



Instituto de  
Ciencias  
Nucleares  
UNAM



# STUDYING THE INTERSTELLAR MAGNETIC FIELD MEASURING THE ANISOTROPY IN VELOCITY

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*in collaboration with*

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# THE INTERSTELLAR MEDIUM (ISM) IS TURBULENT

- Theory

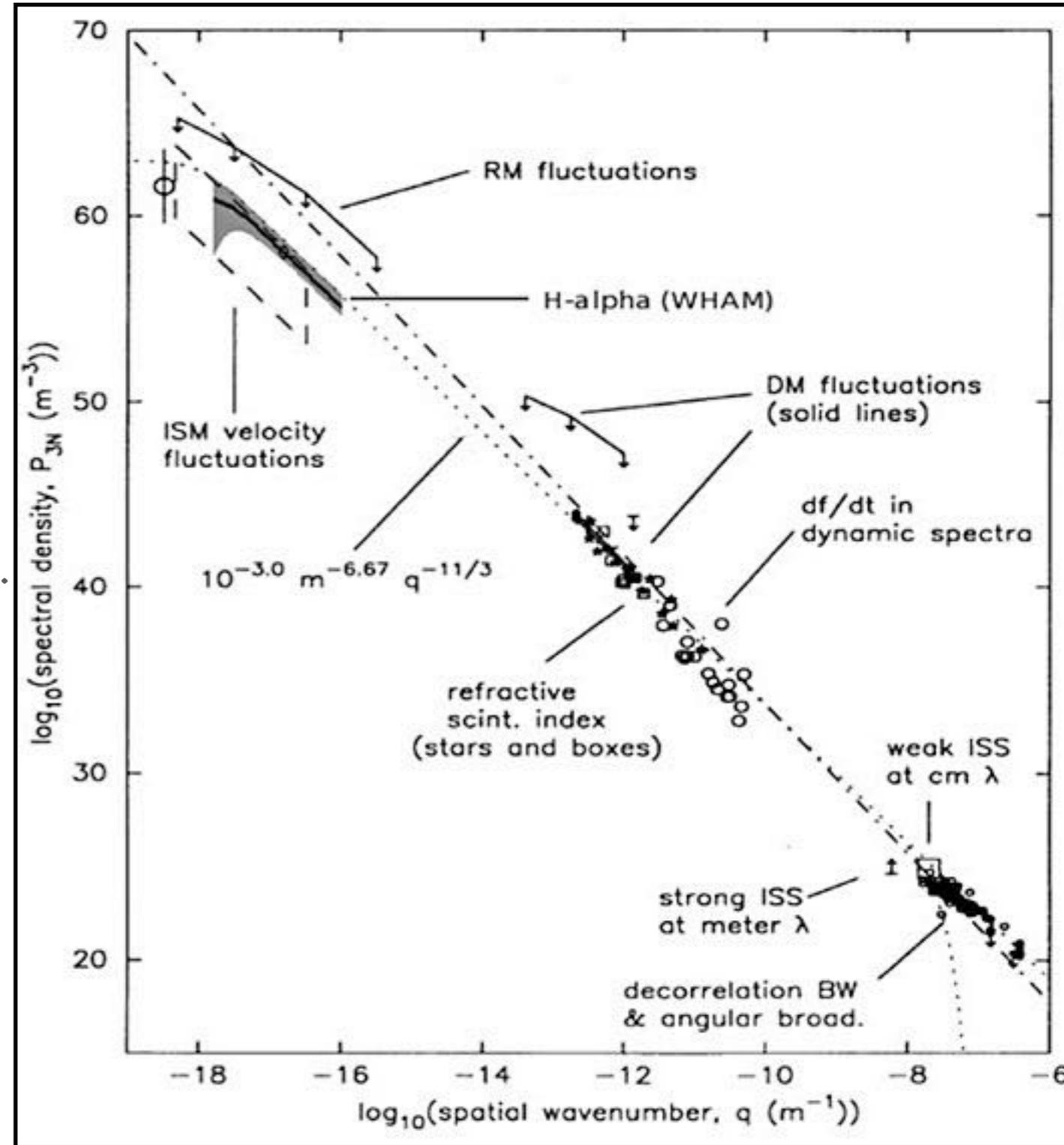
- The Reynolds number in the ISM:  
 $Re > 10^8$  (current computers can only reach  $\sim 10^4$ )

- Observations

- Line widths show a “non-thermal” component.
- Density/velocity/magnetic field fluctuations show a self-similar structure.

- Important for:

- cosmic ray scattering and acceleration,
- molecular cloud dynamics,
- star formation,
- mixing of elements,
- magnetic field generation,
- accretion processes,
- ... virtually any transport process in the ISM.



*“The great power law in the sky”  
Chepurnov & Lazarian (2010)*

# HOW DO WE STUDY TURBULENCE?

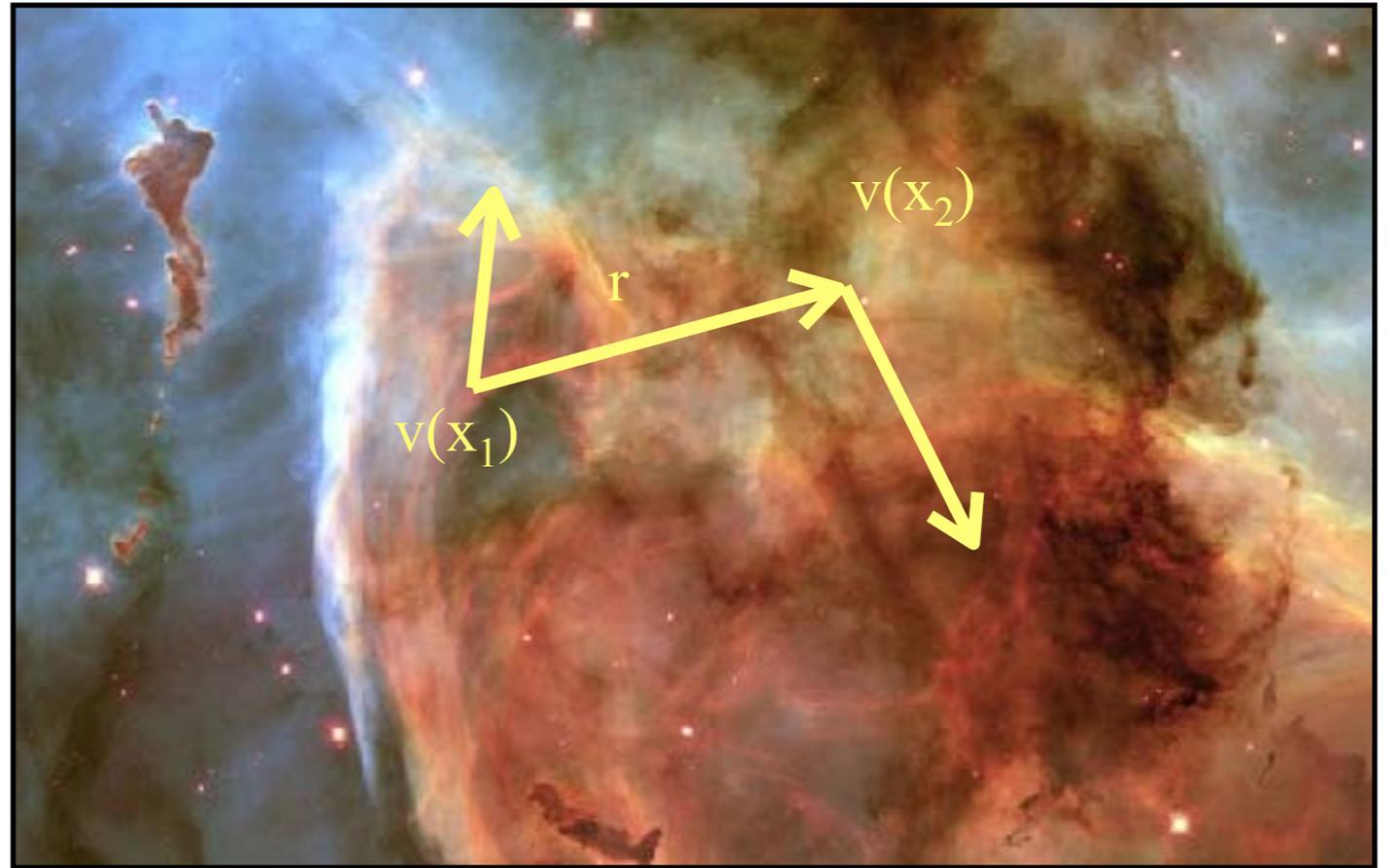
- Statistical tools, such as:
  - structure/correlation functions,

$$SF(r) = \langle [v(x_1) - v(x_2)]^2 \rangle$$

$$r = |x_2 - x_1|$$

$$CF(r) = \langle v(x_1) \cdot v(x_2) \rangle$$

$$SF(r) = 2[CF(0) - CF(r)]$$

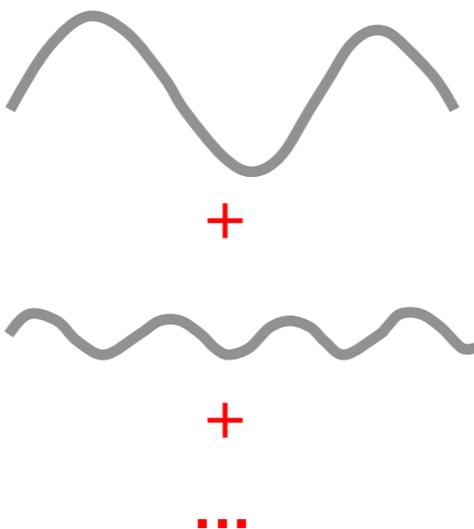


- or power spectra

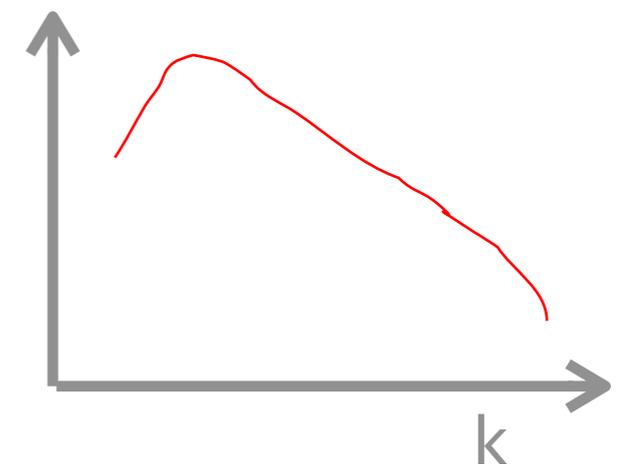
$v(x), \rho(x), \dots$



Fourier analysis



$P(k)$



# KOLMOGOROV MODEL OF TURBULENCE

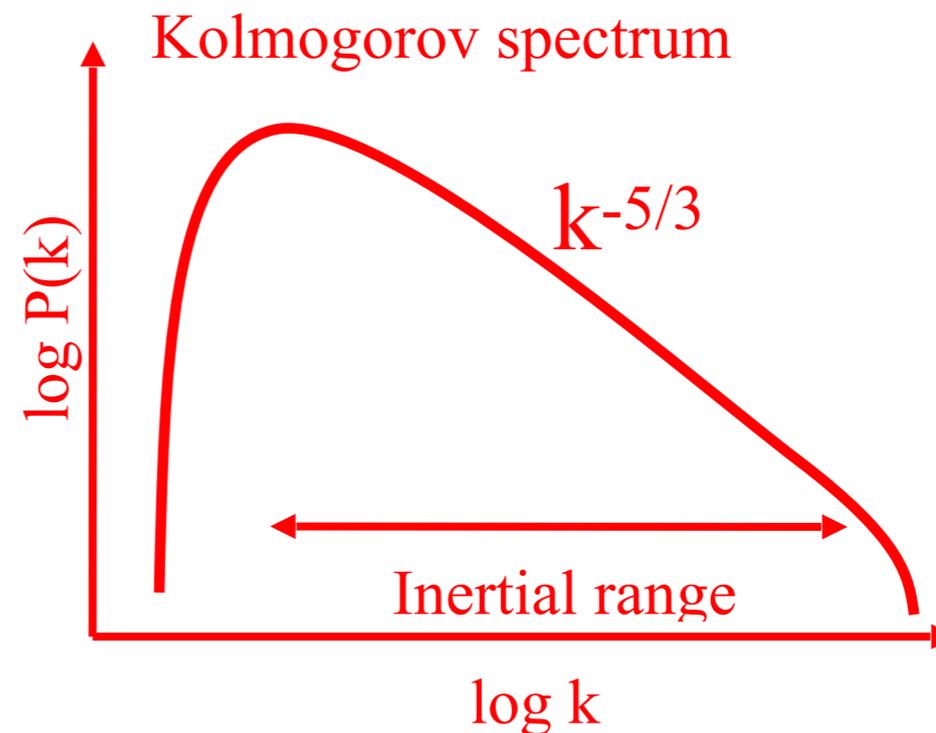
- Model for incompressible HD turbulence
  - Energy injected at large scales, cascades without losses until dissipation takes over (inertial range).
  - Constant Energy transfer rate  $\dot{\mathcal{E}} \approx \frac{\rho v_\ell^2}{\tau} \sim \frac{v_\ell^2}{\ell/v_\ell} \sim \frac{v_\ell^3}{\ell} \Rightarrow v_\ell \propto \ell^{1/3} \sim k^{-1/3}$
  - The energy power spectrum  $P(k)$

$$\int P(k) dk \propto v_\ell^2 \Rightarrow P(k) \propto \frac{k^{-2/3}}{k} \sim k^{-5/3}$$

In 1D:  $P(k) \propto k^{-5/3}$

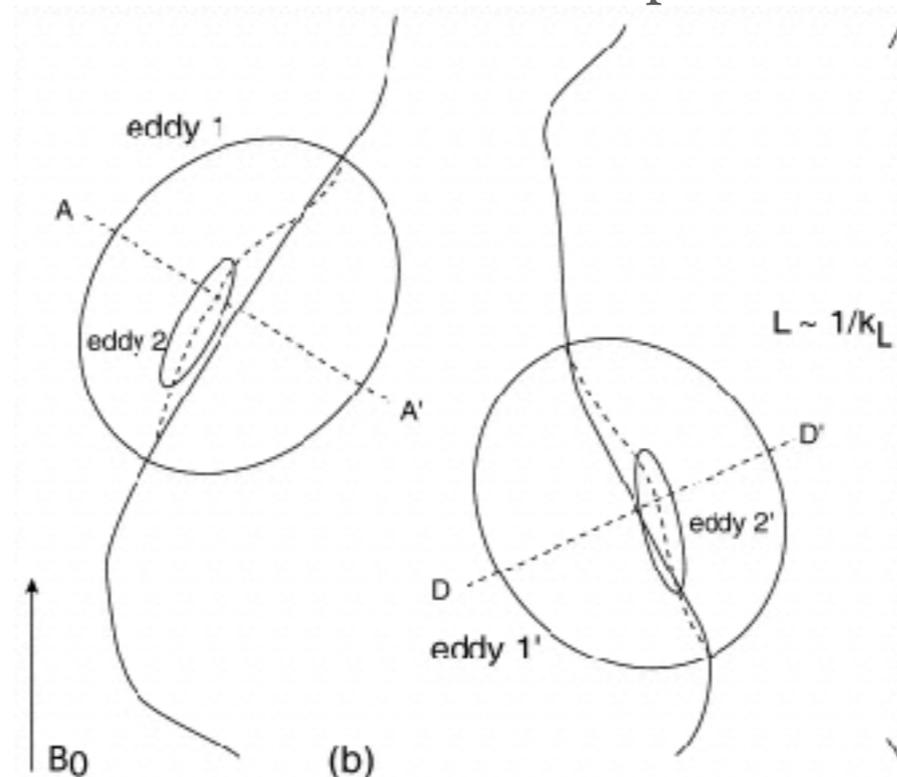
In 2D:  $P(k) \propto k^{-8/3}$

In 3D:  $P(k) \propto k^{-11/3}$



# MHD TURBULENCE IS ANISOTROPIC (AND SCALE DEPENDENT)

- Goldreich & Sridhar (1995) MHD turbulence model
  - Anisotropic cascade:
    - Motions perpendicular to B follow a Kolmogorov type cascade  $v_l \propto l_{\perp}^{1/3}$
    - Parallel B are dominated by Alfvénic perturbations
    - A *critical balance* condition relate the two  $\frac{v_{\perp}}{l_{\perp}} \sim \frac{v_A}{l_{\parallel}} \Rightarrow l_{\parallel} \sim l_{\perp}^{2/3}$
  - Later confirmed with numerical simulations (Cho & Vishniac 2000, Maron & Goldreich 2001)
    - The anisotropy should be measured with respect to the *local* magnetic field



Cho, Lazarian & Vishniac (2002)

# GRID OF MHD MODELS

- Ideal 3D MHD simulations of fully developed (driven at large scales) turbulence.
- Isothermal, in a periodic Cartesian grid.
- The parameters that control the simulations are the Alfvén and the sonic Mach numbers.

$$M_s \equiv \frac{v_L}{c_s}; \quad M_A = \frac{v_L}{v_A}$$

- where

$$c_s = \sqrt{\frac{P}{\rho}}; \quad v_A = \frac{B}{\sqrt{4\pi\rho}}$$

- $B_0$  is in the  $x$  direction.
- We take the output of the simulations to create synthetic spectroscopic observations of optically thin media.

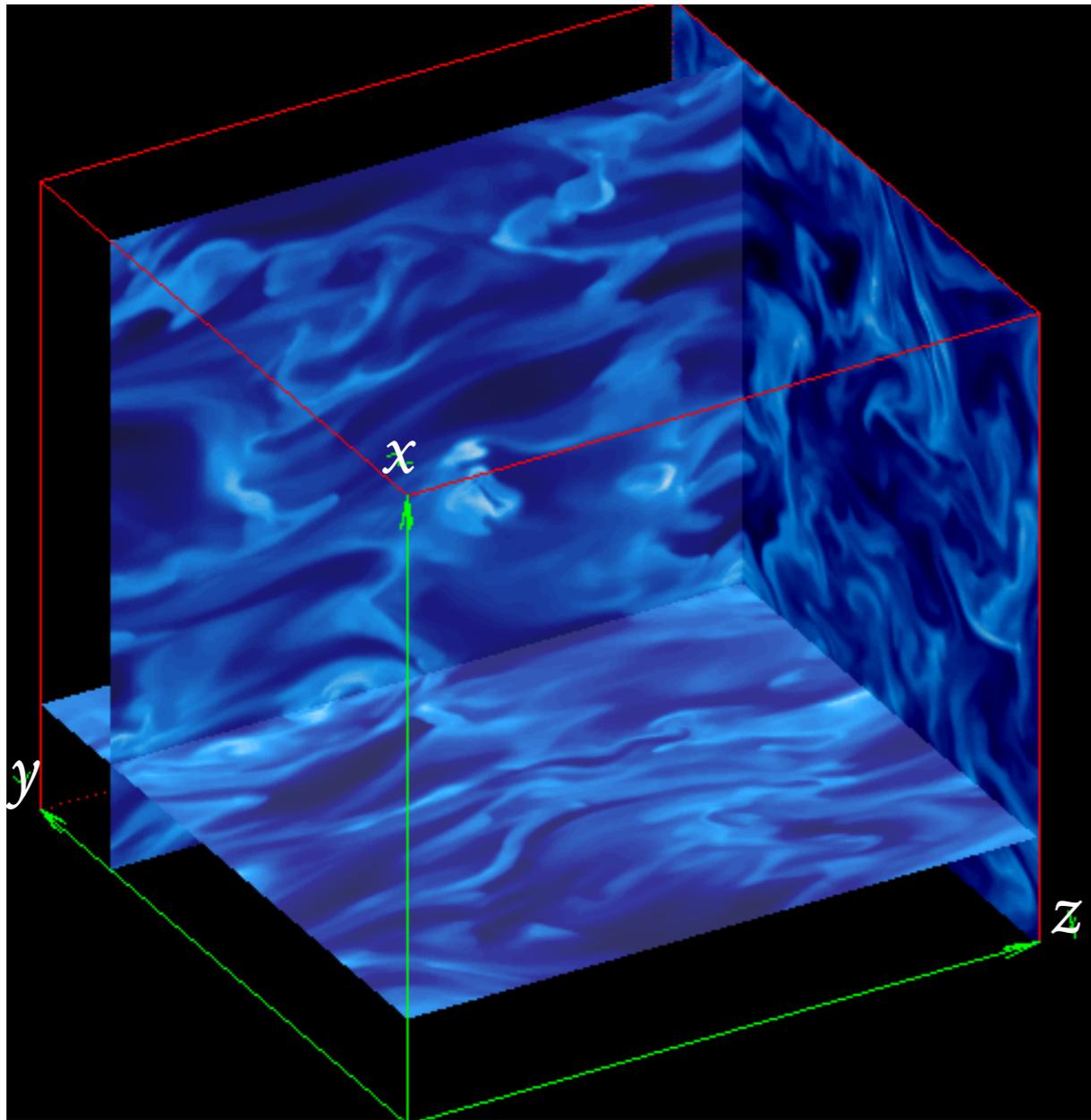
Model	$v_{A,0}$	$P_{\text{gas},0}$	$\mathcal{M}_A$	$\mathcal{M}_s$	$\beta$
M1	0.1	0.0049	$\sim 8.3$	$\sim 11.9$	$\sim 0.49$
M2	0.1	0.0100	$\sim 7.7$	$\sim 7.7$	$\sim 1.0$
M3	0.1	0.0250	$\sim 7.4$	$\sim 4.7$	$\sim 2.5$
M4	0.1	0.0500	$\sim 7.6$	$\sim 3.4$	$\sim 5.0$
M5	0.1	0.1000	$\sim 8.2$	$\sim 2.6$	$\sim 10.0$
M6	0.1	0.7000	$\sim 7.6$	$\sim 0.9$	$\sim 70.0$
M7	0.1	2.0000	$\sim 7.0$	$\sim 0.5$	$\sim 200.0$
M8	1.0	0.0049	$\sim 0.8$	$\sim 10.8$	$\sim 0.0049$
M9	1.0	0.0077	$\sim 0.8$	$\sim 8.6$	$\sim 0.0077$
M10	1.0	0.0100	$\sim 0.7$	$\sim 7.4$	$\sim 0.01$
M11	1.0	0.0250	$\sim 0.8$	$\sim 4.8$	$\sim 0.025$
M12	1.0	0.0500	$\sim 0.8$	$\sim 3.4$	$\sim 0.05$
M13	1.0	0.1000	$\sim 0.8$	$\sim 2.7$	$\sim 0.1$
M14	1.0	0.7000	$\sim 0.8$	$\sim 1.0$	$\sim 0.7$
M15	1.0	2.0000	$\sim 0.7$	$\sim 0.5$	$\sim 2.0$
M16	2.0	0.0100	$\sim 0.4$	$\sim 7.6$	$\sim 0.0025$
M17	2.0	0.1000	$\sim 0.4$	$\sim 2.7$	$\sim 0.025$
M18	2.0	1.0000	$\sim 0.5$	$\sim 1.0$	$\sim 0.25$
M19	3.0	0.0100	$\sim 0.3$	$\sim 8.2$	$\sim 0.001$
M20	3.0	0.1000	$\sim 0.3$	$\sim 2.6$	$\sim 0.01$
M21	3.0	1.0000	$\sim 0.3$	$\sim 1.0$	$\sim 0.1$
M22	5.0	0.0100	$\sim 0.2$	$\sim 9.0$	$\sim 0.0004$
M23	5.0	0.1000	$\sim 0.2$	$\sim 2.7$	$\sim 0.004$
M24	5.0	1.0000	$\sim 0.2$	$\sim 0.9$	$\sim 0.04$

# SYNTHETIC OBSERVATIONS

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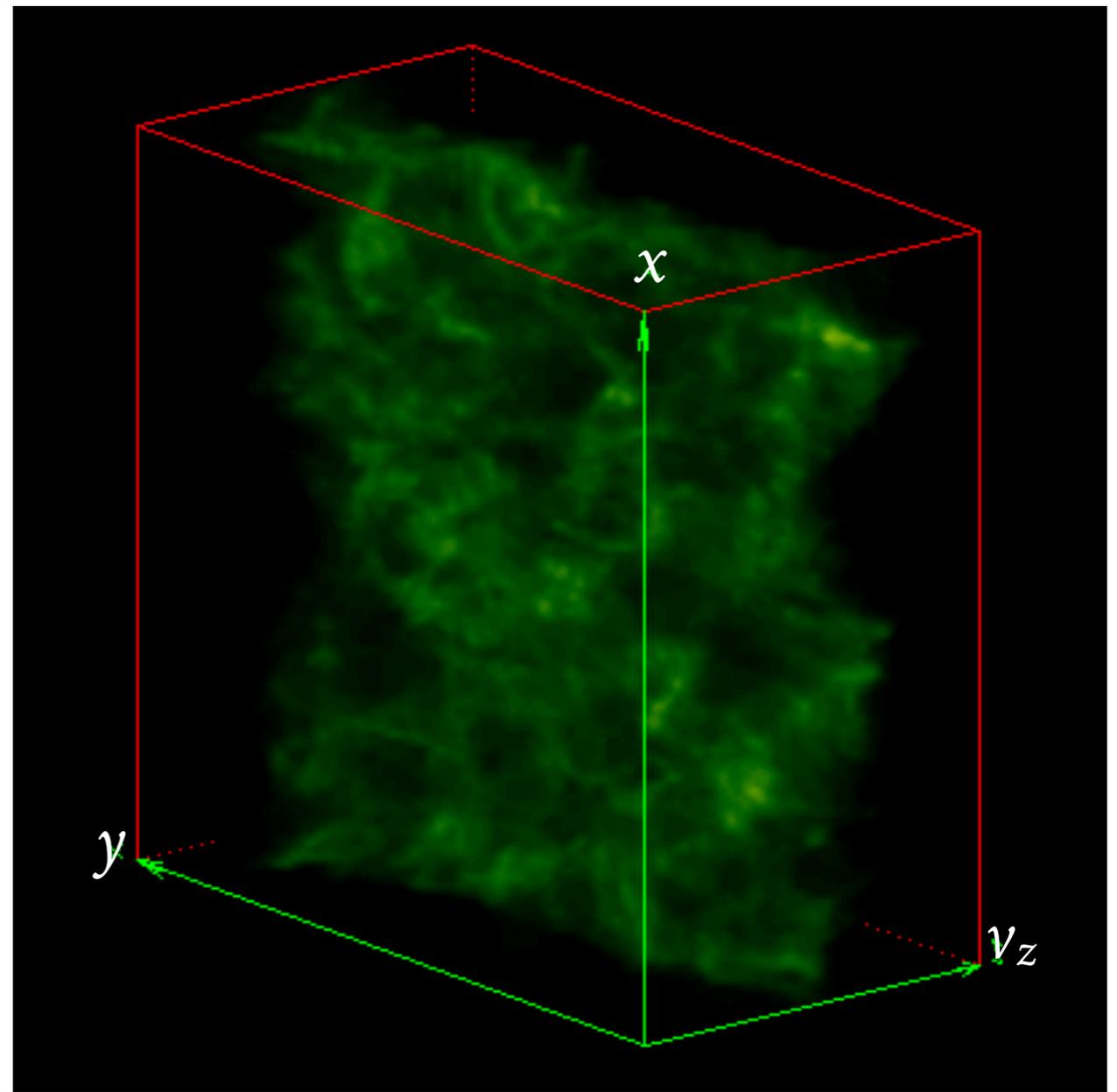
Model M13 (with  $M_s=2.7$  and  $M_A=0.8$ )

Simulation PPP (x,y,z) space



*Density cuts*

Synthetic Observations  
PPV (x,y,v<sub>z</sub>) space

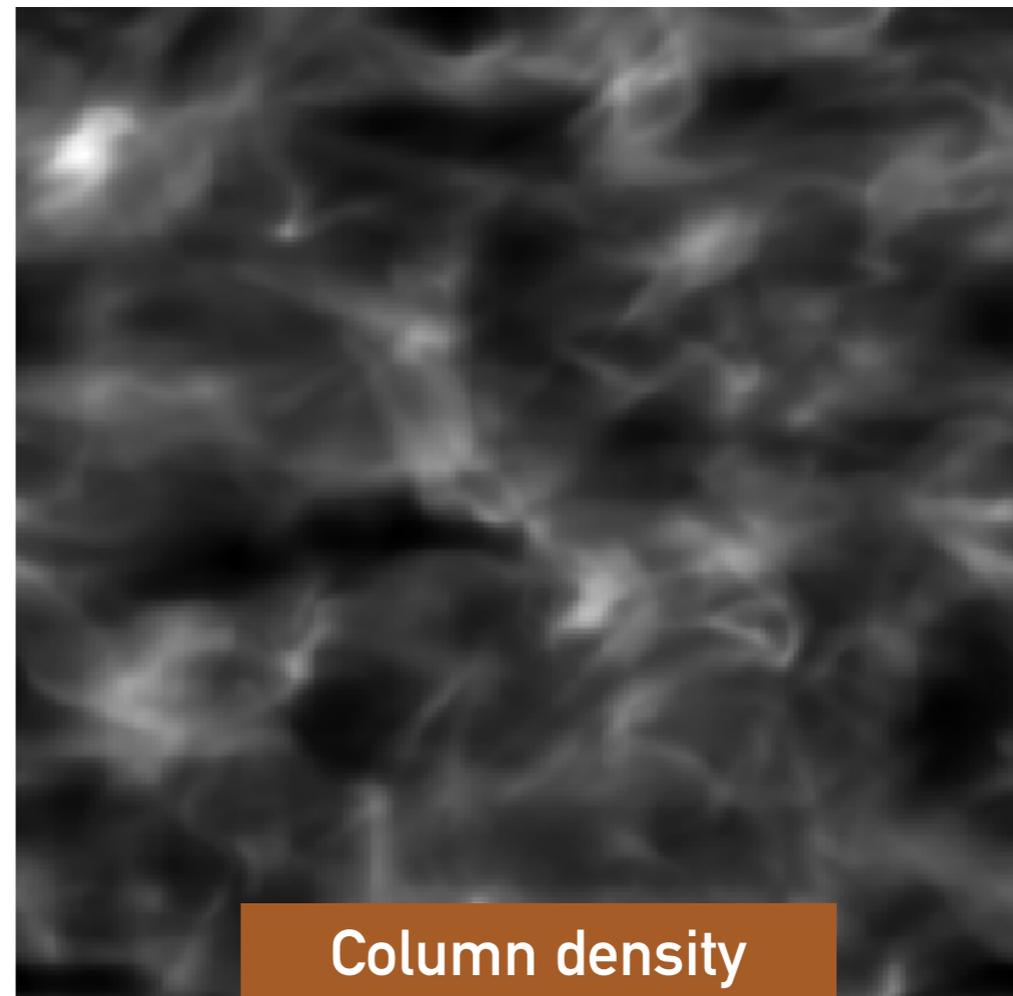
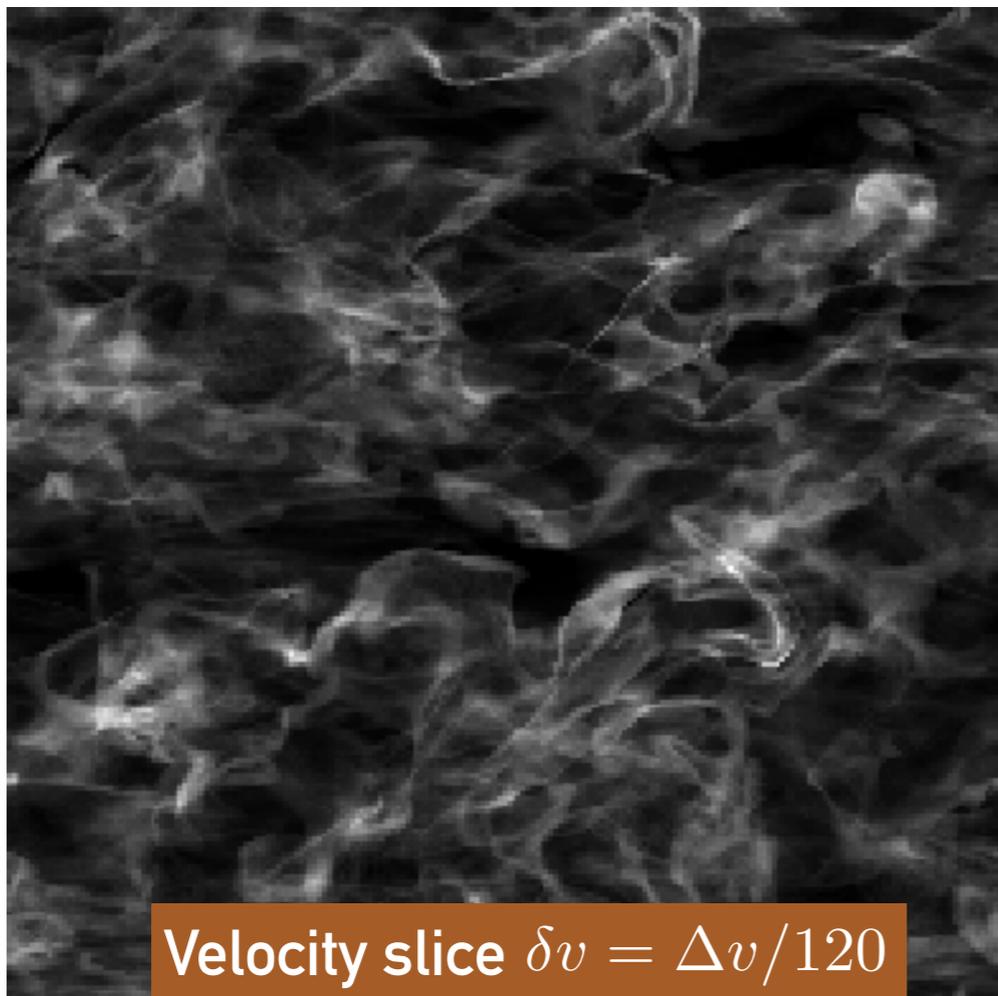


*Intensity  $\propto \rho$*

# PPV DATA: THE EFFECT OF VARYING RESOLUTION

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- Emissivity in PPV data depends on density and velocity at the same time.
- Lazarian & Pogosyan (2000) study the effect of varying the thickness in velocity channels (velocity resolution) to obtain the velocity spectral index from observations.
- As we lower the velocity resolution, the contribution of density becomes more prominent. In thinner velocity channels the velocity can dominate the spectrum.



# (SYNTHETIC) OBSERVATIONAL DATA

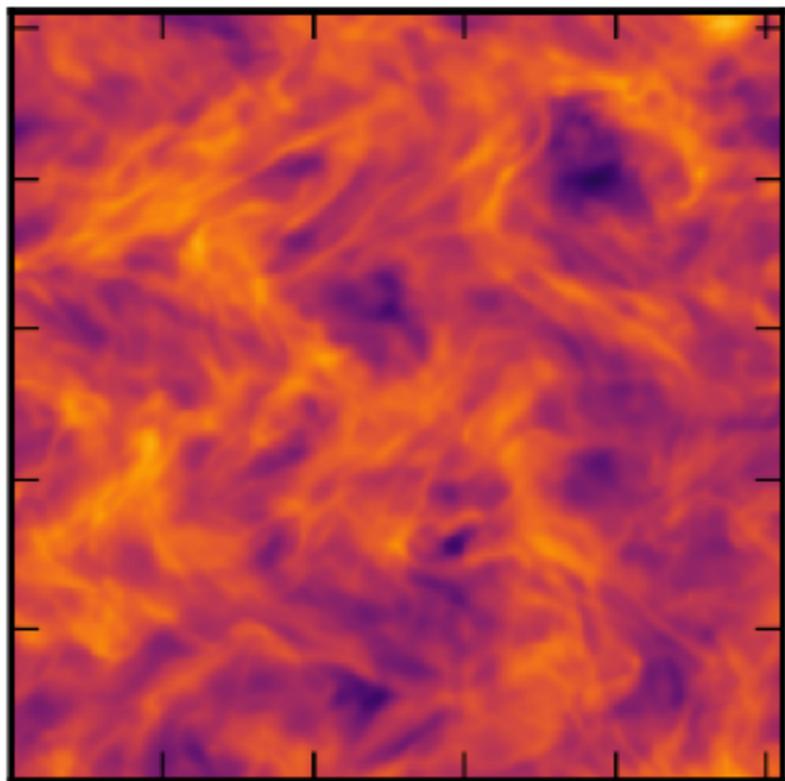
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Column density

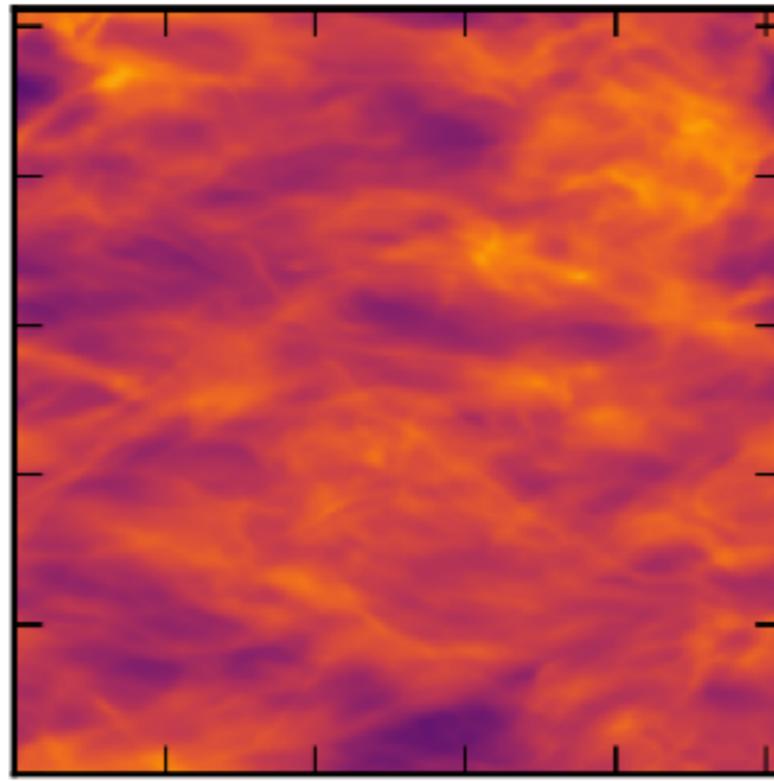
$$N(x, y) = \int I(x, y, v_{\text{los}}) dv_{\text{los}},$$

$$N(x, y) = \int \rho(x, y, z) dz \quad (\text{Optically thin media, LOS}=z)$$

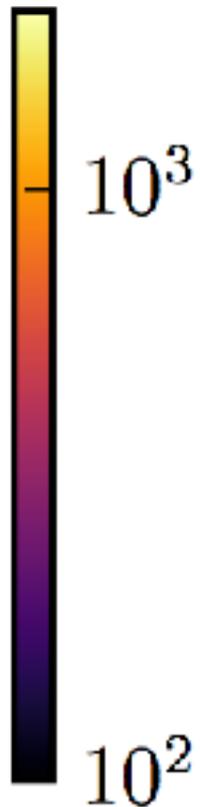
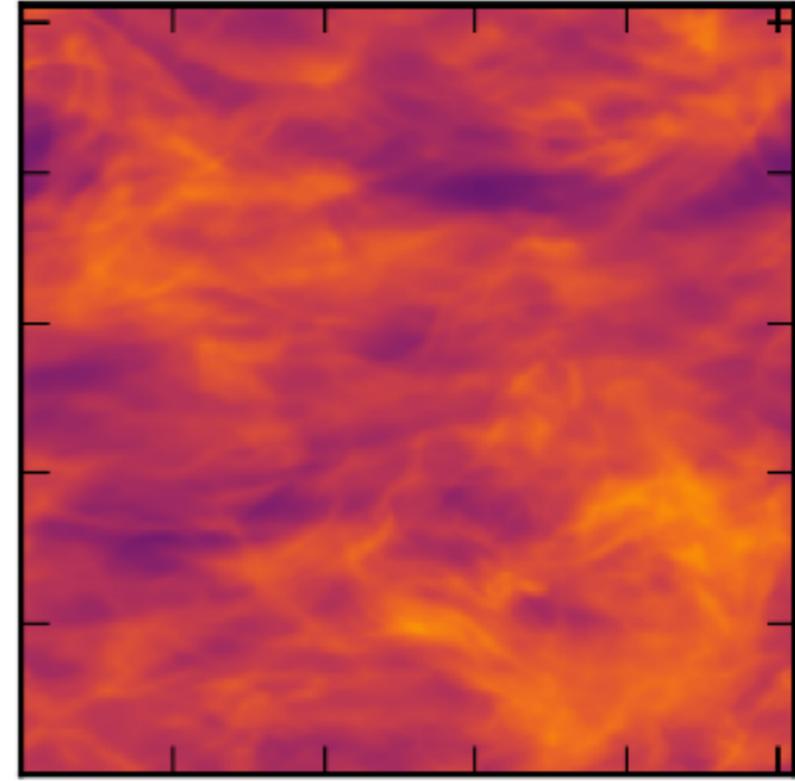
column density LOS=X



column density LOS=Y



column density LOS=Z



# (SYNTHETIC) OBSERVATIONAL DATA

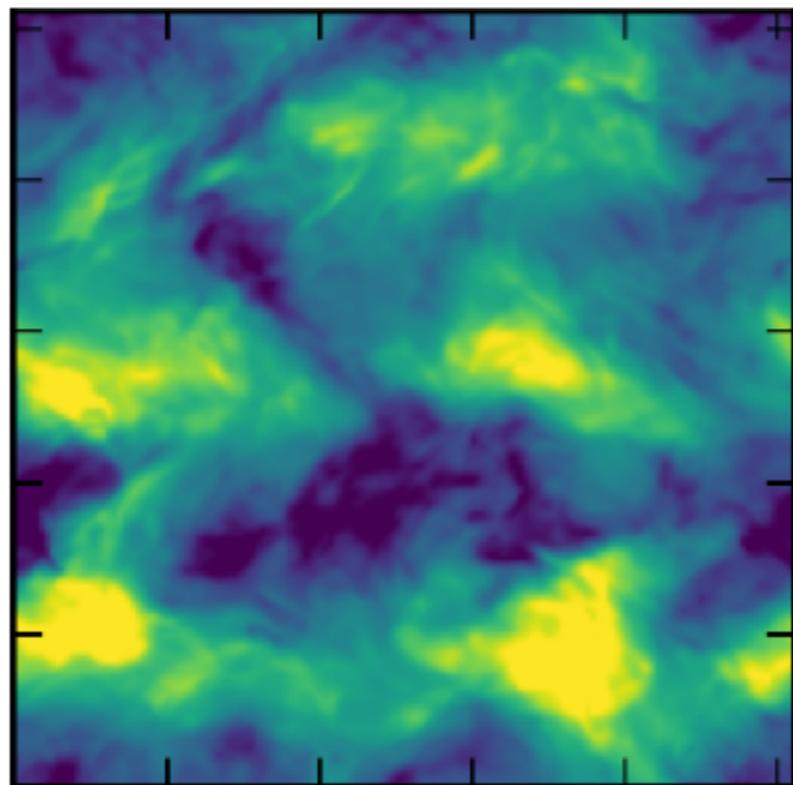
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Velocity Centroids  
(unnormalized)

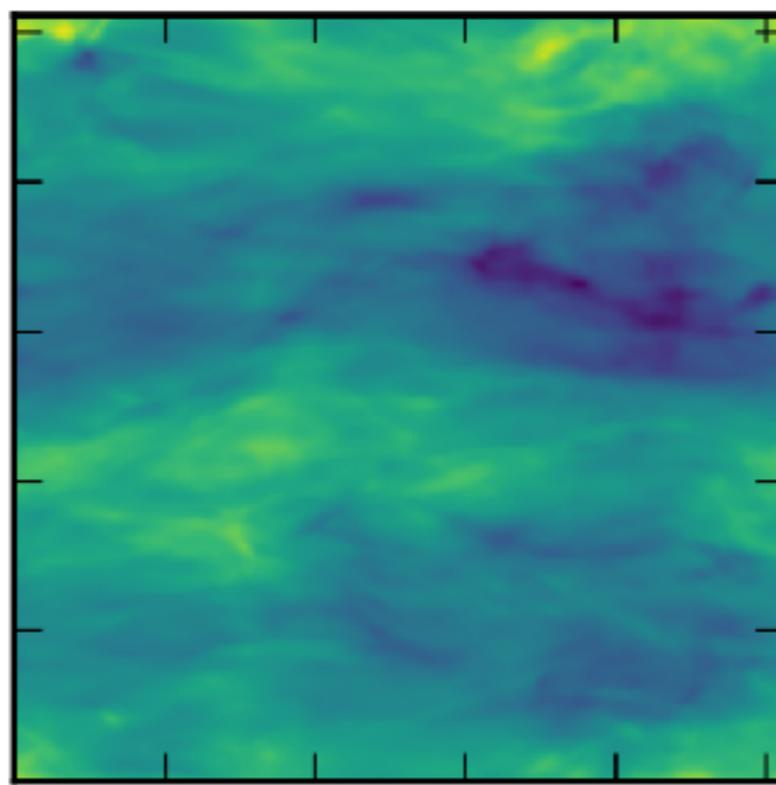
$$C_{\text{los}}(x, y) = \int I(x, y, v_{\text{los}}) v_{\text{los}} dv_{\text{los}},$$

$$C_z(x, y) = \int \rho(x, y, z) v_z(x, y, z) dz \quad (\text{Optically thin media, LOS}=z)$$

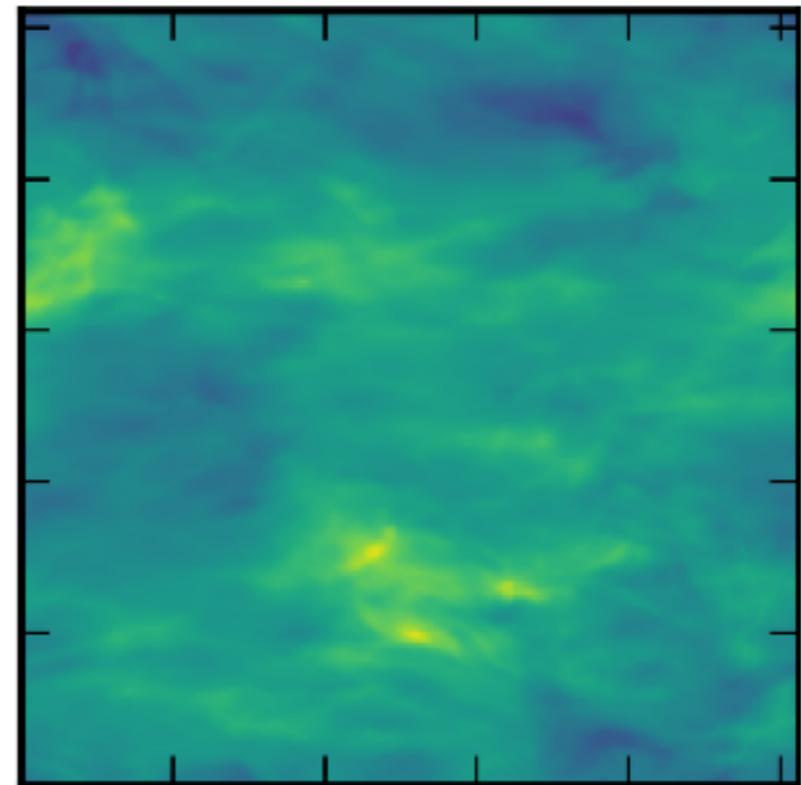
centroids LOS=X



centroids LOS=Y



centroids LOS=Z



500

400

300

200

100

0

-100

-200

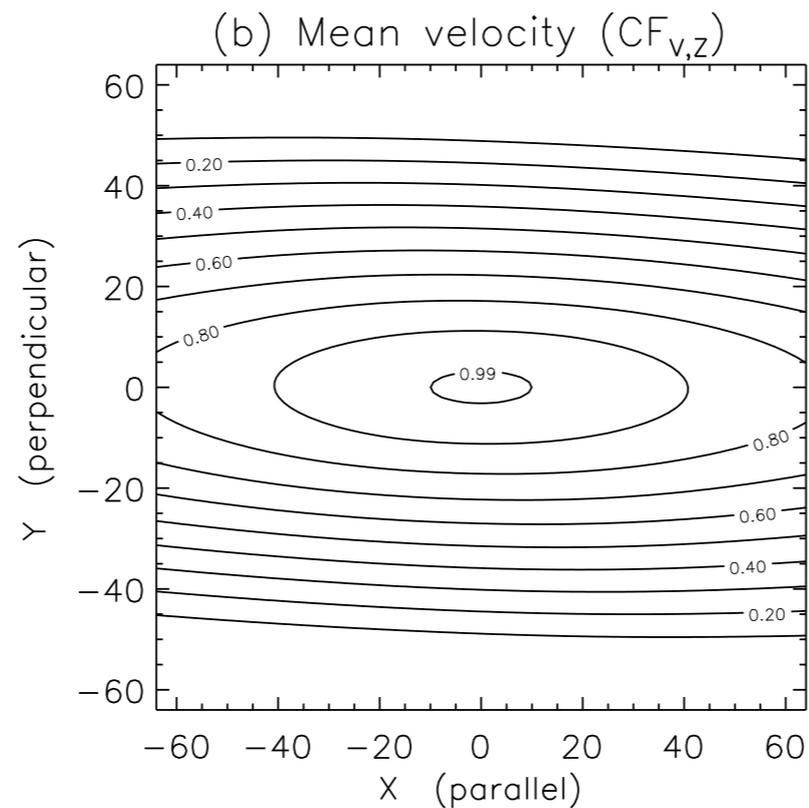
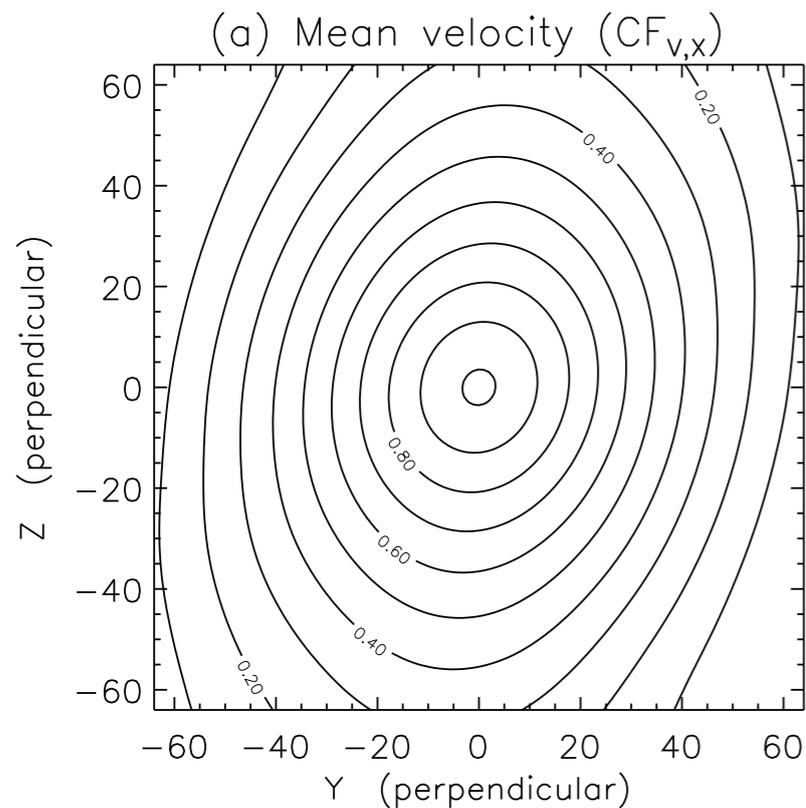
-300

-400

-500

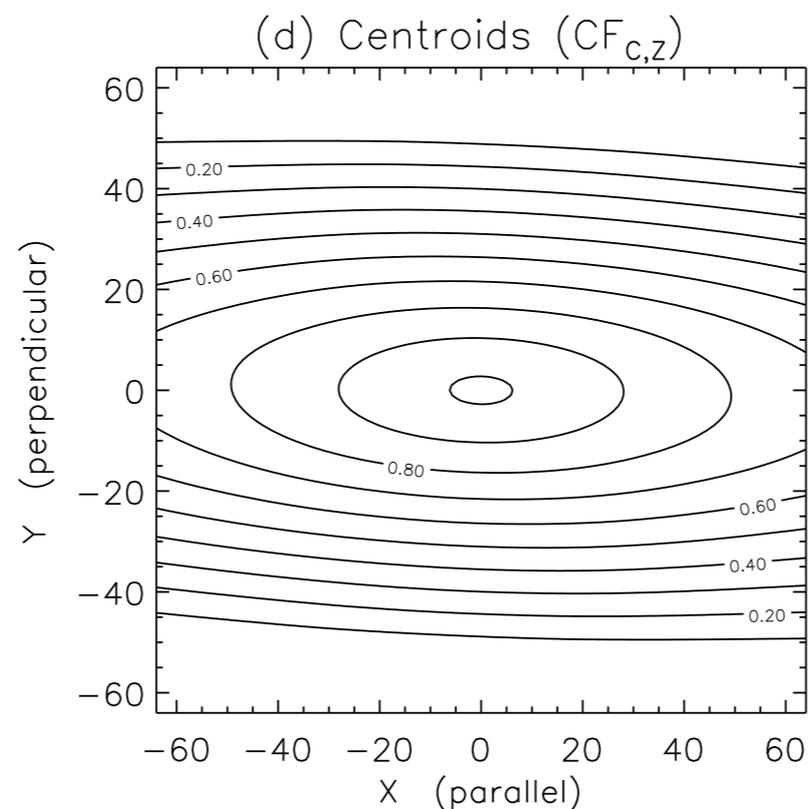
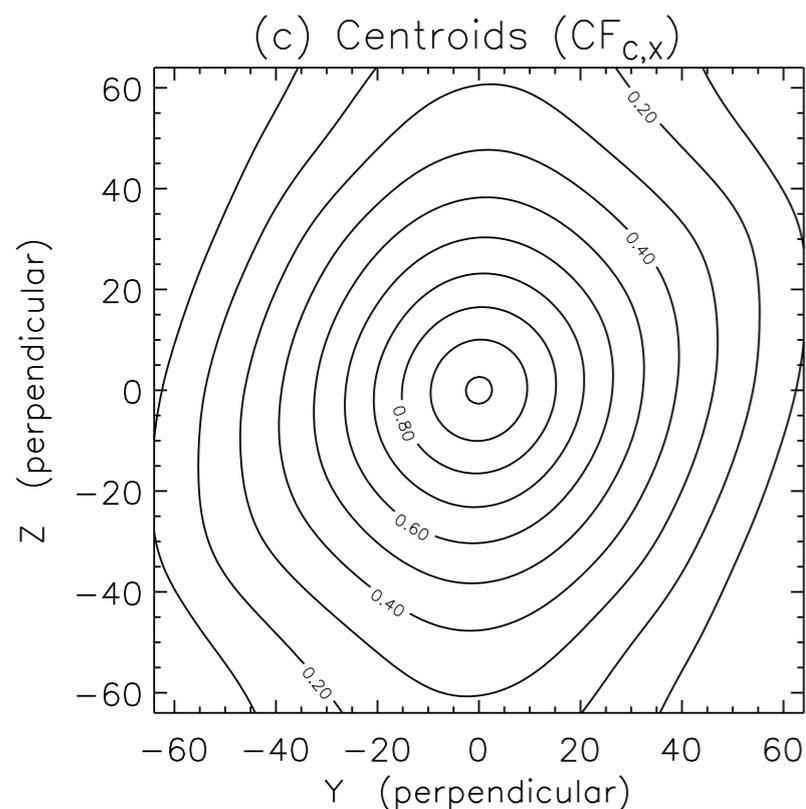
# ANISOTROPY IN VELOCITY CENTROIDS

- The structure function of velocity centroids  $SF(\mathbf{R}) = \langle [C(\mathbf{X}) - C(\mathbf{X} + \mathbf{R})]^2 \rangle$ ,



Velocity centroids are a combination of density and velocity fluctuations. To isolate velocity one can use from the simulations maps of mean LOS velocity, e.g.

$$V_z(x, y) = \frac{1}{N_z} \int v(x, y, z) dz$$

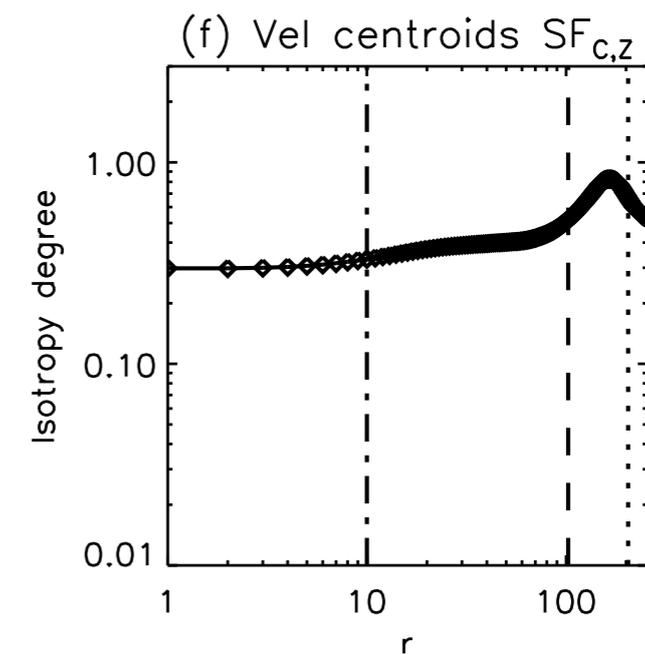
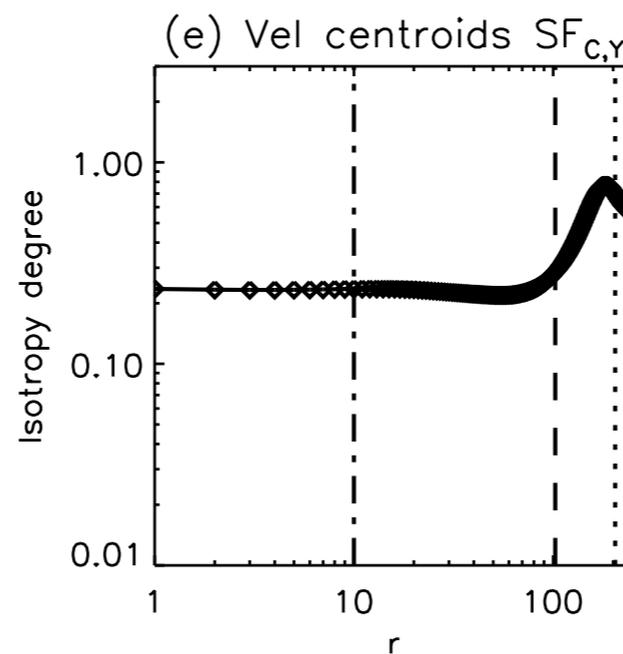
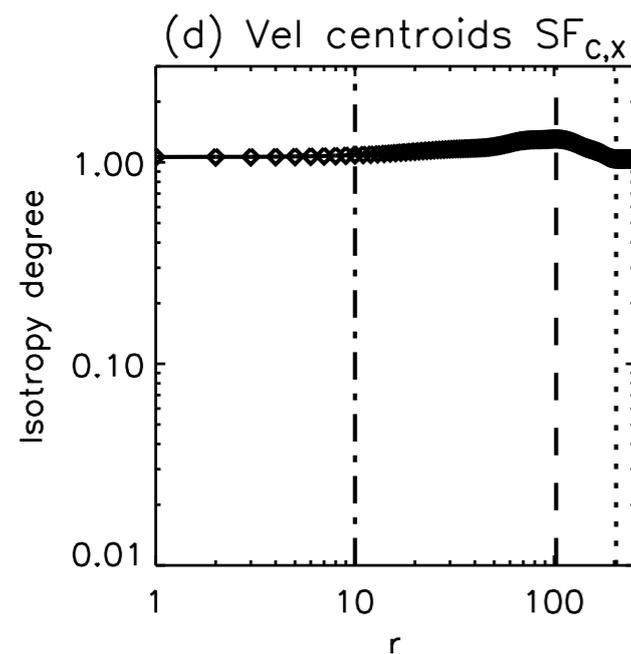
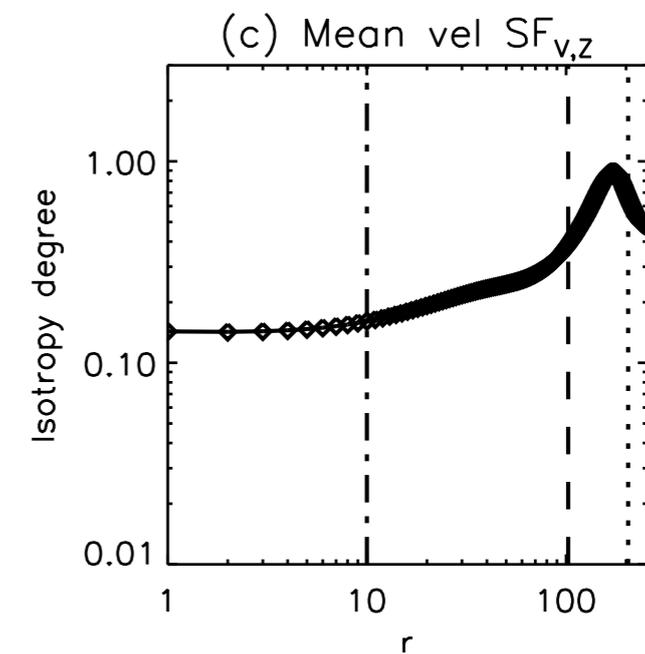
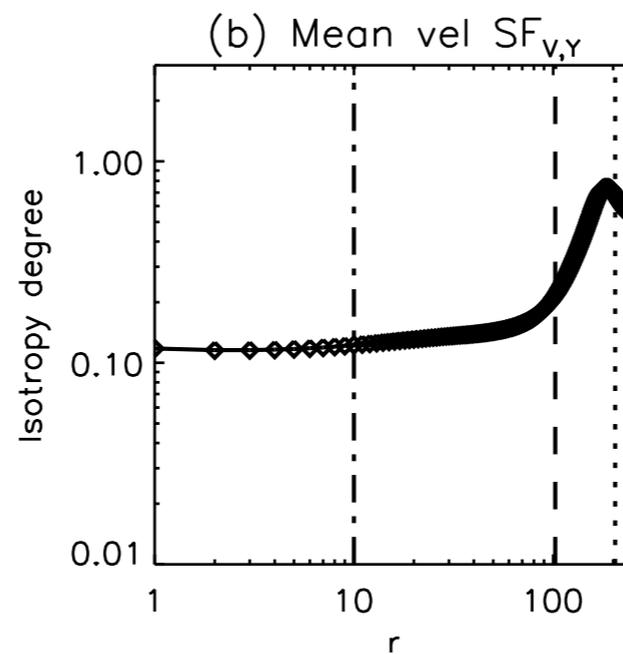
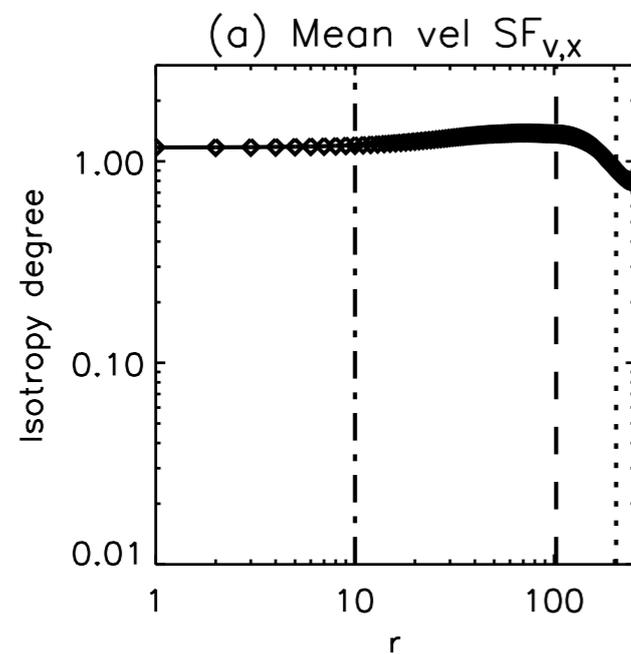


*The SFs are elongated in the direction of the mean magnetic field.*

*Esquivel & Lazarian (2011)*

# HOW DOES THE ANISOTROPY OF CENTROIDS DEPENDS ON SCALE

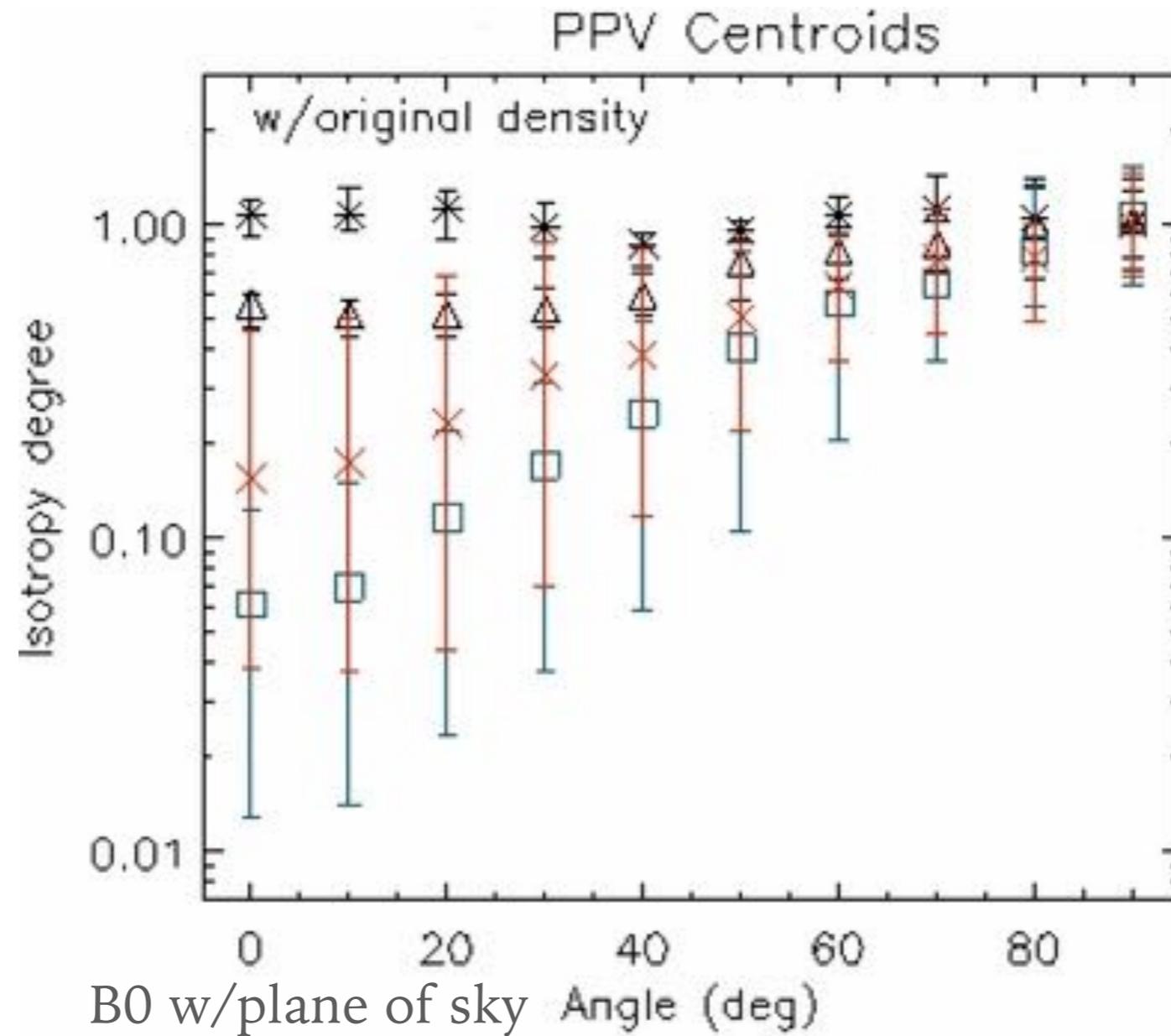
- Velocity centroids sample the entire LOS at a given velocity, thus one probe the mean magnetic field (as opposed to the local one)
  - Isotropy degree( $\ell$ ) =  $\frac{SF(R_{\parallel})}{SF(R_{\perp})}$ .
- velocity centroids anisotropy is (mostly) SCALE INDEPENDENT.



# AVERAGE ISOTROPY DEGREE

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*Higher magnetization → more elongated structure functions, but...*



*Burkhart et al. (2014)*

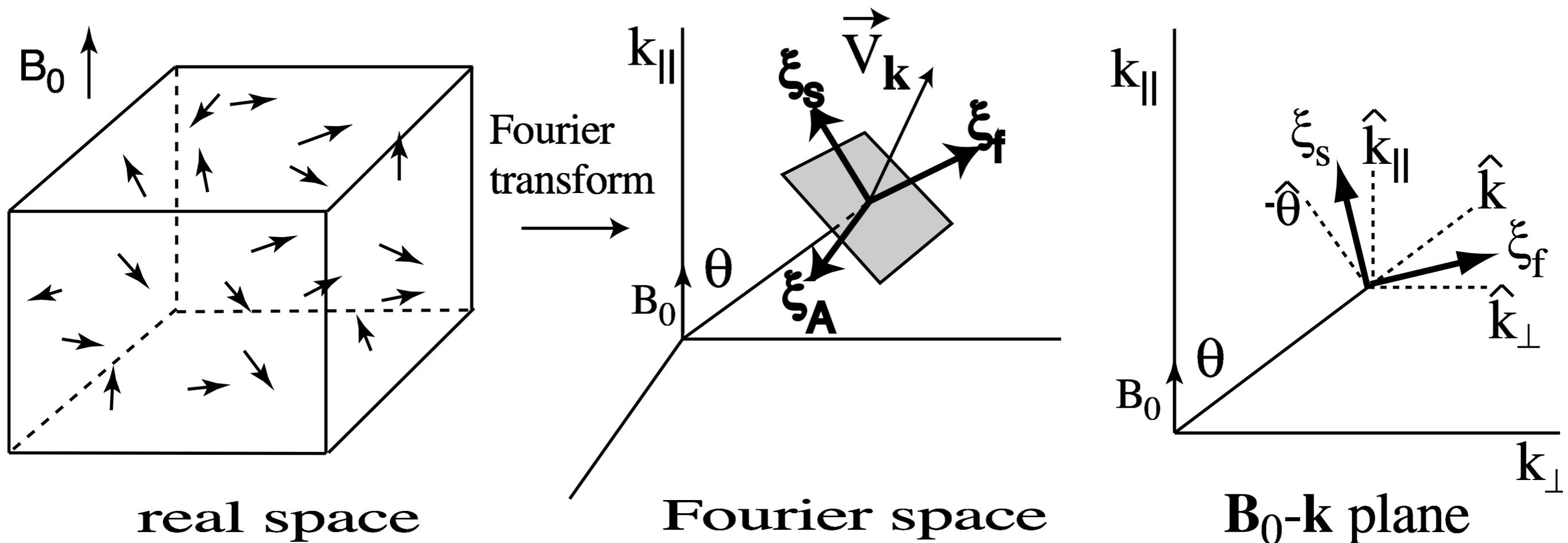
# ADVANTAGE OF CENTROIDS: WE HAVE THEORY BEHIND THEM

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- Recently Kandel, Lazarian & Pogosyan (MNRAS, 2016a,b) extended the VCA formalism to study the anisotropy in PPV and velocity centroids.
- They study the level of anisotropy in the different velocity modes in the SF of velocity centroids (at a constant density)
  - Alfvén Mode:  
anisotropic at low and high  $\beta$ , with more pronounced anisotropy at small  $M_A$ .
  - Slow mode:  
Same general behavior as the Alfvén mode, but with vanishing signal at an angle perpendicular to  $B_0$ .
  - Fast mode:  
Isotropic in high- $\beta$ , but anisotropic in low- $\beta$

# DIFFERENT MHD MODES ARE ALL PRESENT IN THE VELOCITY

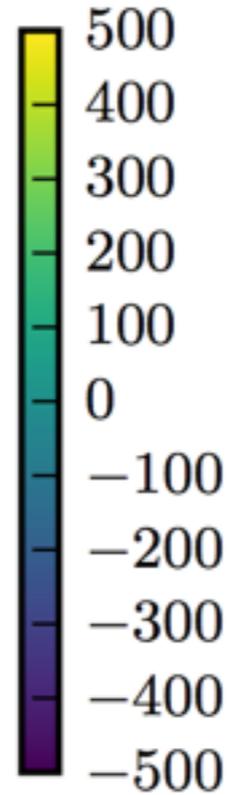
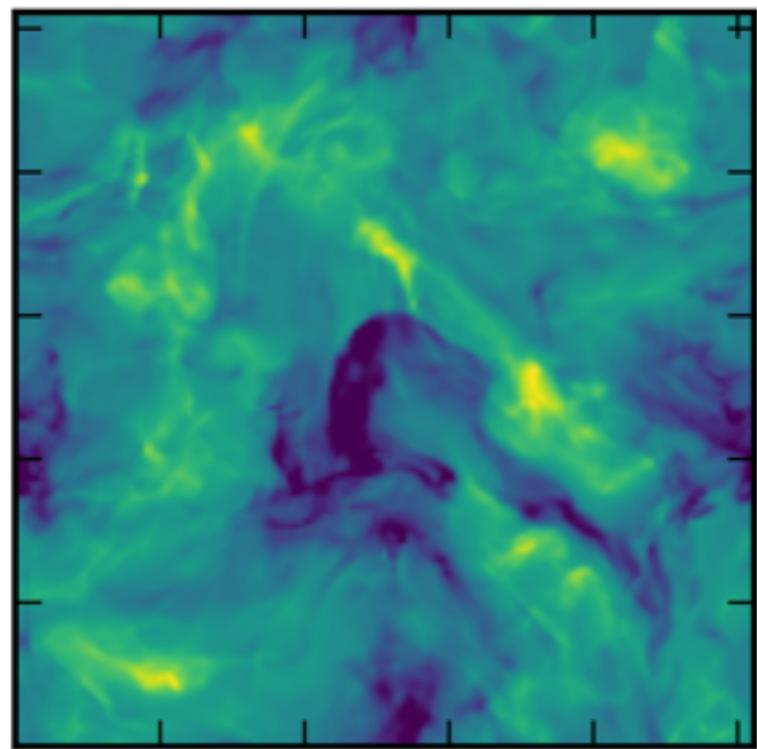
- We decompose the velocity into Alfvén, Fast and Slow magneto-sonic modes.
- We take each of the velocity fields and compute the mean LOS velocity, and centroids combining such velocities with the original velocity.



# EXAMPLE OF DECOMPOSED MODES MAPS (PARALLEL TO $B_0$ )

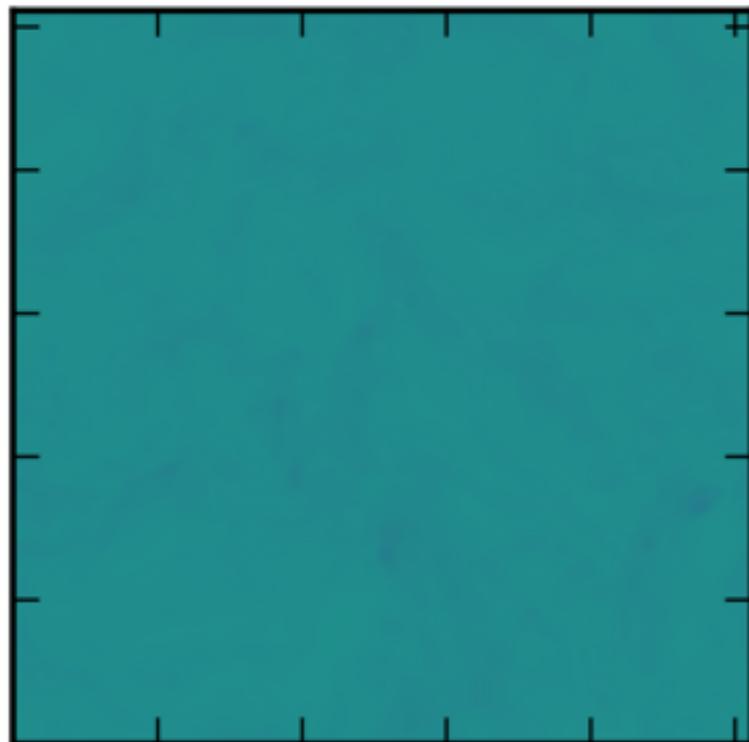
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centroids LOS=X

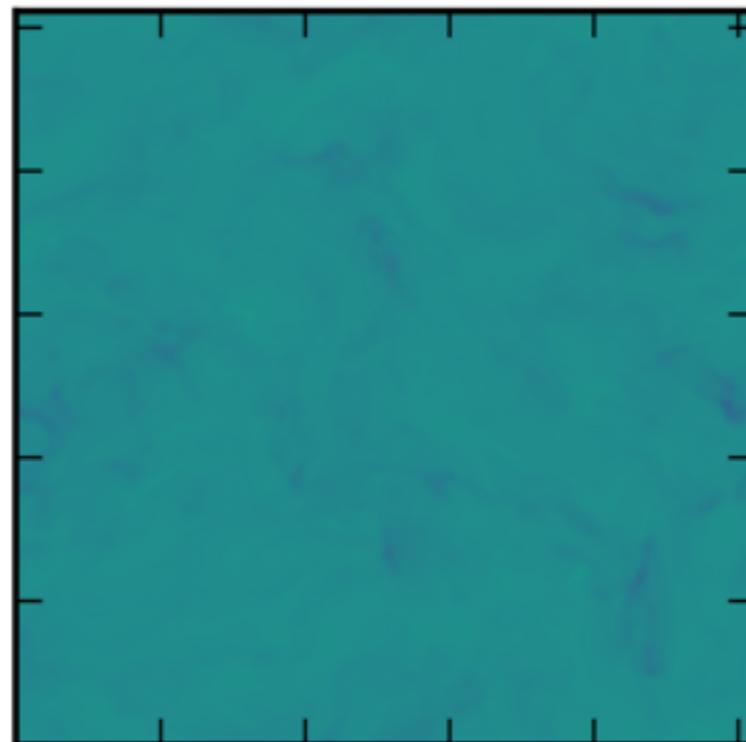


$$C_x(y, z) = \int \rho(x, y, z) v_x(x, y, z) dx$$

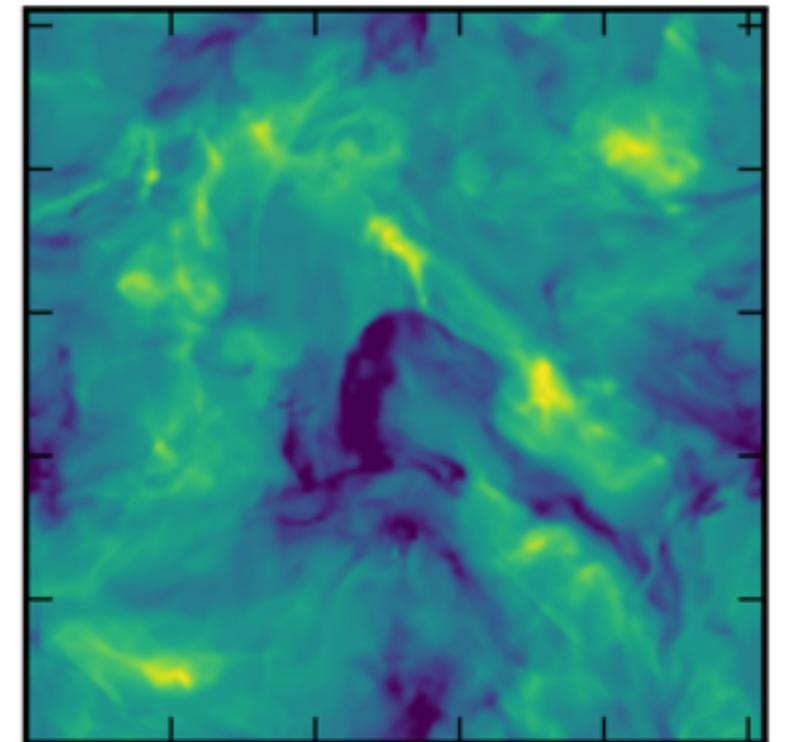
centroids (Alfvén) LOS=X



centroids (fast) LOS=X



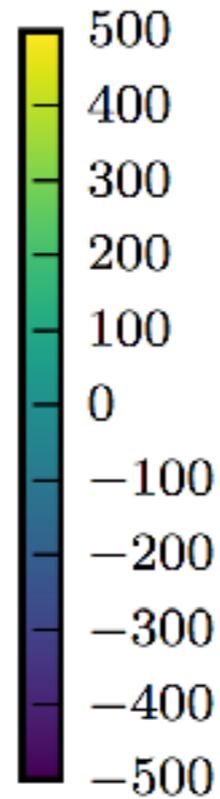
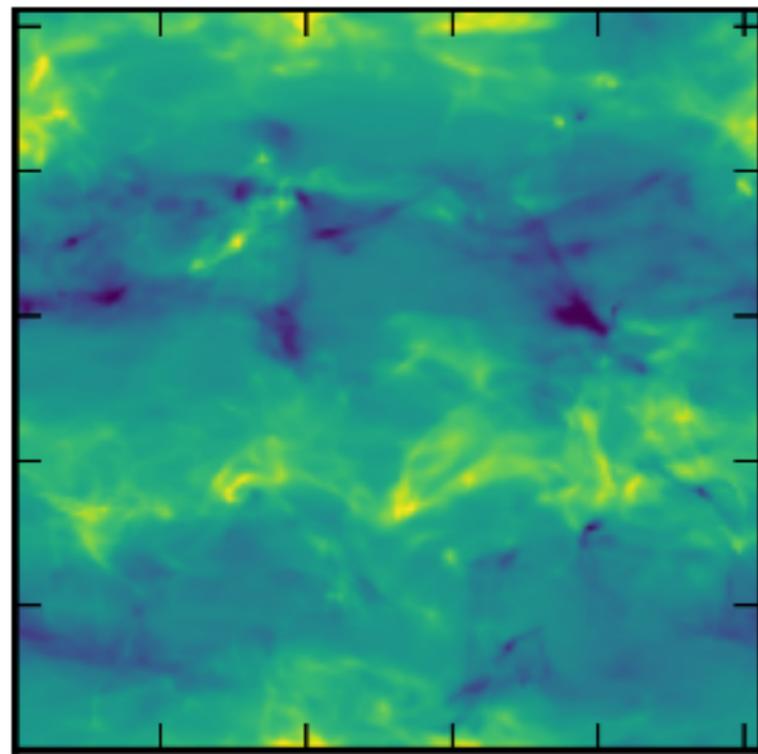
centroids (slow) LOS=X



# EXAMPLE OF DECOMPOSED MODES MAPS (PERPENDICULAR TO $B_0$ )

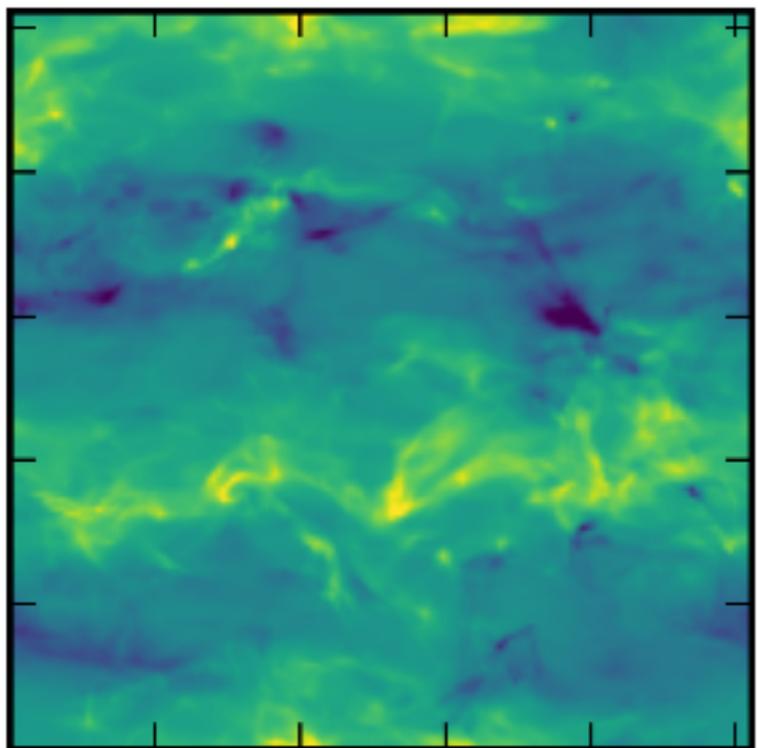
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centroids LOS=Y

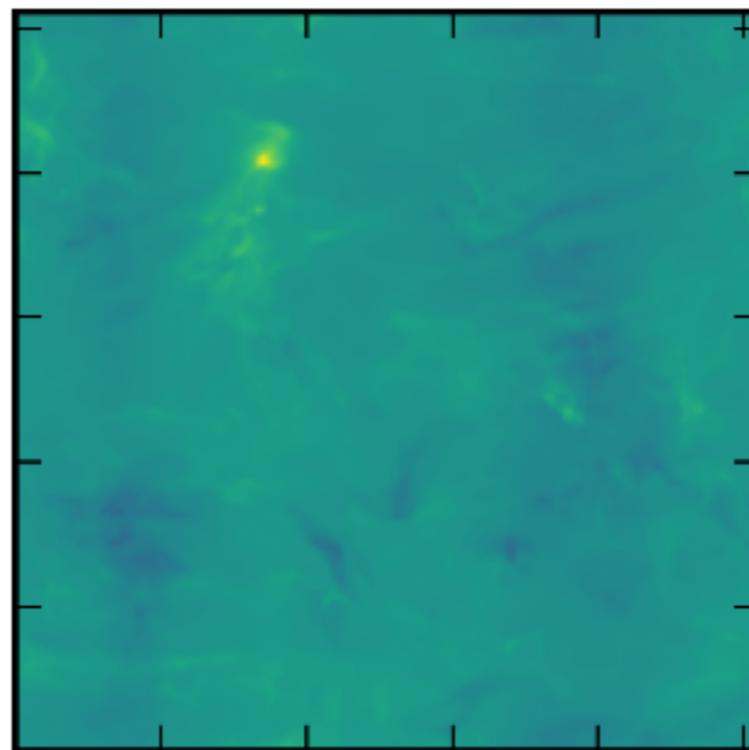


$$C_y(x, z) = \int \rho(x, y, z) v_y(x, y, z) dy$$

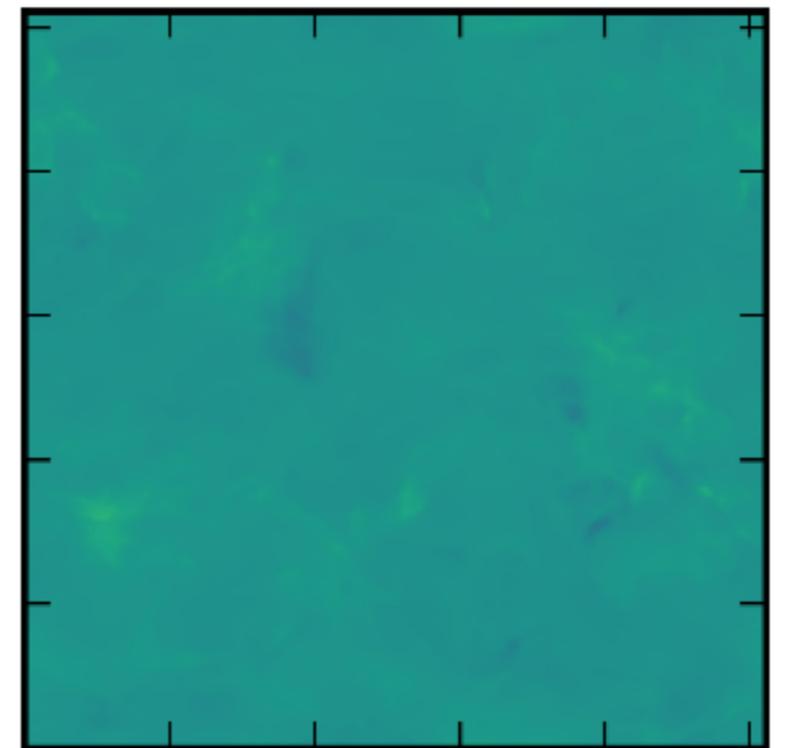
centroids (Alfvén) LOS=Y



centroids (fast) LOS=Y

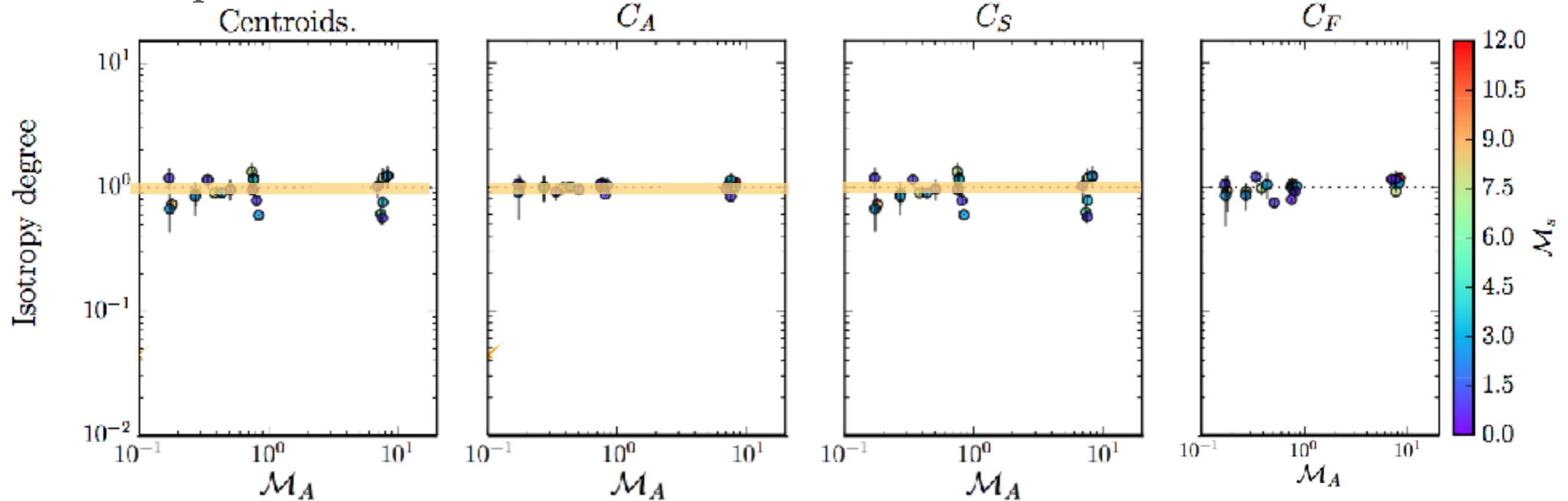


centroids (slow) LOS=Y

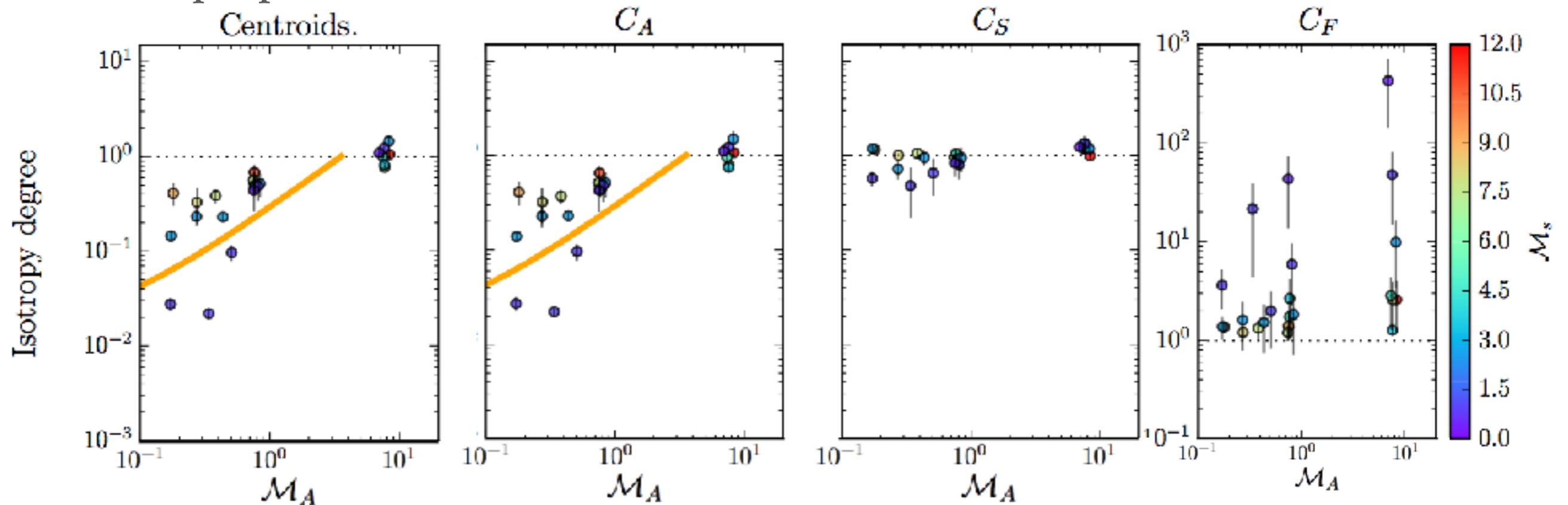


# AVERAGE ISOTROPY DEGREE FOR ALL MODES

LOS parallel to  $B_0$



LOS perpendicular to  $B_0$



# SUMMARY/CONCLUSIONS

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- Structure functions in velocity centroids are anisotropic.
- Such anisotropy points in the direction of the plane of the sky B field.
- The degree of anisotropy increases with the strength of  $B_0$  (i.e.  $\sim 1/M_A$ ), with a secondary dependence on the sonic Mach number ( $M_S$ ).
  - Thus given an estimate of  $M_S$  one can infer an upper bound on the Alfvénic Mach number.
  - With help of other techniques/measurements for the LOS component one could determine  $M_A$ .
- The Alfvén mode is the dominant contribution to the centroids map, and thus their structure function when the LOS is perpendicular to  $B_0$  (maximum anisotropy).
- The slow mode dominates in the case of LOS parallel to  $B_0$ , but the SFs are isotropic from that point of view.
- These results are consistent with those previously obtained with velocity centroids (Esquivel & Lazarian 2011, Burkhart et al. 2014).
- Also consistent with analytical predictions for the Alfvén modes by Kandel et al. 2016.