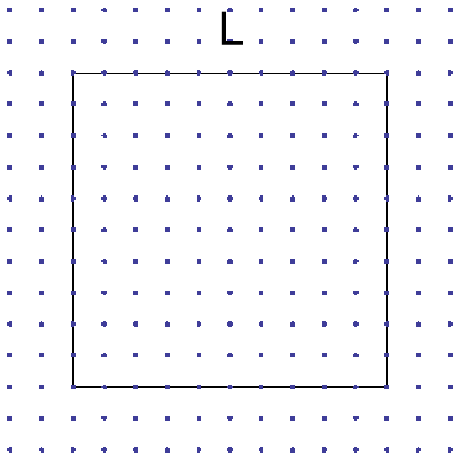
Massless (gapless) scalar field model. Vacuum (fundamental) state in a square lattice
Similar to phonons in a solid

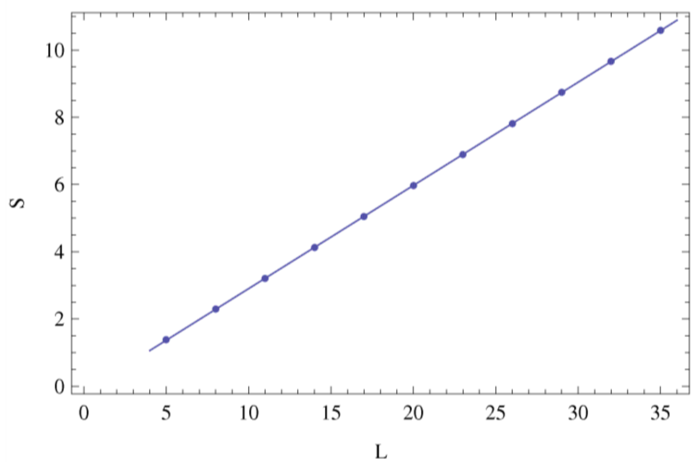


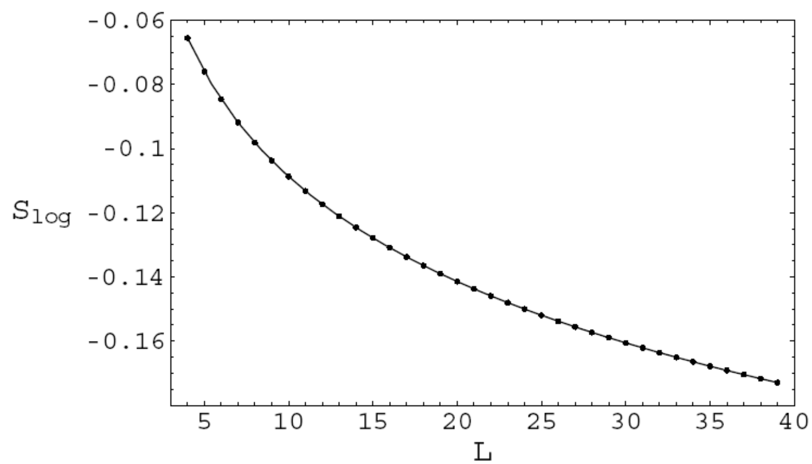
$$H = \frac{1}{2}\int d^2x \left(\dot\phi(x)^2 + (\nabla\phi(x))^2\right)$$

$$\to H = \frac{1}{2}\sum_i \epsilon^2 \left(\dot{\phi_i}^2 + \sum_{j\sim i}\frac{(\phi_i-\phi_j)^2}{\epsilon^2}\right)$$

For interacting spin systems the Hilbert space dimension grows as 2^N

For coupled Harmonic oscillators we have only to diagonalize matrices of $N \times N$

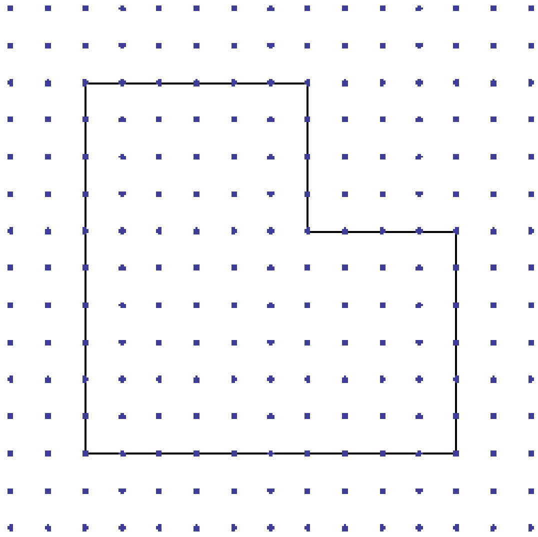




S=.075 (4 L/ε)-0.047 Log[L/ε] +const=.075 (perimeter/ε)-0.047 Log[L/ ε]+const

We have an «area» term and a logarithmic correction. These are divergent as ε ->0

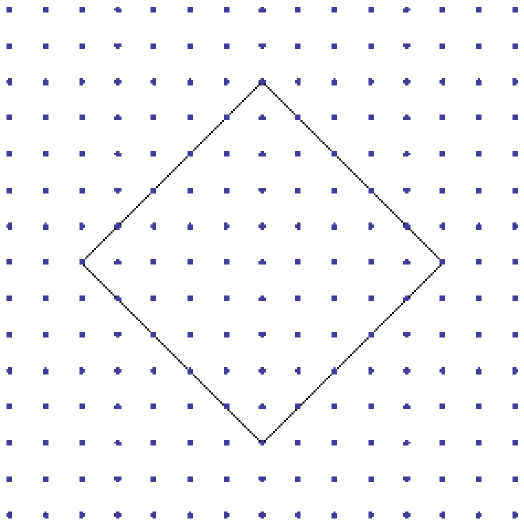
S=.075 (perimeter/ε)-(6/4) 0.047 Log[L/ε]+const



The same «area» term. A logarithmic coefficient growing
with the number of vertices.
(All vertices have the same angle S(A)=S(-A) for a global pure state)

In general:

$$S(A) = c_1\,(\text{perimeter}/\epsilon) - \sum_{\text{vertices}} c_{\log}(\theta)\log(R/\epsilon) + \text{const}$$

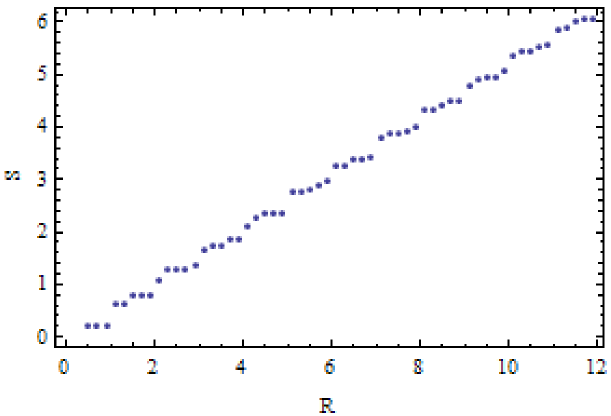


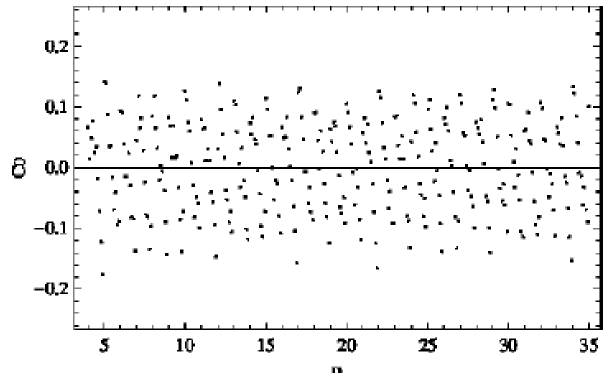
S=.085 (perimeter/ε)-0.047 Log[L/ε]+const

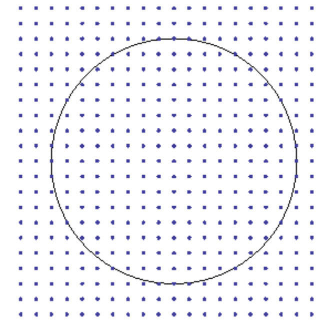
Bad: area term does not have the rotational
symmetry of the theory in the continuum limit

Good: the logarithmic term does not notice
the lattice

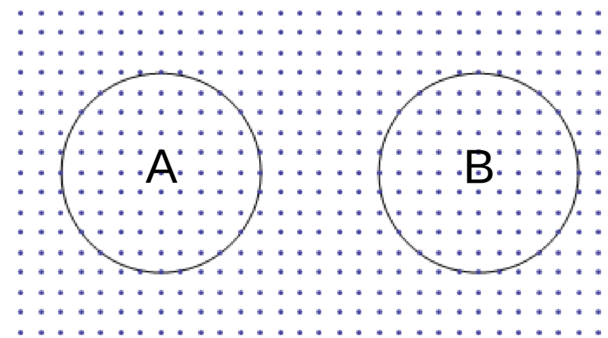
How to extract unambiguous information from the finite term?





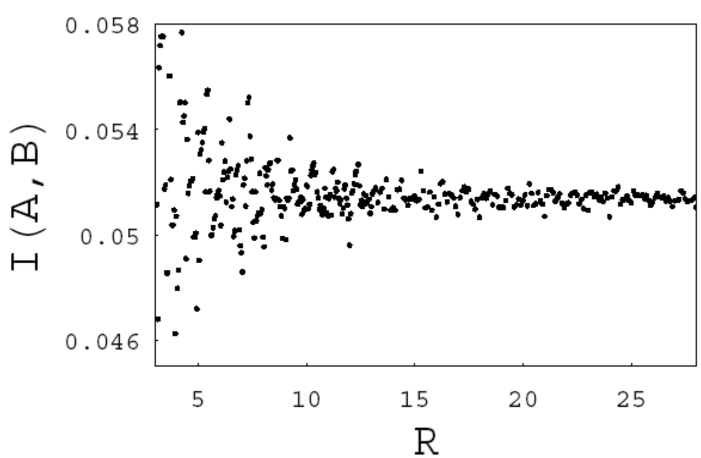


Circles in a square lattice (no log term): $S(R) = c_1 R + c_0$



Mutual information $I(A,B) = S(A) + S(B) - S(A \cup B)$

The boundary divergences cancel out in the combination.



Very little large distance entanglement
Compare I(A,B)=0.05 with log(2)=0.69
Less than 1/10 bit for infinitely many degree of freedom!

A lot of short distance entanglement:
I(A,B) diverges when A and B touch each other.
This reflects the locality of the theory

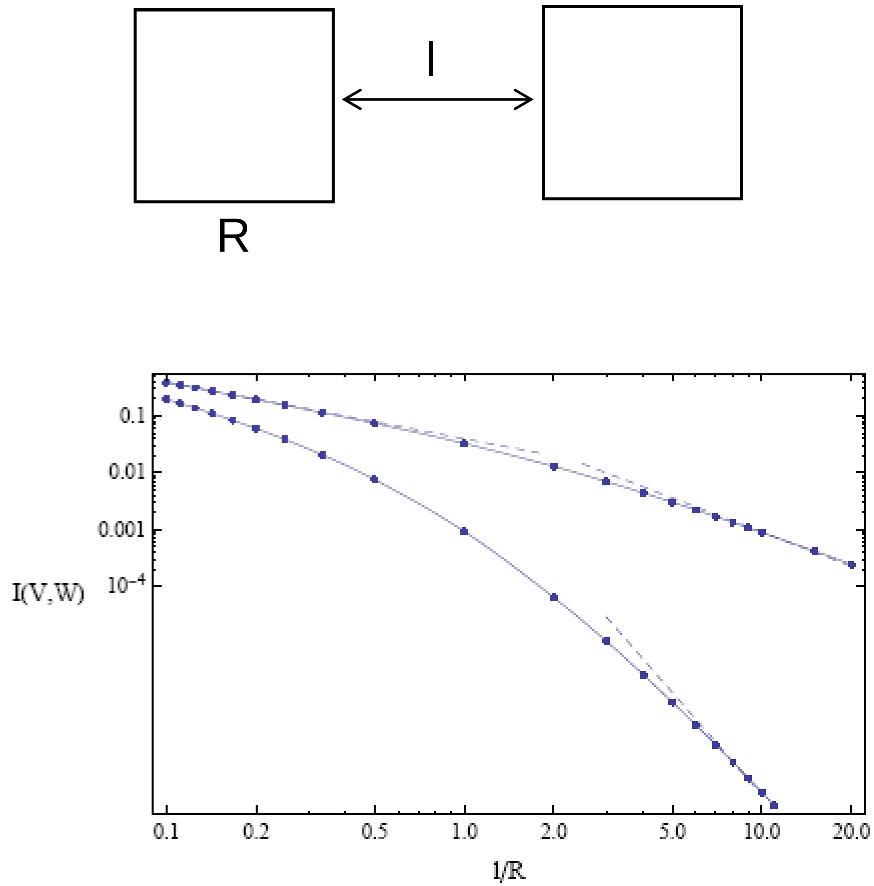
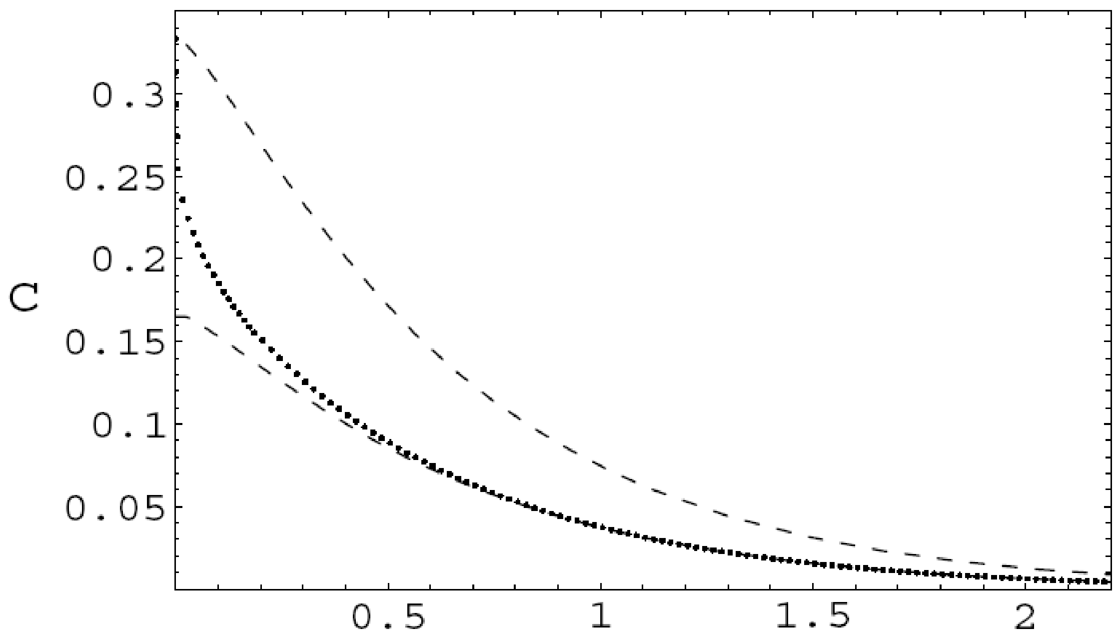
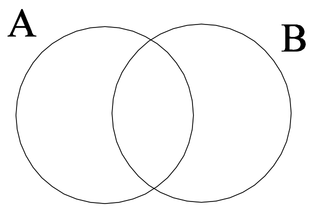


Figure 9: Log log plot of the mutual information for two squares of side R separated by a distance l, as a function of l/R. The curve at the top is the mutual information for the scalar and the lower one in the mutual information of the gauge model. The dashed lines are asymptotic behaviors. For small l/R we expect $I(V,W) \sim .0397R/l$ for both models, while for large distances we expect $I(V,W) \sim (l/R)^2$ for the scalar and $I(V,W) \sim (l/R)^6$ for the Maxwell field.

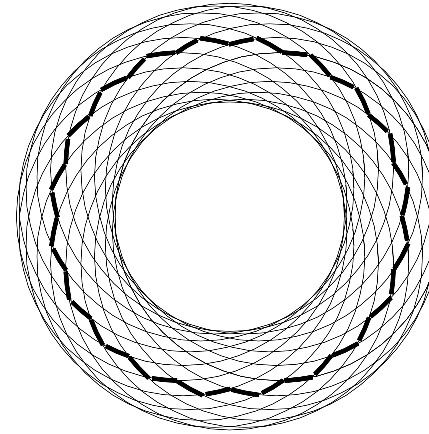

C
0.3
0.25
0.2
0.15
0.1
0.05
0.5
1
1.5
2

C-theorem in d=3 from SSA for circles

H.C, M. Huerta 2012

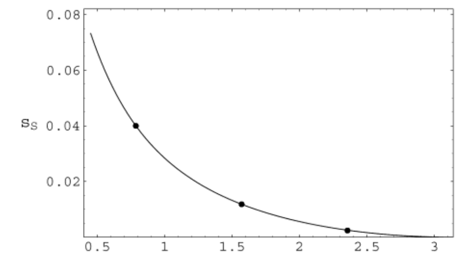


Two problems: different shapes
and log divergent angle contributions
Use many rotated regions for first problem



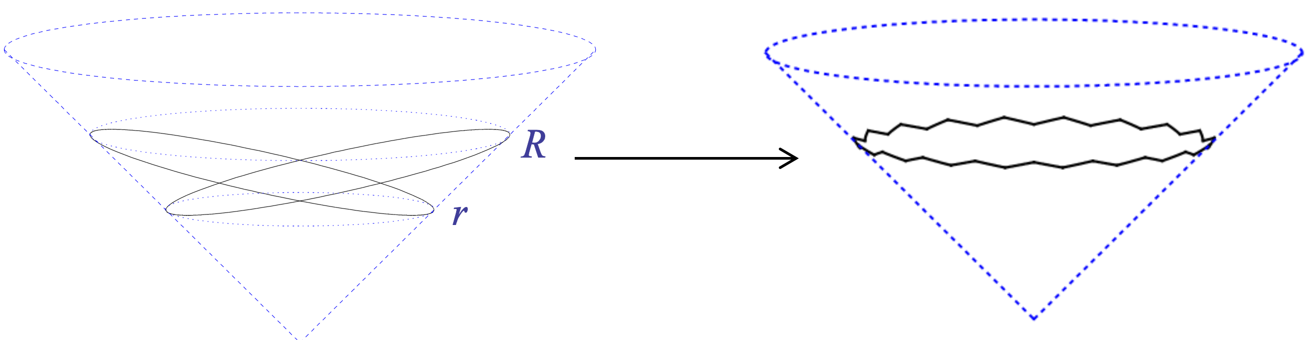
From SSA:

$$\sum_i S(X_i) \geq S(\cup_i X_i) + S(\cup_{\{ij\}}(X_i \cap X_j)) + S(\cup_{\{ijk\}}(X_i \cap X_j \cap X_k)) + \ldots + S(\cap_i X_i)$$



Coefficient of the logarithmically divergent term for a free scalar field

Log divergent terms cannot appear for «angles» on a null plane
since the feature does not have any local geometric measure



$$S(\sqrt{Rr}) \geq \frac{1}{\pi}\int_0^{\pi} dz\, S\left(\frac{2rR}{R+r-(R-r)\cos(z)}\right) \Rightarrow S'' \leq 0$$

$$c_0(r) = rS'(r) - S(r) \Rightarrow c_0(r)' \leq 0$$

Dimensionless and decreasing

At fixed points

$$c_0(r) = c_0$$

$$S(R) = c_1 R - c_0$$

Is the constant term of the entropy of the circle