Low-dimensional physics perspective on holography

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Strings @ Dunes
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Previously in Strings@Dunes

Holographic milestones

- predictions of $\mathcal{N} = 4$ SYM [lectures of H. Nastase]

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

- hadrons [lectures of J. Sonnenschein]
- strongly correlated fermions, superconductors *etc.* [lectures of C. Hoyos]

$$G(w,k) \sim \frac{Z(\omega)}{\omega - v_f k_\perp + \Sigma(\omega)}$$

- holographic entanglement entropy [lectures of H. Casini]

$$S_{\text{EE}}(A) = \frac{\gamma(A)}{4G_N}$$

- new look on higher spins, $\text{AdS}_3/\text{CFT}_2$ [lectures of D. Grumiller]

Today

More (specific) results in 2D and 3D

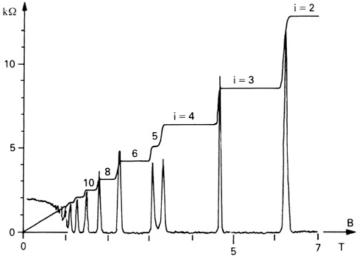
- system in external magnetic field: view on quantum Hall
- AdS/BCFT: (boundary) entropies
- towards AdS_4/CFT_3
- gravity and Chern/Simons: quantum Hall again

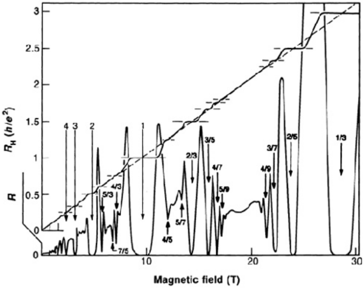
2-Dimensional Electrons in Magnetic Field

Quantum Hall Effect

Experiment

[von Klitzing *et al.*'80]
[Störmer,Tsui'85]





The Hall conductivity is quantized

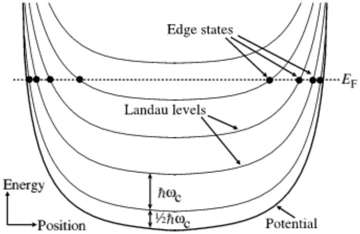
$$\sigma_H = \frac{\rho}{B} = \nu \, \frac{e^2}{h}$$

Fractional plateaux are due to electron interactions

Quantum Hall Effect

Basic theory

At a plateau 2D electrons form a gapped state (zero DC conductivity), equivalently, an incompressible fluid



Ingredients (IQHE)

- Discrete Landau levels
- Disorder allowing to vary the chemical potential in the gap between the levels
- Every filled Landau level contributes a unit of conductivity [Thouless *et al*'82]

A common explanation of FQHE is the *composite fermion* model

Quantum Hall Effect

Chern-Simons description of the QHE [Zhang,Hansson,Kivelson'89]

Quantum Hall state can be described by

$$j_i = \sigma_{ij} E_j \,, \qquad \sigma = \begin{pmatrix} 0 & \sigma_H \\ -\sigma_H & 0 \end{pmatrix}$$

This can follow from the Chern-Simons action

$$S = \frac{k}{4\pi} \int d^3x \, \epsilon^{ijk} A_i \partial_j A_k \,, \qquad k = 2\pi\sigma_H$$

? How can k be consistently quantized?

- $\nu = \frac{1}{2n+1}$ – Laughlin series
- $\nu = \frac{n}{2n\pm 1}$ – FQHE principal series
- $\nu = 5/2, \ldots$ – exotic fractions

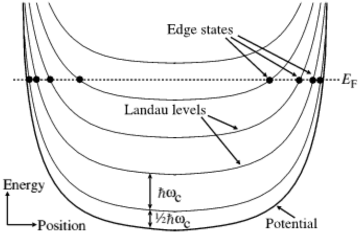
Quantum Hall Effect

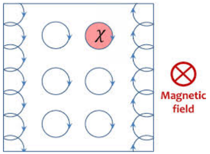
Edge states

- No bulk transport, all transport occurs along the edges (edge currents)
- bulk/boundary correspondence (edge modes encode the state in the bulk)
- boundary theory is typically a 1D CFT (Luttinger liquid)

$$\sigma_H = e\,\frac{\partial J}{\partial \mu}$$

Each edge mode contributes a quantized value to the conductivity





Quantum Hall Effect

Thermal transport

In the integer QHE we expect Wiedemann-Franz law to be satisfied

$$\frac{\kappa_H}{\sigma_H} = LT\,, \qquad L = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2$$

In the interacting case, from the bulk/boundary correspondence it is expected that

$$\kappa_H = c\, g_0\,, \qquad g_0 = \frac{\pi^2}{3}\frac{k_B^2}{h}\,T$$

where c – is a central charge of the boundary CFT [Kane,Fisher'96;Read,Green'00]

QHE from black holes

Holographic Models of QHE

Vast literature, including top-down approaches

- Keski-Vakkuri, Kraus'08; Davis, Kraus, Shah'08 Construction of an effective Chern-Simons theory. D-brane theory of plateau transitions
- Fujita, Ryu, Takayanagi'09 D-brane engineering of an effective Chern-Simons theory in low dimensions. Model massless edge modes and stripes of states with different ν. Proposal hierarchical FQHEs, using IIA string on C^2/Z_n
- Bergman, Jokela, Lifschytz, Lippert'10 D-brane engineering. Model a gapped system with massless edge modes. Quantization of conductivity as a result of quantization of a flux through a compact manifold. Irrational filling fractions

Black Holes

Black holes in AdS-space

$$ds^2 = \frac{L^2}{z^2}\left(-f(z)\mathrm{d}t^2 + \sum_i \mathrm{d}x_i^2 + \frac{\mathrm{d}z^2}{f(z)}\right), \qquad f(z) = 1 - \frac{z^d}{z_h^d}$$

In AdS_4

$$T = \frac{3}{4\pi z_h}$$

$$S = \frac{L^2 \Delta x \Delta y}{4 G z_h^2} \qquad \text{– Bekenstein-Hawking entropy}$$

- Black holes are thermodynamical systems
- TD quantities are typically defined for an infinitely remote observer

Black Holes

Introducing charge density

$$ds^2 = \frac{L^2}{z^2}\left(-f(z)dt^2 + \sum_i dx_i^2 + \frac{dz^2}{f(z)}\right)$$

Charge density/Chemical potential $\longleftrightarrow$ bulk gauge field

$$A_0 = \mu - \langle\rho\rangle z^{d-2}, \qquad \mu = \langle\rho\rangle z_h^{d-2}, \qquad f(z) = 1 - \left(1 + Q^2\right)\frac{z^d}{z_h^d} + Q^2\frac{z^{2d-2}}{z_h^{2d-2}}$$

In AdS_4

$$T = \frac{3 - Q^2}{4\pi z_h}$$

Black Holes

Magnetic field [Hartnoll,Kovtun'07]

Dyonic AdS black hole

$$\frac{\mathrm{d}s^2}{L^2} = \frac{\alpha^2}{z^2}\left(-f(z)\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2\right) + \frac{\mathrm{d}z^2}{z^2 f(z)}$$

$$A_t = \mu - \frac{\rho}{\chi\alpha} z, \qquad A_x = -By$$

Regularity at the horizon $A_t = 0$ relates $\mu = \mu(\rho)$

$$T = \alpha\,\frac{3 - Q^2}{4\pi}, \qquad Q^2 = \frac{\rho^2 + \chi^2 B^2}{\chi^2\alpha^4} \qquad S = \frac{L^2}{4G}\,\pi\alpha^2\,\Delta x \Delta y$$

- Extremal ($T = 0$) black hole: $(\rho^2 + \chi^2 B^2) = 3\chi^2\alpha^4$ has $S \neq 0$
- Quantity $\chi = \frac{L^2}{4G}$ can be related to the central charge c of the dual CFT

Transport

Green's functions [Hartnoll,Kovtun'07]

Find the response of the system to a small perturbation of electric field and temperature gradient. The holographic prescription for calculation of correlators gives the following for the retarded Green's functions:

- for 2 currents $\langle[\mathcal{J}_i(t), \mathcal{J}_j(0)]\rangle_R$

$$G^R_{ij}(\omega) = -i\omega\epsilon_{ij}\,\frac{\rho}{B}$$

  By Kubo formula

$$\sigma_{ij} = -\lim_{\omega\to 0}\frac{\mathrm{Im}\,G^R_{ij}(\omega)}{\omega} = \epsilon_{ij}\sigma_H\,, \qquad \sigma_H = \frac{\rho}{B}$$

  σ_{ij} is antisymmetric, but not quantized. ρ and B so far independent

- for $\langle[\mathcal{J}_i(t), \mathcal{T}_{tj}(0)]\rangle_R$ and $\langle[\mathcal{T}_{ti}(t), \mathcal{T}_{tj}(0)]\rangle_R$

$$G^R_{i\pi_j}(\omega) = -i\omega\epsilon_{ij}\,\frac{3\varepsilon}{2B}\,, \qquad G^R_{\pi_i\pi_j}(\omega) = \frac{\chi s^2 T^2\, i\omega\delta_{ij}}{\rho^2+\chi^2B^2} - \frac{9\rho\varepsilon^2\, i\omega\epsilon_{ij}}{4B\left(\rho^2+\chi^2B^2\right)}$$

Transport

Thermal conductivities [Hartnoll et al'07][DM,Orazi,Sodano'12]

Low temperature expansions of the conductivities yield

$$\alpha_{xx} = \alpha_{yy} = 0\,, \qquad \alpha_{xy} = -\alpha_{yx} = \frac{s}{B} = \frac{\pi}{\sqrt{3}}\sqrt{1+\sigma_H^2} + O(T)$$

$$\kappa_{xx} = \kappa_{yy} = \frac{\chi s^2 T}{\rho^2 + \chi^2 B^2} \to \chi\frac{\pi^2}{3}T + O(T^2)$$

$$\kappa_{xy} = -\kappa_{yx} = \frac{\rho s^2 T}{B(\rho^2 + \chi^2 B^2)} \to \sigma_H\frac{\pi^2}{3}T + O(T^2)$$

Wiedemann-Franz law

$$\frac{\kappa_H}{\sigma_H} = \mathcal{L}T\,, \qquad \mathcal{L} = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2$$

Quantum Hall vs Black Hole

Dyonic black hole

- is dual to a phase similar to a quantum Hall system at a plateau:

$$\sigma_{ab} \;=\; \epsilon_{ab}\,\frac{\rho}{B}$$

- does not impose quantization of σ_H
- does not exhibit a gap in the geometry
- yields an interesting result for heat conductivities, which alludes to a presence of a boundary

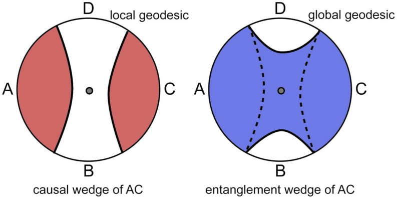
$$\kappa_{\|} \;=\; c\,\frac{\pi^2}{3}\,T, \qquad \kappa_{\perp} \;=\; \sigma_H\,\frac{\pi^2}{3}\,T$$

AdS and edges

Edges

If the system has an edge, what kind of geometry must encode that?

- causal wedge
- entanglement wedge
- dynamical principle



Edges

AdS/BCFT [Takayanagi'11]

$$S = \frac{1}{2\kappa}\int_N \mathrm{d}^{d+1}x\,\sqrt{-g}\,(R-2\Lambda) + \frac{1}{\kappa}\int_{\partial N}\mathrm{d}^d x\,\sqrt{-h}K + S_{\partial N}[\text{matter}]$$

h_{ab}-induced metric on ∂N, K-extrinsic curvature, $K_{ab} = h_a^\mu h_b^\nu \nabla_\mu n_\nu$

$$\delta S = \frac{1}{2\kappa}\int_{\partial N}\mathrm{d}^d x\,\sqrt{-h}\,(K_{ab} - K h_{ab} + \Sigma h_{ab} - T_{ab})\,\delta h^{ab}$$

Neumann boundary conditions [Compere,Marolf'08]

$$K_{ab} - (K-\Sigma)h_{ab} = T_{ab}$$

This is a dynamical equation that determines h_{ab} and the shape of the wedge

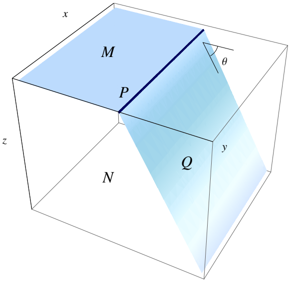
Edges

Simple example: AdS_3 [Takayanagi'11]

$$h_{ab}: \quad \frac{L^2}{z^2}\left(-\mathrm{d}t^2 + (x'(z)^2 + 1)\mathrm{d}z^2\right)$$

Solving "no-fluid" ($T_{ab} = 0$) Neumann boundary conditions gives

$$Q: \quad x(z) = z\cot\theta\,, \qquad \Sigma L = \cos\theta$$



Edges

AdS/BCFT at finite T

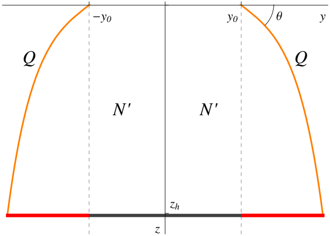
[Takayanagi'11]

$$ds^2 = \frac{L^2}{z^2}\left(-f(z)dt^2 + dx^2 + \frac{dz^2}{f(z)}\right)$$

$$f(z) = 1 - z^2/z_h^2$$

The same exercise yields

$$x(z) - x(0) = z_h \operatorname{arcsinh}\left(\frac{z}{z_h}\cot\theta\right)$$



Boundary Entropy

Thermodynamics [Takayanagi'11]

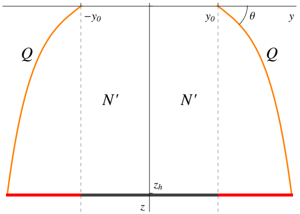
From

$$F = T\, I_{\text{GH}}$$

(I_{GH} – Euclidean action) thermal entropy is

$$S = \frac{c}{3}\left(\pi T \Delta x + \text{arcsinh}(\cot\theta)\right)$$

assuming $T\Delta x \gg 1$



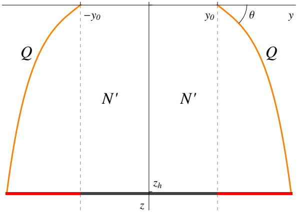
- Consists of 'bulk' and 'boundary' Bekenstein-Hawking contributions
- Consistent with entanglement entropy $S_{\text{EE}} = \frac{c}{6}\log\frac{l}{\epsilon} + \log g$

Higher dimensions

Profiles

- Empty AdS works similarly
- Black hole metrics are generally incompatible with the boundary condition

$$K_{ab} - (K - \Sigma)h_{ab} = 0$$



Which T_{ab} are compatible with the planar black hole metrics?

$$K_{ab} - (K - \Sigma)h_{ab} = T_{ab}$$

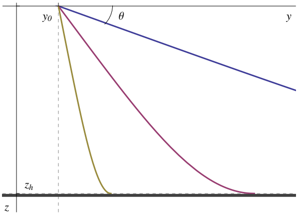
Higher dimensions

Boundary T_{ab} [Magán,DM,Silva'14]

Assuming generic $x = x(z)$ and planar BH geometry

$$T_{ab} = \begin{pmatrix} \epsilon h_{tt} & & \\ & p_y h_{yy} & \\ & & p_z h_{zz} \end{pmatrix}$$

generally non-fluid-like stress tensor



Condition $p_y = p_z$ uniquely defines the profile $x(z)$

$$x(z) - x(0) = \int_0^z \mathrm{d}\zeta \, \frac{\cot\theta}{\sqrt{f(\zeta)}} \, , \qquad f = 1 - z^d / z_h^d$$

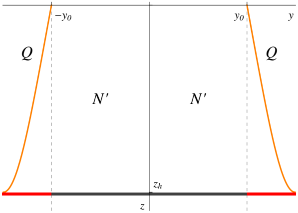
Higher dimensions

Boundary entropy

[Magán,DM,Silva'14]

Two prescriptions

- Compute thermodynamic potential (free energy) as Euclidean action
- Integrate the relation $\epsilon + p = Ts$ over Q



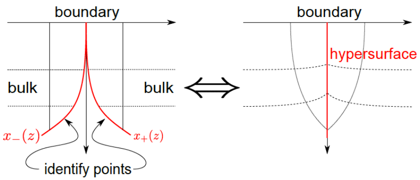
Entropy $S = -\partial F/\partial T$ $\qquad$ $(c = L^2/4G)$

$$S = S_{\text{bulk}} + 2S_{\text{bdry}} = \frac{16\pi^2 c}{9} T^2 \Delta x \Delta y + \frac{32\pi c}{9} T \Delta y \cot\theta$$

Boundary entropy is not BH, $S_{\text{bdry}} \simeq 0.95 S_{\text{BH}}$

Edges as Defects

Refinement of the Takayanagi's proposal [Erdmenger,Flory,Newrzella'14]



A holographic Kondo model (ask Carlos) [Hoyos *et al*'13-16]

- Entaglement entropy in the presence of impurity
- The length scale ξ in the boundary entropy – correlation length

$$S_{\text{bdry}} = \frac{c}{3} \operatorname{arcsinh}(\cot\theta) = \frac{L}{2G}\frac{\xi}{z_h}$$

Edges of Quantum Hall

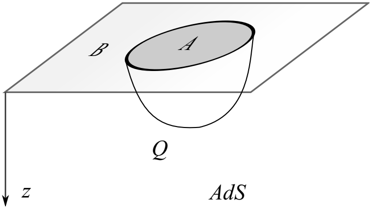
Adding gauge fields

[Fujita,Kaminski,Karch'12][DM,Orazi,Sodano'12]

$$c_1 \int_N d^4x \sqrt{-g} F_{\mu\nu}^2 + c_2 \int_N F \wedge F + k \int_Q A \wedge F - k \int_P d^2x \, A_x A_t$$

Neumann boundary conditions imply

$$c_1 F + (c_2 + k) * F|_Q = 0$$



- Density and magnetic field are locked together $\rho \sim B$

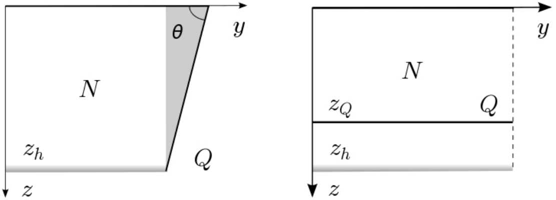
Edges of Quantum Hall

Edge Currents [DM,Orazi,Sodano'12]

Calculate current as the response to a variation of the external field

$$\langle \mathcal{J}^x(x^i) \rangle = \frac{\delta S_k}{\delta A_x(x^i)} = -\frac{k}{2\pi}\,\mu \left(\delta(y) - \frac{\Theta(y + \cot\theta)}{\cot\theta} \right)$$

For $\theta < \pi/2$ there is a non-zero current, which is maximal for $\theta = 0$



Geometrically $\theta = 0$ corresponds to a gap, independent of the position. For $\theta > 0$ edge current diffuses and disappears for $\theta = \pi/2$ (no gap).

Low-Dimensional AdS/CFT

Chern-Simons and CFT

Precursors [Belavin,Polyakov,Zamolodchikov'84]

Conformal symmetry in $1+1$D highly contrains the theory

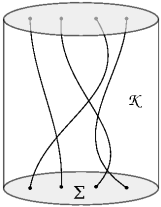
- Conformal blocks

$$\langle \prod_{i=1}^{n} \phi_{R_i}(z_i) \rangle = \sum_{\lambda} |\mathcal{F}_{\lambda;R_1,\ldots,R_n}(z_1,\ldots,z_n)|^2$$

can be reconstructed from the constraints imposed by the symmetry

Quantization of Chern-Simons theory in 2+1D [Witten'89]

- States in the Hilbert space of CS $\longleftrightarrow$ conformal blocks
- Knot invariants are Wilson line correlators in CS



Chern-Simons and CFT

TQFT vocabulary

- Partition function $\partial\mathcal{M} = 0 \longleftrightarrow$ C-number

$$Z = \int \mathcal{D}A \, \exp(-S_{\text{CS}})$$

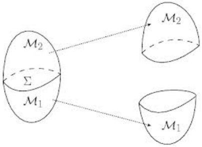
- Wilson loop $\longleftrightarrow$ topological invariant

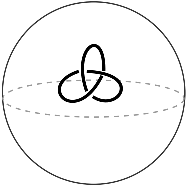
$$\langle \, W_R(C) \, \rangle = \left\langle \mathrm{Tr}_R \, \mathrm{P} \exp \int_C A \, \mathrm{d}x \right\rangle_{\text{CS}}$$

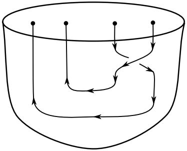
- Partition function $\partial\mathcal{M} \neq 0 \longleftrightarrow$ wavefunction

All the bulk information of CS is encoded in the boundary (conformal block):

$\Rightarrow$ holographic correspondence





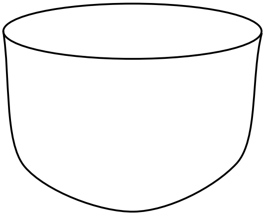


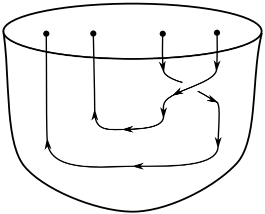
CFT and Quantum Hall

Conformal blocks [Moore,Read'91]

Wavefunctions of QHE $\Longleftrightarrow$ conformal blocks of CFT's

$$\Psi = \prod_{i<j} (z_i - z_j)^{1/\nu} \exp\left(-1/4\ell^2 \sum_i |z_i|^2 \right)$$





Low-dimensional AdS/CFT

Duality triangle

Chern-Simons

$\swarrow \qquad \searrow$

AdS_3 gravity $\quad \overset{AdS/CFT}{\longleftrightarrow} \quad$ CFT_{1+1}

$\Rightarrow$ In low dimensions AdS/CFT exists without strings (at least for classical gravity)

$\Rightarrow$ Chern-Simons provide a compact setup to study $\mathrm{AdS}_3/\mathrm{CFT}_2$

[lectures of D. Grumiller]

AdS$_3$ and Chern-Simons

3d Gravity as Chern-Simons [Witten'88]

$$A = \omega + \frac{1}{\ell} e \qquad \bar{A} = \omega - \frac{1}{\ell} e$$

$$S = S_{\rm CS}[A] - S_{\rm CS}[\bar{A}]$$

where $A, \bar{A}$ are $SL(2,R)$-valued flat connections

for $SL(N,R) \times SL(N,R)$ one also obtains higher spin fields $s \leq N$

$$g_{\mu\nu} = \text{Tr } \left(e_\mu e_\nu\right) \qquad \phi_{\mu\nu\rho} = \text{Tr } \left(e_{(\mu} e_\nu e_{\rho)}\right)$$

Flat connections are mapped to solutions of Einstein eqs. Gauge transforms become diffeos

AdS_3 and Chern-Simons

Black holes from flat connections

Gauge transformation ($w = t + i\phi$) $\qquad L_0, L_{\pm 1} \in sl(2)$

$$A = b^{-1} a b + b^{-1}\,\mathrm{d}b \qquad b = \exp(-L_0 \rho) \qquad a = a_w dw + a_{\bar{w}} d\bar{w}$$

If one chooses

$$a_w = L_1 + M L_{-1}\,, \qquad \bar{a}_{\bar{w}} = L_{-1} + M L_1$$

one gets

$$\frac{\mathrm{d}s^2}{\ell^2} = \mathrm{d}\rho^2 - \left(e^{\rho} - M e^{-\rho}\right)^2 \mathrm{d}t^2 + \left(e^{\rho} + M e^{-\rho}\right)^2 \mathrm{d}\phi^2$$

Correlators and Wilson lines

Entaglement entropy [Ryu,Takayanagi'06]

Holographic formula for computing entanglement entropy

$$S_{\mathrm{EE}}(A) = \frac{\mathrm{Area}(\gamma(A))}{4G}\,, \qquad \gamma(A)\ -\ \text{minimal area surface}$$

In AdS_3 it reproduces the known CFT_2 result [Calabrese,Cardy'04]

$$S_{\mathrm{EE}} = \frac{c}{6}\log\frac{\sqrt{\epsilon^2 + x^2/4} + x/2}{\sqrt{\epsilon^2 + x^2/4} - x/2} \to \frac{c}{3}\log\frac{x}{\epsilon}$$

- The relation opens up a rich source of speculations about the meaning of quantum geometry

Correlators and Wilson lines

Entanglement entropy from Chern-Simons [de Boer,Jottar'13][Ammon,Castro,Iqbal'13]

Natural observables in Chern-Simons theory are (vevs of) Wilson loops

$$W_R(C) = \mathrm{Tr}_R \, \mathrm{P} \exp \oint_C A$$

– gauge invariants, topological invariants.

Less obvious – Wilson lines: looking at the data defining W_R one can guess

$$W_R(x_i, x_f) \sim \exp\left(-\sqrt{2c_2(R)} L(x_i, x_f)\right)$$

Wilson line computes the proper geodesic distance for a particle of mass $m^2 = 2c_2$

Correlators and Wilson lines

Example $$ds^2 = (d\tau^2 + dx^2 + du^2)/u^2$$

$$W(C) = \mathrm{Tr}\,\mathbf{P}\exp\left(-\int_{\bar{C}} A\right)\mathbf{P}\exp\left(-\int_{C} \bar{A}\right)$$

Wilson line between points $(u, -x/2, 0)$ and $(u, x/2, 0)$

$$A_x = \begin{pmatrix} 0 & 1/u \\ 0 & 0 \end{pmatrix}, \qquad \mathbf{P}\exp\int_{-x/2}^{x/2} A_x dx = \exp A_x \cdot x = \begin{pmatrix} 1 & x/u \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{P}\exp\int_{-x/2}^{x/2} A_x dx\, \mathbf{P}\exp\int_{x/2}^{-x/2} \bar{A}_x dx = \begin{pmatrix} 1 + x^2/u^2 & x/u \\ x/u & 1 \end{pmatrix}$$

Correlators and Wilson lines

(AdS/)CFT interpretation

Wilson lines compute the coupling of a probe particle of mass $m = \sqrt{2c_2(R)}$ to the classical background provided by the connection A, $\bar{A}$. From the AdS/CFT point of view this is

$$\langle\, O_H(\infty) O_L(0) O_L(w) O_H(1)\,\rangle = \langle\, O_H \,|\, O_L(0) O_L(w) \,|\, O_H\,\rangle$$

For O_L corresponding to the ρ-primary one gets the von Neumann entropy

Correlators and Wilson lines

General behavior [Hegde,Kraus,Perlmutter'15]

$SL(N)$, any representation $w = t + i\phi$

$$W_R(C) \xrightarrow[\epsilon \to 0]{} \langle \mathrm{hw}_R | W | \mathrm{hw}_R \rangle$$
$$= e^{-4h_R} \langle \mathrm{hw}_R | e^{-a_w w - a_{\bar{w}} \bar{w}} | -\mathrm{hw}_R \rangle \langle -\mathrm{hw}_R | e^{\bar{a}_w w + \bar{a}_{\bar{w}} \bar{w}} | \mathrm{hw}_R \rangle$$

- Entanglement entropy case corresponds to $\mathrm{hw}_R = \rho$
- For general R the Wilson line computes a semiclassical ($c \to \infty$) conformal block

Integrability connection

From matrix elements to tau-functions [DM,Mironov,Morozov'16]

Calculation of Wilson lines reduces to determination of matrix elements

$$\langle -\mathrm{hw}_R | \mathrm{e}^{a_w w + a_{\bar w}\bar w} | \mathrm{hw}_R \rangle \,, \qquad a_w = L_{-1} + \sum_{s=2}^{N} Q_s L^{(s)}_{s-1}$$

It turns out that physically interesting matrix elements are described by special τ-functions

$$\tau^{(k)}(s,\bar s|G) = \langle \mathrm{hw}_k | e^H \, G \, e^{\bar H} | \, \mathrm{hw}_k \rangle \,, \qquad e^H = \exp \sum_{i=1}^{s} s_s R_k(L^s_{-(s-1)})$$

Toda recursion relation

$$\tau^{(k)} \partial_1 \bar\partial_1 \tau^{(k)} - \partial_1 \tau^{(k)} \bar\partial_1 \tau^{(k)} = \tau^{(k+1)} \tau^{(k)}$$

Integrability connection

Skew tau-function [DM,Mironov,Morozov'16]

$$\tau_-^{(k)}(s,G) = \langle\, \mathrm{hw}_k \,|\, e^H\, G \,|\, -\mathrm{hw}_k \,\rangle = \left(\frac{\partial}{\partial \bar{s}_1}\right)^{k(N-k)} \tau^{(k)}(s,\bar{s},G)$$

Recursion relation

$$\tau_-^{(k)} \frac{\partial^2 \tau_-^{(k)}}{\partial t^2} - \left(\frac{\partial \tau_-^{(k)}}{\partial t}\right)^2 = \tau_-^{(k+1)}\, \tau_-^{(k-1)}$$

3D Gravity and QHE

Is the 3D gravity useful for QHE?

Recent progress in the understanding of the relations between (higher spin) 3d gravity, (SL(N)) Chern-Simons theories and (W_N) CFT's open new perspectives on the QHE

- rational CFT's as theories for the edge states?
- black holes as QHE quasiparticle excitations?
- characters of minimal models as QHE wavefunctions?

3D Gravity and QHE

New proposals [Vafa'15]

$$Q = i\left(\sqrt{2\nu} - \frac{1}{\sqrt{2\nu}}\right) \qquad c = 1 + 6Q^2 = 1 - 3\,\frac{(2\nu-1)^2}{\nu}$$

For rational $\nu = n/m$ this corresponds to $(2n, m)$ minimal models. Since m must be odd:

- unitarity fixes $m = 2n \pm 1$

$$\nu = \frac{n}{2n \pm 1} \qquad - \text{ FQHE principal series}$$

Conclusions

This talk is a subjective recollection of results in low-dimensional holographic models and their connection to various topics of mathematical physics. It is aimed to underline the following points

- Symmetries are powerful in low dimensions so that AdS/CFT conjecture can be tested
- The correspondence can work (be justified) beyond string (top-down) formulation. In particular, there is no particular need for large N – large c is enough
- Predictions of holographic models can be relevant for real-life phenomena.