t^3}$$

$$N_{B-L} = -\frac{0.13}{8\pi M_*^2 c^4}\int_{t_{rad}^*}^{\infty}\frac{\hbar^2}{t^3}\,dt = -\frac{0.13\hbar^2}{16\pi M_*^2 c^4 (t_{rad}^*)^2}$$

n_B and s and η

- The baryon number density is

$$n_B = \frac{|N_{B-L}|}{Volume} = \frac{0.39 H^3 \hbar^2}{64\pi^2 M_*^2 c^7 (t_{rad}^*)^2}$$

- The entropy density is

$$s = \frac{\rho + pc}{k_B T} \approx \frac{2\pi^2}{45(\hbar c)^3} g_* (k_B T)^3 \to \frac{2\pi^2}{45(\hbar c)^3} g_* (k_B T_{FRW})^3 = \frac{H^3}{180 c^3 \pi} g_*$$

- g* is the number of boson/fermion degrees of freedom. g*~ 100

n_B and s and η

- Using n_B and s we now obtain η

$$\eta = \frac{n_B}{s} \approx \frac{17.6\hbar^2}{16\pi g_* M_*^2 c^4 (t^*_{rad})^2} \approx \frac{\hbar^2}{100\pi M_*^2 c^4 (t^*_{rad})^2}$$

- To obtain observed η we only need to set t^*_{rad} *to* $\sim 10^{-39}$ *sec*

- This can be accomplished by setting $K_0 \sim 10^6$

Conclusions

- Generic particle creation (*a la* Prigogine *et al*) can drive inflationary expansion. Here generic particle creation → Hawking-like radiation.
- Explains large entropy production and avoids initial singularity.
- Gives a natural exit and (maybe) entrance to inflation.
- Same mechanism can give rise to baryogenesis.

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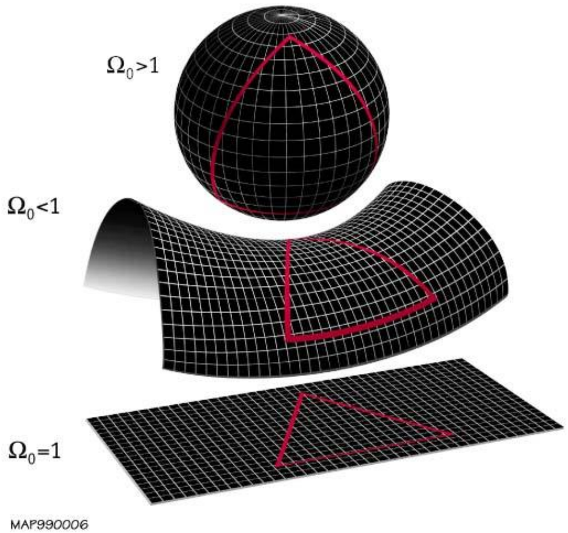
Reasons for Inflation

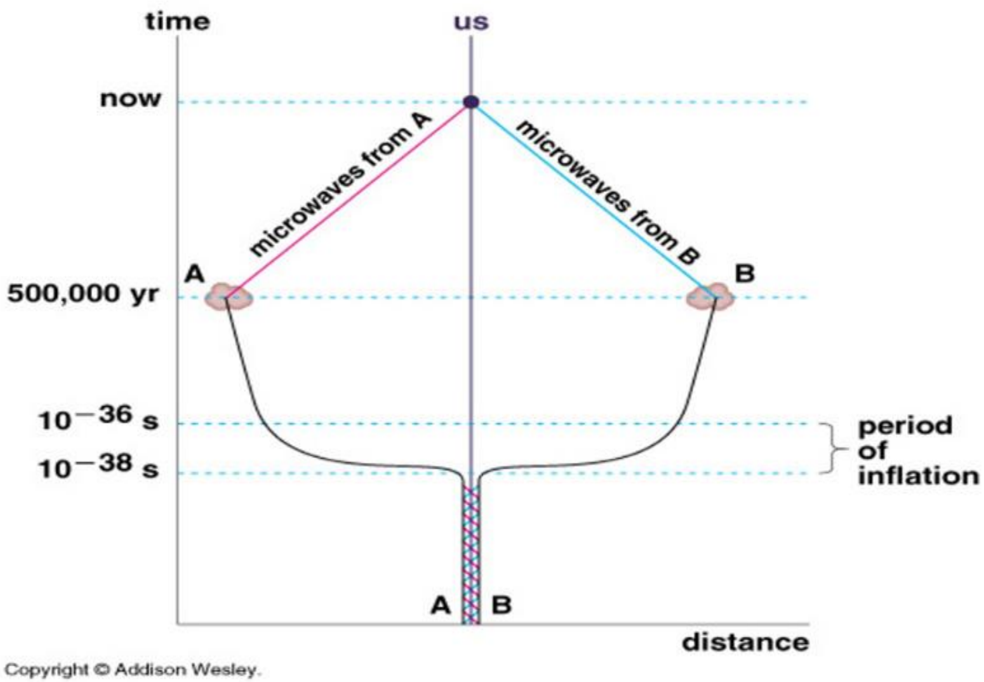
Inflation Explains/Solves

1. *(Spatial) flatness problem*



2. *Horizon problem*

3. *Monopole problem*



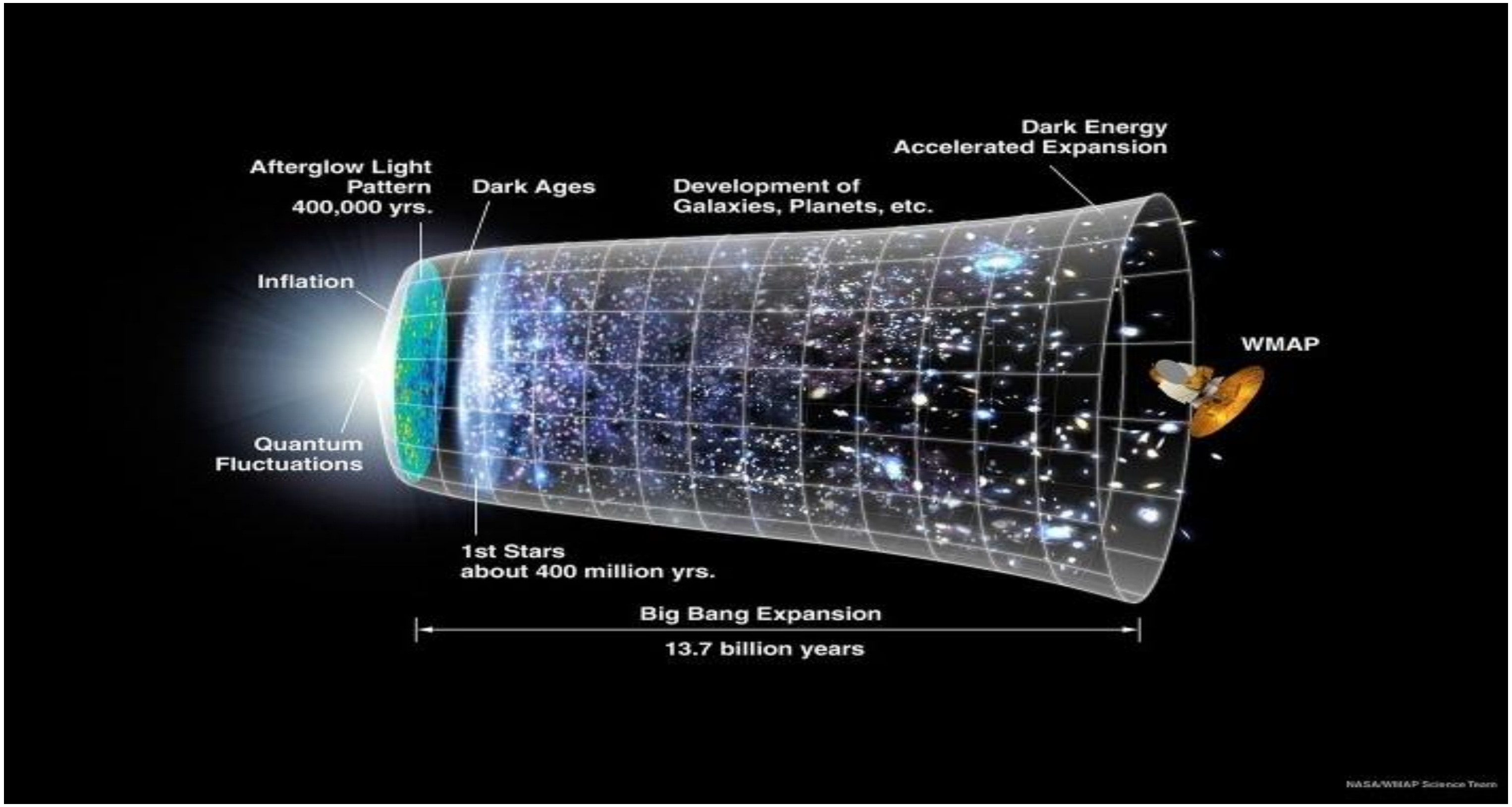
- Exact integration of a(t)

$$\sqrt{\alpha D+\frac{4}{3}a^4}+\sqrt{\alpha D}\ln\left[\frac{a^2}{2\sqrt{3}\left(\sqrt{\alpha D}+\sqrt{\alpha D+\frac{4}{3}a^4}\right)}\right]=\frac{8}{3}\sqrt{\frac{2\pi GD}{c^2}}t-(K-1)\sqrt{\alpha D}$$

Inflation

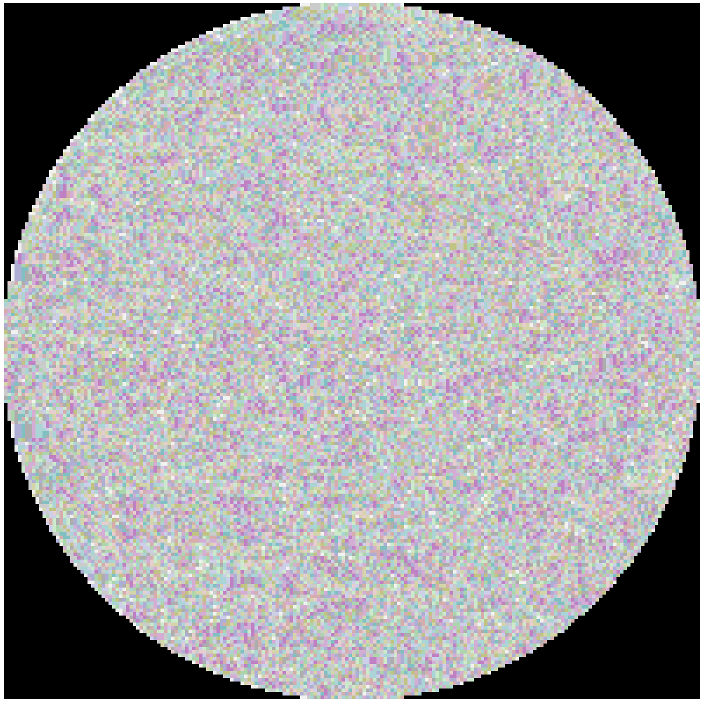
- Rapid (exponential) expansion in the early Universe

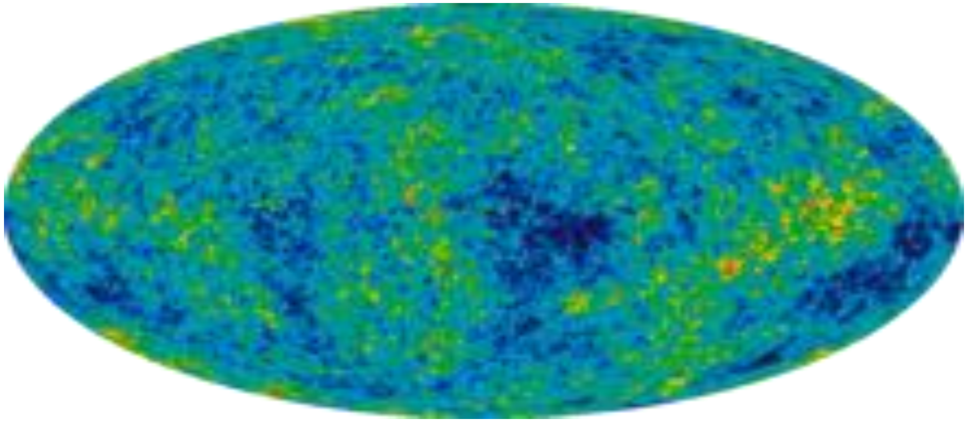
- Questions: how does inflation start/stop ? What is mechanism for inflation?



Small fluctuations: Inflation vs. topological defects

- Inflation is better at creating the ~1 degree sized temperature fluctuations vs. cosmic strings





Influence of particle production/decay on cosmology

- Idea that particle production influences cosmological evolution goes back to Schrodinger *[Physica **6**, 899 (1930)]*
- Others (Parker and Brout, Englert, Gunzig) considered this idea.
- Our approach is most similar to that of Prigogine, Geheniau, Gunzig, Nardone in *Gen. Rel. Grav. **21**, 767 (1989)*
- The above work considers *generic* particle creation.