Effectiveness and naturalness of the spectral action for theories on foliated space-times

Aleksandr Pinzul
Universidade de Brasília

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Measure theory and topology

- Commutative Von Neumann algebra ⬄ σ-finite measure space
- This motivates to *define* the non-commutative measure theory via general (non-commutative) von Neumann algebras
- Commutative C*-algebra ⬄ locally compact topological space
- This motivates to *define* the non-commutative topology via general (non-commutative) C*-algebras

Measure theory and topology

Topology	Algebra
$C_0(X)$	*C*-algebra A*
proper map	*morphism*
homeomorphism	*automorphism*
measure	*positive functional*
compact	*unital*
σ-compact	*σ-unital*
open subset	*ideal*
open dense subset	*essential ideal*
closed subset	*quotient*
compactification	*unitization*
$C(\alpha X)$	*A~*
$C(\beta X)$	*M(A)*
…	…

Differential Calculus

Classical	Quantum
Complex variable	*Operator in* $\mathscr{H}$
Real variable	*Selfadjoint operator in* $\mathscr{H}$
Infinitesimal	*Compact operator in* $\mathscr{H}$
Infinitesimal of order of α	*Compact operator in* $\mathscr{H}$ *whose characteristic values behave as* $\mu_n = \mathcal{O}(n^{-\alpha})$ *when* $n \to \infty$
Differential of real or complex variable	$df = i[F, f]$
Integral of infinitesimal of order 1	*Dixmier trace,* $Tr_\omega(T)$

Geometry

Commutative case (reconstruction theorem, Connes 2008)

A spectral triple: $(\mathcal{A},\mathcal{H},\mathcal{D})$, $\mathcal{A}$ is commutative and

i) $\mu_n(R(\mathcal{D}))=\mathcal{O}(n^{-1/p})$

ii) $[[\mathcal{D},a],b]=0$ for any $a,b\in\mathcal{A}$

iii) For any $a\in\mathcal{A}$, a and $[\mathcal{D},a]$ belong to Dom(δ^m) where $\delta^m(T)=[|\mathcal{D}|^m,T]$ is a derivation

iv) Exists a Hochschild p-cycle c: $\pi_{\mathcal{D}}(c)=1$ for p odd or $\pi_{\mathcal{D}}(c)=\gamma$ – Z_2 grading for p even

v) $\mathcal{A}$-module $\mathcal{H}_\infty=\bigcap$ Dom($\mathcal{D}^m$) is finite and projective and the Hermitian structure (.|.) defined by $<\xi,a\eta>=:⨍ a(\xi|\eta)|\mathcal{D}|^{-p}$

Then $(\mathcal{A},\mathcal{H},\mathcal{D})$ is in one-to-one correspondence with smooth oriented compact manifold.

Geometry

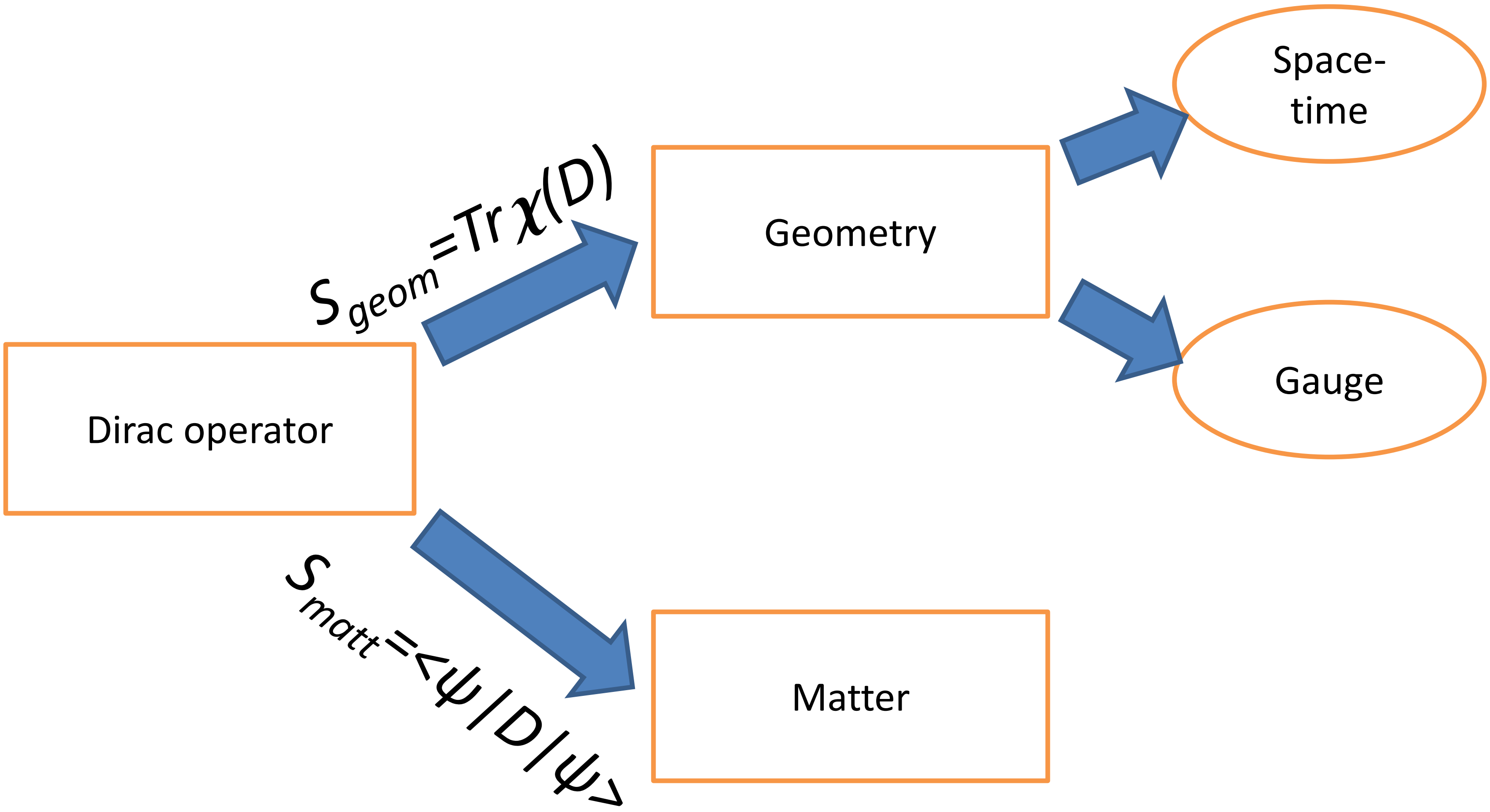
$\mathcal{A}$=$C^\infty(M)$, $\mathcal{H}$=$L^2(M,S)$, $\mathcal{D}$=$\gamma^\mu(\partial_\mu+\omega_\mu)$

- $d(x,y) = \sup\left\{ | f(x)\text{-}f(y)| : f \in C(M), \left\| [D, f] \right\| \leq 1 \right\}$
- $N_{|D|}(\lambda) \underset{\lambda\to\infty}{\longrightarrow} \frac{2^m \Omega_n}{n(2\pi)^n} Vol(M)\lambda^n$
- $Tr^+(f\,|D|^{-n}) = \frac{2^m \Omega_n}{n(2\pi)^n} \int_M f \nu_g$, where $Tr^+ A = \lim_{N\to\infty} \frac{\sigma_N(A)}{\log N}$
- $Tr\chi\left(\frac{D^2}{m_0^2}\right) = \frac{N}{48\pi^2}\Big[12 m_0^4 f_0 \int d^4x\sqrt{g} + m_0^2 f_2 \int d^4x \sqrt{g} R +$

$$+ f_2 \int d^4x\sqrt{g}\left(-\frac{3}{20} C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \frac{1}{10} R_{;\mu}{}^{\mu} + \frac{11}{20} R^* R^* \right) + O\left(\frac{1}{m_0^2}\right)\Big]$$

Geometry

We take the generalized spectral triple *(A,D,H)* as the *definition* of non-commutative geometry.



Dirac operator
$S_{geom} = Tr\,\chi(D)$
Geometry
Space-
time
Gauge
$S_{matt} = \langle\psi|D|\psi\rangle$
Matter

Examples. I – AC geometry and SM

Usual geometry:

$\mathcal{A}=C^\infty(M)$, $\mathcal{H}=L^2(M,S)$, $\mathcal{D}=\gamma^\mu(\partial_\mu+\omega_\mu)$; J_M, γ_M

Finite (matrix) geometry:

$\mathcal{A}_F$, $\mathcal{H}_F$, $\mathcal{D}_F$; J_F, γ_F

Almost commutative (AC) geometry:

$M\mathrm{x}F=(C^\infty(M,\mathcal{A}_F)$, $\mathcal{H}=L^2(S\otimes(M\mathrm{x}\mathcal{H}_F))$,

$\mathcal{D}\otimes 1+\gamma_M\otimes\mathcal{D}_F$; $J_M\otimes J_F$, $\gamma_M\otimes\gamma_F$)

Take the finite (matrix) geometry as follows:

$\mathcal{A}_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$

$\mathcal{H}_F = (H_\ell \oplus H_{\ell^*} \oplus H_q \oplus H_{q^*})^{\oplus 3}$

$\mathcal{D}_F =$ 'Yukawa mass matrices'

$J_F =$ 'charge conjugation'

$\gamma_F =$ left-handed particles are eigenvectors with +1, right-handed with -1

Gauge fields come from the fluctuation of the full Dirac operator: $D_A = D + JAJ^{-1}$, where $A = \Sigma a_j [D, b_j]$

Then the spectral action $Tr\chi\left(\frac{D^2}{m_0^2}\right)$ produces the full action of the Standard Model!

Why do we “deform” geometry?

1) Problems with the quantization of gravity

$[\lambda] = \delta$ in momentum units

$D = d - (d/2 - 1)E - n\delta$

D - superficia l degree of divergence

d - space - time dimension

E - numberf of the external legs

n - number of vertices

We can expect renormaliz ability only when $\delta \geq 0$

Why do we "deform" geometry?

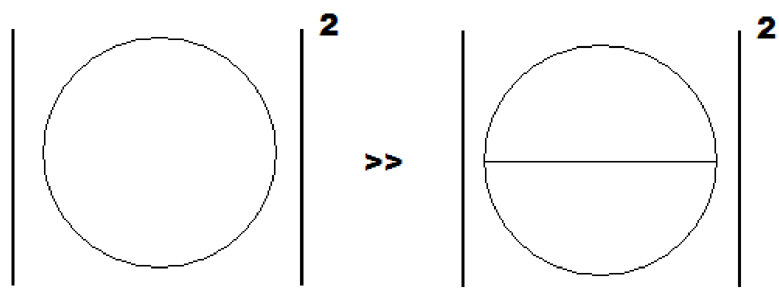
$$S_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R \quad => \quad \delta \equiv [G] = 2-d$$

for $d = 4, \delta = -2 < 0$

As the result, the effective dimensionless constant is given by

$$GE^2 := \left(\frac{E}{M_P}\right)^2 \quad \text{where } M_P = \sqrt{\frac{\hbar c}{G}} = 1.22\times 10^{19} GeV$$

i.e. when $E << M_p$



Why do we “deform” geometry?

Possible solutions

i) (Super)string theory: contains a spin-2 massles mode => has to describe gravity. GR is recovered in long-wave regime. But, the predictive power is quite poor: the string theory landscape has 10^{500} vacua.
ii) Loop quantum gravity: one can perform non-perturbative quantization. Among problems, the difficulty of the recovery quasiclassical space.
iii) Some other approaches treat gravity as an emergent phenomenon (e.g., entropic gravity).

Why do we "deform" geometry?

2) General arguments that the notion of a space-time as a classical manifold should be abandoned Doplicher, Fredenhagen and Roberts 1995

Examples. II – Horava-Lifshits models

- Lifshitz model (Lifshitz 1941)

$$S = \int dt d^n x \left(\dot{\phi}^2 + g(\Delta\phi)^2 - c^2 \phi \Delta \phi\right)$$

$$[x] = -1, [t] = -2, [\mathrm{c}] = 1$$

The propagator has the form :

$$G(\omega, \vec{k}) \propto \frac{1}{\omega^2 - c^2 \vec{k}^2 - g\vec{k}^4}$$

$$\mathrm{UV}:\ \frac{1}{\omega^2 - c^2\vec{k}^2 - g^2\vec{k}^4} = \frac{1}{\omega^2 - g^2\vec{k}^4} + \frac{1}{\omega^2 - g^2\vec{k}^4} c^2\vec{k}^2 \frac{1}{\omega^2 - g^2\vec{k}^4} + \ldots$$

$$\mathrm{IR}:\ \frac{1}{\omega^2 - c^2\vec{k}^2 - g^2\vec{k}^4} = \frac{1}{\omega^2 - c^2\vec{k}^2} + \frac{1}{\omega^2 - c^2\vec{k}^2} g^2\vec{k}^4 \frac{1}{\omega^2 - c^2\vec{k}^2} + \ldots$$

I.e. we have two fixed points: UV, which corresponds to z=2 and has significantly improved behavior and IR, in which by the time rescaling we can set c=1 and restore relativistic invariance, z=1

Why to break Lorentz invariance?

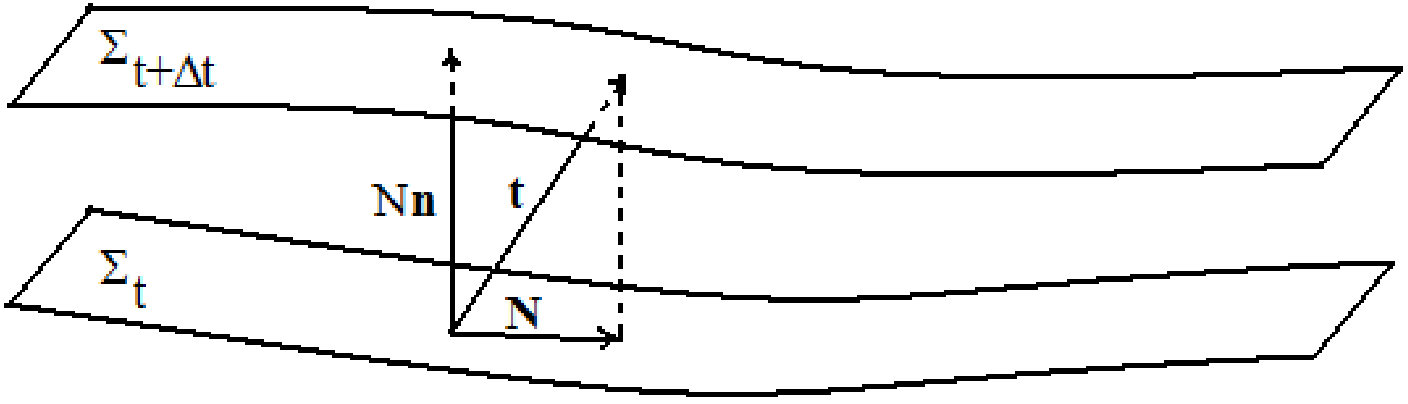
Let us consider the same type of the modification, but when the higher derivatives are added in the Lorentz invariant way.

$$S = \int d^4x \left(\partial_\mu \phi \partial^\mu \phi + g (\partial^\mu \partial_\mu \phi)^2 \right)$$

The propagator takes the form :

$$G(\omega, \vec{k}) \propto \frac{1}{k^2 - gk^4} = \frac{1}{k^2(1 - gk^2)} = \frac{1}{k^2} - \frac{1}{k^2 - 1/g}$$

ADM



$$ds^2 = g_{ij}(dx^i + N^i dt)(dx^j + N^j dt) - (Ncdt)^2$$

$$S_{EH} = \frac{1}{16\pi G}\int dt d^3x N\sqrt{g}\left(K_{ij}K^{ij} - K^2 + {}^3R\right)$$

where $K_{ij} = \frac{1}{2N}\left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$ - second fundamenta l form

We take ADM slicing as fundamental, i.e. instead of considering just a manifold, we endow it with the foliation structure:

$$\tilde{x}^i = \tilde{x}^i(\vec{x}, t), \ \tilde{t} = \tilde{t}(t)$$

These are foliation-preserving diffeos or FDiffs

Also, we introduce anisotropic scaling between *x* and *t*:

$$\vec{x} \to \alpha \vec{x}, \ t \to \alpha^z t \quad \text{or} \ [\vec{x}] = -1, \ [t] = -z$$

This is equivalent to prescribing the following dimensions:

$$[c] = z - 1, \ [N] = [g_{ij}] = 0, \ [N_i] = z - 1 \Rightarrow [G] = 3 - z$$

- Projectable FDiff gravity (Horava 2009)

$$N = N(t), \quad N \to N \frac{\partial t}{\partial \tilde{t}}$$

$$S = \frac{M_P^2}{2} \int d^3x dt \sqrt{g} N \left(K_{ij} K^{ij} - \lambda K^2 - V_P \right)$$

$$V_P = 2\Lambda - \xi R + M_*^{-2} \left(A_1 R^2 + A_2 R_{ij} R^{ij} \right) +$$

$$+ M_*^{-4} \left(B_1 R \Delta R + B_2 R_{ij} R^{jk} R_k^i + B_3 \nabla_i R_{jk} \nabla^i R^{jk} + B_4 R R^{jk} R_{jk} + B_5 R^3 \right)$$

- Non-projectable FDiff gravity (Blas et al. 2010)

$$N = N(t, \vec{x}), \quad a_i := N^{-1} \nabla_i N$$

$$S = \frac{M_P^2}{2} \int d^3x dt \sqrt{g} N \left(K_{ij} K^{ij} - \lambda K^2 - V_{NP} \right)$$

$$V_{NP} = V_P - \alpha a_i a^i + M_*^{-2} \left(C_1 a_i \Delta a^i + C_2 (a_i a^i)^2 + C_3 a_i a_j R^{ij} \cdots \right) +$$

$$+ M_*^{-4} \left(D_1 a_i \Delta^2 a^i + D_2 (a_i a^i)^3 + D_3 a_k a^k a_i a_j R^{ij} \cdots \right)$$

Some properties

- Broken 4d diffeos => Lorentz violation
- Extra scalar mode in addition to two graviton polarizations
- In general the scalar mode does not decouple in IR, this can endanger the renormalizability
- The model with the detailed balance condition does not pass the Solar system tests
- The healthy extension (with a_i) has A LOT of free parameters and some of them still require fine tuning
- …

IR limit

Keeping only the terms with lowest derivatives, we arrive at IR limit

$$N = N(t, \vec{x}), \quad a_i := N^{-1}\nabla_i N$$

$$S_{IR} = \frac{M_P^2}{2}\int d^3x dt \sqrt{g} N\left(-K_{ij}K^{ij} + \lambda K^2 + \alpha a_i a^i + \xi {}^{(3)}R + \Lambda_C\right)$$

This action is used to study the gravitational equations of motion (Barausse & Sotiriou 2013)

Spectral dimension

(AP 2010)

- The choice of the Dirac operator in the form $D=\gamma^\mu(\partial_\mu+\omega_\mu)$ is not natural anymore
- The foliation structure dictates the following (schematic) form for D (for z=3) $D= \partial_t+\sigma^\mu\partial_\mu\Delta+M_*\Delta+M_*^2\sigma^\mu\partial_\mu$
- This D should be used to obtain "physical" geometry instead of auxiliary 3+1 dimensional

(AP 2010, Gregory & AP 2012)

Model calculation

- $M = S^1 \times T^3$, $D^2 = \partial_t^2 + \Delta^3 + M_*^2 \Delta^2 + M_*^4 \Delta$
- $sp(D^2) = \{n^2 + (n_1^2 + n_2^2 + n_3^2)^3 + M_*^2 (n_1^2 + n_2^2 + n_3^2)^2 + M_*^4 (n_1^2 + n_2^2 + n_3^2)$, $n_i \in Z\}$
- $N_{|D|}(\lambda) = \{\#$ eigenvalues $< \lambda\}$
- when $\lambda << M_*^6$ the last term dominates:

$$N_{|D|}(\lambda) \cong \int_0^{\lambda} dn \int_0^{(\lambda^2 - n^2)^{1/2}} 4\pi\rho^2 d\rho \propto \lambda^4 \quad \Rightarrow \quad d = 4$$

when $\lambda >> M_*^6$ the first term dominates:

$$N_{|D|}(\lambda) \cong \int_0^{\lambda} dn \int_0^{(\lambda^2 - n^2)^{1/6}} 4\pi\rho^2 d\rho \propto \lambda^2 \quad \Rightarrow \quad d = 2$$

One can do better and go beyond the flat case.

- Define a generalized ζ-function

$$\zeta_\Delta(s) := \mathrm{Tr}(\Delta^{-s})$$

- Now Δ can be any generalized elliptic operator.
- ζ-function can be extended to a meromorphic function on the whole complex plane with the only poles given by

$$\frac{n-p+zp}{2z}, \frac{n-p+zp-1}{2z}, \ldots, \frac{n-p+zp-k}{2z}, \ldots$$

- The first pole is related to the analytic dimension

$$\frac{n-p+zp}{2z} = \frac{n_a}{2}$$

- n=D+1, p=1 (co-dimension) we have $n_a = 1 + \dfrac{D}{z}$

Spectral Action

Part I $\quad Tr\chi\left(\frac{D^2}{m_0^2}\right) = \text{Horava - Lifshitz gravity?}$

- Dirac operator is very complicated:

$$D^2 = \Delta_\tau + f(\Delta_x),$$

$$\text{where } \Delta_\tau = -\frac{1}{N\sqrt{g}}\partial_\tau\left(\frac{\sqrt{g}}{N}\partial_\tau\right) \text{ and } \Delta_x = \frac{1}{N\sqrt{g}}\partial_i\left(N\sqrt{g}\,g^{ij}\partial_j\right)$$

- To calculate the trace of this operator one has to find the heat kernel

$$\begin{cases} (\partial_s + D^2)K(x,x';s) = 0 \\ K(x,x';+0) = \delta(x,x') \end{cases}$$

Even the flat case is not trivial (Mamiya & AP 2013)

$$K(x-x';\tau) = \frac{1}{z(4\pi)^2\tau^{\frac{1}{2}(1+3/z)}} \mathrm{e}^{-\frac{(t-t')^2}{4\tau}} \sum_{\{j_k\}=0}^{\infty} \left(\prod_{k=0}^{z-1} \frac{(-\tau\gamma_k)^{j_k}}{j_k!}\right) (\tau\gamma_z)^{-\sum_k k j_k/z} \times$$

$$\times {}_1\Psi_1\left[\left((3/2+\textstyle\sum_k k j_k)/z, 1/z\right); (3/2,1); -\frac{|\vec{x}-\vec{x}'|^2}{4(\tau\gamma_z)^{1/z}}\right] .$$

- This allows to perform a completely analytical study of the spectral dimension flow:

$$d_S = 1 + \frac{3}{z} + 2\gamma\gamma_z^{-\frac{k}{z}}\tau^{1-\frac{k}{z}}\left(1-\frac{k}{z}\right) \frac{{}_1\Psi_0\left[\left(\frac{3+2k}{2z}, \frac{k}{z}\right); -\gamma\gamma_z^{-\frac{k}{z}}\tau^{1-\frac{k}{z}}\right]}{{}_1\Psi_0\left[\left(\frac{3}{2z}, \frac{k}{z}\right); -\gamma\gamma_z^{-\frac{k}{z}}\tau^{1-\frac{k}{z}}\right]}$$

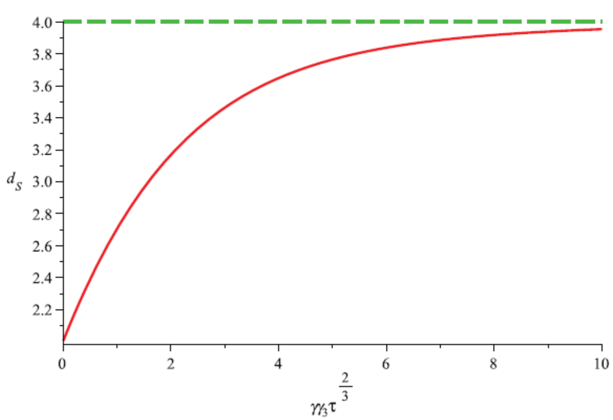


FIG. 1. The example of a smooth transition from the UV to IR regime for $z = 3$ and $k = 1$.

Part II Matter

- The matter coupling to geometry is restricted only by FPDiff.
- This permits inclusion of the higher spatial derivatives in S_{matter}
- There is no guiding principle on how to proceed except the control over the amount of Lorentz violation (Pospelov&Shang 2010, Kimpton&Padilla 2013)
- The spectral action approach has the second part (Chamsedinne&Connes 1996)

$$S_{matter} \propto \langle \psi | D | \psi \rangle$$

- The operator D is the same that was used for the gravity part!

- What happens to the geodesic motion?

$$\nabla_{\mu} T^{\mu\nu} = 0 \;\Rightarrow\; \text{geodesic motion}$$

(Dixon 1970, Hawking&Ellis 1973)

- Now we DO NOT have $\nabla_{\mu} T^{\mu\nu} = 0$

  Instead we do have $h_{\lambda\nu} \nabla_{\mu} T^{\mu\nu} = 0, \text{ where } T^{\mu\nu} \propto \frac{\delta S_{matt}}{\delta g_{\mu\nu}}$

- Alternative way to get geodesics:
  - Write a field theory
  - Find field equations
  - Restrict to the 1-particle sector
  - Do quasi-classical analysis
  - Hamilton-Jacobi => geodesic motion

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\left(g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + \frac{m^2c^2}{\hbar^2}\phi^2\right)$$

$$\Box\phi - \frac{m^2c^2}{\hbar^2}\phi = 0$$

$$\phi = A\mathrm{e}^{\frac{i}{\hbar}S}$$

$$\begin{cases} 2\nabla_\mu A\nabla^\mu S + A\Box S = 0 \;\Rightarrow\; \nabla_\mu(A^2\nabla^\mu S) = 0 \\ \nabla_\mu S\nabla^\mu S + m^2c^2 = \hbar^2\frac{\Box A}{A} \end{cases}$$

$$H = g^{\mu\nu}p_\mu p_\nu + m^2c^2$$

$$\begin{cases} g^{\mu\nu}p_\mu p_\nu + m^2c^2 = 0 \\ \dot{x}^\mu = 2N(\tau)g^{\mu\nu}p_\nu \\ \dot{p}_\mu = -N(\tau)\frac{\partial g^{\nu\lambda}}{\partial x^\mu}p_\nu p_\lambda \end{cases}$$

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\lambda}\frac{dx^\nu}{d\tau}\frac{dx^\lambda}{d\tau} = 0\ ,\ \tau \text{ is a proper time}$$

- Immediate result is that “geodesics” change

Applications: IR limit

$$S_{IR} = \frac{M_P^2}{2}\int d^3x dt\sqrt{g}N\left(-K_{ij}K^{ij} + \lambda K^2 + \alpha a_i a^i + \xi^{(3)}R + \Lambda_C\right)$$

The most general FPDiff covariant generalized operator in IR limit takes the form (AP 2014)

$$D = \gamma^0 D_n + c_1{}^{(3)}D + c_2\gamma^0 K + c_3\gamma^\alpha a_\alpha + c_4 K + c_5\gamma^0\gamma^\alpha a_\alpha$$

To the Diff covariant case correspond the following values:

$$c_1 = 1, c_2 = -\frac{1}{2}, c_3 = \frac{1}{2}, c_4 = c_5 = 0$$

i)Geodesic motion

- The approach based on Hamilton-Jacobi equation or
- Calculating the spectral distance based on the deformed Dirac operator lead to the same result: The geodesic motion of a point test particle is the same as for a (pseudo)Riemannian manifold with the effective metric

$$\tilde{g}_{\mu\nu} = -n_\mu n_\nu + \frac{1}{c_1^2} h_{\mu\nu}$$

ii) Spectral action (Lopes, Mamiya & AP 2015)

Using the heat kernel expansion for the deformed Dirac operator $D = \gamma^0 D_n + c_1 {}^{(3)}D + c_2 \gamma^0 K + c_3 \gamma^\alpha a_\alpha + c_4 K + c_5 \gamma^0 \gamma^\alpha a_\alpha$

one arrives at

$$S_{IR} = \frac{M_P^2}{2} \int d^3x dt \sqrt{h} N \left(- K_{ij} K^{ij} + \lambda K^2 + \alpha a_i a^i + \xi {}^{(3)}R + \Lambda_C \right)$$

where

$$\frac{M_P^2}{2} = f_2 \left(\frac{1}{4\pi} \right)^2 \frac{\mathrm{Tr}\, \mathbf{1}}{12}, \ \xi = \sqrt{c_1}, \ \lambda = 1 - 36 c_4^2, \ \alpha = 12 c_5^2, \ \Lambda_C = \frac{12 f_0 \Lambda^2}{f_2}$$

iii) Matter coupling (Lopes, Mamiya & AP 2015)

$$S_{matter} \propto \langle \psi | D | \psi \rangle \;=>$$

$$S_{matter} = \int dt d^3x \sqrt{h} N \overline{\psi} \left(\gamma^0 D_n + c_1 {}^{(3)}D + c_2 \gamma^0 K + c_3 \gamma^\alpha a_\alpha + c_4 K + c_5 \gamma^0 \gamma^\alpha a_\alpha \right) \psi$$

This should be compared with (Kostelecky 2004)

$$S_{LV} = \int d^4x \sqrt{g} \left(e_a^\mu \overline{\psi} \Gamma^a D_\mu \psi + \overline{\psi} M \psi \right), \text{ where}$$

$$\Gamma^a = \gamma^a - \tilde{c}_{\mu\nu} e^{\nu a} e_b^\mu \gamma^b - \tilde{d}_{\mu\nu} e^{\nu a} e_b^\mu \gamma^b \gamma^5 - \tilde{e}_\mu e^{\mu a} - i \tilde{f}_\mu e^{\mu a} \gamma^5 - \frac{1}{2} \tilde{g}_{\lambda\mu\nu} e^{\nu a} e_b^\lambda e_c^\mu \sigma^{bc}$$

$$M = m + i m_5 \gamma^5 + \tilde{a}_\mu e_b^\mu \gamma^b + \tilde{b}_\mu e_b^\mu \gamma^5 \gamma^b + \frac{1}{2} \tilde{H}_{\mu\nu} e_a^\mu e_b^\nu \sigma^{ab}$$

Using our Dirac operator we can express the Lorentz violating parameters as some combinations of c_i

$$c_{\mu\nu} = (\xi^2 - 1)h_{\mu\nu}\ ,\ m_\mu = \left(\left(\frac{\xi^2}{2} + c_2\right)K\ ,\ \left(c_3 - \frac{1}{2}\right)a_\alpha\right)\ ,\ m = \frac{1}{6}\sqrt{1-\lambda}K\ ,\ H_{0\alpha} = \sqrt{\frac{\eta}{3}}a_\alpha$$

This, in principle, could allow to put the bounds on the parameters of the gravity action that will be much more restricting that the ones coming from gravitational experiments.

Gravitational $|\beta| \leq 10^{-3}$

LV matter $|\beta| \leq 10^{-15}$

Here $\beta = \dfrac{\xi - 1}{\xi}$

Conclusions/Discussions

- Horava-Lifshitz could provide a UV completion of GR
- For this the original proposal should be modified ("healthy" extension?)
- It would be good to have a more geometrical approach to construct the theory
- At least in IR, the geodesic motion is still in some effective commutative geometry
- The spectral action allows to calculate both, gravity and matter parts

- A lot of fine tuning happens automatically due to the fact that both parts are defined by the same Dirac operator
- Bounds on LV parameters on the matter side could be used to bound the gravity action (and vice versa)
- Gauge sector, matter content (it is more natural now to have fields in reps of SO(3))
- Methods of spectral geometry plus spectral action principle have proven to be useful though we do expect much more complicated situation for the fully deformed theory